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Impact of the mass and other parameters of charged particles on the results of laser resonance acceleration

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Abstract. Theoretical and numerical analyses are presented concerning the conditions at which the charged particles of different masses can be accelerated to significant kinetic energy in the circularly polarized laser or maser beams and a static magnetic field. The studies are carried out using the analytical derivations of the particles dynamics and theirs kinetic energy. The presented illustrations enabled interpretation of the complex motion of particles and the possibilities of their acceleration. At the examples of an electron, proton and deuteron, the velocity, kinetic energy and trajectory as a function of the acceleration time at the resonance condition are illustrated in the appropriate graphs. The particles with larger masses require the application of enhanced magnetic field intensity at the resonance condition. However, this field intensity can be significantly reduced if the particles are preaccelerated.
Keywords: optoelectronics, acceleration of charged particles, laser, maser, relativistic dynamics, kinetic energy of a particle, electron, proton, deuteron

1. Introduction

The progress in practical development of ultra-intense lasers has led to the increase in their use as acceleration devices. The laser based accelerating devices are already capable of generating narrowly shaped beams of charged particles moving with relativistic velocity of electrons, positrons, and also particles of higher masses as protons and ions. Many successful experimental and theoretical achievements resulting in gaining by the charged particles the relativistic energies gave an additional stimulus for these studies. This direction of investigation is very interesting because of considerable progress in high power laser construction which was used to perform verification of the new models of particle acceleration. No doubt these accelerating devices begin to be alternative accelerators to the ones used nowadays. The accelerated particles can make an essential influence in many areas of technology and science [1-7].

Many experiments have shown the possibility of particles acceleration as a result of their interaction with the laser or maser beams of different parameters [8-29]. In the last years, at the laboratories there were used the lasers with radiation power density of the level 10^{22} W/cm^2, which corresponds to the electric field intensity amplitude 10^{14} V/m. There are known presentations of new types of charged particle accelerators and a different approach leading to acceleration of charged particles to relativistic energies [30-35].

The main purpose of this paper is to show the conditions at which the particles of different masses and under influence of various parameters of electromagnetic and magnetic fields in a circularly polarized laser or maser beams can be accelerated to large kinetic energy. Another purpose is to indicate the significance of the initial velocity of the particles of different masses on the resonance acceleration process.

2. Efficiency of energy transfer to the charged particle

The energy transfer rate is proportional as it follows from energy equation of conservation

$$\frac{d\gamma}{dt} = \frac{q}{m_0 c}\vec{\beta} \cdot \vec{E},$$

to $\vec{\beta} \cdot \vec{E}$ product where $\vec{\beta}c$ is the particles' velocity vector, and $\vec{E}$ is the radiation electric field. The equation shows that the energy transferred to the particle will be maximized if the vectors $\vec{\beta}$ and $\vec{E}$ are parallel, and retain their orientation over many oscillations [37]. In this magnetic field, the particle's velocity vector and the electric field vector rotate in the same sense if the radiation is circularly polarized, with positive helicity. $\vec{E}$ oscillates so rapidly that $\left|\vec{\beta} \cdot \vec{E}\right|$ nearly averages to zero over any macroscopic time scale. The key to achieving large energy transfers is to make $\vec{\beta}$ rotate rapidly (synchronously with $\vec{E}$) for long time. If the laser radiation is circularly polarized, it is possible to choose initial parameters such that $\vec{\beta}$ and $\vec{E}$ retain their relative orientation through many oscillations and it allows a significant energy transfer to occur. For most choices of initial parameters, the energy is an oscillatory function of time. The particle alternately gains and losses energy. If the parameters are such that $\vec{\beta}$ and $\vec{E}$ rotate synchronously, the particle's energy is not a periodic function but increases or decreases monotonically. In this region, efficient coupling between the particle and radiation field can occur and the interaction is resonant. Such synchronization can be achieved by combination of the particle initial velocity, static longitudinal magnetic induction, and the laser radiation frequency. The sign depending on the initial orientation of $\vec{\beta}$ and $\vec{E}$ i.e. on the angle $\theta(t)$ between the rotating electric field vector and the rotating perpendicular component of the particle's velocity vector

$$\vec{\beta}_{\perp} \cdot \vec{E} = \beta_{\perp} E \cos\theta.$$

As the orientation of these vectors changes in time, the energy of the particle must change, too. In such a case we will observe a periodic function of particle's kinetic energy in time. Only in the resonance case, when $\theta = 0$, $(\cos\theta = 1)$, or $\theta = \pi$ for electron, i.e. as the rate of rotation of $\vec{\beta}$ approaches to that $\vec{E}$, the particle can get the maximum energy from the radiation electric field and efficient energy transfer is possible. In the spatial case, if B_z is chosen such that $\Delta\omega = \dfrac{qB_z}{\gamma_0 mc} - (1 - \beta_{z0})\omega = 0$ (the resonance condition), θ does not oscillate, but (for electron) evolves monotonically toward π. So, if a homogeneous, monoenergetic beam of electrons is injected into the interaction region with $\Delta\omega = 0$, the energy change of a particular electron depends on its initial position within of a wavelength of light: those with θ_0 between $\pi/2$ and $3\pi/2$ are accelerated, the others are decelerated. If $\theta_0 = \pi$ and $\Delta\omega = 0$, than $\theta(z)$ is constant, and $\vec{\beta}$ and $\vec{E}$ rotate at exactly the same rate. In this case, the electron initially experiences and continues the greatest acceleration. Therefore, if two particles are injected with the same energy but at different angles, the particle with the smaller angle θ_0 will be accelerated longer and to the higher energy. So, if the charged particle is injected into the resonance interaction region with the velocity $\vec{V}_{\perp}$, the energy of the particle in the (x, y) plane depends on the their initial velocity direction.

3. Equations for charged particle trajectories and kinetic energy

In this section we derive analytic trajectories for a free electron moving in a plane-wave laser field and an externally applied strong uniform magnetic field aligned in the direction of propagation of the laser field. The electron dynamics resulting from interaction with these fields we have analyzed in the case of a circular polarized electromagnetic monochromatic wave in the lossless vacuum conditions [38]. We have neglected the quantum effects. In some cases, the electron spin may impact the motion of this particle in the magnetic field. There exists interaction between the external magnetic field and the electron magnetic momentum. If we look at the accepted theory on this effect, we can find the following statement: if a particle moves through a homogeneous magnetic field, the forces exerted on opposite ends of the dipole cancel each other out and the trajectory of the particle is unaffected. However, if the magnetic field is inhomogeneous, then the force on one end of the dipole will be slightly greater than the opposing force on the other end, so that there is a net force which deflects the particle's trajectory. Additionally, the interaction does not depend on the magnetic field intensity. In the case of laser acceleration, the resultant magnetic field is slightly inhomogeneous especially at the beam boundary. In our studies we assume it as a homogeneous one. In other words, it may be neglected in the case of an electron, since there appear two forces — one attractive and the second repulsive (N-S poles) and they both cancel each other because they act almost at the same point (extremely small dimension of the electron).

Dynamical relativistic equation and continuous equation of the normalized energy γ in this case have the following forms

$$\begin{aligned} \frac{d\vec{p}}{dt} &= q\vec{E} + q\left[\vec{V} \times \left(\vec{B} + \vec{B}_z\right)\right], \\ \frac{d\gamma}{dt} &= \frac{q}{m_0 c^2}\vec{V} \cdot \vec{E}, \end{aligned} \tag{1}$$

where p, q, and m_0 are the momentum, charge, and the rest mass of the particle, E and B are the electric and magnetic field strengths of the electromagnetic wave, c is the velocity of the electromagnetic wave, V is the particle velocity, and B_z is the external static magnetic induction in direction along the z coordinate and

$$\begin{aligned} &\gamma = \left(1 - \beta^2\right)^{-\frac{1}{2}}, \quad \vec{\beta} = \frac{\vec{V}}{c}, \quad \vec{p} = \gamma \cdot m_0 c \vec{\beta}, \\ &\beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2, \quad \beta_{x,y,z} = \frac{V_{x,y,z}}{c}, \end{aligned} \tag{2}$$

where $\beta = V/c$.
The solving procedure of these equations are presented in [34, 35]. As a result, we have obtained uncomplicated expressions for trajectory coordinates in the form of

$$\begin{aligned} x &= \eta \sin\varphi + (\eta - \xi)\sin\chi\varphi, \\ y &= \eta\cos\varphi - (\eta - \xi)\cos\chi\varphi - \xi, \end{aligned} \tag{3}$$

$$z = \frac{c}{2\omega\gamma_0^2\left(1 - \beta_{0z}\right)^2}\left\{\left[1 + 2\vartheta^2 - \gamma_0^2\left(1 - \beta_{0z}\right)^2\right]\varphi - \frac{2\vartheta^2}{1+\chi}\sin\left(1+\chi\right)\varphi\right\}, \tag{4}$$

where $\eta, \chi, \xi, \vartheta$ are the constant parameters defined as

$$\eta = \frac{\chi\xi}{1+\chi}, \ \chi = \frac{a\alpha}{\gamma_0\left(1-\beta_{0z}\right)}, \ \ \xi = \frac{c}{a\omega}, \ \ \vartheta = \frac{\alpha}{1+\chi}, \tag{5}$$

and the phase of the field φ stands for combination $\omega t - kz$ and has a form

$$\varphi = \omega\left[t - \frac{z(t)}{c}\right]. \tag{6}$$

The components of particle velocity have a form

$$\begin{aligned} V_x &= \frac{c\vartheta}{\gamma}\left(\cos\varphi - \cos\chi\varphi\right), \\ V_y &= -\frac{c\vartheta}{\gamma}\left(\sin\varphi + \sin\chi\varphi\right), \\ V_z &= c\frac{1+2\vartheta^2\left[1-\cos\left(1+\chi\right)\varphi\right]}{2\gamma_0\gamma\left(1-\beta_{0z}\right)} - \frac{1}{2\gamma}c\gamma_0\left(1-\beta_{0z}\right). \end{aligned} \tag{7}$$

4. Prove of the resonance occurrence for non zero and zero initial conditions

Let us define the extremes of the $\gamma(\varphi)$ function for non zero initial velocity $\beta_{0z} \neq 0$. These extremes can be found from the condition

$$\frac{d\gamma}{d\varphi} = 0.$$

According to [34] we get

$$\gamma = \frac{1+2\vartheta^2\left[1-\cos\left(1+\chi\right)\varphi\right]}{2\gamma_0\left(1-\beta_{0z}\right)} + \frac{1}{2}\gamma_0\left(1-\beta_{0z}\right), \tag{8}$$

and differentiating the last expression we get

$$\frac{d\gamma}{d\varphi} = \frac{\vartheta^2\left(1+\chi\right)}{\gamma_0\left(1-\beta_{0z}\right)}\sin\left(1+\chi\right)\varphi = 0.$$

The action on a particle of combined electric and magnetic fields in case of circular polarization is rather complicated to describe. The $\vec{E}$ vector of the electric field of the laser radiation rotates around the z axis. As a result of this, acceleration of the particle occurs. The particle starts moving along the direction of the circularly rotating $\vec{E}$ vector. From this moment, on the particle will start to act the bending effect (without the acceleration effect) connected with the Lorentz force by the magnetic induction vector $\vec{B}$ of the laser field. The $\vec{B}$ vector will start to bend the particle trajectory in the co-axial direction (the z axis). This will create the V_z component of the particle's velocity and due to it the rotation of the particle around the z axis occurs. While the external static magnetic field B_z will push the circularly rotating particle rather in the radial direction, the particle will move along the helix winding around the z axis with rising the radius. The result of these combined actions is the helical trajectory with superimposed wiggles. The wiggles disappear at the resonance conditions.

It follows from the last expression that extremes could be defined on the basis of the following equation

$$\sin(1+\chi)\varphi = 0,$$

and they occur in the positions for which

$$\varphi_n = \pm\frac{n\pi}{1+\chi}. \tag{9}$$

Taking this and Eq. (8) into account it may be seen that

$$\begin{aligned}
\gamma_{\min} &= \frac{1}{2\gamma_0(1-\beta_{0z})} + \frac{1}{2}\gamma_0(1-\beta_{0z}), \quad \textit{for } n = 0,2,4,\ldots \quad a) \\
\gamma_{\max} &= \frac{1+4\vartheta^2}{2\gamma_0(1-\beta_{0z})} + \frac{1}{2}\gamma_0(1-\beta_{0z}) \quad \textit{for } n = 1,3,5,\ldots \quad b).
\end{aligned} \tag{10}$$

For non zeros initial conditions, the parameter

$$\vartheta = \frac{B_0}{\dfrac{B_z}{\gamma_0(1-\beta_{0z})} - B_c}.$$

Thus, Eq. (10b) can be presented in the following form

$$\gamma_{\max} = \frac{2B_0^2}{\left[B_z - \gamma_0(1-\beta_{0z})B_c\right]^2} + \frac{1}{2\gamma_0(1-\beta_{0z})} + \frac{1}{2}\gamma_0(1-\beta_{0z}) \quad \textit{for } n = 1,3,5,\ldots$$

It results from the above that when $B_z \to \gamma_0(1-\beta_{0z})B_c$, then $\gamma_{\max} \to \infty$.

In the case of zeros initial conditions, i.e. if $\gamma_0\left(1-\beta_{0z}\right)=1$, this expression takes the form of the equations which were derived earlier i.e.

$$\gamma_{\min}=1$$
$$\gamma_{\max}=1+2\vartheta^2.$$

The described here analysis shows that the kinetic energy of the particle is fluctuating in the function of the phase φ achieving repeatedly maximum equal values corresponding to the normalized $\gamma_{\max}$ value. It also results from the last equation, that in a case of an increase in the constant co-axial magnetic induction B_z, also a value of the parameter ϑ as well as a value of the $\gamma_{\max}$ factor are growing, which, when $B_z \to B_c$, is heading for the infinity, $\gamma_{\max} \to \infty$. Using the analogous expression as Eq. (9), locating the place of extremes of the function $\gamma(\varphi)$, it is possible to show that the distances between the subsequent extremes as in the case of zero initial conditions can be determined based on the following equation

$$\Delta\varphi=\varphi_n-\varphi_{n-1}=\frac{\pi}{1+\chi}=\frac{\pi}{1-B_z/B_c}.$$

It results from this equation that when the magnetic induction B_z grows, the phase difference between subsequent maxima of the function $\gamma(\varphi)$ grows, too. And if B_z will approach the resonance value, $B_z \to B_c$, these differences are becoming infinitely large, that is $\Delta\varphi \to \infty$.

5. Resonance acceleration process of an electron

In Figs. 1 a, b, c, d, there are presented the graphs illustrating the change of kinetic energy of the electron in time for selected electric fields strengths: $E_0 = 10^{11}$, 10^{12} and 10^{13} V/m and for the wavelength of the laser radiation 10 μm. In all cases, the initial velocity $V_{0z} = 0.5$ c what allowed to reduce the value of the resonance magnetic field strength from $B_z = -1070$ T to about -600 T. It results from the graphs a, b, c, that in relatively long time of the influence, in about 1 μs, the increase in the electric field from intensity 10^{11} V/m to 10^{13} V/m resulted in the rise of the kinetic energy of an electron from the level GeV to a few TeV. In the range of femto-seconds (Fig. 1c), the kinetic energy gained by the electron after 400 fs action of the laser radiation in the electric field of the intensity $E_0 = 10^{13}$ V/m, is considerably smaller and does not exceed 150 MeV.

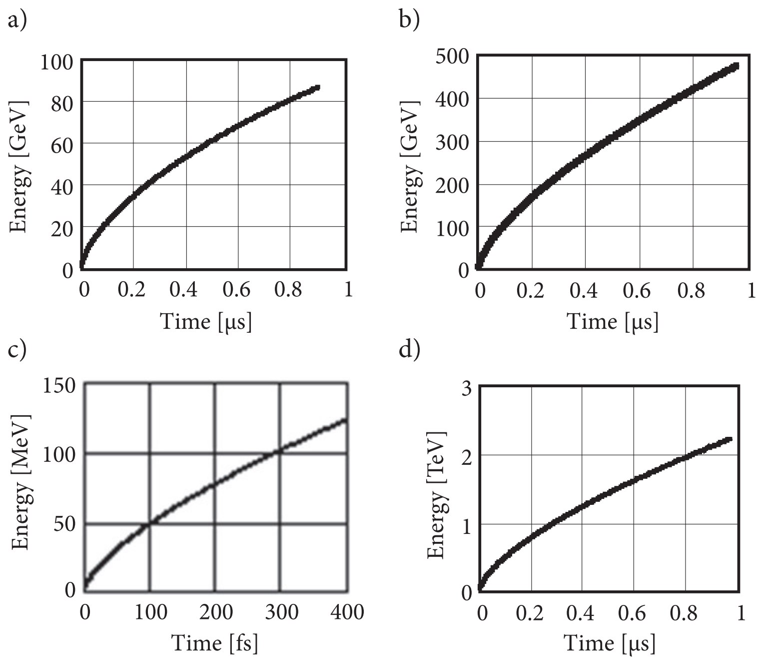
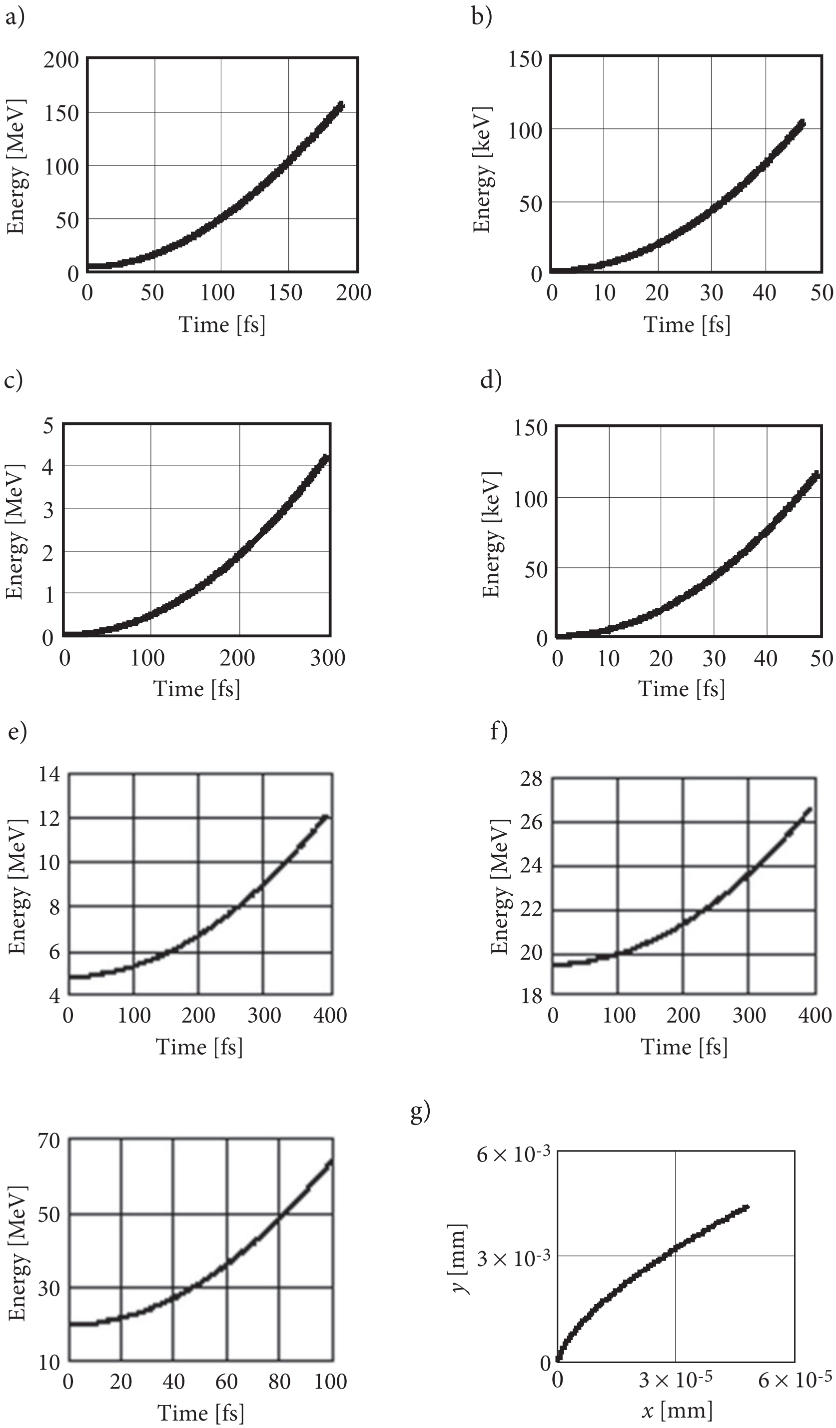


Fig. 1. Electron's kinetic energy variation in time for laser wavelength λ = 10 μm with resonance magnetic field induction $B_z = -617$ T and the initial velocity $V_{0z} = 0.5$ c (0.0791 MeV) when electric field intensities are: a) $E_0 = 10^{11}$ V/m; b) 10^{12} V/m; c) and d) 10^{13} V/m.

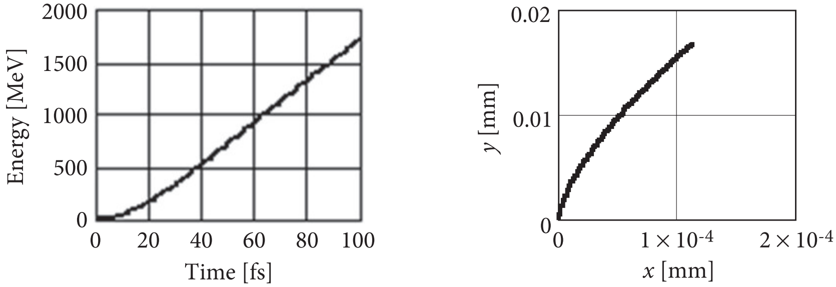
6. Resonance acceleration process of a proton

Figures 2 (a, b) show the change of a proton kinetic energy during the acceleration in a maser beam of the wavelength 10 cm and the electric field intensities $E_0 = 10^{13}$ V/m (Fig. a) and 10^{12} V/m (Fig. b). Figure (a) shows the $E_k(t)$ at the resonance conditions when before the starting of the acceleration process, a proton has been preaccelerated to the velocity $V_{0z} = 0.1$ c with additionally applied the co-axial magnetic field of the induction $B_z = 177$ T. The $E_k(t)$ graph (b) has been obtained with the initial velocity $V_{0z} = 0$ and the resonance magnetic field $B_z = 196$ T. The comparison of the two last graphs shows that even comparatively small initial velocity of a proton results in the reduction of the resonance magnetic field by about 20 T.

Figures 2 (c, d) show the change of a proton kinetic energy during 300 fs (Fig. c) and 50 fs (Fig. d) of acceleration in a maser beam of wavelength 1 cm and the electric field intensity $E_0 = 10^{12}$ V/m. In the both cases, $V_{0z} = 0$ and the resonance magnetic field $B_z = 1957$ T. It follows from the figures that increase in duration of the maser action from 50 fs to 300 fs results in the gained energy rise from about 120 keV to MeV with the maintained electric field intensity.

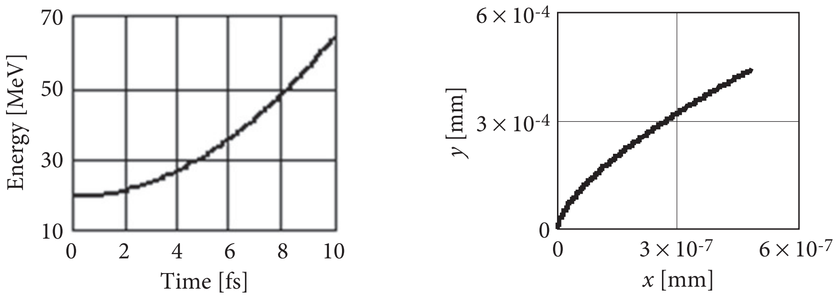

a)
Energy [MeV]
200
150
100
50
0
0
50
100
150
200
Time [fs]
b)
Energy [keV]
150
100
50
0
0
10
20
30
40
50
Time [fs]
c)
Energy [MeV]
5
4
3
2
1
0
0
100
200
300
Time [fs]
d)
Energy [keV]
150
100
50
0
0
10
20
30
40
50
Time [fs]
e)
Energy [MeV]
14
12
10
8
6
4
0
100
200
300
400
Time [fs]
f)
Energy [MeV]
28
26
24
22
20
18
0
100
200
300
400
Time [fs]
Energy [MeV]
70
50
30
10
0
20
40
60
80
100
Time [fs]
g)
y [mm]
6 × 10-3
3 × 10-3
0
0
3 × 10-5
6 × 10-5
x [mm]

h)



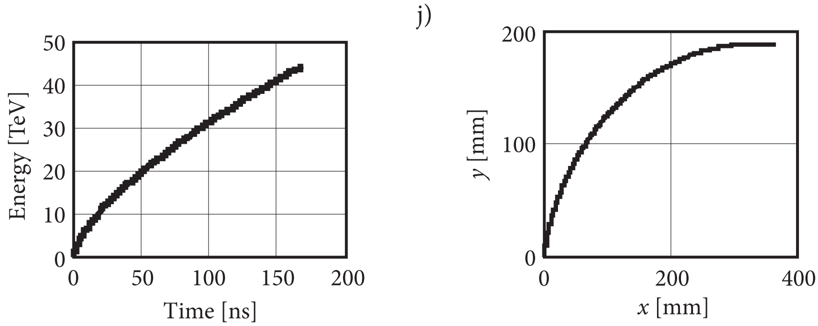
Energy [MeV]
2000
1500
1000
500
0
0 20 40 60 80 100
Time [fs]
y [mm]
0.02
0.01
0
0
1 × 10^-4
2 × 10^-4
x [mm]

i)



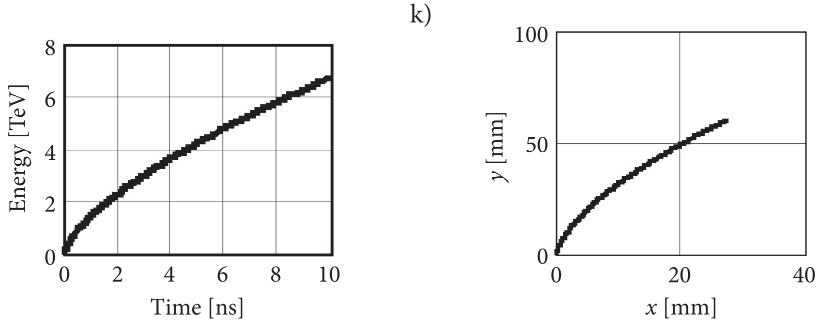
Energy [MeV]
70
50
30
10
0 2 4 6 8 10
Time [fs]
y [mm]
6 × 10^-4
3 × 10^-4
0
0
3 × 10^-7
6 × 10^-7
x [mm]

j)



Energy [TeV]
50
40
30
20
10
0
0 50 100 150 200
Time [ns]
y [mm]
200
100
0
0 200 400
x [mm]

k)



Energy [TeV]
8
6
4
2
0
0 2 4 6 8 10
Time [ns]
y [mm]
100
50
0
0 20 40
x [mm]

l)

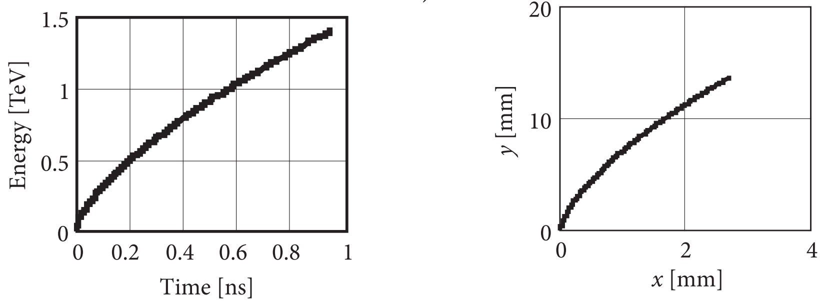


Fig. 2. A proton's kinetic energy variation in time for maser wavelength λ = 10 cm with resonance magnetic field induction B_z = 177 T and the initial velocity V_{0z} = 0.1 c (a) when electric field intensities are: (a) $E_0 = 10^{13}$ V/m and (b) 10^{12} V/m; (b) graph of the function $E_k(t)$ when V_{0z} = 0 and the resonance magnetic field induction B_z = 196 T; (c, d) graph of the function $E_k(t)$ when $E_0 = 10^{12}$ V/m, λ = 1 cm, duration of the acceleration process about 300 fs (c) and 50 fs (d) for V_{0z} = 0 and the resonance magnetic field induction B_z = 1957 T; (e, f) graph of the function $E_k(t)$ when $E_0 = 10^{12}$ V/m, λ = 1 cm and the reduced resonance magnetic field induction B_z = 1770 T at V_{0z} = 0.1 c (e) and B_z = 1598 T at V_{0z} = 0.2 c (f); (g, h) kinetic energy of a proton due to the 100 fs acceleration in maser beam with λ = 1 cm and B_z = 1598 T when V_{0z} = 0.2 c $E_0 = 10^{13}$ V/m (g) and $E_0 = 10^{14}$ V/m (h); (i) kinetic energy of a proton due to the reduced time of acceleration in maser beam with λ = 1 cm and B_z = 1598 T when V_{0z} = 0.2 c $E_0 = 10^{14}$ V/m; (j, k, l) kinetic energy of a proton due to the 200 ns (j) 10 ns (k) and 1 ns (l) acceleration in a maser beam with λ = 1 cm when V_{0z} = 0.2 c $E_0 = 10^{14}$ V/m and B_z = 1598 T

Figures 2 e and f show that the resonance conditions could be realized at considerably reduced the resonance magnetic field induction B_z, provided the initial velocity of a proton $V_{0z} \neq 0$. For example, when V_{0z} = 0, the resonance magnetic field induction B_z = 1957 T (Fig. c), while the preacceleration of a proton to V_{0z} = 0.1 c reduces this value to B_z = 1770 T (Fig. e). The increase in the initial velocity to V_{0z} = 0.2 c reduces the resonance magnetic field of over 350 T (Fig. f). Further increase in the initial velocity results in still larger decrease in the resonance magnetic field. The subsequent figures (g and h) illustrate the impact of the electric field E of a λ = 1 cm maser radiation on the proton kinetic energy. The presented figures show that a proton with V_{0z} = 0.2 c as a result of the 100 fs acceleration gains the energy about 60 MeV at the electric field $E_0 = 10^{13}$ V/m (Fig. g) and about 1.7 GeV at $E_0 = 10^{14}$ V/m (Fig. h). After this time of acceleration, a proton is not subjected to displacement more than a few micrometers. The reduction of the maser action on the proton to 10 fs (Fig. i) results in the decrease in the proton energy to about 60 MeV. If this time is very short, the energy was found to be reduced more significantly which may be important for a maser producing very short pulses. Figures (j, k, and l) are presented in order to show the process of the energy gain of a proton subjected to acceleration during much longer time: 200 ns (Fig. j), 10 ns (Fig. k), and 1 ns (Fig. l) at the wavelength λ = 1 cm, V_{0z} = 0.2 c and $E_0 = 10^{14}$ V/m. It follows from the presented figures that reduction of the acceleration time in a maser beam from 100 ns to 1 ns results in the gained energy reduction from the level of a few tenths of TeV to a few TeV.

7. Resonance acceleration process of a deuteron

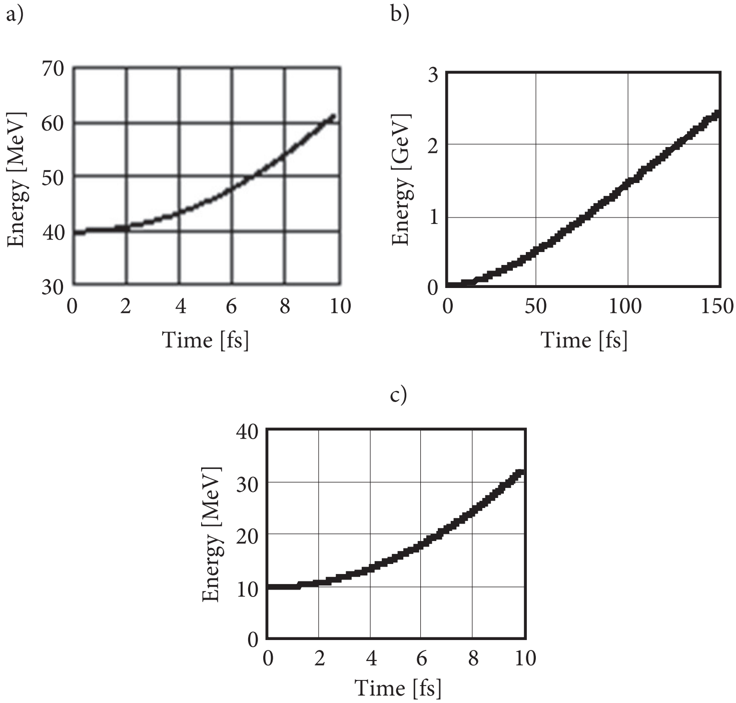


Fig. 3. The energy gain by a deuteron accelerated in maser beam of $E_0 = 10^{14}$ V/m, $\lambda = 1$ cm (a) and $\lambda = 10$ cm (b, c), preaccelerated to $V_{0z} = 0.1$ c (c) and to $V_{0z} = 0.2$ c (a and b) for resonance magnetic fields $B_z = 3259$ T (a), $B_z = 361$ T (c), and 326 T (b)

Figure 3 presents variation of the energy gained by a deuteron, a mass of a particle about two fold as much as a proton. The presented example concerns the acceleration of a deuteron in a maser beam of the electric field intensity $E_0 = 10^{14}$ V/m and the wavelengths $\lambda = 1$ cm and $\lambda = 10$ cm with deuterons of different initial velocities. All presented figures were obtained at the resonance magnetic fields B_z. Similarly as for a proton, the presented in Fig. 3 graphs show that reduction of the resonance magnetic field can be achieved by the increase in the initial velocity of the particle. For instance, the velocity from $V_{0z} = 0.1$ c (Fig. c) to $V_{0z} = 0.2$ c (Fig. b) results in the decrease in the magnetic field from $B_z = 361$ T to 326 T. Moreover, as it follows from Figs. (a and b), much significant reduction in the resonance magnetic field may be achieved by changing the maser radiation wavelength from 1 cm to 10 cm. This change enables us to reduce the resonance magnetic field from $B_z = 3259$ T (Fig. a) to $B_z = 326$ T (Fig. b). From comparison of Figs. (a) and (b) it follows an additional conclusion that increase in kinetic energy of a deuteron, during relatively short time, of about 10 fs was found to be larger for the wavelength λ = 10 cm than for λ = 1 cm. It should be also pointed out that at λ = 10 cm the resonance magnetic field was found to be an order of magnitude smaller than at λ = 1 cm. For a deuteron, the different initial velocities (V_{0z} = 0.1 c, (c) and V_{0z} = 0.2 c (a)) affect the resonance magnetic field B_z in a minor level.

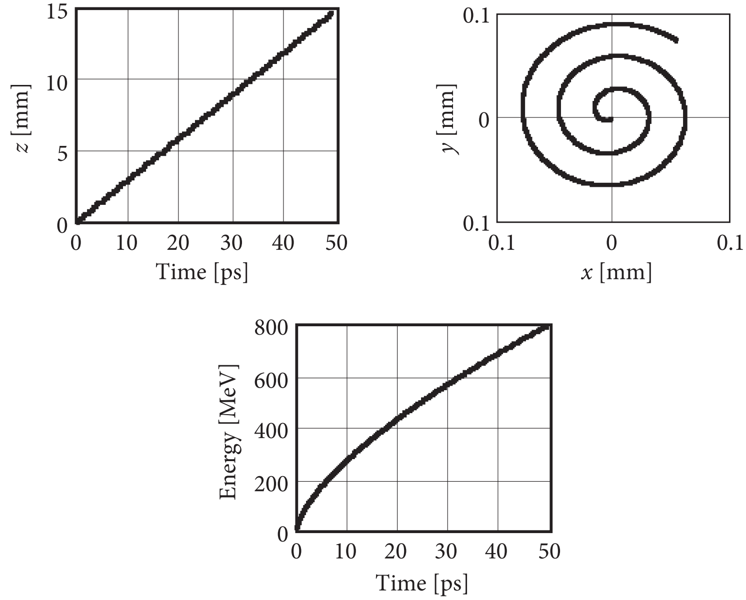
8. Impact of mass and other parameters of charged particle on its motion and the gained energy

Using the examples of an electron and a proton accelerated in the laser or maser beam with the external static co-axial magnetic field, we present the sets of graphs which, more precisely than the above ones, characterize the motion and the energy gaining process within the resonance mode. Particular attention was paid to the influence of the acceleration duration onto the particles preaccelerated to the different velocities V_{0z} in different intensity of the electric field and different wavelengths of laser or maser radiation.

The graphs presented in Fig. 4 show the impact of the increasing electron initial velocity V_{0z} on the reduction of the resonance magnetic field intensity B_z^{rez}. As an example, the laser electric field intensity E_0 = 10^{12} V/m, the wavelength λ = 10 μm, and the acceleration duration 50 ps was chosen. Theoretical prediction of decrease in the resonance magnetic field intensity B_z^{rez} with increase in the electron initial velocity is confirmed. When one increases the initial velocity from V_{0z} = 0 to V_{0z} = 0.7 c, the resonance magnetic field can be reduced from B_z^{rez} = –1071 T to B_z^{rez} = –450 T.

From the presented graphs it is evident that the projections of the trajectory onto the (x, y) plane are subjected to the variation at different initial velocities of an electron. The length of the trajectory decreases with increase in the initial velocity. It is the result of the reduction of the magnetic field which means that the mean component velocity $V_\perp$ in the plane (x, y) decreases, provided that the acceleration duration is the same in each graph. However, the presented results show that the process of gaining the kinetic energy by an electron is different for different initial velocities. The higher the initial velocity, the lower kinetic energy is gained by an electron during the same acceleration time. For instance, after 50 ps, E_k = 800 MeV for V_{0z} = 0 c and E_k = 600 MeV for V_{0z} = 0.7 c.

Figure 5 presents the illustration of the process of gaining kinetic energy during reduced acceleration time to 1 ps and the projection of the electron trajectory onto the (x, y) plain at different electron initial velocity V_{0z} and the resonance magnetic field intensity B_z^{rez}. As an example, the laser electric field intensity E_0 = 10^{12} V/m and the wavelength λ = 10 μm were chosen. The presented results show again that the process of gaining the kinetic energy by an electron is different for different a)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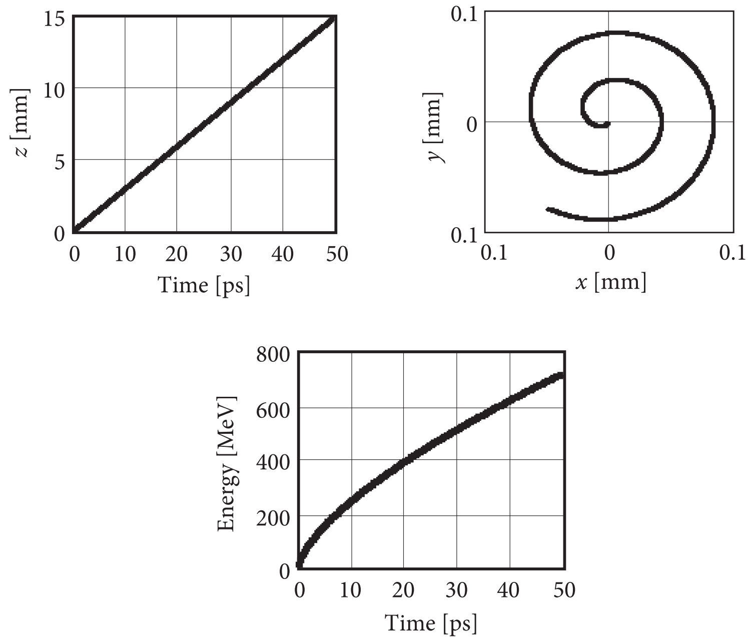
b)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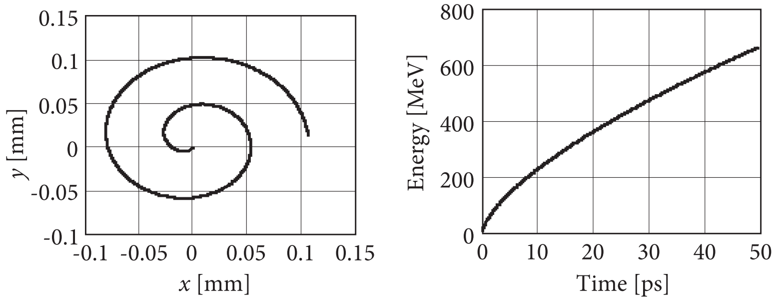

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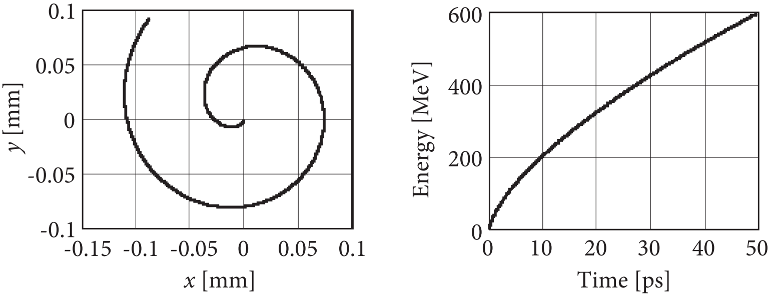


Fig. 4. Impact of the increasing electron initial velocity V_{0z} on the reduction of the resonance magnetic field intensity B_z^{rez}. Parameters: electric field intensity $E_0 = 10^{12}$ V/m, wavelength $\lambda = 10$ μm, time of acceleration 50 ps. a) $V_{0z} = 0$, $B_z^{rez} = -1071$ T, b) $V_{0z} = 0.3$ c, $B_z^{rez} = -786$ T, c) $V_{0z} = 0.5$ c, $B_z^{rez} = -618$ T, d) $V_{0z} = 0.7$ c, $B_z^{rez} = -450$ T

initial velocities. The higher the initial velocity, the lower kinetic energy is gained by an electron after the same acceleration time. For instance, after 1 ps $E_k = 58$ MeV for $V_{0z} = 0$ c and $E_k = 43$ MeV for $V_{0z} = 0.7$ c. The kinetic energies in Fig. 5 are lower than these in Fig. 4 since the 50 fold reduced acceleration time.

Graphs in Fig. 6 present the process of gaining the kinetic energy and the shape of the projection of the trajectory onto the (x, y) plain at the enhanced electric field intensity to $E_0 = 10^{13}$ V/m during acceleration time of 50 ps. Different the electron initial velocities V_{0z} and the appropriate the resonance magnetic field intensities B_z^{rez} are chosen. The laser radiation wavelength is $\lambda = 10$ μm. As it could be expected, due to the 10-fold rise of the electric field, the kinetic energy gained by an electron was found to be significantly higher more than 4-fold that was obtained at 10^{12} V/m. Now again, the presented results show that the process of gaining the kinetic energy by an electron is different for different initial velocities. The higher the initial velocity, the lower kinetic energy is gained by an electron after the same acceleration time. For instance, after 50 ps, $E_k = 3.8$ GeV at $V_{0z} = 0$ c and it falls to $E_k = 2.8$ GeV for $V_{0z} = 0.7$ c.

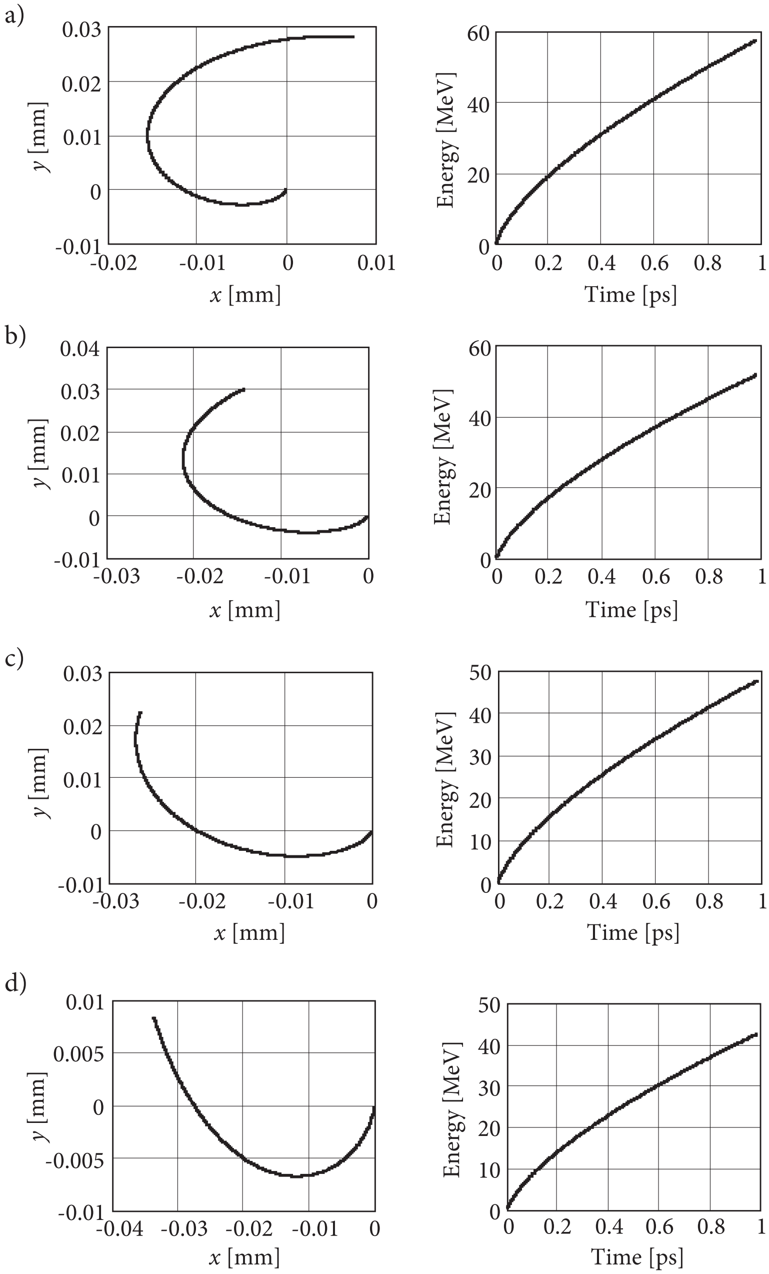


Fig. 5. Impact of the increasing electron initial velocity V_{0z} and the reduced resonance magnetic field intensity B_z^{rez} during 1 ps acceleration time on the projection of the trajectory onto the (x, y) plain and the process of gaining the kinetic energy. Parameters: electric field intensity $E_0 = 10^{12}$ V/m, wavelength $\lambda = 10$ µm. a) $V_{0z} = 0$, $B_z^{rez} = -1071$ T; b) $V_{0z} = 0.3$ c, $B_z^{rez} = -786$ T; c) $V_{0z} = 0.5$ c, $B_z^{rez} = -618$ T; d) $V_{0z} = 0.7$ c, $B_z^{rez} = -450$ T

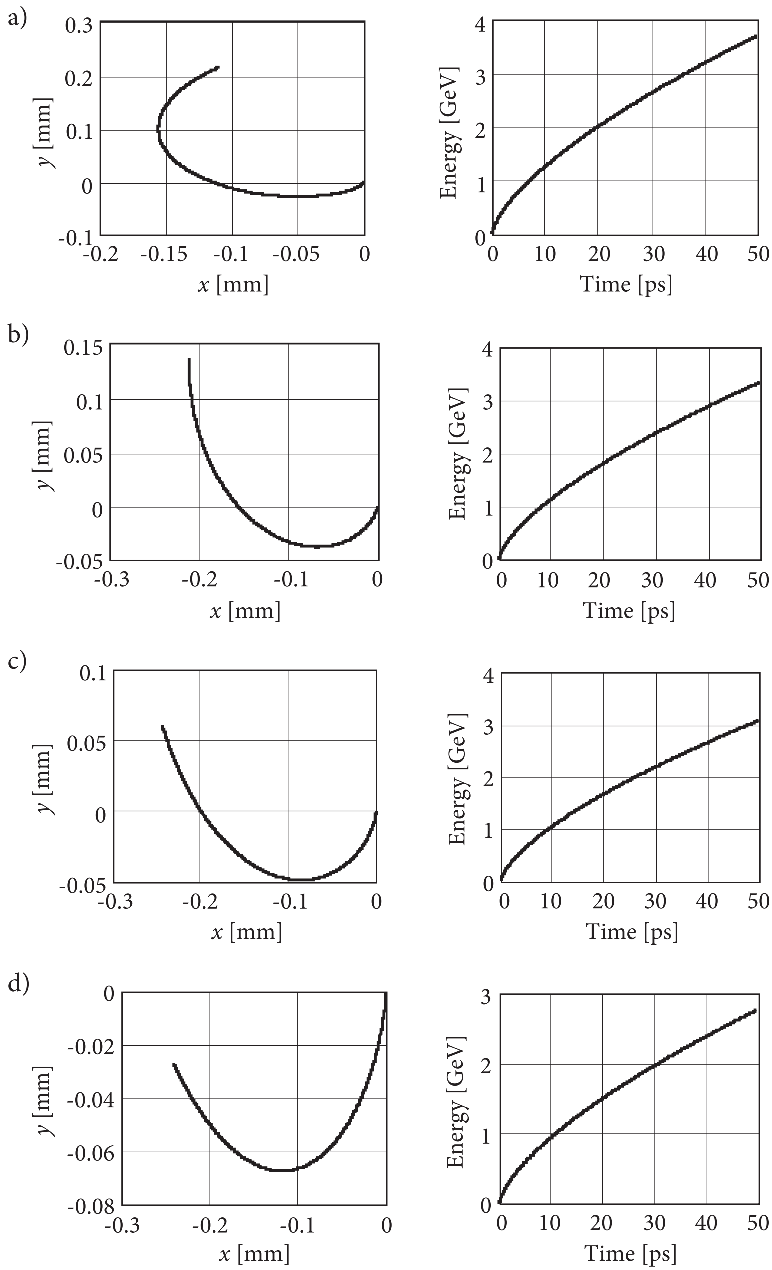


Fig. 6. Impact of the enhanced electric field intensity to $E_0 = 10^{13}$ V/m during acceleration time of 50 ps on the process of gaining the kinetic energy and the projection of the trajectory onto the (x, y) plain, for different the electron initial velocities V_{0z} and the resonance magnetic field intensities B_z^{rez}. Parameters: wavelength $\lambda = 10$ µm, a) $V_{0z} = 0$, $B_z^{rez} = -1071$ T; b) $V_{0z} = 0.3$ c, $B_z^{rez} = -786$ T; c) $V_{0z} = 0.5$ c, $B_z^{rez} = -618$ T; d) $V_{0z} = 0.7$ c, $B_z^{rez} = -450$ T

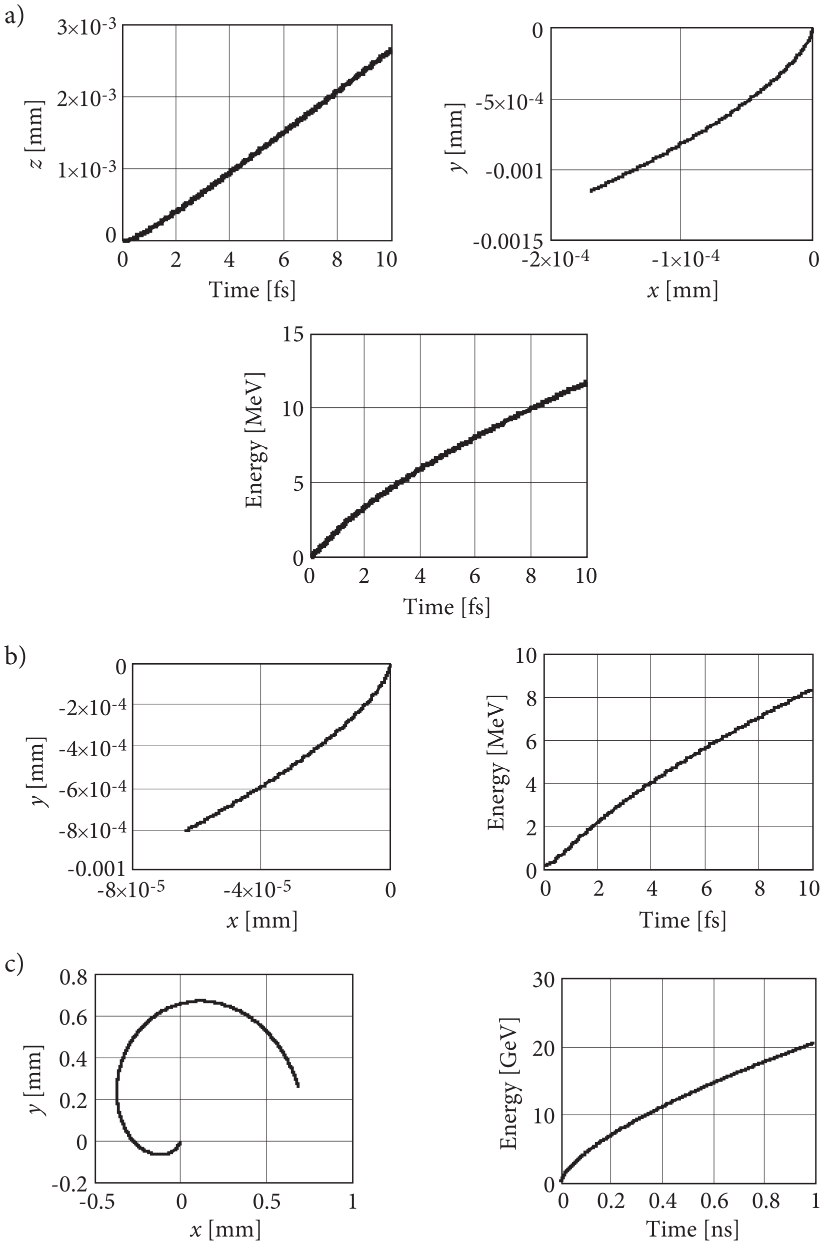


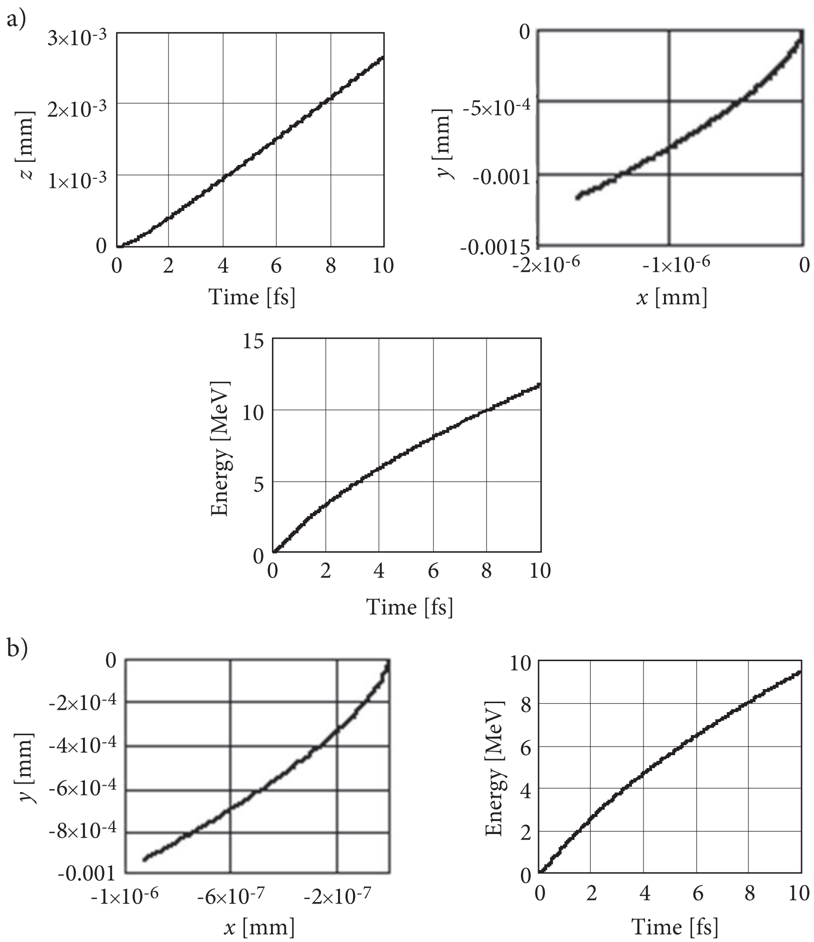
Fig. 7. Process of gaining the kinetic energy and the projection of the trajectory onto the (x, y) plain at the enhanced electric field intensity to $E_0 = 10^{13}$ V/m for laser radiation of the wavelength $\lambda = 10$ µm during short and long acceleration times 10 fs (a, b) and 1 ns (c) respectively, for two different the electron initial velocities V_{0z} and the appropriate resonance magnetic field intensities. a) $V_{0z} = 0$ c, $B_z^{rez} = -1071$ T, acceleration time 10 fs; b) $V_{0z} = 0.7$ c (0.2046 MeV), $B_z^{rez} = -450$ T, 10 fs; c) $V_{0z} = 0$ c, $B_z^{rez} = -1071$ T, 1 ns

Figures 7 show the simulation results concerning the process of gaining the kinetic energy and the projection of the electron trajectory onto the (x, y) plain at the enhanced electric field intensity to $E_0 = 10^{13}$ V/m of the laser radiation wavelength $\lambda = 10$ μm during short and long acceleration times 10 fs (a, b) and 1 ns (c), respectively, for two different the electron initial velocities V_{0z} and the appropriate resonance magnetic field intensities B_z^{rez}. Again, the presented results show that the process of gaining the kinetic energy by an electron is different for different initial velocities. The higher the initial velocity, the lower kinetic energy is gained by an electron after the same acceleration time. For instance, after 10 fs $E_k = 12$ MeV at $V_{0z} = 0$ c and it falls to $E_k = 8$ MeV for $V_{0z} = 0.7$ c. The longer duration of acceleration, the higher the kinetic energy is gained by an electron. This is always true if the resonance conditions are maintained during the acceleration process. The rise of the process time from 10 fs to 1 ns for $V_{0z} = 0$ c, resulted in the rise of the kinetic energy from 12 MeV to 20 GeV. At this point, it should be mentioned that when the initial velocity of the electron changes it results in the change in the values of acceleration parameters. The change is connected with the variation of the resonance magnetic field at different initial velocities.

Figure 8 presents simulation of acceleration process in a maser radiation beam. Due to the 100-fold increase in the wavelength, the resonance conditions are achieved at significant reduced intensity of the magnetic field. The attached graphs show the impact of the increasing electron initial velocity V_{0z} on further reduction of the resonance magnetic field intensity B_z^{rez} for maser radiation beam of the wavelength $\lambda = 1$ mm for $V_{0z} = 0$ and $V_{0z} = 0.5$ c the magnetic field was found to be $B_z^{rez} = -10.71$ T and $B_z^{rez} = -6.19$ T, respectively. The maser radiation intensity is $E_0 = 10^{13}$ V/m for all the graphs. The simulations have been carried out for acceleration duration of 10 fs (Fig. 8a, b) and 10 ps (c, d). Comparison between the laser $\lambda = 10$ μm and the maser $\lambda = 1$ mm shows that the resonance intensity of the magnetic field can be significantly reduced from $B_z^{rez} = -1071$ T to $B_z^{rez} = -10.71$ T for $V_{0z} = 0$ and from $B_z^{rez} = -618$ T to $B_z^{rez} = -6.18$ T for $V_{0z} = 0.5$ c. It means that the reduction of B_z^{rez} was found to be also 100-fold. Due to the acceleration process, at resonance conditions, the longer duration of acceleration, the higher the kinetic energy is gained by an electron. The rise of the process time from 10 fs to 10 ps for $V_{0z} = 0$ c, resulted in the rise of the kinetic energy from 12 MeV to 1.2 GeV, respectively, and for $V_{0z} = 0.5$ c the rise of acceleration time from 10 fs to 10 ps, resulted in the rise of the kinetic energy from 10 MeV to 1 GeV, respectively. For a maser, like for a laser, the presented results show that the process of gaining the kinetic energy by an electron is different for different initial velocities. The higher the initial velocity, the lower kinetic energy is gained by an electron after the same acceleration time. For instance, after 10 fs, $E_k = 12$ MeV at $V_{0z} = 0$ and it falls to $E_k = 10$ MeV for $V_{0z} = 0.5$ c and after 10 ps, $E_k = 1.3$ GeV at $V_{0z} = 0$ and it falls to $E_k = 1.1$ GeV for $V_{0z} = 0.5$ c.

As it can be seen in Fig. 9, due to the increased mass of the particle, the acceleration process compared to the case of a lighter particle is subjected to the significant changes. In order to remain within a realistic intensities of a resonance magnetic field, the simulation has been performed using a maser radiation beam of the wavelength 1 cm and the electric field intensity $E_0 = 10^{13}$ V/m. $B_z^{rez} = +1967$ T for $V_{0z} = 0$ c and $B_z^{rez} = +1135$ T for $V_{0z} = 0.5$ c.

The graphs attached in Fig. 9 confirm theoretical prediction that also for particles of higher mass the increase in the initial velocity V_{0z} results in the reduction of the resonance magnetic field intensity B_z^{rez}. Due to the increase in the initial velocity of the proton from $V_{0z} = 0$ to $V_{0z} = 0.5$ c, the resonance magnetic field can be reduced from $B_z^{rez} = +\ 1967$ T to $B_z^{rez} = +\ 1135$ T. From the presented graphs, it is evident that the projections of the trajectory onto the (x, y) plane are subjected



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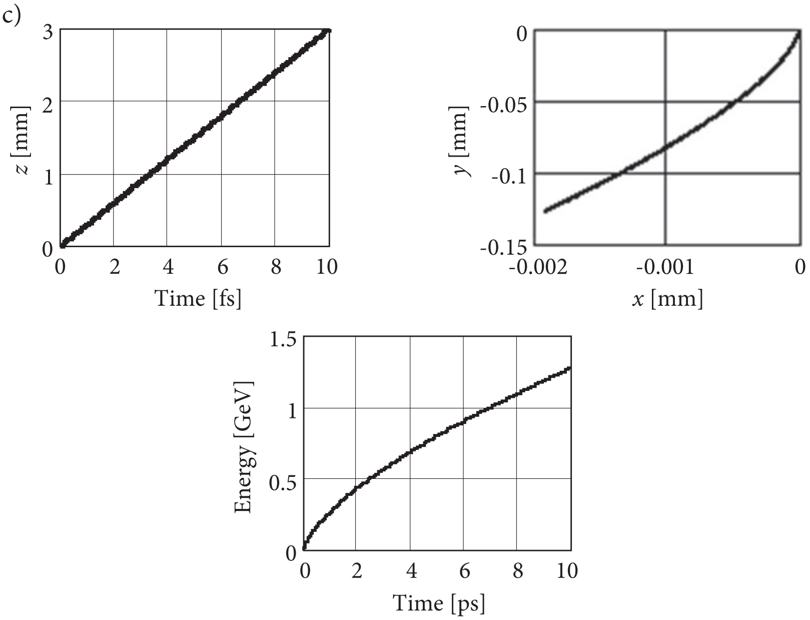

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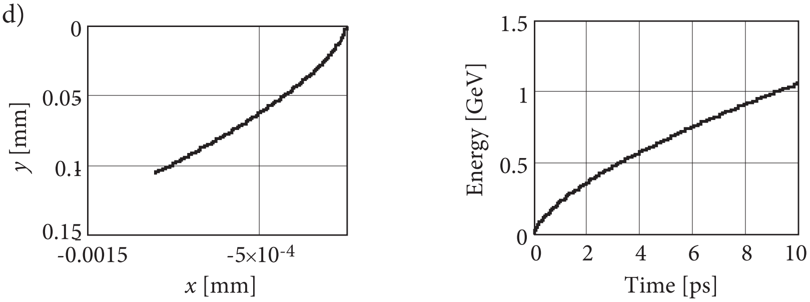
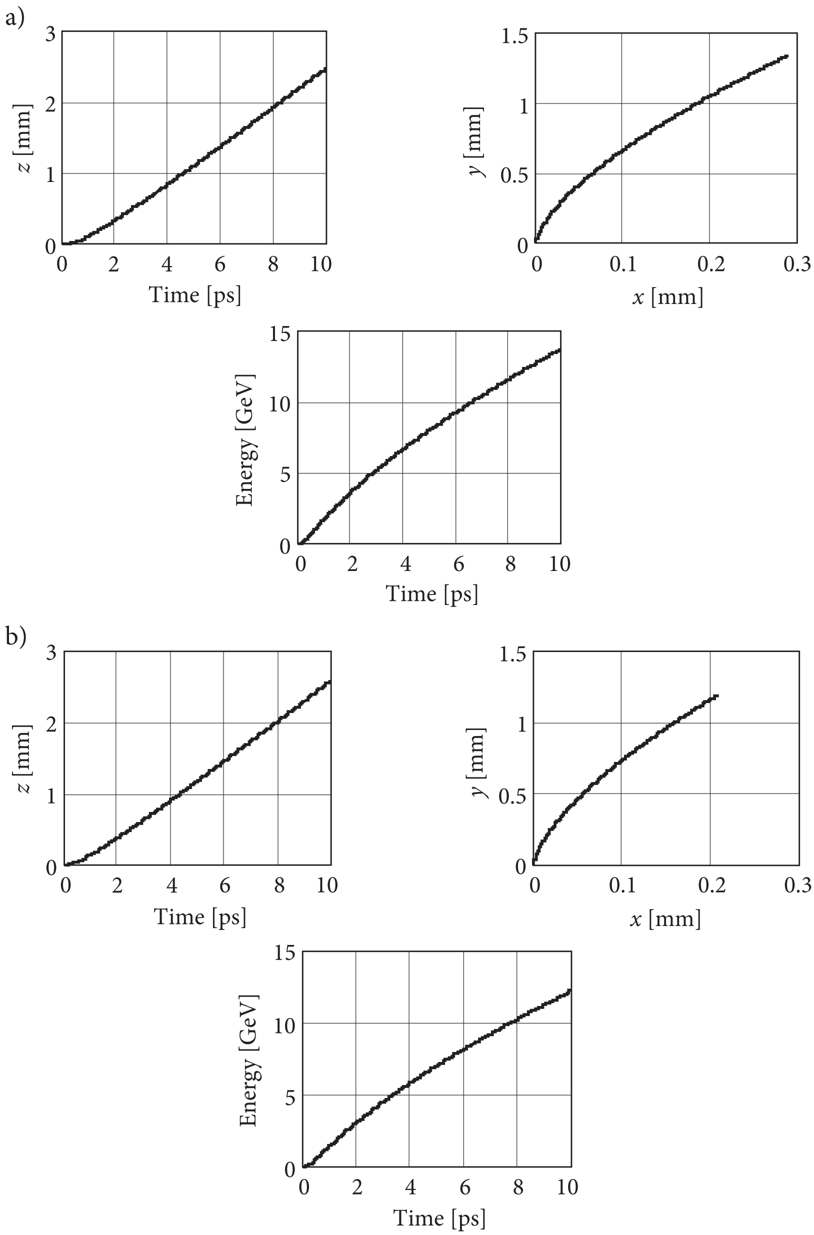


Fig. 8. Impact of the increasing electron initial velocity V_{0z} on the reduction of the resonance magnetic field intensity B_z^{rez} for maser radiation beam of the wavelength λ = 1 mm. Parameters: electric field intensity $E_0 = 10^{13}$ V/m: a) $V_{0z} = 0$, $B_z^{rez} = -10.7$ T, acceleration time 10 fs; b) $V_{0z} = 0.5$ c (0.0791 MeV), $B_z^{rez} = -6.19$ T, acceleration time 10 fs; c) $V_{0z} = 0$, $B_z^{rez} = -10.7$ T, acceleration time10 ps; d) $V_{0z} = 0.5$ c, $B_z^{rez} = -6.19$ T, acceleration time 10 ps

to the variation at different initial velocities of a proton. The length of the trajectory decreases with the increase in the initial velocity. It is the result of the reduction of the magnetic field which means that the mean component velocity $V_\perp$ in the plane (x, y) decreases, provided that the acceleration duration is the same in each graph. However, the presented results show that the process of gaining the kinetic energy by a proton is different for different initial velocities. The higher the initial velocity, the lower kinetic energy is gained by a proton after the same acceleration time. For instance, after 10 ps E_k = 14 GeV for V_{0z} = 0 c and E_k = 11 GeV for V_{0z} = 0.5 c. Due to the acceleration process, maintained at resonance conditions, the longer duration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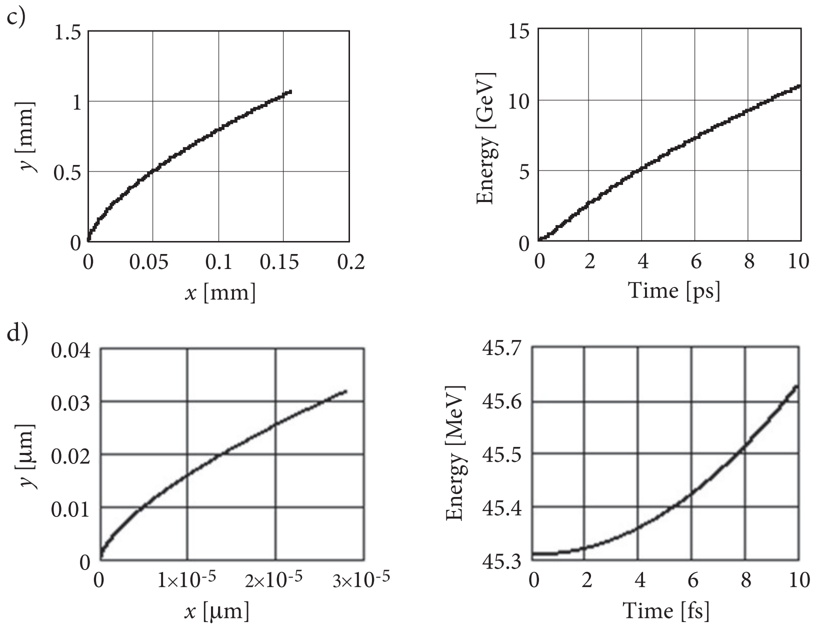

Fig. 9. Impact of the increasing proton initial velocity V_{0z} on the reduction of the resonance magnetic field intensity B_z^{rez} and the course of the kinetic energy gaining process as a result of acceleration in a maser beam. Parameters: electric field intensity $E_0 = 10^{13}$ V/m, wavelength $\lambda = 1$ cm, acceleration times 10 ps (a, b, c) and 10 fs (d). a) $V_{0z} = 0$, $B_z^{rez} = +1967$ T; b) $V_{0z} = 0.3$ c, $B_z^{rez} = +1443$ T; c) $V_{0z} = 0.5$ c, $B_z^{rez} = +1135$ T; d) $V_{0z} = 0.3$ c, $B_z^{rez} = +1443$ T

of acceleration, the higher the kinetic energy is gained by a proton. The rise of the process time from 10 fs to 10 ps for $V_{0z} = 0$ c, resulted in the rise of the kinetic energy from 0.44 MeV (not shown in figures) to 12 GeV and for $V_{0z} = 0.3$ c the rise of acceleration time from 10 fs to 10 ps, resulted in the rise of the kinetic energy from 45 MeV to 12 GeV. For a proton, like for an electron, the presented results show that the process of gaining the kinetic energy by a proton is different for different initial velocities. The higher the initial velocity, the lower kinetic energy is gained by a proton after the same acceleration time. For instance, after 12 ps $E_k = 14$ GeV at $V_{0z} = 0$ and it falls to $E_k = 12$ GeV for $V_{0z} = 0.3$ c and after 10 fs $E_k = 0.48$ MeV at $V_{0z} = 0$ and it rises to $E_k = 45$ MeV for $V_{0z} = 0.3$ c. The last example shows that for a proton, when acceleration duration is selected to be short, the gaining process behaves reversely than that for an electron, i.e., the higher the initial velocity, the higher kinetic energy is gained by a proton after the same acceleration time.

9. Summary

The above sets of figures illustrate the impact of increase in the initial velocity of the particle on decrease in the value of the resonance intensity of the magnetic field for the intensity of the electric field of the laser radiation $E_0 = 10^{12}$ V/m, $E_0 = 10^{13}$ V/m, and $E_0 = 10^{14}$ V/m, the wavelengths of the radiation, $\lambda = 10$ μm, 1 mm, and 1 cm and for different durations of interaction of the field with the electrons, protons, and deuterons. It has been illustrated the impact of the length of the interaction time of the field with an electron, a proton, and deuteron on the trajectory shape of the particle in a plane (x, y) and the course of variation of kinetic energy of particles with time and on the value of this energy for different initial velocities V_{0z} corresponding to the resonant values of the intensity of the magnetic field B_z^{rez} for durations 10 fs, 1 ps, 10 ps, 50 ps, and 1 ns. Presented charts illustrate the impact of the intensity of the electric field $E_0 = 10^{12}$ V/m and $E_0 = 10^{13}$ V/m, on the motion of the electron in the plane (x, y) and the developments of kinetic energy of particles with time and on the value of this energy for different initial velocities V_{0z} of the particles and corresponding values of the intensity of the magnetic field B_z^{rez}. An analysis of the impact of the change of the wavelength $\lambda = 10$ μm and 1 mm on the value of the intensity of the magnetic field B_z^{rez}, at the intensity of electric field of the maser radiation $E_0 = 10^{13}$ V/m has been performed. Using the example of a proton interacting with the maser radiation of the wavelength of $\lambda = 1$ cm and intensity of the electric field radiation $E_0 = 10^{13}$ V/m, it is shown the influence of the particles mass on the resonance magnetic field intensity B_z^{rez} value and on its kinetic energy. Presented graphs illustrate the impact of the initial velocity V_{0z} increase in a proton on the reduction of the resonant magnetic field B_z^{rez} when a proton interacts with the electric field of radiation with the wavelength of $\lambda = 1$ cm and for examples of the interaction of the particles after 10 fs and 10 ps. Graphs of the particles' trajectories, presented at the first set of drawings projected onto a plane (x, y), show that with the increasing initial velocity of the particle V_{0z}, the length of the spiral along which the particle advances is reduced. The reason for this reduction is the decline in the value of the resonant intensity of the magnetic field B_z^{rez}. This means that with the increase in the velocity V_{0z} of the particle, its total velocity $V_\perp$ decreases in the projection on a plane (x, y).

At the same time, the graphs of variation of the kinetic energy of the particle with time indicate a decline in the value of this energy with increasing initial velocity of the particle V_{0z}. This means that an increase in the initial velocity V_{0z} leads to a fall in the value of the kinetic energy of the particles. This is the cause of a fall in the value of the kinetic energy. It may also mean that the kinetic energy, associated with its movement along the z-coordinate, is maintained. Changes in the value of the kinetic energy of the particles with the time, also in the case of shorter periods, for example the ps and fs durations of interaction, show the decline in a value of kinetic energy with the increasing initial velocity V_{0z} of the particle. Presented graphs confirm also the obvious fact that the shorter duration of interaction of the particle with laser or maser radiations absorbs less kinetic energy. Changes in the value of the kinetic energy of the particle with time, at the increased value of the electric field to $E_0 = 10^{13}$ V/m, also show the reduction of this energy with increasing initial velocity of the particle V_{0z}. In the case of an electron, this reduction lies in the range of few GeV after interaction time of 50 ps. The graphs show that the increase in the intensity of the electric field of one order of magnitude leads to the over four-fold increase in the kinetic energy absorbed by an electron during 50 ps. Similar conclusions apply to the characteristics of the motion of the electron interacting with the field at the time much shorter 10 fs. The increase in the interaction duration from 10 fs to 1 ns between the particle and with the laser radiation field leads to the increase in the kinetic energy of the particle more than three orders of magnitude. The presented data show that the change in the wavelength from $\lambda = 10$ μm to $\lambda = 1$ mm drastically reduces the resonant intensity of the magnetic field B_z^{rez} by two orders of magnitude. The another set of diagrams characterizing the proton interaction with the maser radiation field of the wavelength $\lambda = 1$ cm shows strong increase in the particles mass, which significantly increases the intensity of the resonant magnetic field B_z^{rez} which also in this case might be significantly reduced by the increasing initial velocity V_{0z} of the particle, regardless of duration of its impact with the field of radiation. The increase in the duration of the maser radiation field interaction with a proton, as in the case of the electron, results in the increase in the value of the kinetic energy of the particle. Graphs of variation in the kinetic energy of a proton with time show the decline in a value of this energy with the increasing initial velocity V_{0z} of the particle for interaction duration of the order of ps and the increase in this energy with the increasing initial velocity V_{0z} of the particle for interaction duration of the order of fs. In the case of a proton, at shorter durations of interaction with a maser radiation, an increase in kinetic energy of the particle with the increasing initial velocity V_{0z} occurs and so, it appears to be the opposite than it does in the case of longer times of the interaction of a proton or an electron with a laser or maser radiation. Such an effect is not observed in the case of the electron.

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Appendix

Dynamical relativistic equation and the continuous equation of the normalized energy γ has the following form

$$\frac{d\vec{p}}{dt} = q\vec{E} + q\left(\vec{V} \times \vec{B}\right), \tag{A1}$$

$$\frac{d\gamma}{dt} = \frac{q}{m_0 c^2} \vec{V} \cdot \vec{E}. \tag{A2}$$

Multiplying both sides of the relativistic Lorentz force equation (A1) by $\vec{V}$, and neglecting the term

$$\vec{V}\cdot\left(\vec{V}\times\vec{B}\right)\equiv 0,$$

we get equation (A1) in the form:

$$\vec{V}\cdot\frac{d\vec{p}}{dt}=q\vec{V}\cdot\vec{E}. \tag{A3}$$

Equation for momentum has the following relativistic form:

$$\vec{p}=\gamma m_0\vec{V}. \tag{A4}$$

After differentiation of equation (A4) we obtain:

$$\frac{d\vec{p}}{dt}=m_0\vec{V}\frac{d\gamma}{dt}+m_0\gamma\frac{d\vec{V}}{dt}.$$

Multiplying both sides of the above equation by $\vec{V}$ the equation gives:

$$\vec{V}\cdot\frac{d\vec{p}}{dt}=m_0V^2\frac{d\gamma}{dt}+m_0\gamma\vec{V}\cdot\frac{d\vec{V}}{dt}. \tag{A5}$$

According to (A3), equation (A5) can be rewritten as

$$V^2\frac{d\gamma}{dt}+\gamma\vec{V}\cdot\frac{d\vec{V}}{dt}=\frac{q}{m_0}\vec{V}\cdot\vec{E}, \tag{A6}$$

or

$$\frac{d\gamma}{dt}+\frac{\gamma}{V^2}\vec{V}\cdot\frac{d\vec{V}}{dt}=\frac{q}{m_0V^2}\vec{V}\cdot\vec{E}, \tag{A7}$$

Equation (A7) on the left side has two terms and differs from equation of energy conservation (A2):

$$\frac{d\gamma}{dt}=\frac{q}{m_0c^2}\vec{V}\cdot\vec{E}.$$

Equation (A6) can be transformed to the following form:

$$c^2\frac{V^2}{c^2}\frac{d\gamma}{dt}+\frac{1}{2}\gamma c^2\frac{d\left(V^2/c^2\right)}{dt}=\frac{q}{m_0}\vec{V}\cdot\vec{E}. \tag{A8}$$

From the relation

$$y=\left(1-\frac{V^2}{c^2}\right)^{-1/2},$$

we can obtain the following formula:

$$\frac{V^2}{c^2}=1-\frac{1}{\gamma^2}. \tag{A9}$$

Inserting (A9) into (A8) we get:

$$c^2\left(1-\frac{1}{\gamma^2}\right)\frac{d\gamma}{dt}+\frac{1}{2}\gamma c^2\frac{d}{dt}\left(1-\frac{1}{\gamma^2}\right)=\frac{q}{m_0}\vec{V}\cdot\vec{E}, \tag{A10}$$

or after differentiation of the second term on the left side of equation (A10) we obtain

$$\frac{1}{2}\gamma c^2\frac{d}{dt}\left(1-\frac{1}{\gamma^2}\right)=c^2\gamma\gamma^{-3}\frac{d\gamma}{dt}=c^2\gamma^{-2}\frac{d\gamma}{dt}.$$

Equation (A10), after dividing it by c^2 can be written as:

$$\left(1-\gamma^{-2}\right)\frac{d\gamma}{dt}+\gamma^{-2}\frac{d\gamma}{dt}=\frac{q}{m_0c^2}\vec{V}\cdot\vec{E},$$

and finally

$$\frac{d\gamma}{dt}=\frac{q}{m_0c^2}\vec{V}\cdot\vec{E}.$$

A. DUBIK, M.J. MAŁACHOWSKI

Wpływ masy i innych parametrów naładowanej cząstki na wynik laserowej rezonansowej akceleracji

Streszczenie. Stosując metody teoretyczną i numeryczną przebadano warunki, w których naładowane cząstki o różnych masach można przyspieszać do znacznej energii w kołowo spolaryzowanej laserowej bądź maserowej wiązce z dodatkowym statycznym polem magnetycznym. Badania przeprowadzono za pomocą wyprowadzonych analitycznych relacji dotyczących dynamiki i kinetycznej energii cząstek. Dzięki stosunkowo licznym wykresom stała się możliwa interpretacja dość złożonego ruchu cząstek oraz przebiegu ich akceleracji. Na przykładach elektronu, protonu i deuteronu zostały zilustrowane zależności od czasu trwania akceleracji takich wielkości jak kształt trajektorii oraz kinetyczna energia. Wszystkie ilustracje dotyczą warunku rezonansu, czyli synchronizacji ruchów obrotowych cząstki

i wektora natężenia pola elektrycznego. Czym większa masa cząstki, tym większe natężenie stałego pola magnetycznego jest niezbędne do uzyskania warunku synchronizacji. Jednak to natężenie można znacznie zredukować, jeśli cząstka będzie posiadała prędkość początkową.
Słowa kluczowe: optoelektronika, akceleracja naładowanych cząstek, laser, maser, dynamika relatywistyczna, energia kinetyczna cząstki, elektron, proton, deuteron