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Petri Nets of the Materials Flow at the Steel Plant

1. Introduction

The Petri nets are the tools that are useful for production [1, 4, 6, 9], design [8] and service process modelling. One may find interesting Petri net applications to the issues related to the information flows or capital flows [2, 3].

The Petri nets are strict records of all the allowed courses of a flow process [4]. They allow to present not only the precedence relationships imposed on particular activities related to the flow process, but also to take into account the concurrence of many processes, or the availability of the resources that fulfil the flow processes. The nets allow us to test the process being modelled, with respect to the existence of cycles, blockades, specific action sequences and the relations between the actions; thus, they allow us to examine the correctness of the course of the process being modelled. The nets are capable of determining the optimum routing of a process, with current flow monitoring. The Petri nets are the tools designed for the simulation of the modelled system's behaviour [7].

A classical Petri net may be expanded in such a way that it becomes suitable for materials flow modelling, for example the flow of materials at the steel plant.

In this paper, we have proposed expansions of each net vertex, using sets of attributes, including such as: the cost of performing an operation, the time of performing an operation, the value of the material used at the particular stage of the order execution, the logical re-lationships fulfilling the roles of additional conditions that are required to be met when completing the subsequent process stage and the procedure of updating such attributes. Such an expanded Petri net displays many advantages. The mathematical formalism of a Petri net is simple to implement in a computer. A Petri net allows the user to check the relationships between various important properties (costs, delivery dates, or process bottlenecks) at each stage of the process performance, and determine how the changes and interruptions in supplies are spread in the system and how that will affect the whole chain of the materials flow.

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2. A time Petri net for the materials flow

The Petri net designed for the materials flow may be defined as an ordered eleven (1) which is an expansion of the classical Petri net definitions of the location/passage or the condition/event types:

$$PNMF = (P, T, O, D, L, E, F, K, W, \tau, \overline{M_0}) \tag{1}$$

where:

P, T – non-empty, finite sets of locations, $P = \{p_1, p_2, \ldots, p_n\}$, and of passages (transitions), $T = \{t_1, t_2, \ldots, t_m\}$, which meet the following conditions: $P \cup T \neq \emptyset, P \cap T = \emptyset$.
A graphic location is represented by a circle and a transition by a thick line.

E – incidence relationship fulfilling the condition: $\mathrm{dom}(E) \cup \mathrm{cod}(E) = P \cup T$, where: $\mathrm{dom}(E)$ and $\mathrm{cod}(E)$ mean the domain and the anti-domain of the E relationship, respectively. The incidence relationship is presented graphically with a directed arc which connects the location and the transition, or the transition with a location. A set of arcs is also marked by E.

K – location capacity function, $K\colon P \to N$, N – a set of natural numbers.

W – arc multiplicity function, $W\colon E \to N_0$, $N_0 = N \cup \{0\}$.

O^i – a set of the attributes which describe each location p_i of the net, $O^i = \{o_1^{\,i}, o_2^{\,i}, \ldots, o_o^{\,i}\}$.

D^i – a set of the attributes which characterise the transition t_i, $D^i = \{d_1^{\,i}, d_2^{\,i}, \ldots, d_d^{\,i}\}$.

L^i – a set of the logical rules which additionally condition the firing of a given transition $L^i = \{l_1^{\,i}, l_2^{\,i}, \ldots, l_l^{\,i}\}$.

F^i – a set of the functions and procedures that update the sets of attribute, $F^i = \{f_1^{\,i}, f_2^{\,i}, \ldots, f_f^{\,i}\}$. Transition t_i may be expressed by the three components of: (D^i, L^i, F^i).

τ – is the function which assigns the time of passage to each transition or passage. $\tau\colon T \to R^+$.

$\overline{M_0}$ – is the initial marking function which assigns a pair of natural numbers, $\overline{M_0} : P \to N{\times}N$, to each location. The first pair coordinate, the so called $crd^1\ \overline{M_0}\,(p)$, means the condition status or occupancy of the location p. The second coordinate, $crd^2\ \overline{M_0}\,(p)$, is the status of the location reservation (the number of activated passages, or the number of initiated transition firings which will arrive at the location p. The initial marking function fulfils the condition:

$$(\forall p \in P)\left(crd^1\overline{M}\ (p) + crd^2\overline{M}\,(p) \leq K(p)\right) \tag{2}$$

Initial marking, similarly to marking at each process status, represents graphically the markers which are placed in the respective net locations.

The flow operation performance rules, represented by the transitions, have the following form:

- in all the input locations, there are proper numbers of markers:

$$(\forall p \in {}^{*}t)\Big(crd^{1}\overline{M}(p) \geq W(p,t)\Big) \tag{3}$$

$$(\forall p \in t^{*})\Big(crd^{1}\overline{M}\ (p) + crd^{2}\overline{M}(p) \leq K(p)\Big) \tag{4}$$

where: $^{*}t = \{p|\ (p, t) \in E\}$ – a set of input locations of the transitions t;

$t^{*} = \{p|\ (t, p) \in E\}$ – a set of output locations of the transitions t.

- the respective logical rules of the vector L^{i} are fulfilled.

Each Petri net of the location/passage is equivalent to the Petri net of the condition/event type. The marker placed in the selected location of such a net means that the condition of the specific transition performance has been fulfilled. Consequently, there can exist a double interpretation of the vertexes of he Petri net that models any production, service or flow system. In the modelling net, the condition/event transition is the event which starts or ends a given production or flow operation assigned to the given location. In a Petri net, the location/passage transition represents an operation, while the location represents the status of the process performance or the resource status.

The Petri net dynamics is determined by the passage function. That function allows us to determine the marking status $M_{\ i+1}$ achievable directly from the marking status M_{i}. The performance of the passage function – or the transition firing – takes place in two stages. In the first stage, the net passes from the marking status $\overline{M}(p)$ to the activation status $\overline{M}'(p)$ (formula (5)), and afterwards, in the second stage, after the termination of the activation status (or the transition duration, being the operation performance time), $\hat{o}$ passes into the status which is determined by the marking $\overline{M}''(p)$ (formula (6)).

$$\overline{M}'(p) = \begin{cases} \Big(crd^{1}\overline{M}(p) - W(p,t), crd^{2}\overline{M}\ (p)\Big) & \text{for } p_i \in {}^{*}t \\ \Big(crd^{1}\overline{M}(p), crd^{2}\overline{M}(p) - W(t,p)\Big) & \text{for } p_i \in t^{*} \\ \overline{M}(p) & \text{in the remaining cases} \end{cases} \tag{5}$$

$$\overline{M}''(p) = \begin{cases} \Big(\overline{M}'(p)\Big) & \text{for } p_i \in {}^{*}t \\ \Big(crd^{1}\overline{M}'(p) + W(t,\ p),\ crd^{2}\overline{M}'(p) - W(t,\ p)\Big) & \text{for } p_i \in t^{*} \\ \overline{M}(p) & \text{in the remaining cases} \end{cases} \tag{6}$$

The update of the location feature status after the completion of a selected transition is determined by formula (7).

$$[O^1, O^2, \dots, O^n] \cdot M^{i+1} = F\{[O^1, O^2, \dots, O^n] \cdot M^{i},\ [D^1, D^2, \dots, D^m] \cdot M^{i} \tag{7}$$

where $M^{i} = crd^1 \overline{M^{i}}(p)$.

What is important in the analysis of the operation of the systems modelled with Petri nets is the set of the so-called achievable states, marked by S. The set $S(M^{i})$ of the given marking status M^{i} is a set of all the marking states that are achievable by firing all possible transition sequences of that state. The analysis of the set S allows us to discover non-beneficial phenomena that may occur in the process being modelled. For example, when one of the achievable states is empty, the net is blocked. Then, we say that the net is unused or blocked. Lack of the initial marking status M^{0} in one of the S states indicates the existence of the so-called inactive net in which it is not possible to achieve the initial net status. That means for example that the termination of production at the steel plant will be associated with liquid steel remaining in the system.

The deterministic time Petri net should be applied whenever there is a possibility of delayed performance times of particular operations or flows, which causes the change of the operation performance sequences. In a general case, the time Petri net allows us to consider certain sets of allowed set solutions of the materials flow process course. The problem of selection of the best solution may be resolved for example by application of the computer simulation method. During such a simulation, we examine the model's behaviour for various allowed solutions at various rules of the priorities of transition firing. The rule for which the best solution is found, based on the assumed system operation criterion, determines the option of the process performance that we are looking for.

3. Petri net at steel plant

A synthesis of a Petri net applicable to a production process or a materials flow process should be conducted in accordance with the following algorithm:

Step 1. Identify all operations and all resources that are indispensable to perform a production or flow process.

Step 2. Order the operations, in accordance with the precedence relationship required in the process.

Step 3. For each operation, carry out the following sequence:

- Create and describe the locations representing particular operations or flows.
- Add the transitions being the events which commence the operations, with input arcs to those locations.
- Add the transitions being the events which end the operations, with output arcs to those locations. The transition which ends a certain action is at the same time the one which commences the subsequent action or actions.

Step 4. For each operation, carry out the following sequence:
- Select the resources that are indispensable for the operation performance, ed, create them.
- Using arcs, connect all the respective locations representing resources with the respective transitions which model the events commencing the actions and with the transitions of the events which end the actions.

The steel plant Petri net is a time net. It contains both time transitions (e.g.: transport of steel between particular working positions), and time locations in which process operations are conducted (e.g.: steel melting in converters, argon treatment, or continuous steel casting).

The timeliness of the materials flow processes may generally occur as follows:

- During the passage performance: the transition firing times are different from zero.
- At the locations which model the operations or flows: the markers which enter the given location are achievable only after certain time (Fig. 1a).
- On the arcs modelling a transportation flow or operation that lead from the locations to the transitions: the markers have to wait in the locations for the assumed transportation operation time to expire (Fig. 1b).
- At the arcs modelling a transportation flow or operation that lead from the transition to locations: the markers generated by the transition must wait to feed the location (Fig. 1c).

One can prove that each deterministic Petri net may be transformed into one of the deterministic Petri nets with time transitions or time locations or time arcs.

Figure 1 presents the method of transforming the net with time locations and time arcs into one with time transitions.

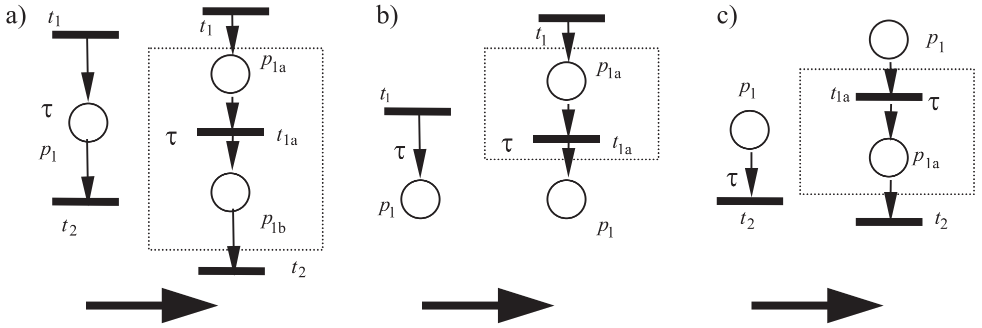


Fig. 1. Transformation of the time locations and arcs of the Petri network: a) location; b) arc from the transition to a location; c) arc from the location to a transition

The materials flow processes occurring at the continuous steel casting department became an object of a quantitative analysis, e.g. in [5]. That analysis allowed to make an attempt at the presentation of those processes for the purpose of using a model built on the basis of deterministic time Petri nets. The analysis was applied to the system composed of two oxygen converters (always one of the three converters is stopped for repairs) and two argon treatment positions from which liquid steel is transferred either to the furnace ladle or to two ingot casting positions. Two continuous steel casting machines receive steel either directly from the furnace ladle or from the machine in which the process of vacuum steel degassing is taking place. The final products are blooms, billets or ingots.

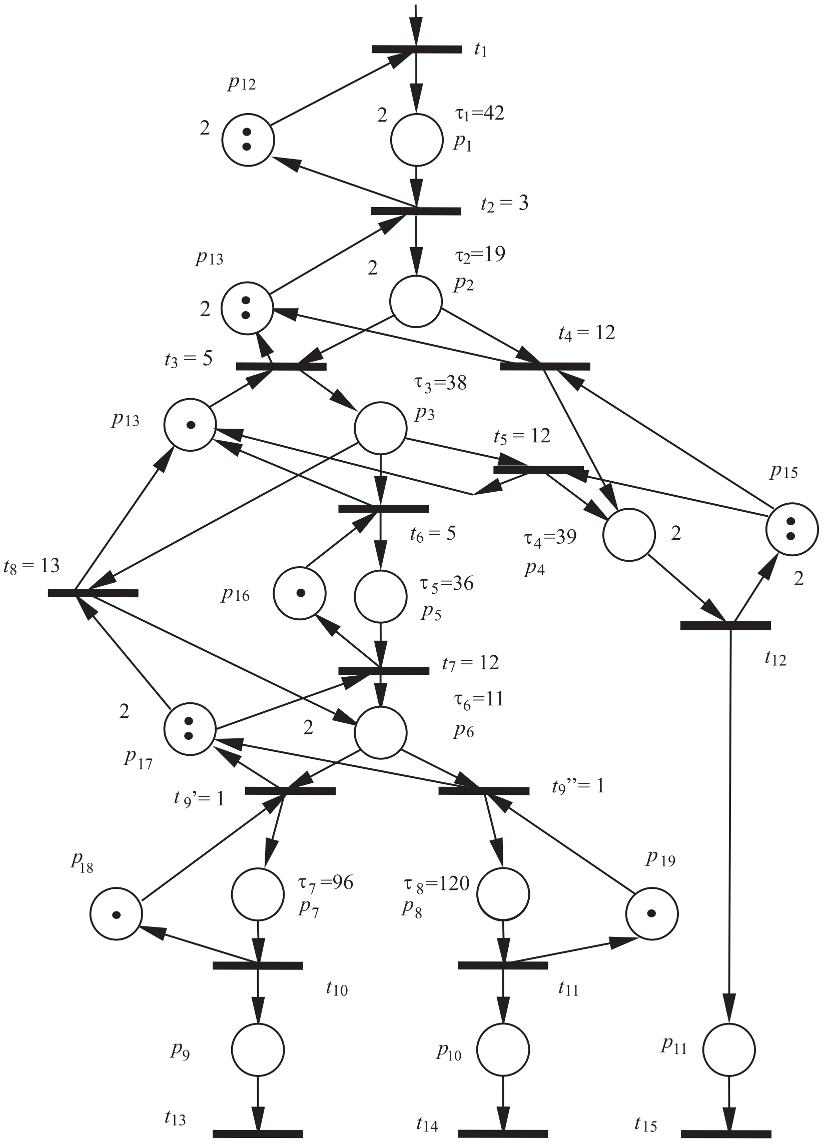


Fig. 2. Deterministic time Petri net at the steel plant

The steel plant Petri net is presented in Figure 2. Certain reductions were made in the net. They affect the course of the process analysis and the process simulation results. The converters are recorded in one net vertex, given the capacity of 2 (since two converters are operational). Similar decision concerned two argon treatment positions, two billeting positions and two rotating towers.

During the steel plant operation analysis, as modelled by a Petri net, we used the following markings:

Locations: p_1 – steel in the converter, p_2 – steel in argon treatment, p_3 – steel in furnace ladle, p_4 – steel in ingot casting, p_5 – steel in degassing, p_6 – steel at the rotating tower, p_7 – steel in Caster 1, p_8 – steel in Caster 2, p_9 – steel in blooms leaving the system, p_{10} – steel in billets leaving the system, p_{11} – steel in ingots leaving the system, p_{12} – converter occupation status, p_{13} – argon treatment position occupation status, p_{14} – furnace ladle occupation status, p_{15} – ingot mould occupation status, p_{16} – degassing position occupation status, p_{17} – rotating tower occupation status, p_{18} – Caster 1 occupation status, p_{19} – Caster 2 occupation status.

Transitions: t_1 – casting start in the converter, t_2 – casting end in the converter and argon treatment start, t_3 – argon treatment end and furnace ladle process start, t_4 – argon treatment end and casting process start, t_5 – furnace ladle process end and casting start, t_6 – furnace ladle process end and steel degassing start, t_7 – steel degassing process end and rotating tower process start, t_8 – furnace ladle process end and rotating tower process start, t_9' – rotating tower process end and Caster 1 process start, t_9'' – rotating tower process end and Caster 2 process start, t_{10} – Caster 1 process end and bloom transport process start, t_{11} – Caster 2 process end and billet transport process start, t_{12} – ingot casting process end and ingot transport process start, t_{13} – blooms leaving the system, t_{14} – billets leaving the system, t_{15} – ingots leaving the system.

Each net vertex contains the attributes which, among others, include the cost (value) and availability time, the time of performing the transport or process operation (Tab. 1).

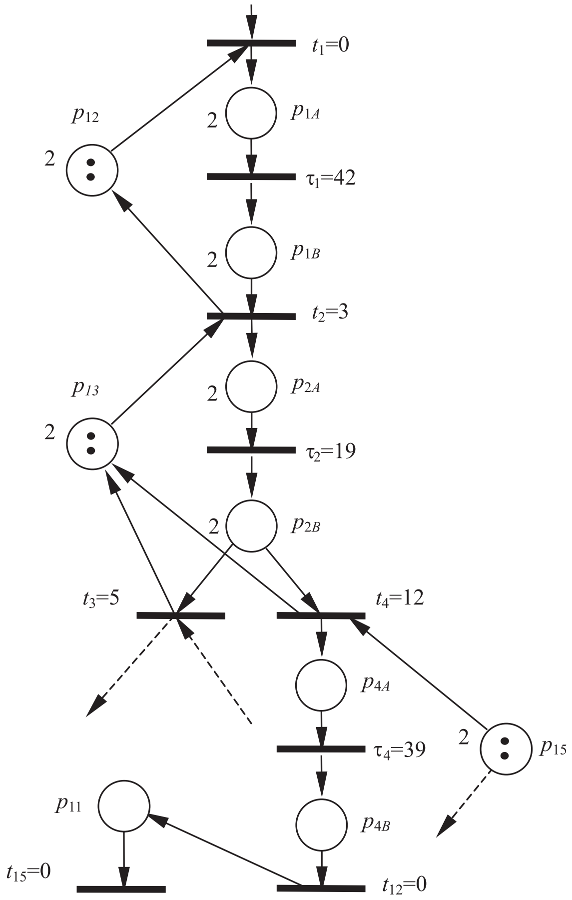
Table 1
Average operation times at the steel plant

Stay time at locations		Transport operation time	
p_1	42	t_2	3
p_2	19	t_3	5
p_3	38	t_4	12
p_4	39	t_5	12
p_5	36	t_6	5
p_6	11	t_7	12
p_7	96	t_8	13
p_8	120	t_9'	1
		t_9''	1

The deterministic time net of the steel plant is presented in Figure 2.

The achievable marker graph analysis leads to the following conclusions:

- The net is living which means that no position blockades are occurring in the system, resulting from poor departmental structure.
- The net is active which indicates the possibility of achieving the initial department load status, the status in which the converters do not produce steel. The status may be achieved without the need to leave steel which is not processed completely.
- There are conflicts in the net. They consist in the possibility of the process performance by several methods leading to obtaining various final products. For example, the steel which has passed through argon treatment may be transported either to the ingot casting position or subjected to further processing in the furnace ladle. Such and other options may be used by application of the logical rules *L* (formula (7)), proposed in this paper, which rules strictly condition the proper continuation of the production process.



Rys. 3. Modified fragment of Petri net presented in Figure 2

The application of the transformation laws (Fig. 1) to the net in which, in addition to the transitions, also locations are of time nature, will lead to the generation of a new net. The Petri net modified in that way contains only time transitions and, owing to that, a classical steel plant net analysis is possible. Figure 3 contains only a fragment of a highly developed net. Our analysis is limited to the sub-net made up of the following vertexes: p_1, p_2, p_4, p_{11}, p_{12}, p_{15}, t_1, t_2, t_4, t_{12}, t_{15}, as well as the respective arcs. The marker graph of that sub-net is shown in Figure 4.

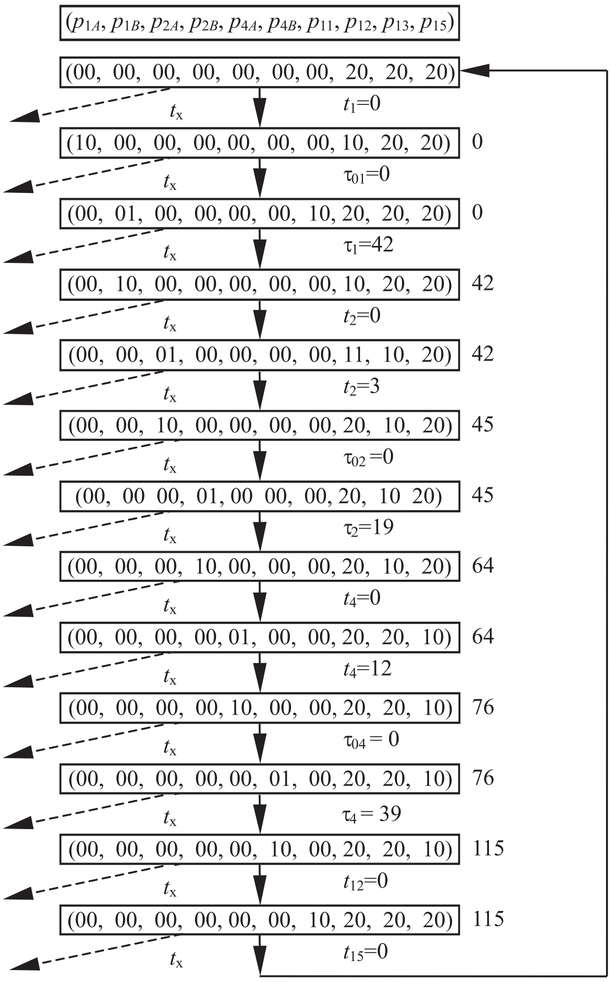


Fig. 4. Fragment of the marker graph of the Petri net presented in Figure 3

The net analysis and the simulation of its behaviour allows us, among others, to check the production cycle duration. For example, if the operation sequence is such as in Figure 4, the moment of ingot production completion will happen after 115 time units.

Modelling of complex production and flow systems requires the application of a modular approach. Usually, the process runs in accordance with the following procedure:

Step 1. Decomposition of the system into smaller modules that are easier to model and analyse.

Step 2. Building a Petri net separately for each module. That requires:

- Introduction of additional transitions in the modules to which streams of e.g. raw materials, semi-products, parts or tools are delivered.
- Introduction of additional transitions in the modules from which such streams are channelled.
- Introduction of the locations modelling particular module connections.

Step 3. Checking the particular module net properties with respect to blockades, activity and flexibility.

Step 4. Analysis of each module's properties, capacity and load.

Step 5. Integration of the modules, observing the module's properties.

Step 6. Determination of the whole proper integrated system.

In modelling, we can also apply a hierarchic approach which consists in building first a general Petri net, followed by sub-nets for distinguished transitions. Each sub-net can be composed of even more detailed subsequent nets. The essence of that approach is presented in Figure 5.

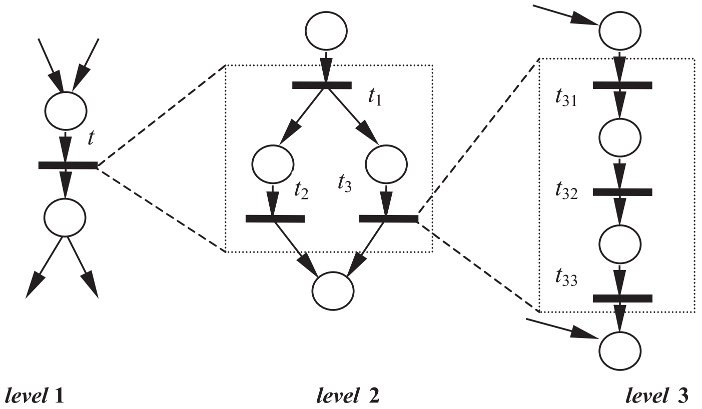


Fig. 5. Hierarchic Petri net structure

In many cases, the operation performance time is not deterministic, and it is characterised by the random variable probability distribution function. In such cases, it is necessary to define and apply stochastic Petri nets [1].

4. Conclusions

The Petri nets applied to the material flow net analysis in the steel plant turned out to be very effective research tools. They model in a simple way the whole flow net, including also the net which takes into account a common use of the resources in various processes. The nets allow us to model concurrent processes and flows. The nets can be flexibly expanded, with analysis of particular subsystems and creation of the so-called hyper nets which operate on the sets of changing sub-nets. That formalised approach is easy to apply in he computer implementation and computer visualisation of flows. The expansion of classical Petri nets by the vectors of the attributes of locations and transitions, logical rules of transition firing and the procedures that update the values of those attributes is extremely important from the user's viewpoint. Owing to such expansion, it is possible to observe the properties of the streams flowing through the system, determine easily for example the project cost and time, simulate the effects of the system changes and analyse the effects of the unexpected disturbances and interruptions occurring in the supply chain.

The issue of the application of the Petri nets to the materials flow analysis is not exhausted. Future research should concentrate on the development of effective methods of the integration of particular Petri nets that model the fragments of the flow nets. One of the possible methods of such integration can be the agent systems in which each agent is, in a sense, an independent sub-net of materials flow. Another issue which requires intense research is the development of the methods for redesigning of the poorly or unreliably operating systems.

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