Integrated Scheduling in a Customer Driven Supply Chain

1. Introduction

One of the main issues of supply chain management is how to coordinate manufacturing and supply of raw materials and production and distribution of finished products, e.g., Erenguc et al.(1999), Kolisch (2000), Shapiro (2001), Chen and Vairaktarakis (2005), Kaczmarczyk et al. (2006).

In the customer driven supply chains, the procurement policies are dependent on the production schedule that is driven by the customer orders for the finished products. Hence, it is necessary to coordinate the procurement and production policies and to determine a joint raw material manufacturing and supply schedule and the finished products production schedule. The integrated schedule should aim at reaching the highest customer service level and the best tradeoff between total supply chain inventory holding cost and both the producer and the supplier set-up and shipping costs.

Majority of research on supply chain coordination is devoted to developing joint economic lot size models, where the objective is to determine joint economic ordering and production lot sizes to simultaneously minimize total cost of material ordering and holding and manufacturing setup and finished products holding, e.g. Goyal and Deshmukh (1992), Lee (2005). The models are based on various simplifying assumptions such as a single finished product with a constant or a piece-wise linear demand pattern. Therefore, they cannot be directly applied in a complex, multi-product make-to-order environment, with an arbitrary demand pattern for different finished products.

In this paper a mixed integer programming approach is proposed for the multi-objective, integrated scheduling of material manufacturing, material supply and product assembly in a customer driven supply chain.

In the literature on production planning and scheduling the integer programming models have been widely used, e.g. Bradley and Arntzen (1999), Kolisch (2000), Pochet and Wolsey (2006), Sawik (2006, 2007a, 2007b, 2007c). For example, a mixed integer programming formulation for the integrated assembly scheduling and fabrication lot sizing in make-to order production is proposed in Kolisch (2000), with the objective function that minimizes the total inventory holding and fabrication setup cost. A hierarchical framework

^{*} Department of Operations Research and Information Technology, AGH University of Science and Technology, Cracow

and integer programming formulations for a long-term assignment of customer orders to planning periods and a short-term machine assignment and scheduling of production lots in a flexible flowshop are proposed in Sawik (2006).

The paper is organized as follows. In Section 2 description of the integrated scheduling in a customer driven supply chain is provided. The mixed integer programming formulation for the multi-objective integrated scheduling of parts manufacturing and supply and of products assembly is presented in Section 3. Numerical examples and some computational results are reported in Section 4. Conclusions are made in the last section.

2. Problem description

The supply chain consists of three distinct stages: manufacturer/supplier of product-specific materials (parts), producer where finished products are assembled according to customer orders and a set of customers which generates final demand for the products.

In the manufacturing stage product-specific parts are manufactured for all product types that are assembled at the producer stage. The manufacturing stage is made up of m_M identical production lines in parallel and an unlimited output buffer for storing the parts waiting for shipment to producer. Let K be the set of product-specific part types and q_k processing time of part type $k \in K$. The planning horizon consists of t_h planning periods (e.g. business days) of equal length l (e.g. hours or minutes) and let $T = \{1, ..., t_h\}$ be the set of planning periods. In each period at most one part type can be manufactured on each production line. When a production line switches from one part type to another a start-up time should be considered at the beginning of the period. The start-up times are sequence-independent and are assumed to be equal for all part types. Let σ be the start-up time of each production line.

The manufactured parts are next transported to producer at most once per period. Different part types can be shipped together so that a shipping cost arises only once per shipment. The size of each shipment is limited by the minimum and the maximum allowed

shipping lot, respectively \underline{V} and \overline{V} . The transportation time is assumed to be constant and equal to one period for every shipping lot. Parts manufactured in period t can be transported to the producer stage in the same period and can be used for products assembly in period t+1, at the earliest.

The producer stage is a flexible assembly line made up of m_A assembly stages in series, an unlimited input buffer for storing the supplied parts waiting for assembly and an unlimited output buffer for storing the finished products waiting for delivery to the customers. In this stage various types of products are assembled in a make-to-order environment responding directly to customer orders. Let J be the set of customer orders known ahead of time, K – the set of product types (identical with the set of product-specific part types) and J_k the subset of orders for product type $k \in K$ (i.e. requiring part type k to be assembled). Each order $j \in J$ is described by a triple (r_j, d_j, o_j) , where r_j is the order ready date (e.g. the earliest period of material availability), d_j is the customer requested due date (e.g. customer required shipping date), and o_j is the size of order (quantity of ordered products of a specific type).

Each order requires processing in various assembly stages, however some orders may bypass some stages. Let $p_{ij} \ge 0$ be the processing time in stage $i \in A$ of each product in order $j \in J$ and c_{it} – the total processing time available in stage $i \in A$ in period $t \in T$.

It is assumed that each product requires one unit of the corresponding product-specific part (e.g. one printed wiring board of a specific design per one electronic device of the corresponding type). Furthermore, the manufacturing and supplies of common materials for different product types are not explicitly taken into account. As a result, for each order j the required quantity of product-specific parts type k (such that $j \in J_k$) equals the quantity o_j of the ordered products k. The above assumptions make the inventory calculations clearer and can be relaxed by introducing for each type of product the associated unit requirements for each type of part, e.g. Sawik (2005). Accordingly, the models proposed in this paper can be further enhanced to simultaneously consider both product-specific and common materials.

The overall problem is how to coordinate manufacturing and supply of parts and assembly of products with respect to limited capacities and required service level such that inventory holding, production line start-up and parts shipping costs are minimized.

3. The integrated scheduling of manufacturing, supply and assembly

In this section the mixed integer program **SAMS** is presented for integrated scheduling of products assembly and parts manufacturing and supply in the customer driven supply chain. The notation used is presented in Table 1.

3.1. Supply chain inventory

The total supply chain inventory I(t) in every period t is the sum of the supplier output inventory of manufactured parts, the producer input inventory of supplied parts, and the producer output inventory of finished products, that is,

$$I(t) = \sum_{k \in K} (I_{k0}^1 + I_{k0}^2 + \sum_{\tau=1}^t u_{k\tau} - \sum_{\tau=1}^t \sum_{j \in J_k : r_j \le \tau \le d_j \le t} o_j x_{j\tau}; \quad t \in T$$
 (1)

where I_{k0}^1, I_{k0}^2 is the beginning inventory of part type k at supplier, producer, respectively.

The maximum level of the total producer inventory can be implicitly minimized by minimizing the maximum earliness $E_{\rm max}$ of early orders (Sawik, 2007b), subject to the required service level constraints. If no tardy orders are allowed (i.e., 100% of service level), the total producer inventory decreases with $E_{\rm max}$, i.e. both the input inventory of product-specific parts and the output inventory of finished products can be reduced when ready periods and due dates of customer orders are closer. However, when ready dates (i.e., material availability periods) and due dates are closer than $E_{\rm max}$, the limited order earliness due to later material availability restricts reallocation of orders to the earlier periods with surplus of capacity, which may result in tardy orders or even infeasible schedules with some customer orders unscheduled during the planning horizon.

In the sequel, product-specific parts for each customer order $j \in J$ are assumed to be delivered not later than E_{\max} periods ahead of the order due date d_j , i.e., the customer ready date r_j is

$$r_i = \max\{1, d_i - E_{\max}\}\tag{2}$$

where E_{max} can be determined by the mixed integer program proposed in Sawik (2007b).

3.2. Mixed integer programming formulation

The following two variables used in model **SAMS** need to be additionally explained (for definitions of all decision variables, see Tab. 1):

Table 1 Notation

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Indices  i = \text{assembly stage, } i \in A = \{1, ..., m_A\}   j = \text{customer order, } j \in J = \{1, ..., n\}   k = \text{product type or product-specific part type } k \in K = \{1, ..., g\}   t = \text{planning period, } t \in T = \{1, ..., t_h\}
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Input parameters

 c_{it} = processing time available in assembly stage i in period t

l = length of each planning period

 m_M = number of parallel production lines in the manufacturing stage

 p_{ij} = processing time in stage i of each product in order j

 q_k = processing time for one unit of part type k

 r_i , d_i , o_i = ready date, due date, size of order j

 σ = the start-up time of production line in the manufacturing stage

 J_k = subset of customer orders for product type k (i.e. requiring part type k)

 $\underline{V}, \overline{V}$ = the minimum, the maximum shipping lot, respectively

Decision variables

 u_{kt} = manufacturing lot of part type k in period t

 v_{kt} = transportation lot of part type k in period t

 $w_t = 1$, if a shipment of parts is scheduled for period t, otherwise $w_t = 0$

 $x_{jt} = 1$, if customer order j is assigned to planning period t; otherwise $x_{jt} = 0$ (order assignment variable)

 y_{kt} = number of parallel production lines setup for processing part type k in period t

 z_{kt} = number of parallel production lines started up in period t to process part type k after processing another part type

- 1) The set-up variable y_{kt} that describes a state of manufacturing system in each planning period, i.e. the number of production lines set up for manufacturing part type k.
- 2) The start-up variable z_{kt} that represents the number of production lines which are started up to manufacture part type k in period t, i.e. the number of lines set up in period t for part type k, which has not been set up for this part type in period t-1. The start-up variable can take a positive value only if the corresponding set-up variable has in period t higher value than in period t-1, i.e.

$$z_{kt} = \max\{0, y_{kt} - y_{kt-1}\}.$$

The objective function (3) represents the weighted sum of the maximum level of total inventory, the number of start ups of production lines in the manufacturing stage and the number of part shipments from that stage to the producer stage. The weights λ_1 , λ_2 represent the relative importance in the objective function of material shipments and production line start ups and with respect to maximum level of parts and products inventory.

Model **SAMS**: Scheduling assembly, manufacturing and supply Minimize

$$I_{\max} + \lambda_1 \sum_{t \in T} w_t + \lambda_2 \sum_{k \in K} \sum_{t \in T} z_{kt}$$
(3)

subject to:

- 1. Customer order non-delayed assignment constraints
 - each customer order is assigned to exactly one planning period not later than its due date,

$$\sum_{t \in T: \, r_j \le t \le d_j} x_{jt} = 1; \quad j \in J \tag{4}$$

- 2. Assembly capacity constraints
 - in every period the demand on capacity at each assembly stage cannot be greater than the maximum available capacity in this period,
 - in period t = 1 the demand on each part type k cannot be greater than the initial inventory of this part type,

$$\sum_{j \in J} p_{ij} o_j x_{jt} \le c_{it}; \quad i \in A, t \in T$$

$$\tag{5}$$

$$\sum_{j \in J_k: r_i = 1} o_j x_{j1} \le I_{k0}^2; \quad k \in K$$
 (6)

- 3. Manufacturing line set-up and start-up constraints
 - in every period total number of production lines set up for manufacturing different part types is not greater than total number m_M of available lines,
 - all production lines set up for part type k in period 1 should be started up to manufacture this part type,

- in every period t > 1, the number of production lines started up for part type k cannot be less than the difference between the number of lines set up for this part type in periods t and t - 1,

- in every period t > 1, the number of production lines started up for part type k cannot be greater than the number of lines set up for part type k in this period and cannot be greater than the number of lines set up for the other part types or idle in period t - 1,

$$\sum_{k \in K} y_{kt} \le m_M; \quad t \in T \tag{7}$$

$$z_{k1} = y_{k1}; \quad k \in K \tag{8}$$

$$z_{kt} \ge y_{kt} - y_{k,t-1}; \quad k \in K, t \in T: t > 1$$
 (9)

$$z_{kt} \le y_{kt}; \quad k \in K, t \in T: t > 1 \tag{10}$$

$$z_{kt} \le m_M - y_{k,t-1}; \quad k \in K, t \in T : t > 1$$
 (11)

- 4. Manufacturing capacity constraints
 - in every period t the production volume of part type k cannot be greater than the maximum volume and less than the minimum volume corresponding to the capacity assigned to part type k in this period,

$$u_{kt} \le |(l - \sigma)/q_k| |z_{kt} + |l/q_k| |(y_{kt} - z_{kt}); \quad k \in K, t \in T$$
 (12)

$$u_{kt} \ge \lfloor (l - \sigma)/q_k \rfloor y_{kt}; \quad k \in K, t \in T$$
 (13)

where $\lfloor a \rfloor$ is the greatest integer not greater than a.

- 5. Material manufacturing and shipment constraints
 - parts can be supplied only in periods scheduled for shipment, and each shipping lot is limited by its minimum and maximum allowed size, respectively \underline{V} and \overline{V} ,
 - for each part type k and period t the cumulative shipping lots in periods 1 through t cannot be greater than the initial stocks and the cumulative production of this part type in periods 1 through t,

$$v_{kt} \le \overline{V}w_t; \quad k \in K, t \in T \tag{14}$$

$$\sum_{k \in K} v_{kt} \le \overline{V}; \quad t \in T \tag{15}$$

$$\sum_{k \in K} v_{kt} \ge \underline{V}w_t; \quad t \in T \tag{16}$$

$$\sum_{\tau=1}^{t} v_{k\tau} \le I_{k0}^{1} + \sum_{\tau=1}^{t} u_{k\tau}; \quad k \in K, t \in T$$
(17)

- 6. Material demand satisfaction constraints
 - for every period t the cumulative production of product type k in periods 1 through t cannot be greater than the initial stocks and the cumulative shipping lots of part type k in periods 1 through t-1,

$$\sum_{j \in J_k} \sum_{\tau=1}^t o_j x_{j\tau} \le I_{k0}^2 + \sum_{\tau=1}^{t-1} v_{k\tau}; \quad k \in K, t \in T$$
 (18)

7. Maximum inventory constraints

$$\sum_{k \in K} \left(I_{k0}^1 + I_{k0}^2 + \sum_{\tau=1}^t u_{k\tau} - \sum_{\tau=1}^t \sum_{j \in J_k: r_j \le \tau \le d_j \le t} o_j x_{j\tau} \right) \le I_{\max}; \quad t \in T \quad (19)$$

8. Variable nonnegativity and integrality constraints

$$u_{kt} \ge 0; \quad k \in K, t \in T \tag{20}$$

$$v_{kt} \ge 0; \quad k \in K, t \in T \tag{21}$$

$$w_t \in \{0, 1\}; \quad t \in T$$
 (22)

$$x_{jt} \in \{0,1\}; \quad j \in J, t \in T : r_j \le t \le d_j$$
 (23)

$$y_{kt} \ge 0$$
, integer; $k \in K, t \in T$ (24)

$$z_{kt} \ge 0$$
, integer; $k \in K, t \in T$ (25)

The mixed integer program **SAMS** contains some cutting constraints: (10), (11), (13), (14). In order to further tighten the formulation additional cutting constraints can be identified and introduced into the model, e.g. Sawik (2006, 2007a, 2007b).

4. Computational examples

In this section some computational examples are presented to illustrate possible applications of the proposed approach. The examples are modeled after a real world supply chain of high-tech products. A brief description of the manufacturing and assembly stages of the supply chain, parts, products and the customer orders is given below.

- 1. Planning horizon: $t_h = 30$ days, each of length $l = 2 \times 9$ hours.
- 2. Manufacturing
 - $-m_M$ = 22 parallel production lines, each to be set-up at most once per planning period, with identical start-up time σ = 9000 seconds.

- Parts
 - g = 10 product-specific part types,
 - processing times q_k (in seconds) for part types k = 1, ..., 10: $q_1 = 65, q_2 = 70, q_3 = 80, q_4 = 85, q_5 = 90, q_6 = 95, q_7 = 65, q_8 = 80, q_9 = 90, q_{10} = 85.$

3. Supply

- at most one shipment of various parts per period,
- the minimum and the maximum allowed shipping lot: $\underline{V} = 1000$, and $\overline{V} = 50~000$ parts,
- transportation time constant and equal to one period for every shipping lot.

4. Assembly

- $-m_A=6$ assembly stages in series with parallel machines: $m_i=10$ parallel machines in each stage i=1,2; $m_i=20$ parallel machines in each stage i=3,4,5; and $m_i=10$ parallel machines in stage i=6,
- for each assembly stage i the available processing time c_{it} is the same in every period t:

$$c_{it} = c_i = \alpha_i l m_i$$
; $i = 1, ..., 6, t = 1, ..., 30$

where parameter $\alpha_i \in (0, 1)$ reflects the idle time of each machine waiting for the first production lot from upstream stages 1, ..., i-1 and the idle time during processing of the last production lot at downstream stages $i+1, ..., m_A$, assuming that all production lots of each customer order are completed in a single period (Sawik, 2007a).

- Products

- g = 10 product types of three product groups, each to be processed on a separate group of machines (in stage 3 or 4 or 5),
- processing times (in seconds) for product types:

product type/stage 1		2	3	4	5	6
1	20	0	120	0	0	15
2	20	0	140	0	0	15
3	10	0	160	0	0	10
4	15	5	0	120	0	15
5	15	10	0	140	0	15
6	10	5	0	160	0	10
7	15	10	0	180	0	15
8	20	5	0	0	120	15
9	15	0	0	0	140	10
10	15	0	0	0	160	10

• n = 805 customer orders ranging from 5 to 9620 products, with various due dates d_j and ready dates $r_j = \max\{1, d_j - 2\}$ (2), each to be completed in a single period. The total demand is 535 000 products.

5. Initial inventory

 The initial supplier stocks of manufactured parts are equal to an average two-day manufacturing volumes

$$I_{k0}^{1} = 2 | lm_{M} / gq_{k} |; k = 1, ..., 10.$$

 The initial producer stocks of supplied parts ensure an average two-day as-sembly of the finished products

$$I_{k0}^2 = 2 \max_{i \in A, j \in J_k, k \in K} \left[c_i / g p_{ij} \right]; \quad k = 1, ..., 10.$$

Table 2Computational results

Model	Var.	Bin.	Int.	Cons.	Solution values ^(a)	CPU(GAP) ^(b)
SAMS(0.1,0.1) ^(c)	3508	2307	601	3725	$I_{\text{max}} = 120348, W_{sum} = 24, Z_{sum} = 40$	195
SAMS(1,1) ^(c)	3509	2307	601	3725	$I_{\text{max}} = 120348, W_{sum} = 19, Z_{sum} = 32$	369
SAMS(10,10) ^(c)	3509	2307	601	3725	$I_{\text{max}} = 120348, W_{sum} = 19, Z_{sum} = 32$	(0.10%)
SAMS(100,100) ^(c)	3509	2307	601	3725	$I_{\text{max}} = 120348, W_{sum} = 19, Z_{sum} = 32$	(0.99%)

⁽a) $W_{sum} = \sum_{t \in T} w_t$ - total number of shipments,

$$Z_{sum} = \sum_{k \in K} \sum_{t \in T} z_{kt}$$
 – total number of setups.

The characteristics of mixed integer program are summarized in Table 2. The size of mixed integer program **SAMS** is represented by the total number of variables, Var., number of binary variables, Bin., number of integer variables, Int., and number of constraints, Cons. The last two columns of Table 2 present the solution values: I_{max} – maximum total inventory level, $W_{\text{sum}} = \sum_{t \in T} w_t$ – total number of shipments, $Z_{\text{sum}} = \sum_{k \in K} \sum_{t \in T} z_{kt}$ – total number of start ups, and CPU time in seconds required to prove optimality of the solution (or % GAP after 1800 seconds of CPU time, if optimality is not proven). The computational experiments were performed using AMPL programming language and the CPLEX v.9.1 solver (with the default settings) on a laptop with a Pentium IV processor running at 1.8 GHz and with 1GB RAM.

The solution results are additionally illustrated in Figures 1–4. Manufacturing, supply and assembly schedules for various weights λ_1 and λ_2 are shown in Figure 1.

⁽b) CPU seconds for proving optimality on a PC Pentium IV, 1.8GHz, RAM 1GB/CPLEX v.9.1 or GAP% if optimality not proven in 1800 CPU seconds.

⁽c) weights λ_1 , λ_2 .

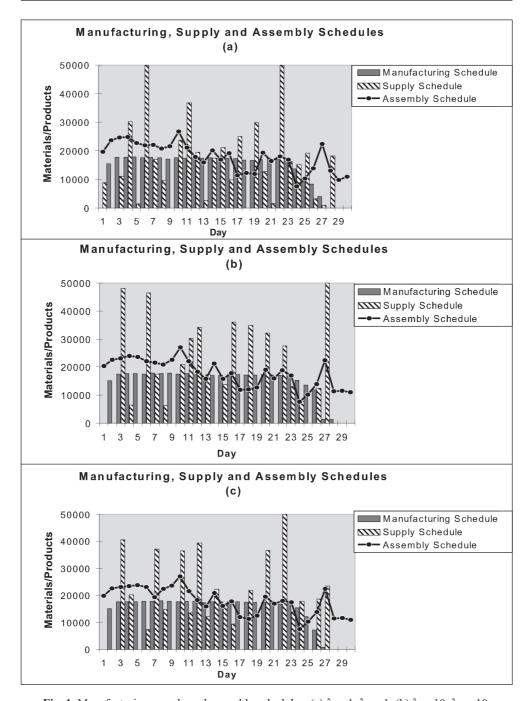


Fig. 1. Manufacturing, supply and assembly schedules: (a) λ_1 =1, λ_2 = 1, (b) λ_1 =10, λ_2 = 10, (c) λ_1 =100, λ_2 = 100

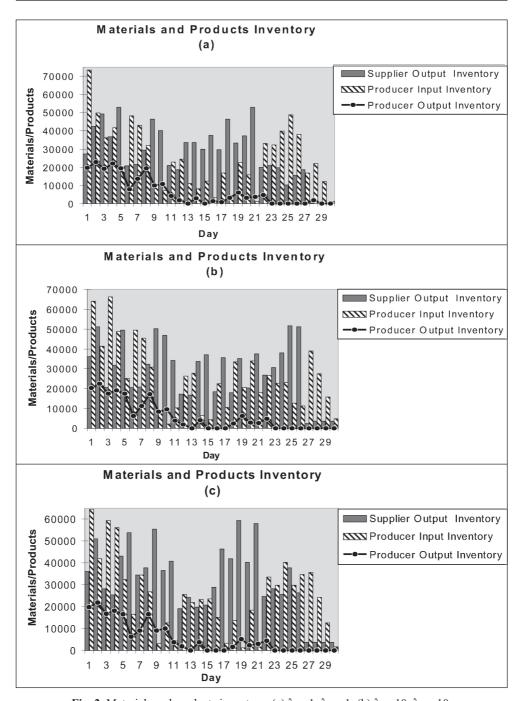


Fig. 2. Materials and products inventory: (a) λ_1 =1, λ_2 = 1, (b) λ_1 =10, λ_2 = 10, (c) λ_1 =100, λ_2 = 100

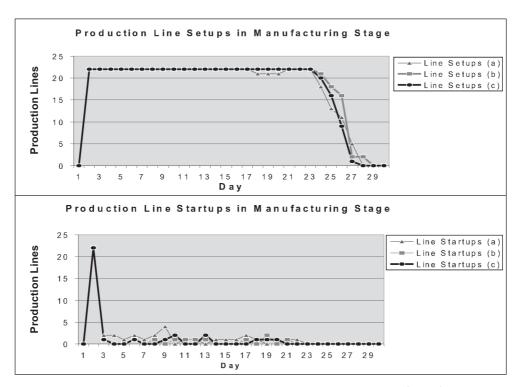


Fig. 3. Setups and startups of production lines in manufacturing stage: (a) λ_1 =1, λ_2 = 1, (b) λ_1 =10, λ_2 = 10, (c) λ_1 =100, λ_2 = 100

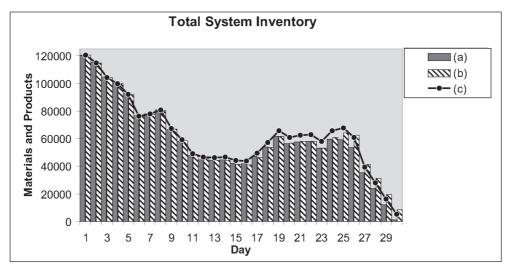


Fig. 4. Total system inventory: (a) λ_1 =1, λ_2 = 1, (b) λ_1 =10, λ_2 = 10, (c) λ_1 =100, λ_2 = 100

The corresponding supplier output inventory of manufactured parts, producer input inventory of delivered parts and producer output inventory of finished products are presented in Figure 2. The figures indicate that the assembly schedules and the finished products inventories are virtually independent on the selected values of weights. The greater λ_2 the less frequent are the shipments of parts to producer and the smaller are differences among the shipping lots. The greater weights λ_1 and λ_2 , the smaller fluctuations of line set ups and start ups over the horizon, see Figure 3. Figure 4 shows that the total inventory level varies over the horizon virtually independently on the weights λ_1 and λ_2 . The results indicate that the objective function aims at leveling of production at manufacturing stage (and thereby leveling of line set ups and reduction of line start ups) at the expense of greater fluctuations and higher levels of parts inventories at supplier and producer and larger shipping lots.

The proposed mixed integer programming approach can also be applied for a reactive planning on the rolling horizon basis (e.g. Sawik, 2007d), in response to the various disruptions in manufacturing, supply and assembly that may occur in practice.

5. Conclusion

In this paper the integration of supply, production and distribution in a customer driven supply chain is considered and mixed integer programming formulations are proposed for the long term scheduling and coordination of parts manufacturing and supply and finished products assembly.

The computational experiments modeled after a real world customer driven supply chain in the electronics industry have indicated that the proposed approach is capable of finding good integrated long-term schedules for large size problems, in a reasonable computation time, using commercially available software for integer programming.

The proposed approach can be modified or enhanced to consider hierarchical planning frameworks, different objective functions and various additional features of the customer driven supply chain, such as the manufacturing and supplies of both product-specific and common parts, the manufacturing and supplies by different suppliers at different locations and with different costs, e.g. Kaczmarczyk *et al.* (2006), etc.

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