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# Analysis of Electrical Impedance Tomography Sensitive Field Based on Multi-Terminal Network

## 1. Introduction

As a non-intrusive internal visualizing methodology, Electrical Impedance Tomography (EIT) has been developed rapidly. A typical EIT system has 3 main parts: sensor, data acquisition unit and image reconstruction software. The sensor consists of a set of electrodes mounted around the interested object. When voltages or currents are applied to the electrodes and the resulting currents or voltages are measured, the internal impedance distribution can be obtained based on these boundary data by a suitable image reconstruction algorithm. To minimize the effect of inherent ill-posed problem, we must try to acquire more independent measurements and improve the uniformity of the spatial sensitivity distribution. EIT sensor is usually regarded as a static or constant current field. Finite Element Method (FEM) is commonly used to compute the sensitivity matrix and to optimize the sensor structure. Because only the lumped circuit parameters can be measured by the data acquisition unit, it is better to give a multi terminal network model of the EIT sensor is introduced and the relationships of different lumped circuit parameters are analyzed. An optimum single source excitation strategy so-called quasi opposite excitation, that has the advantages of more symmetrical sensitive distribution and more independent measurements, is proposed.

## 2. Multi-terminal network

The EIT sensitive field can be regarded as a passive multi terminal network which is a special multi port network. The multi port network (Fig. 1a) has *m* pairs of terminal or *m* ports, and the currents inject into and flow out of the two ends of each port are equal.

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The relationship of the voltages and the currents of an $m$-port network is given by

$$\boldsymbol{U} = \boldsymbol{Z}\boldsymbol{I} \tag{1}$$

where $\boldsymbol{U} = \begin{bmatrix} U_1 & U_2 & \cdots & U_m \end{bmatrix}^T$ is the voltage vector, $\boldsymbol{I} = \begin{bmatrix} I_1 & I_2 & \cdots & I_m \end{bmatrix}^T$ is the current vector, $\boldsymbol{Z}$ is the open circuit impedance matrix. The element of $\boldsymbol{Z}$

$$Z_{kj} = \left.\frac{U_k}{I_j}\right|_{I_l=0, l\neq j} \tag{2}$$

equals to the voltage of the port $k$ with the port $j$ excited by unit current and other ports opened, i.e., the open circuit impedance. The diagonal elements are open circuit driving point impedance and the non-diagonal elements are open circuit transfer impedance.

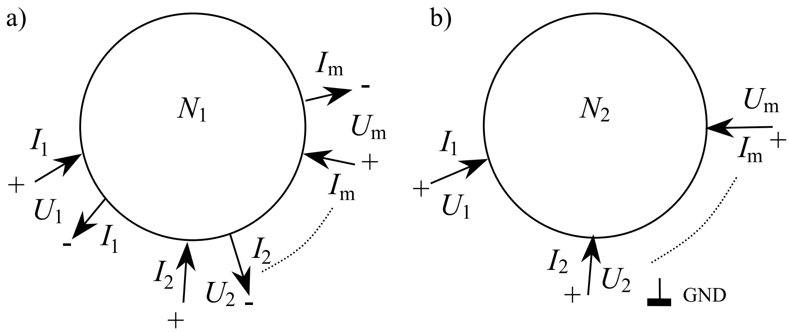


**Fig. 1.** Network modal: a) Multi-port network; b) Multi-terminal network

The passive $m$-port network equation can also be expressed as

$$\boldsymbol{I} = \boldsymbol{Y}\boldsymbol{U} \tag{3}$$

where $\boldsymbol{Y} = \boldsymbol{Z}^{-1}$, with the diagonal elements are short circuit driving point admittance and the non-diagonal elements are short circuit transfer admittance.

The practical EIT sensor is not the multi port network mentioned above because the ports are not fixed and the currents are not coupled. EIT sensor can be considered as multi terminal network as shown in Figure 1b. There are two types of multi terminal network, common node network and common loop network. The former refers to the multi port network in which every terminal is the *in* ends and a float node is the common *out* end, while, the latter refers to the multi port network in which every adjacent terminal pair is considered as a port, e.g., (1, 2), (2, 3) …

Electrical Capacitance Tomography (ECT) and Electrical Resistance Tomography (ERT) are the two most developed EIT technologies. Usually, voltage excitation and current measurement is used in ECT that is a common node multi terminal network as shown in Figure 2(a). The fifteen capacitances which derived from the areas divided by the electrical lines are in parallel.

The network equation is

$$\boldsymbol{I} = \boldsymbol{Y}_i\boldsymbol{U} \tag{4}$$

where $\boldsymbol{Y}_i$ is called indefinite admittance matrix. Because, according to the circuit network principle [1], $\boldsymbol{Y}_i$ is a singular matrix, a column and a row can be deleted without information loss. If the row $m$ and the column $m$ are removed, that means the terminal $m$ is the common node for its potential is always zero, the network is converted to an ($m$–1)-port network, and the indefinite matrix $\boldsymbol{Y}_i$ is changed to nonsingular definite matrix $\boldsymbol{Y}_d$ that is a symmetry $(m-1)\times(m-1)$ short circuit admittance matrix with $m\,(m-1)/2$ independent parameters. Taking the terminal $m$ as the common node, all the parameters can be obtained by applying voltage source to other $(m-1)$ terminals sequentially and measuring the result current with the help of following equation

$$Y_{kj} = \left.\frac{I_k}{U_j}\right|_{U_l=0, l\neq j} \tag{5}$$

So, only ($m$–1) independent excitations are necessary for an $m$-terminal network.

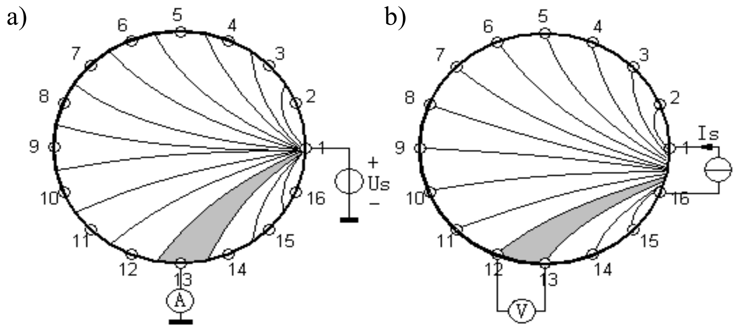


**Fig. 2.** Typical ECT and ERT sensor models: a) ECT; b) ERT

$$\boldsymbol{U} = \boldsymbol{Z}_i\boldsymbol{I} \tag{6}$$

Similarly, ERT sensor shown in Figure 2b is a common loop multi terminal network. The fifteen resistances which derived from the areas divided by the equipotential lines are in series. The definite open circuit impedance matrix $\boldsymbol{Z}_d$ has $m\,(m-1)/2$ independent parameters and $(m-1)$ independent excitations are necessary. The indefinite admittance matrix and indefinite impedance matrix are singular, so the inverse matrixes do not exist. The definite admittance matrix $\boldsymbol{Y}_d$ of common node network and the definite impedance matrix $\boldsymbol{Z}_d$ of common loop network are not the inverse matrix of each other. According to electrical network theory, the relationship of $\boldsymbol{Z}_d$ and $\boldsymbol{Y}_d$ is given by

$$\boldsymbol{Z}_d = (\boldsymbol{W}\boldsymbol{Y}_d\boldsymbol{W}^T)^{-1} \tag{7}$$

where $\boldsymbol{W}$ is a $(m-1)\times(m-1)$ lower triangular matrix that the diagonal and under diagonal elements are 1 while the upper diagonal elements are 0.

## 3. Excitation strategy of EIT

Admittance and impedance parameters can be gotten from the dualistic common node model and common loop model. If current source and voltage source are used in Figures 2a and b respectively, current transfer ratio and voltage transfer ratio can be computed. [6]

The adjacent excitation/measurement strategy is widely used in common loop EIT model currently. Table 1 shows all the electrode pairs of a typical 16-electrode common loop EIT sensor. The electrode pairs of the diagonal are the excitation ports. If the voltages of all the adjacent electrode pairs are measured, e.g., when (1, 2) is excited, (1, 2), (2, 3) … (15, 16), (16, 1) are measured, we can get 16×16 = 256 measurements totally. Obviously, these measurements are linear correlated. Because the driving voltages that can be derived by the other result voltages according to the Kirchhoff's voltage law are not measured, the number of available measurements is 16×15 = 240, i.e., the diagonal elements in Table 1 are removed. Further, all the lower or upper diagonal measurements can be removed due to the availability of the reciprocal theory to the passive network, so the independent measurement is (16×15)/2 = 120. The elements with gray background make up of a set of independent measurement. As mentioned above that $m$-terminal network has ($m$–1) independent excitations, there are no independent measurement when pair (16, 1) is excited. In some applications, the result voltages adjacent to the driving point are also removed for they are easily influenced and the number of left independent measurements is (16×13)/2 = 104.

**Table 1**
Adjacent electrode pairs of a 16 electrode EIT sensor

| | | | | | |
|---|---|---|---|---|---|
| 1-2 | 2-3 | 3-4 | … | 15-16 | 16-1 |
| 1-2 | 2-3 | 3-4 | … | 15-16 | 16-1 |
| 1-2 | 2-3 | 3-4 | … | 15-16 | 16-1 |
| … | … | … | … | … | … |
| 1-2 | 2-3 | 3-4 | … | 15-16 | 16-1 |
| 1-2 | 2-3 | 3-4 | … | 15-16 | 16-1 |

The more flexible excitation strategy in which the two driving terminals are not adjacent is complicated. Taking still the 16-electrode EIT sensor as an example, a general exciting current vector can be represented as

$$\boldsymbol{I} = \begin{bmatrix} I_1 & I_2 & \cdots & I_{16} \end{bmatrix}^T \tag{8}$$

For adjacent excitation strategy, a typical exciting current vector is

$$\boldsymbol{I} = \begin{bmatrix} \cdots & 0 & 1 & -1 & 0 & \cdots \end{bmatrix}^T \tag{9}$$

The whole exciting matrix consists of 16 exciting current vectors can be expressed as

$$\boldsymbol{I}_s = \begin{bmatrix} 1 & & & & -1 \\ -1 & 1 & & 0 & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ 0 & & & -1 & 1 \end{bmatrix}_{16\times 16} \tag{10}$$

$\boldsymbol{I}_s$ is singular and its rank is 15, i.e., 15 independent excitations.

The first 15 vectors compose a set of independent excitations. The number of independent measurements is 120. When we take the every other electrode pair as driving ports, the rank of $\boldsymbol{I}_s$ is 14. Taking the first 14 vectors as the independent excitation, only 119 independent measurements can be obtained.

Generally, for an $N$-electrode EIT sensor, when the interval between the two ends of the driving port is $k$, the exciting current vector is

$$\boldsymbol{I} = \begin{bmatrix} \cdots & 0 & \underset{j}{1} & 0 & \cdots & 0 & \underset{j+k}{-1} & 0 & \cdots \end{bmatrix}^T \tag{11}$$

Similarly, the number of independent excitations equals to the rank of the $N\times N$ exciting matrix $\boldsymbol{I}_s$ which consists of all the exciting current vectors. Supposing the independent excitation is $R$, it can be proved that the first $R$ vectors of $\boldsymbol{I}_s$ are independent. The first excitation can produce $N$–1 independent measurements; the second excitation can produce $N$–2 independent measurements; and so on, the last excitation can produce $N$–$R$ independent measurements. So the total number of independent measurements can be calculated by

$$M = \frac{(2N - R - 1)R}{2} \tag{12}$$

The independent excitation/measurement of a typical 16-electrode EIT is shown in Table 2. We can see that there are the least independent measurements in the case of the electrode interval 8, i.e., opposite excitation.

**Table 2**
Independent excitation/measurement of 16-electrode EIT

| Electrode interval ($K$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Independent excitation ($R$) | 15 | 14 | 15 | 12 | 15 | 14 | 15 | 8 |
| Independent measurement ($M$) | 120 | 119 | 120 | 104 | 120 | 119 | 120 | 92 |

The above conclusion can be verified by Singular Value Decomposition (SVD) method to sensitive coefficient matrix [5]. The inverse problem can be simply expressed as

$$\underset{N\times 1}{\mathbf{g}} = \underset{N\times M}{\mathbf{S}}{}^{T} \underset{M\times 1}{\boldsymbol{\lambda}} \tag{13}$$

Where, $\boldsymbol{g}$ is the gray level of pixels of reconstructed image; $\boldsymbol{S}$ is the sensitive matrix; $\boldsymbol{\lambda}$ is the measurements; $N$ is the number of pixels and $M$ the number of the complete measurements. $M$ is 256 for 16-electrode EIT. If its rank is $p$, $\boldsymbol{S}$ can be decomposed as

$$\boldsymbol{S} = \boldsymbol{WDV}^{T} \tag{14}$$

where, $\boldsymbol{W}$ is an $N\times N$ orthogonal matrix, $\boldsymbol{V}$ is an $M\times M$ orthogonal matrix.

$$\boldsymbol{D} = \mathrm{diag}\begin{bmatrix}\delta_1 & \delta_2 & \cdots & \delta_p & 0 & \cdots\end{bmatrix} \tag{15}$$

$\boldsymbol{D}$ is a $N\times M$ matrix with all components of zero except for the diagonal components, and $\delta_1$, $\delta_2$ …$\delta_3$ are the $p$ non-zero singular values of $\boldsymbol{S}$. For different excitation strategies as shown in table 2, it can be verified that $p$ equals to the number of independent measurements. In another words, the independent measurements out of $\boldsymbol{\lambda}$ is $p$. So it leads to a same conclusion about the independent measurement. Breckon, Pidcock [2] and Dickin [3] showed that, for the opposite strategy, the number of independent measurements $M$ is given by

$$M = \frac{N}{4}\left(\frac{3N}{2} - 1\right) \tag{16}$$

That is the same as (12).

## 4. Conclusion

It is know that there are two major difficulties associated with the inverse problem. Firstly, it is under-determined because the number of unknown impedance distribution, i.e. the number of pixels, is much larger than the number of independent measurements. Secondly, it is ill-conditioned because of the very un-uniformed sensitivity distribution. To improve the reconstructed image, we should get as more independent measurements as possible and make a more uniform sensitivity distribution. About the first aspect, Table 2 gives a guidance to select a suitable excitation strategy. Apparently, adjacent excitation is better than opposite excitation. On the other hand, with respect to the adjacent strategy, the opposite excitation leads to a more uniform sensitivity distribution [4]. Further more, since the impedance between the opposite electrode pair is greater, under a certain driving current, the result voltage is bigger and the signal to noise ratio (SNR) is higher. So, stand on the point of uniform sensitivity distribution, the opposite strategy is the best. Take the two aspects into account, the so-called quasi opposite excitation strategy, in which the interval of the electrode pair is 7, should be the optimum.

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