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A Multi-Objective Assignment of Customer Orders to Planning Periods in Make-to-Order Manufacturing**

1. Introduction

In make-to-order manufacturing one of the basic goals of the long-term scheduling is to maximize customer service level, i.e., to maximize the fraction of customer orders filled on or before their due dates. A typical customer due date performance measure is minimization of the number of tardy orders, e.g. [1, 2].

The purpose of this paper is to propose mixed integer programming formulations for a multi-objective, long-term production scheduling in a flexible flowshop. The scheduling objective is to allocate a set of customer orders with various due dates among planning periods to minimize, first number of tardy orders, then number of early orders, and finally to level production resources over a planning horizon, e.g. [3, 4, 5]. Minimizing the number of early orders help to reduce the inventory of finished products waiting for the delivery to customers, whereas levelling the resource usage reduces unit production costs [6].

In the literature on production planning and scheduling the integer programming models have been widely used. In industrial practice, however, the application of integer programming for scheduling is limited. For example, a hierarchical approach and integer programs for production scheduling in make-to-order company are presented in [1], however computational results are based on developed heuristics. An integer goal programming formulation for production scheduling with a due date related criterion is also presented in [2], and the focus is again on application of heuristic approaches.

The paper is organized as follows. In the next section the description of make-to-order production scheduling in a flexible flowshop is provided. The integer programming formulations for the weighting and the lexicographic approach to multi-objective production scheduling are presented in Section 3. Numerical examples modeled after a real-world make-to-order assembly system and some computational results are provided in Section 4. Conclusions are presented in the last section.

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2. Problem Description

The production system under study is a flexible flowshop that consists of m processing stages in series and each stage $i \in I = \{1, ..., m\}$ is made up of $m_i \geq 1$ identical, parallel machines. In the system various types of products are produced in a make-to-order environment responding directly to customer orders. Let J be the set customer orders. Each order $j \in J$ is described by a triple (a_j, d_j, s_j), where a_j is the order arrival date (e.g. the earliest period of material availability), d_j is the customer due date (e.g. customer required shipping date), and s_j is the size of order (quantity of ordered products). Denote by $J(d)$ the subset of orders with the same due date $d \in D$, where $D = \{d_j : j \in J\}$ is the set of distinct due dates of all customer orders.

Each order requires processing in various processing stages, however some orders may bypass some stages. Let $J_i \subset J$ be the subset of orders that must be processed in stage i, and let $p_{ij} > 0$ be the processing time in stage i of each product in order $j \in J_i$. The orders are processed and transferred among the stages in lots of various size and let b_j be the size of production lot for order j.

The planning horizon consists of h planning periods (e.g. working days).

Let $T = \{1, ..., h\}$ be the set of planning periods and c_{it} the processing time available in period t on each machine in stage i.

It is assumed that each customer order must be fully completed in exactly one planning period and the available capacity is sufficient to schedule all orders during the planning horizon.

The objective of the long-term production scheduling is to assign customer orders to planning periods and to select machines for assignment in every period to minimize numbers of tardy and early orders, respectively as a primary and secondary optimality criterion and to level machine assignments as an auxiliary criterion.

The two approaches will be applied:

1) weighting,
2) lexicographic.

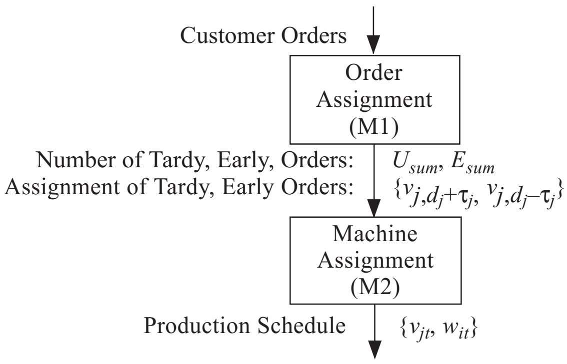

Fig. 1. A lexicographic approach to multi-objective production scheduling

In the weighting approach the triple-objective scheduling problem is reduced to a single objective problem by introducing the weights representing the relative importance of the three objectives. In the lexicographic approach first the customer orders are allocated among planning periods to find the optimal numbers of tardy and early orders and then the final production schedule is determined to level machine assignments over the horizon for a minimum number of tardy and early orders (see Fig. 1).

3. Multi-Objective Production Scheduling

In this section the integer programming formulation are presented for the two approaches to multi-objective production scheduling: **M0** for the weighting approach, and a pair **M1**, **M2** for the lexicographic approach. Decision variables are defined in Table 1.

Table 1
Decision variables

v_{jt}	=	1, if order j is performed in period t; otherwise v_{jt} = 0 (order assignment variable)
w_{it}	=	number of machines selected for assignment in stage i in period t (machine selection variable)
E_{Sum}, U_{sum}	=	number of early orders, tardy orders, respectively
$M_{\max}$	=	maximum number of machine assignments in a single planning period

Model M0: *Customer order and machine assignment*
Minimize

$$\lambda_1 \sum_{j \in J: t > d_j} v_{jt} + \lambda_2 \sum_{j \in J: a_j \le t < d_j} v_{jt} + \lambda_3 M_{\max} \quad (1)$$

subject to:

1. *Order assignment constraints*
   – each customer order is assigned to exactly one planning period:

$$\sum_{t \in T} v_{jt} = 1; \quad j \in J \quad (2)$$

2. *Machine assignment constraints*

   In every period:
   – the number of machines selected for assignment at each stage is not greater than the maximum number of available machines,
   – the number of machines selected for assignment at each stage is not greater than the total number of assigned production lots,

– the total number of machines selected for assignment cannot exceed the maximum number of machine assignments to be minimized,
– the demand on capacity at each processing stage cannot be greater than the total capacity of machines selected for assignment in this period,

$$w_{it} \le m_i; \quad i \in I, t \in T \tag{3}$$

$$w_{it} \le \sum_{j \in J_i} \lceil s_j / b_j \rceil v_{jt}; \quad i \in I, t \in T \tag{4}$$

$$\sum_{i \in I} w_{it} \le M_{\max}; \quad t \in T \tag{5}$$

$$\sum_{j \in J_i} p_{ij} s_j v_{jt} \le c_{it} w_{it}; \quad i \in I, t \in T \tag{6}$$

3. *Variable integrality constraints*

$$v_{jt} \in \{0, 1\}; j \in J, t \in T : t \ge a_j \tag{7}$$

$$w_{it} \ge 0, \textit{integer}; i \in I, t \in T \tag{8}$$

$$M_{\max} \ge 0, \textit{integer} \tag{9}$$

where $\lceil \cdot \rceil$ is the least integer not less than $\cdot$.

In the objective function (1) tardy orders are penalized at a much higher rate than early orders, and the early orders are penalized higher than maximum machine assignments, i.e., $\lambda_1 >> \lambda_2 > \lambda_3$.

Model M1: *Customer order assignment*
Minimize

$$\lambda_1 \sum_{j \in J : t > d_j} v_{jt} + \lambda_2 \sum_{j \in J : a_j \le t < d_j} v_{jt} \tag{10}$$

subject to (2), (7) and

Capacity constraints

– in every period the demand on capacity at each processing stage cannot be greater than the maximum available capacity in this period,

$$\sum_{j \in J_i} p_{ij} s_j v_{jt} \le c_{it} m_i; \quad i \in I, t \in T \tag{11}$$

where $\lambda_1 >> \lambda_2$.

The solution to **M1** (Fig. 1) determines the minimum number of tardy orders u^*_{sum}, minimum subset of tardy orders $JT \subset J$ and their assignment to planning periods $v_{j,d_j+\tau_j} = 1$, $j \in JT$, where $\tau_j \geq 1$ is the tardiness of order $j \in JT$. Simultaneously, the minimum number of early orders E^*_{sum} and the subset of early orders $JE \subset J$ are determined as well as their assignment to planning periods $v_{j,d_j-\tau_j} = 1$, $j \in JE$, where $\tau_j \geq 1$ is the earliness of order $j \in JE$. Implicitly, the subset $JD = J \setminus JE \cup JT$ of orders assigned on due dates is determined, i.e., such that $v_{j,d_j} = 1$, $j \in JD$.

The smaller the earliness for the early orders, the lower the inventory of the finished products waiting for the delivery to customers. Similarly, the smaller the tardiness of the tardy orders, the lower the input inventory of purchased materials waiting for processing. Therefore, at the bottom level problem the assignment of orders $j \in JD$ on due dates should remain unchanged, and the earliness τ_j of the early orders $j \in JE$, and the tardiness τ_j of the tardy orders $j \in JT$ should not be increased.

Given the minimum numbers of early and tardy orders, the subsets of early orders and tardy orders, the maximum earliness for early orders, and the maximum tardiness for tardy orders, the next optimization step is to find production schedule such that levels machine assignments over the planning horizon either for minimum numbers of early and tardy orders or, in addition, for limited order earliness and tardiness.

The mixed integer program **M2** for the bottom level problem can be formulated alternatively, either for minimum numbers of early and tardy orders (order assignment constraints 1a) or for limited order earliness and tardiness (order assignment constraints 1b).

Model M2: *Machine assignment*
Minimize

$$M_{\max} \tag{12}$$

subject to (3)–(9) and:

1a. *Order assignment constraints*

– numbers of tardy and early orders are at minimum:

$$\sum_{j \in J : t > d_j} v_{jt} = U^*_{sum} \tag{13}$$

$$\sum_{j \in J : a_j \leq t < d_j} v_{jt} = E^*_{sum} \tag{14}$$

or

1b. *Order assignment constraints*

– on due date assignment constraints,

$$v_{jd_j} = 1;\ j \in JD \tag{15}$$

– early orders assignment constraints (the earliness cannot be greater than τ_j),

$$\sum_{t\in T:d_j-\tau_j\le t\le d_j} v_{jt} = 1; \quad j\in JE \tag{16}$$

– tardy orders assignment constraints, (the tardiness cannot be greater than τ_j),

$$\sum_{t\in T:d_j<t\le d_j+\tau_j} v_{jt} = 1; \quad j\in JT \tag{17}$$

Denote by **M2a** and **M2b**, model **M2** with order assignment constraints 1a and 1b, respectively.

The solution to **M2** determines the optimal production schedule, i.e., the optimal assignment of customer orders to planning periods, $\{v^*_{jt}\}$ and a leveled machine assignment over the horizon, $\{w^*_{it}\}$ such that numbers of tardy and early orders are at minimum or, in addition (model **M2b**), the corresponding tardiness and earliness are limited.

The mixed integer programs **M0** and **M1** can be enhanced by adding cutting constraints that are derived by relating the demand on required capacity to available capacity for each subset of orders with the same due date, see [5].

4. Computational Examples

In this section numerical examples and some computational results are presented to illustrate possible applications of the proposed formulations and to compare weighting with lexicographic approach (model **M0** with a pair of models **M1**, **M2**). The examples are modeled after a real world distribution center for high-tech products, where finished products are assembled for shipping to customers.

In the computational experiments four types of the test problems are constructed with the following four regular patterns of demand:

1) Increasing, with demand skewed toward the end of the planning horizon.
2) Decreasing, with demand skewed toward the beginning of the planning horizon.
3) Unimodal, where demand peaks in the middle of the planning horizon and falls under available capacity in the first and last days of the horizon.
4) Bimodal, where demand peaks at the beginning and at the end of the planning horizon and slumps in mid-horizon.

Pattern 1) requires some orders to be completed earlier, for pattern 2) a majority of orders must be moved later in time, whereas patterns 3) and 4) require that orders are moved both early and late to reach feasibility.

For each pattern demand, the following three scenarios will be considered with different total capacity ratio $r = \max_{i\in I}\left(\frac{\sum_{j\in J} p_{ij}s_j}{m_i \sum_{t\in T} c_{it}}\right)$ of the total demand on capacity to total available capacity:

– scenario I with low tightness of capacity constraints: $r = 0.553$,
– scenario II with medium tightness of capacity constraints: $r = 0.738$,
– scenario III with high tightness of capacity constraints: $r = 0.923$.

A brief description of the production system, production process, products and customer orders is given below.

1. Production system:
   – six processing stages: 10 machines in each stage $i = 1, 2$; 20 machines in each stage $i = 3, 4, 5$; and 10 machines in stage $i = 6$.
2. Products:
   – 10 product types;
   – 100 customer orders, each consisting of several suborders (customer required shipping volumes); every suborder has a different volume ranging from 5 to 6600 products, the same arrival date (period 1), and different due date, and each suborder is to be completed in a single day, the total number of suborders is ranging from 669 to 816 and the total demand for products from 322130 to 537995 depending on demand pattern and the capacity scenario;
   – production lot sizes: 200, 200, 300, 100, 100, 100, 200, 200, 300, 100, respectively for product type 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
3. Processing times (in seconds) for product types:

product type/stage	1	2	3	4	5	6
1	20	0	120	0	0	15
2	20	0	140	0	0	15
3	10	0	120	0	0	10
4	15	5	0	120	0	15
5	15	10	0	180	0	15
6	10	5	0	120	0	10
7	15	10	0	180	0	15
8	20	5	0	0	100	15
9	15	0	0	0	80	10
10	15	0	0	0	100	10

4. Planning horizon: $h = 30$ days, each of length 2×9 hours.

Notice that the suborders in the computational experiments play the role of orders in the mathematical formulations.

In models **M0** and **M1**, the weights used for tardy orders, early orders and maximum machine assignments are $\lambda_1 = 250$, $\lambda_2 = 10$ and $\lambda_3 = 1$, respectively.

The characteristics of integer programs for the three scenarios and various demand patterns and the solution results are summarized in Tables 2–4. The size of the integer programs **M0**, **M1** (enhanced with the cutting constraints [5]) and **M2** (**M2a**, **M2b**) is represented by the total number of variables, Var., number of binary variables, Bin., number of constraints, Cons., and number of nonzero elements in the constraint matrix, Nonz. The last two columns of Tables 2–4 present the solution values and CPU time in seconds required to prove optimality of the solution.

Table 2
Computational results for scenario I

Demand pattern/Model	Var.	Bin.	Cons.	Nonz.	$U_{sum}, E_{sum}, M_{max}$	CPU*
Increasing/M0	15990	15809	2162	226520	0, 4, 61	2.33
Increasing/M1	15809	15809	1940	170152	0, 4, –	1.39
Increasing/M2a	15990	15809	1201	141920	0, 4, 61	2.75
Increasing/M2b	168	19	96	329	0, 4, 67	0.01
Decreasing/M0	10292	10111	2136	135044	0, 4, 61	1.43
Decreasing/M1	10111	10111	1892	98335	0, 4, –	0.64
Decreasing/M2a	10292	10111	1201	89920	0, 4, 61	1.83
Decreasing/M2b	160	10	38	208	0, 4, 61	0.01
Unimodal/M0	11431	11250	1915	152691	0, 5, 73	2.40
Unimodal/M1	11250	11250	1671	112205	0, 5, –	1.07
Unimodal/M2a	11431	11250	1093	100466	0, 5, 73	2.88
Unimodal/M2b	173	37	91	407	0, 5, 73	0.01
Bimodal/M0	10597	10416	1857	142432	0, 9, 65	2.06
Bimodal/M1	10416	10416	1612	104902	0, 9, –	0.90
Bimodal/M2a	10597	10416	1060	93805	0, 9, 65	3.12
Bimodal/M2b	188	35	119	434	0, 9, 67	0.01

*CPU seconds for proving optimality on a PC Pentium 2.4 GHz, RAM 512MB /CPLEX v.9

The optimal production schedules and machine assignments obtained for various demand patterns in scenario II, using the lexicographic approach with a pair of models M1, **M2a** are shown in Figures 2 and 3 to illustrate the examples.

The computational experiments have been performed using AMPL programming language and the CPLEX v.9 solver on a laptop with Pentium IV at 2.4GHz and 512MB RAM. The results have indicated that for scenario I and II with low and medium tightness of capacity constraints both the weighting and the lexicographic approach are capable of finding proven optimal schedules in a short CPU time. The lexicographic approach with model **M2b** at the bottom level outperforms the weighting approach for scenario III with a high tightness of capacity constraints. However, when model **M2a** is used for that scenario a large computation time is required to find the first feasible solution (Tabl. 3 shows the results for model

M2b only). On the other hand, the weighting approach outperforms the lexicographic approach with model **M2b** at the bottom level with respect to the auxiliary criterion of levelling machine assignments for most test examples.

Table 3
Computational results for scenario II

Demand pattern/Model	Var.	Bin.	Cons.	Nonz.	U_{sum}, E_{sum}, M_{max}	CPU*
Increasing/M0	16037	15856	2160	221880	0, 21, 62	19.56
Increasing/M1	15856	15856	1940	165454	0, 21, –	2.08
Increasing/M2a	16037	15856	1203	142388	0, 21, 62	12.45
Increasing/M2b	405	234	223	1694	0, 21, 68	0.04
DecreasingM0	21786	21605	1623	281371	1, 20, 69	17.07
DecreasingM1	21605	21605	1413	205250	1, 20, –	7.89
0Decreasing/M2a	24541	21605	1204	218158	1, 20, 69	647.00
DecreasingM2b	145	23	79	268	1, 20, 71	0.01
Unimodal/M0	11456	11275	1938	156675	0, 16, 66	4.70
Unimodal/M1	11275	11275	1706	116546	0, 16, –	2.54
Unimodal/M2a	11456	11275	1095	100714	0, 16, 66	6.02
Unimodal/M2b	251	103	120	759	0, 16, 66	0.02
Bimodal/M0	10618	10437	1860	140044	0, 22, 70	4.60
Bimodal/M1	10437	10437	1623	102772	0, 22, –	2.02
Bimodal/M2a	10618	10437	1062	93293	0, 22, 70	10.90
Bimodal/M2b	235	88	138	671	0, 22, 70	0.02

*CPU seconds for proving optimality on a PC Pentium 2.4 GHz, RAM 512MB /CPLEX v.9

Table 4
Computational results for scenario III

Demand pattern/Model	Var.	Bin.	Cons.	Nonz.	U_{sum}, E_{sum}, M_{max}	CPU*
Increasing/M0	16110	15929	2164	220987	0, 44, 71	>3600
Increasing/M1	15929	15929	1949	164519	0, 44–	4.56
Increasing/M2b	411	251	223	1624	0, 44, 71	0.04
Deer easing/ M0	24636	24455	1389	302086	10, 40, 74	>3600
Decreasing/M1	24455	24455	1179	216335	10, 40, –	>3600
Deereasing/M2b	182	44	120	452	10, 40, 75	0.01
Unimodal/M0	18199	18018	1453	239313	2, 38, 69	>3600
Unimodal/M1	18018	18018	1243	176094	2, 38, –	>3600
Unimodal/M2b	189	54	111	428	2, 38, 71	0.03
Bimodal/M0	17768	17587	1391	230434	1, 51, 76	>3600
Bimodal/M1	17587	17587	1181	168888	1, 51, –	>3600
Bimodal/M2b	148	38	90	333	1, 51, 76	0.01

*CPU seconds for proving optimally on a PC Pentium 2.4 GHz, RAM 512MB /CPLEX v.9

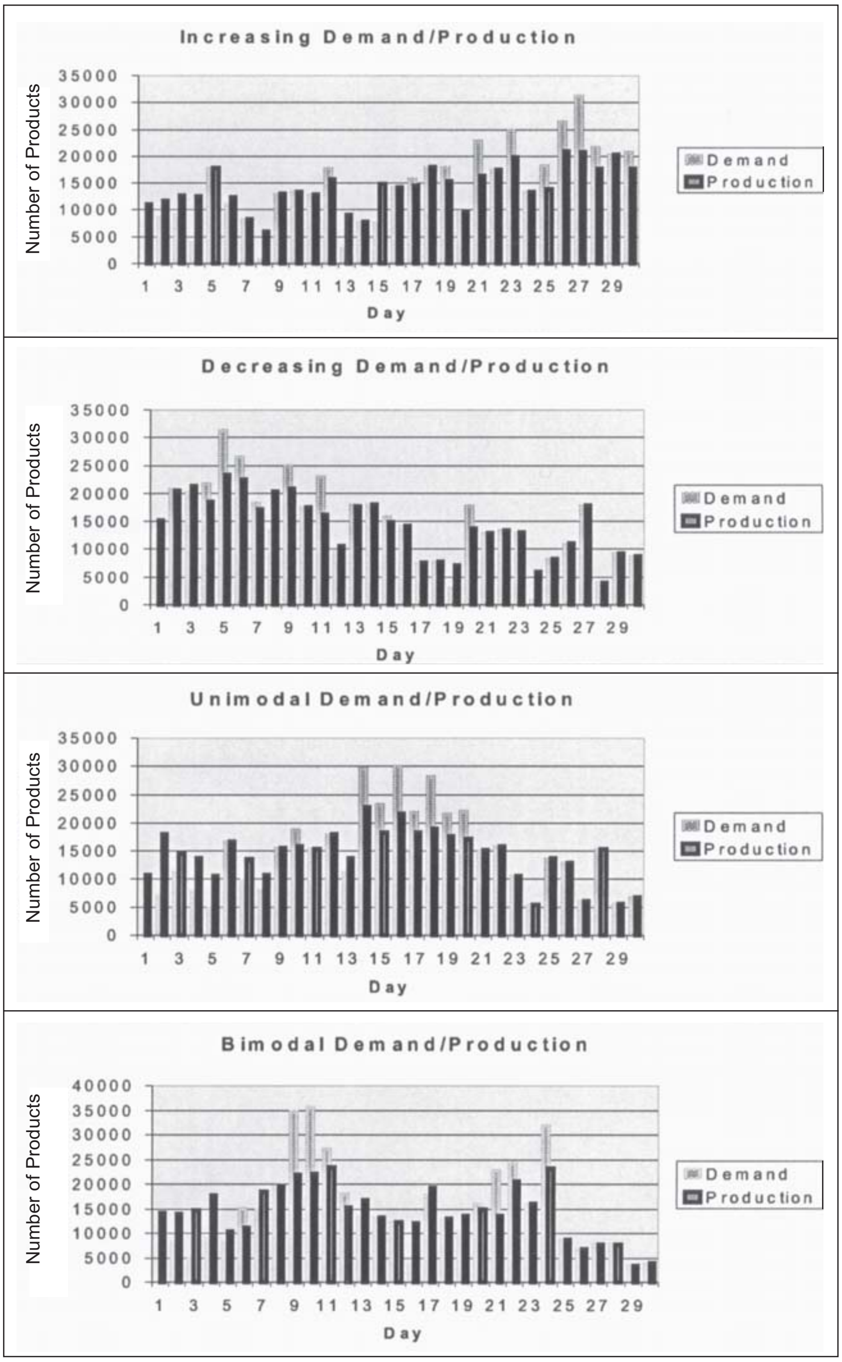

Fig. 2. Production schedules for various demand patterns in scenerio II

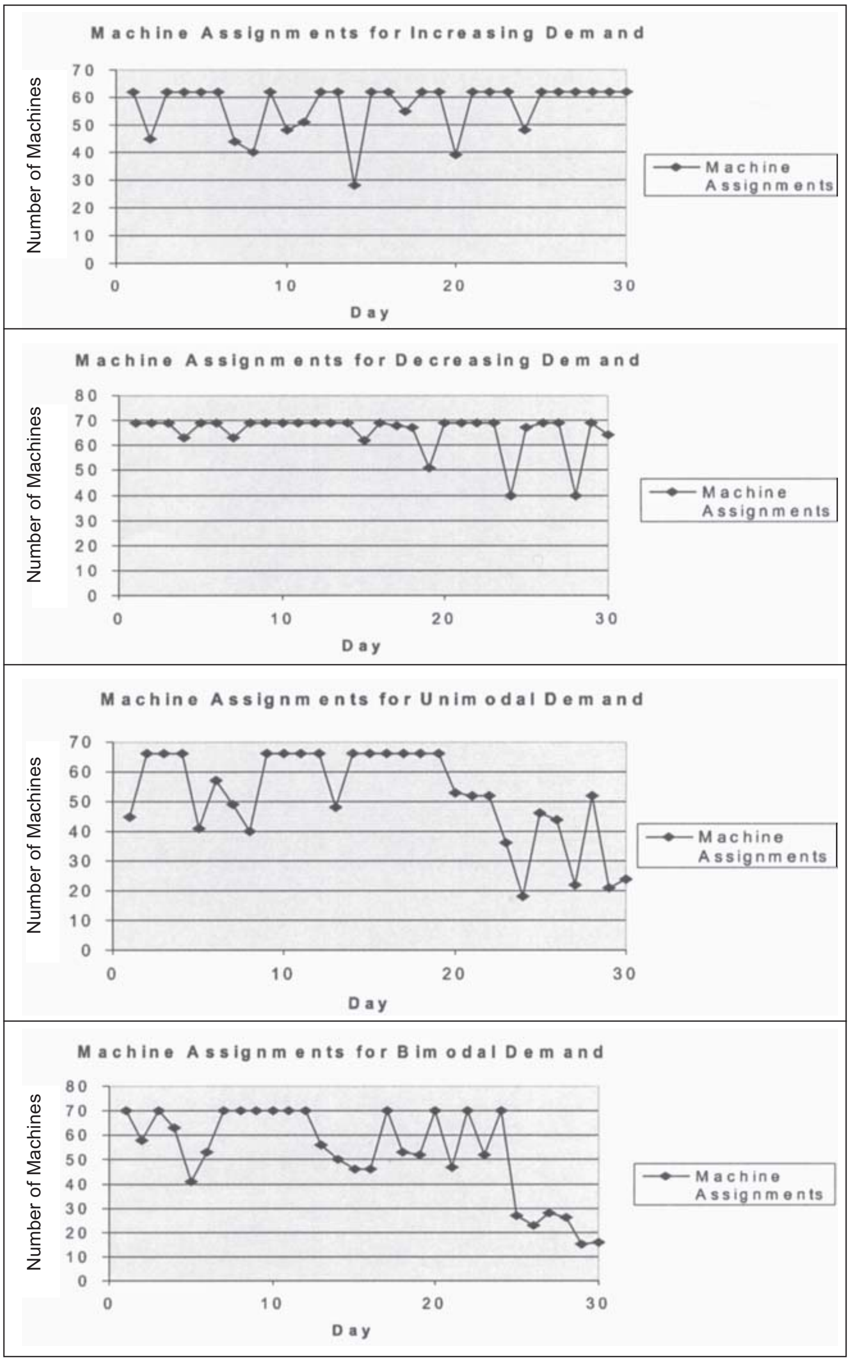

Fig. 3. Machine assignments for various demand patterns in scenerio II

5. Conclusions

This paper has presented and compared the weighting and the lexicographic approach and the corresponding integer programming formulations for the multi-objective production scheduling in make-to-order manufacturing environment. The computational experiments modeled after a real-world make-to-order assembly system in the electronics industry have indicated that the two approaches are capable of finding proven optimal solutions for large size problems in a reasonable computation time using commercially available software for integer programming.

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