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Simple Method of a Dynamical System Asymmetric Transients Detection

1. Introduction

In this paper a simple method of a dynamical system transient characteristics asymmetry detection is presented. One considers a non-linear multi-input single output dynamical system periodic orbit achieved by an appropriate input signals or non-zero initial conditions. The system asymmetry is modelled by an asymmetric static saturation element. Adding an additional state one can easily check the trajectory asymmetry.

The paper is organised as follows. First a dynamical system and asymmetrical saturation element description is given. In Sections 3 and 4 a linear second order oscillator with asymmetrical saturation is analysed. Investigations lead to the system asymmetry measure which is applied to the analyse of a voice signal.

1.1. Mathematical preliminaries

One considers a multi-input one output (in general) (MISO) continuous-time non-linear dynamical system. It can be described by a set of the first-order non-linear differential equations [3]

$$\dot{\mathbf{x}}(t) = \boldsymbol{f}\left[\mathbf{x}(t), \mathbf{u}(t), t\right] \tag{1}$$

and a set of non-linear algebraic output equations

$$y(t) = \boldsymbol{g}\left[\mathbf{x}(t), \mathbf{u}(t), t\right] \tag{2}$$

where:

- $\mathbf{u}(t)$ – the system input vector,
- $\mathbf{x}(t)$ – the system state vector,
- $y(t)$ – the system output signal,
- $\boldsymbol{f}[\mathbf{x}(t), \mathbf{u}(t), t]$ – the state function,
- $\boldsymbol{g}[\mathbf{x}(t), \mathbf{u}(t), t]$ – the output function.

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A block diagram of a system is presented in Figure 1. Note that paths of the vectors **u**(t) and **x**(t) are depicted by wide whereas the output signal by narrow one, respectively. One should mention that formulae (1) and (2) can describe open and closed-loop dynamical systems. Moreover, it is mainly assumed that the description concerns a proper dynamic system operation.

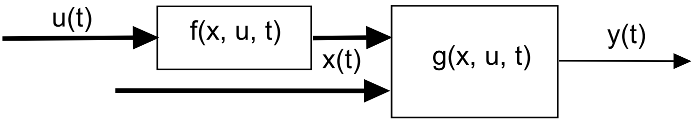


Fig. 1. Block diagram of the dynamic system

Any working device may be partly or even totally subjected to different type defects. Here one can mention dangerous asymmetry in electronic devices such as operational and acoustic amplifiers as well as generators [3]. In rotating mechanical systems very undesirable are non-uniformly distributed resistances [3]. Also in physics plenty of symmetrical oscillators are analysed [3]. Every linear time-invariant dynamical system has a symmetry property.

Dynamical system failures mentioned above can be modeled by a static saturation element characterized by a non-symmetrical characteristic. A block diagram of a defected dynamical system is presented in Figure 2.

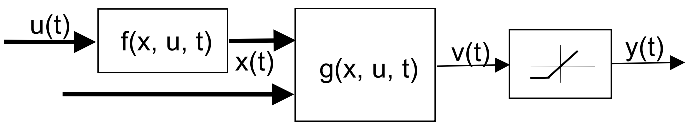


Fig. 2. Block diagram of the dynamic system with non-symmetric saturation element

1.2. Non-symmetric saturation element

Now it is assumed that the dynamical system impairment (failure, lesion) symptoms are modeled by a non-symmetric saturation element connected to the output signal. The element considered in this paper is described by a following static equation

$$y(t)=\begin{cases} v(t) & \text{for} \quad v(t)\geq -A_s \\ -A_s+f_m e^{\frac{A_s-f_m-v(t)}{f_m}} & \text{for} \quad v(t)<-A_s \end{cases} \tag{3}$$

where:

A_s, $-A_s$ – the upper and lower limits of a linear behavior,

f_m – the asymmetric saturation element parameter,

$v(t)$ – the input signal,

$y(t)$ – the output signal.

The asymmetric saturation element static characteristics for different linearity limits is plotted in Figure 3. It should be noted that the proposed non-linear element possess a continuity property which is typical in practice. Realise that commonly considered saturation elements with sharp linearity/non-linearity cross practically doesn't exist. It means that for the considered saturation element its first order derivative of $y(t)$ with respect to $v(t)$

$$\frac{dy}{dv} = \begin{cases} 0 & \text{for} \quad v(t) \geq -A_s \\ f_m e^{\frac{A_s - f_m - v(t)}{f_m}} & \text{for} \quad v(t) < -A_s \end{cases} \tag{4}$$

is a continuous function. Plot of (4) for different saturation levels is given in Figure 4.

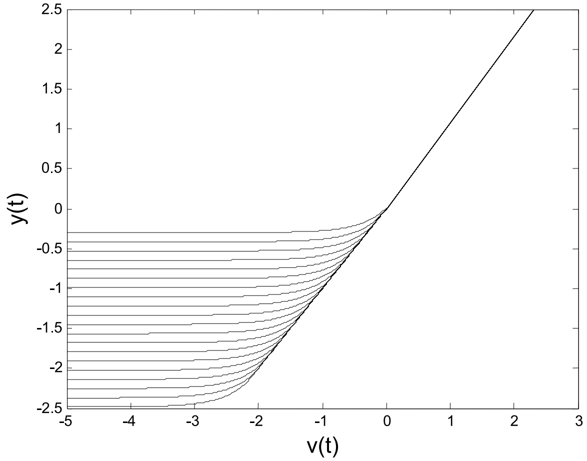


Fig. 3. Static characteristics of the non-symmetric saturation element

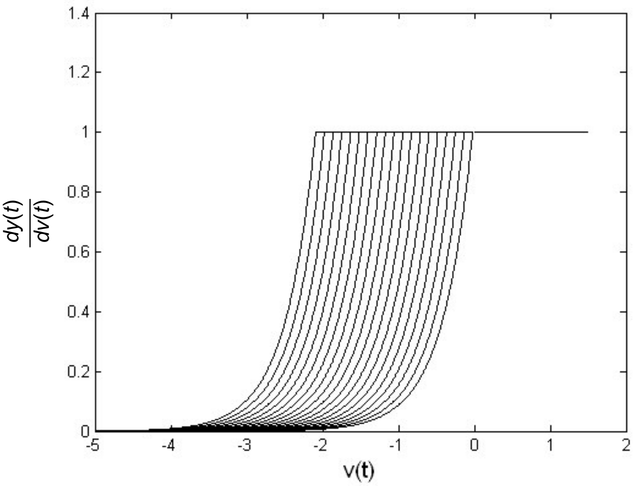


Fig. 4. Characteristic (4) of the non-symmetric saturation element

2. Asymmetry marker

To detect dynamical asymmetry behavior of the system one introduces its additional state by connecting an ideal integrator. The integration can be also performed by numerical integration of the output signal. The shape of this new additional state will indicate a system non symmetry level.

Now one assumes that the system description (1) is transformed to the form where the higher state is a first-order derivative with respect to time of the previous one. Hence a state equation takes a form

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \\ \dot{x}_{n+1}(t) \end{bmatrix} = \begin{bmatrix} f_1\left[x_1(t), x_2(t), \cdots, x_{n+1}(t), u_1(t), u_2(t), \cdots, u_m(t), t\right] \\ f_2\left[x_1(t), x_2(t), \cdots, x_{n+1}(t), u_1(t), u_2(t), \cdots, u_m(t), t\right] \\ \vdots \\ f_n\left[x_1(t), x_2(t), \cdots, x_{n+1}(t), u_1(t), u_2(t), \cdots, u_m(t), t\right] \\ f_{n+1}\left[x_1(t), x_2(t), \cdots, x_{n+1}(t), u_1(t), u_2(t), \cdots, u_m(t), t\right] \end{bmatrix} = \tag{5}$$
$$= \begin{bmatrix} x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \\ f_{n+1}\left[x_1(t), x_2(t), \cdots, x_{n+1}(t), u_1(t), u_2(t), \cdots, u_m(t), t\right] \end{bmatrix}$$

The output equation is

$$y(t) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \\ x_{n+1}(t) \end{bmatrix} \tag{6}$$

This means that

$$x_1(t) = \int_0^t x_2(\tau) d\tau + x_2(0) \tag{7}$$

and the shape of this function can be treated as a dynamic system asymmetry marker. One should mention that

$$x_3(t) = \frac{dx_2(t)}{dt} \tag{8}$$

The states defined above are elements of the so called phase space [5, 6]. The first three may be plotted its 3D subspace which reveals the dynamical system asymmetry property.

3. Linear oscillator analysis

Now one considers a linear, time invariant second order system with additional integrator. It is described as follows

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ x_3(t) \\ -a_0 x_3(t) + a_0 u(t) \end{bmatrix} \tag{9}$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \tag{10}$$

where a_0 – the system parameter.

The plots of $x_1(t)$, $x_2(t)$, $x_3(t)$ in a steady-state evaluated numerically for $u(t) = 0$ and non-zero initial conditions are presented in Figure 5.

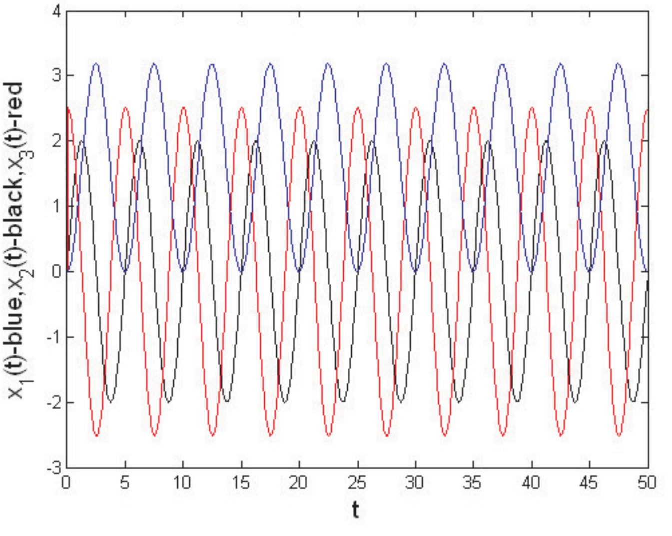


Fig. 5. Transient characteristics of the second order system (9)–(10)

Appropriate 3D plot is given in Figure 6.

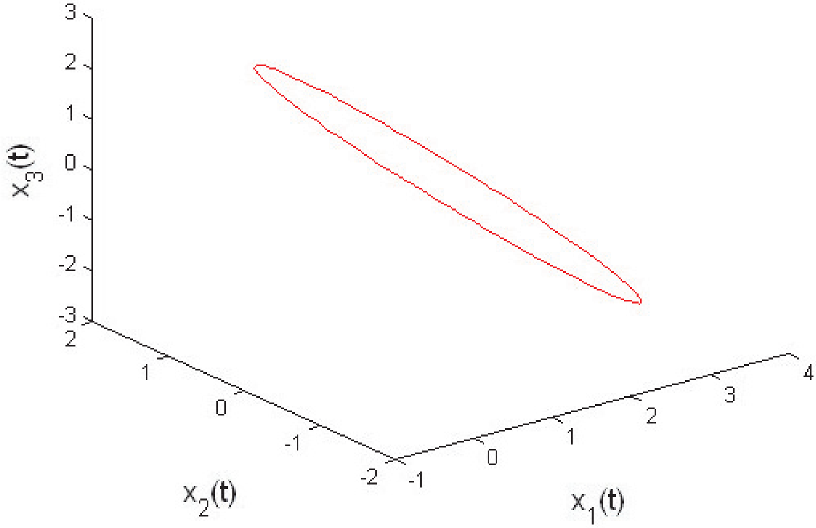


Fig. 6. 3D plot of $x_1(t)$, $x_2(t)$, $x_3(t)$

Now a system fault characterised by a static asymmetry modelled by the saturation element is considered. Analogous transients are presented in Figures 7 and 8a, b, respectively.

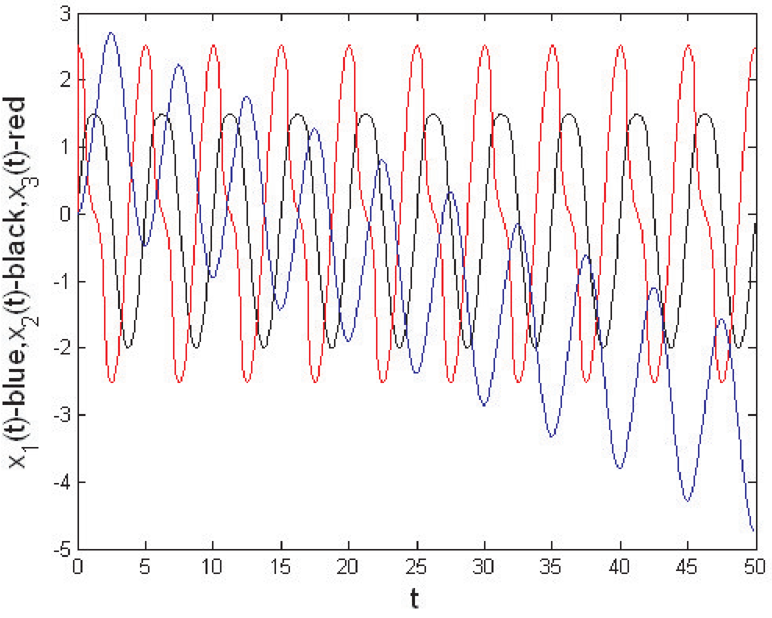
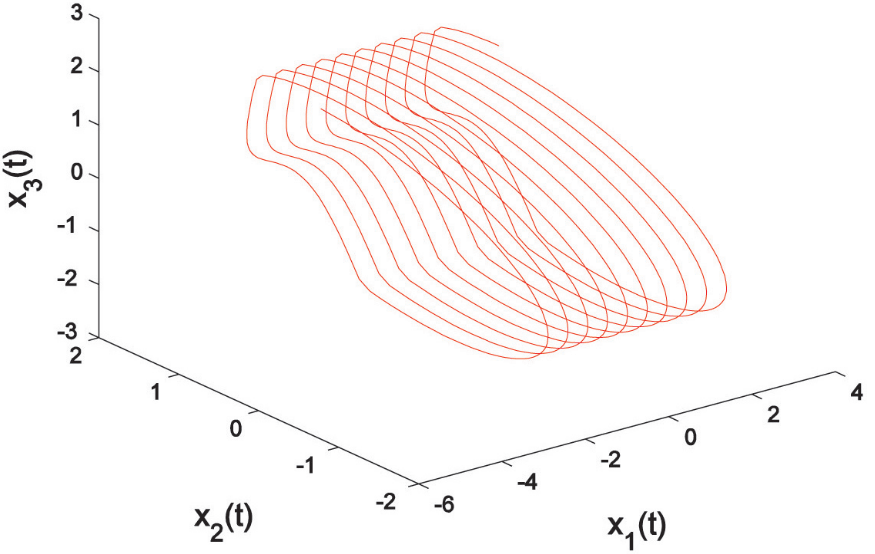


Fig. 7. Transient characteristics of the second order system (9)–(10) with asymmetry

a)



b)

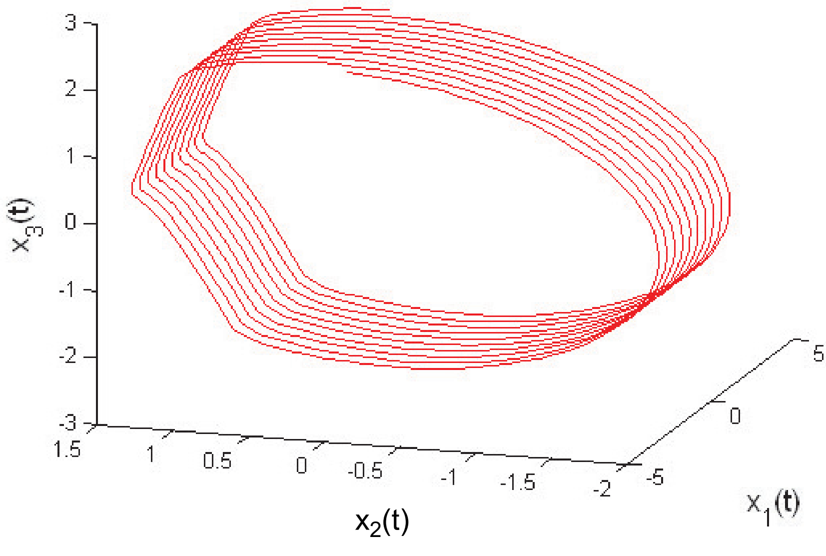


Fig. 8. 3D plot of $x_1(t)$, $x_2(t)$, $x_3(t)$ with asymmetry (a); another point of view (b)

Figure 9 shows the same transients as those which were presented in Figure 7 but in wider time range. Here two black lines define an angle representing an asymmetry level.

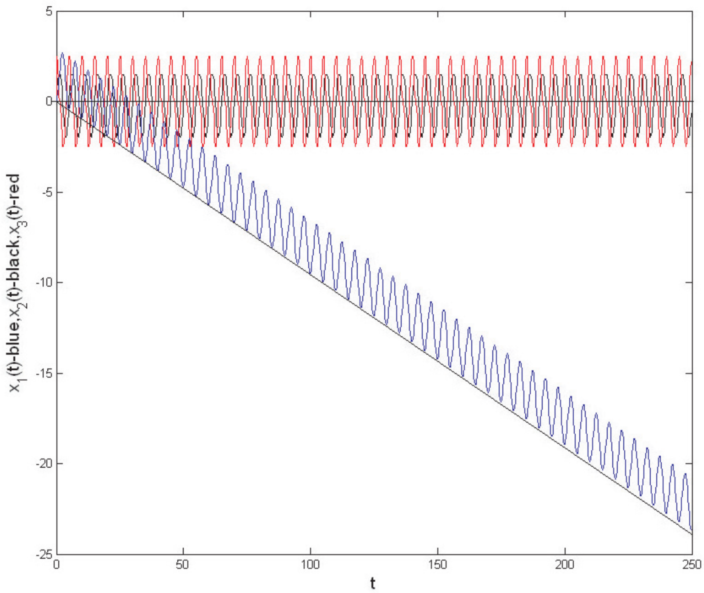


Fig. 9. Non-symmetric transient characteristics of the second order system (9)–(10) with two black lines revealing an angle between $x_2(t)$ and $x_1(t)$

4. Signal asymmetry measure

Now one defines two functions related to the states $x_2(t)$ and $x_1(t)$

$$\begin{aligned} \bar{x}_1(t) &= \frac{1}{t}\int_0^t x_1(\tau)d\tau \\ \bar{x}_2(t) &= \frac{1}{t}\int_0^t x_2(\tau)d\tau = \frac{x_1(t)}{t} \end{aligned} \tag{11}$$

Definition 1

An angle describing a dynamical system asymmetry level is an angle between $\bar{x}_1(t)$ and $\bar{x}_2(t)$ for $t > NT$ where T denotes an oscillation period and N is a number of periods after which system oscillations achieves a steady-state

$$\alpha = \angle\{\bar{x}_1(t), \bar{x}_2(t)\}_{t>NT} \tag{12}$$

It is well known that saturation introduces higher frequencies. Its differentiation leads to a periodic function with high amplitudes. Hence one can define another asymmetry measure. It is formulated in a Definition 2.

Definition 2

A measure of high frequency components in a non-symmetrical periodic signal is a ratio of maximal amplitudes of $x_2(t)$ and $x_3(t)$

$$\beta = \frac{\max\limits_{t>NT} \left| x_3(t) \right|}{\max\limits_{t>NT} \left| x_2(t) \right|} \tag{13}$$

5. Voice signal analysis

Voice, as the carrier of information, constitutes an important element of social communication. It is an acoustic phenomenon produced in larynx due to synergy of aerodynamic and psychoneural mechanisms. Three anatomical and physiological structures play a significant role in voice production:

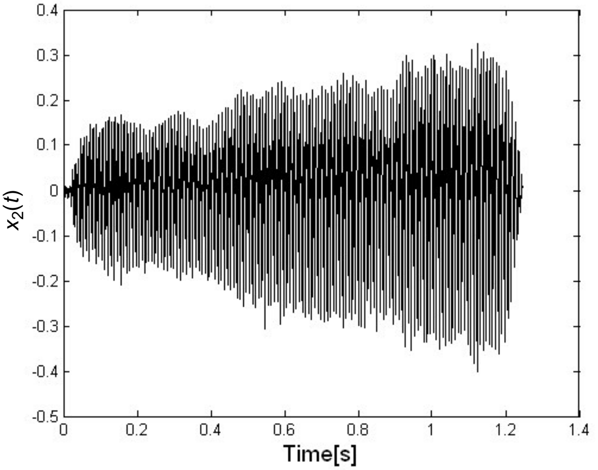
- subglottal air reservoir within the lower respiratory tract (lungs, bronchi, trachea), which issues a high-pressure airstream;
- acoustic energy generator in larynx (glottis and vocal folds), producing laryngeal tone (fundamental frequency);
- resonanse chambers (laryngeal vestibule, pharynx, nasal cavity, oral cavity) which form vocal timbre and auditory elements of speech that is phonemes [9, 11].

Glottis is a part of intermediate cavity of larynx and is built of two vocal folds. A vocal fold consists of mucous membrane, vocalis muscle (cover-body complex) and has a complex, multilayer structure. According to the myoelastic-aerodynamic theory vibrations of vocal folds depend on competition of the forces which clench (increasing subglottal pressure) and close the glottis (contractions of the adductor muscles of vocal folds, elastic forces and viscosity of tissues, the Bernoulli effect). Vocal folds movements during phonation occur in three planes: vertical (opening of glottis upwards), horizontal (backwards) and sagittal (wave motions of vocal fold mucosa along its medial surface) [8, 11, 12].

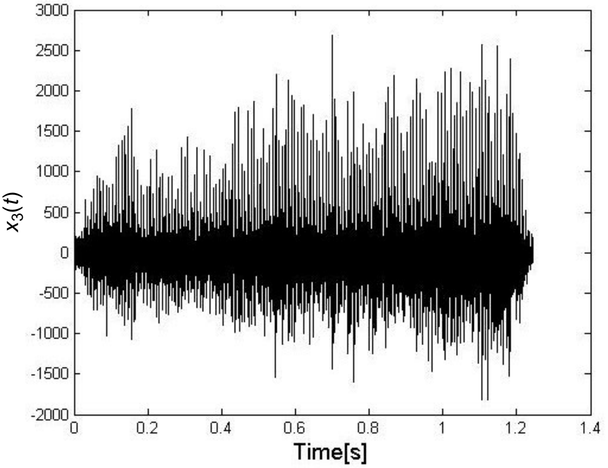
Voice disturbances (dysphonia) occur as a result of motor discoordination in the vocal organ. It leads to irregularities of phonatory vibrations with the lack of the phase of full glottal closure. Proper, laminar flow of the air turns into the turbulent one thus causing murmurs perceived as hoarseness. We distinguish organic dysphonia resulting from primary organic lesion of larynx and functional dysphonia connected with functional disorders without tangible organ lesion (hiper- and hipofunctional dysphonia). The most frequent causes of organic dysphonia are acute and chronic laryngitis, cyst, polyp of vocal fold, neoplasm of larynx, neurological, hormonal or posttraumatic movement disorders of vocal fold and laryngeal disturbances in the course of gastroesophageal reflux. Functional dysphonia develops as a result of incorrect voice production (phonoponosis), psychogenic disorders (phononeurosis) or phono-respiratory-articulative discoordination (phonastenia). It often occurs in professional voice users like teachers, lawyers or actors. Functional dysphonia, due to vocal abuse, can induce secondary organic lesions (vocal nodules) [9, 11].

Complex voice evaluation, according to suggestions of the Committee on Phoniatrics of the European Laryngological Society, includes auditory perceptual estimation, video-laryngostroboscopy, acoustic analysis, aerodynamic assessment (maximum phonation time) and subjective voice evaluation done by patients themselves (e.g. according to a questionnaire determining voice handicap, Voice Handicap Index – VHI) [2].

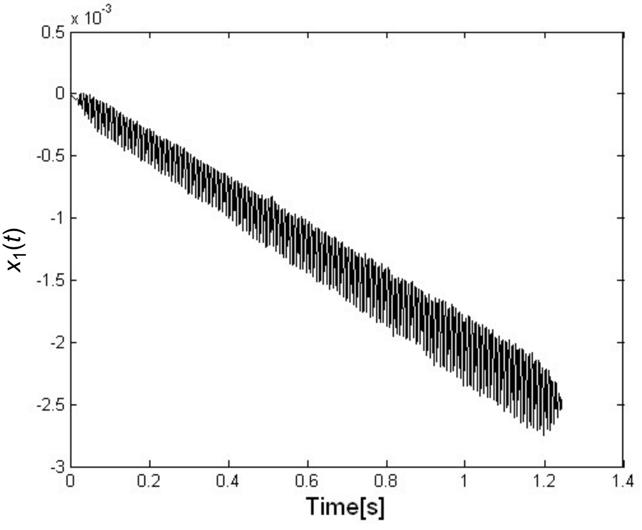
In an experiment a long-lasting vowel "aaa…a" (in Polish) was digitally registered and numerically processed. In Figures 10, 11 and 12 "a" voice signal, its first order derivative and an integral are presented [2, 4, 7, 10]. One can easily find signal asymmetry and calculate an angle (13). Finally in Figure 13 the 3D plot of $x_1(t)$, $x_2(t)$, $x_3(t)$ is presented.



Rys. 10. Plot of a digitally registered long-lasting vowel "aaa…a" (in Polish)



Rys. 11. Plot of a numerically evaluated first-order derivative of a digitally registered vowel "aaa…a" (in Polish)



Rys. 12. Plot of a numerically evaluated integral of a digitally registered vowel "aaa…a" (in Polish)

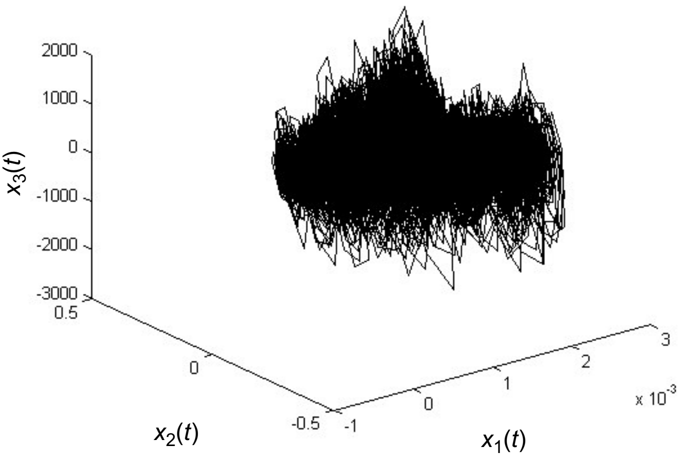


Fig. 13. 3D plot of $x_1(t)$, $x_2(t)$, $x_3(t)$ of a vowel "aaa…a"

6. Final conclusions

The voice analysis show applicability of the proposed method to signal asymmetry detection. The shape of 3D orbit may serve as an indicator of system asymmetry and higher harmonics contents. The angle defined by Formula (12) indicates an asymmetry level.

In future works a relation between the angle (12) and the health state will be established. One may expect that the angle may vary due to a therapy effects. The last statement should be proved by experiments on numerous samples.

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