ARCHIVES OF ELECTRICAL ENGINEERING VOL. 69(3), pp. 487–498 (2020)

DOI 10.24425/aee.2020.133914

Synthesis of active LTV elements

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(Received: 22.12.2019, revised: 02.03.2020)

Abstract: The article proposes the method of synthesis of active elements with time-varying parameters $R(t)$, $C(t)$ and $L(t)$. In order to construct the elements, it is necessary to use operational amplifiers, multipliers and classic *RLC* components. The variability in time of the elements results from applying voltage to control terminals. Assuming that the parameters of elements $R(t)$, $L(t)$, $C(t)$ are exponentially varying, dependencies describing the control voltage waveforms which enable such a parameter variability were determined. The obtained results were illustrated with examples and PSpice simulations.

Key words: parametric capacitor, parametric coil, parametric resistor, time-varying systems, synthesis of parametric elements

1. Introduction

The article is devoted to linear parametric systems, non-stationary systems with parameters varying in time. In short, they are called LTV systems. One can treat those systems as a direct generalization of time-invariant systems. The benefit of their application is the reduction of a transient state and improving the dynamic properties of the systems [1–3].

There are known works focused on the theoretical analysis in the time and in the frequency domain [1, 4, 7–9]. The continuous time-varying system can be treated as an auxiliary tool for analysis of fractional systems [10]. Practical applications can be enumerated as well. Parametric filters can be used in various fields such as: signal processing and sampling systems [11–16] or filtering of signals and noise elimination [1, 2, 12, 13, 17, 18]. In a power network one can encounter LTV systems used for current compensation [19, 20]. Moreover, those systems are applied also as: [21], chaos generators [22], electromagnetic launchers [23] and medicine devices [24].

The literature proposes implementation of LTV systems both in analogue systems (e.g. biquadrate structure of analogue LTV filters [1, 18, 25]) and digital solutions, (e.g. [15, 26, 27]). An element of novelty in the research on the parametric system, introduced by the authors, is the attempt to create the base of R, L, and C elements of the LTV class.

2. General structure of parametric elements

This article is devoted to the method of synthesis of elements $R(t)$, $L(t)$, $C(t)$ with resistance, inductance and capacitance which are variable in time. Such elements can be presented in the form of three poles (Fig. 1(a)) or one-ports with assumed forced control voltages $v_p(t)$ (Fig. 1(b)). In order to construct parametric elements, it is necessary to use operational amplifiers, multipliers, and classical *RLC* elements as well as autonomic sources of controlling voltages.

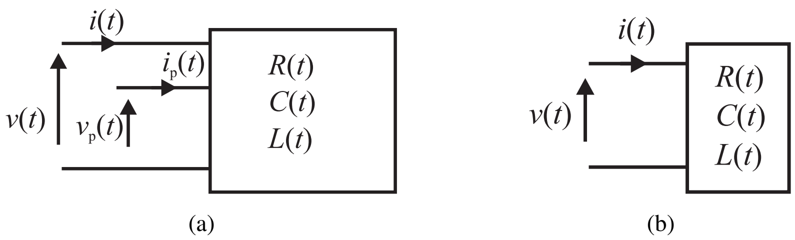

Fig. 1. Equivalent circuit of parametric elements in the forms of: three poles (a); one-port (b)

Those elements can be dedicated to construct many more complex parametric systems with various applications [17–20, 28].

The general structure of the system which can take a function of LTV elements is shown in Fig. 2. This system is composed of an operational amplifier, multipliers and classical *RLC* elements. The usage of one of the options (R, C or L) in the feedback loop results in the creation of a parametric resistor, a parametric capacitor or a parametric inductor, respectively. The in-depth analysis is presented in sections 2.1, 2.2, 2.3.

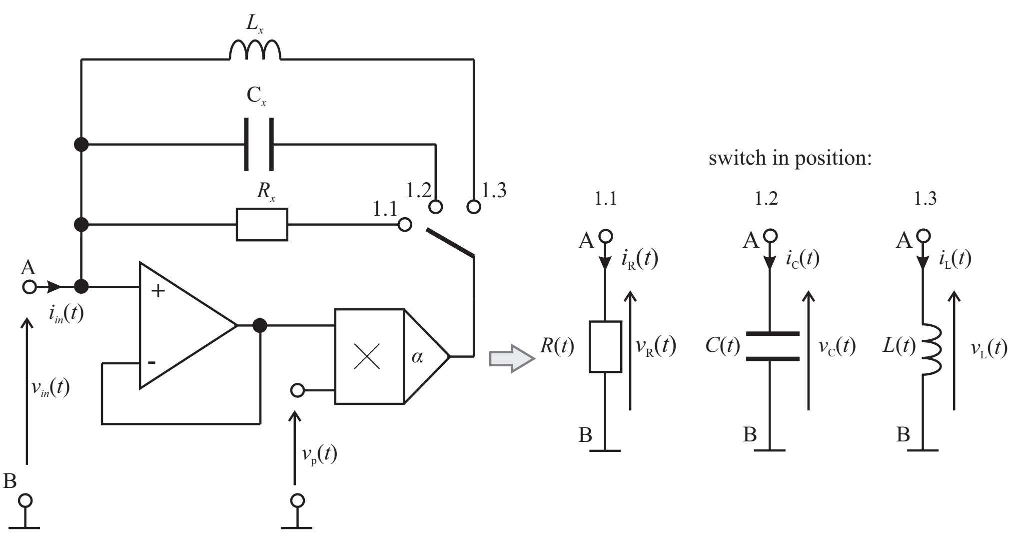

Fig. 2. General scheme for realization of a grounded LTV elements

The article is based on some concept of analogue realization of *RLC* elements of the LTV class. The proposed circuit structures are supported by PSpice analysis with models of ideal elements. The simulations show the current-voltage relation for selected elements, with unit-step excitation and prove the possibility to improve the dynamic properties of systems with LTV elements. The particular choice of structure elements, i.e.an operational amplifier, is strictly connected with the application of the system using LTV elements. The parameters of specific models of elements will be influenced by the variability of a parametric function and a method of its generation. The variability in time of the elements actually depends solely on the waveform of voltage $v_p(t)$ applied to control terminals. The obtained results were illustrated with examples, assuming that variabilities of $R(t)$, $C(t)$, $L(t)$ are exponential functions. For such functions, relations determining waveforms of control voltages enabling such parameter variability were presented. The authors emphasize that they have not optimized the variability of the parametric functions or selected particular elements. Due to the dynamic properties of the system, however, this issue will be considered in the future.

2.1. Parametric resistor

A parametric resistor is an element of the LTV class and its feature is time-dependence of resistance $R(t)$, Fig. 3.

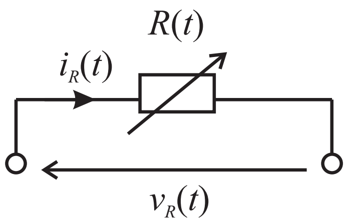

Fig. 3. Model of a LTV-class resistor

The model of the parametric resistor is described by rhe formula:

$$v_R(t) = i_R(t)R(t). \tag{1}$$

Equation (1) describes the relationship between the current and the voltage of the LTV resistor with resistance dependent on time, as waveform $R(t)$. Dependence $R(t)$ is called a parametric function of the system. Fig. 2 with resistor R_x in the feedback-loop presents the system which realizes Formula (1) with $i_{\text{in}}(t) = i_R(t)$ and $v_{\text{in}}(t) = v_R(t)$.

From the AB nodes' point of view, the system can be interpreted as the grounded resistor with the resistance (2) varying according to the variability of voltage $v_p(t)$.

$$R(t) = \frac{R_x}{1 - \alpha v_p(t)}, \tag{2}$$

where α is the gain coefficient of the multiplier.

Articles [3, 19, 29, 30] have revealed benefits of the application of the LTV systems with a parametric function waveform, which is exponential. Due to this fact, function $R(t)$ is expected to be:

$$R(t) = R_0 + \beta e^{-\gamma t}, \tag{3}$$

where R_0, β, γ are the coefficients of the parametric function such as: β, which is the coefficient determining the difference between the resistance initial value of $R(t)$ as well as the value of set resistance R_0, and γ is the coefficient determining the pace of obtaining the set value R_0. In the case the waveform of the parametric function is selected in such a manner, control voltage $v_p(t)$ is in the form of Function (4). The waveforms of the assumed functions, resistance changes and control voltages are presented in Fig. 4.

$$\alpha v_p(t) = 1 - \frac{R_x}{R_0 + \beta e^{-\gamma t}} . \tag{4}$$

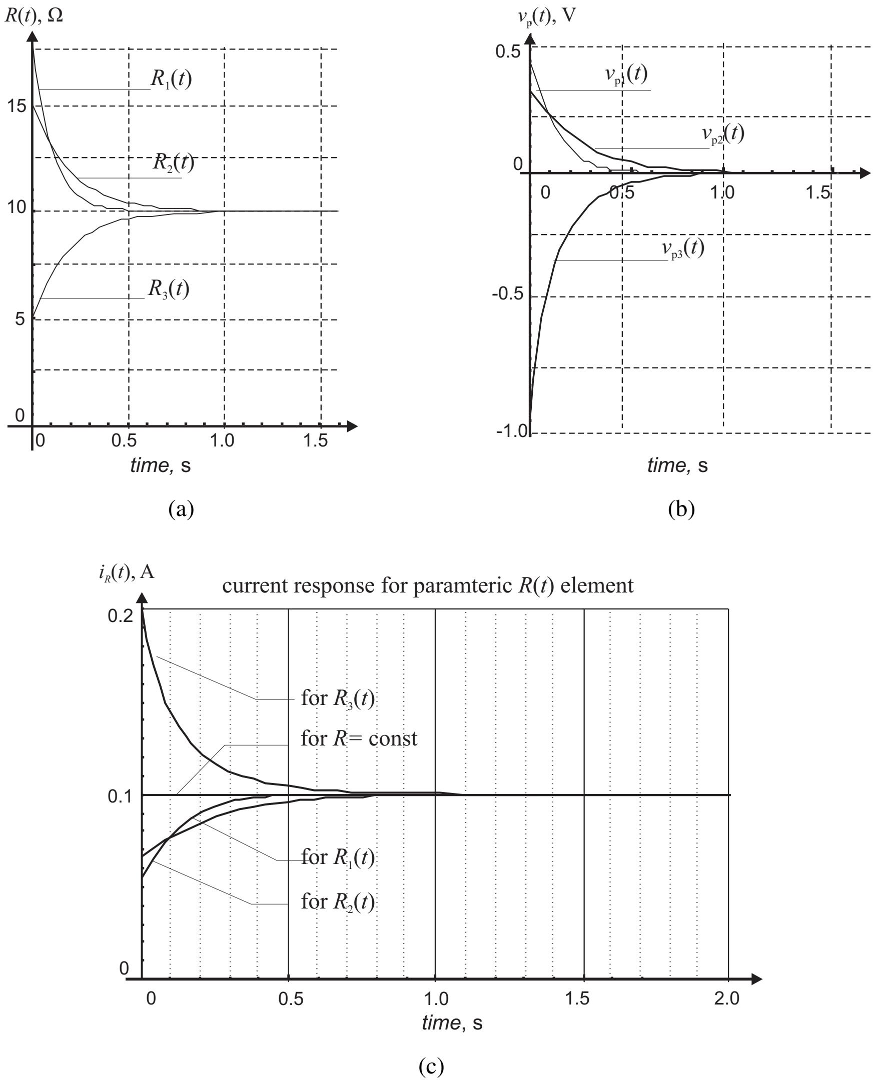

Fig. 4. Simulations for LTV resistor: varying resistance $R(t)$ (a); control voltage $v_p(t)$ (b); current waveforms through the parametric resistor for unit-step voltage (c)

Meeting the condition $\beta > 0$, ensures the realization of control function $v_p(t)$ and the stability of the system [29] containing element $R(t)$.

Moreover, the waveform of the current of a parametric resistor was presented in Fig. 4, assuming that voltage was applied to its input in the form of a unit step signal and the resistance of stationary prototype R_0 was 100 Ω. The current waveforms are in accordance with (1). It is worth to notice that when a parametric function reaches its set value, the parametric resistor becomes equal to the stationary resistor. The introduction of resistance which varies in time gives a benefit in the form of the possibility to regulate the system dynamics.

2.2. Parametric capacitor

The parametric capacitor is an element of the LTV class and its feature is time-dependence of capacitance $C(t)$, Fig. 5.

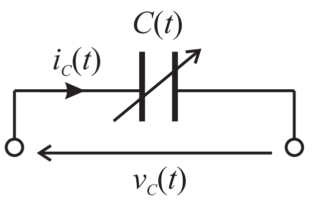

Fig. 5. Model of a LTV- class capacitor

The charge of the parametric capacitor is described by the formula:

$$q(t) = v_C(t)C(t). \tag{5}$$

The current $i_C(t)$ through the LTV capacitor is expressed as:

$$i_C(t) = \frac{\mathrm{d}q(t)}{\mathrm{d}t} = C(t)\frac{\mathrm{d}v_c(t)}{\mathrm{d}t} + v_C(t)\frac{\mathrm{d}C(t)}{\mathrm{d}t}. \tag{6}$$

Equation (6) describes the relationship between the current and the voltage of the LTV capacitor with capacitance dependent on time, as the waveform $C(t)$. The dependence $C(t)$ is called the parametric function of the system. Fig. 2 with capacitor C_x in the feedback-loop presents the system which realizes Formula (6). From the AB nodes' point of view, the system can be interpreted as the capacitor with the capacitance varying according to the variability of the voltage $v_p(t)$. The analysis of the system in Fig. 2 with the switch in 1.2 position, with $i_{\mathrm{in}}(t) = i_{Ct})$ and $v_{\mathrm{in}}(t) = v_{Ct})$ results in:

$$i_C(t) = \left(1 - \alpha v_p(t)\right) C_x \frac{\mathrm{d}v_C(t)}{\mathrm{d}t} - C_x \alpha v_C(t)\frac{\mathrm{d}v_p(t)}{\mathrm{d}t}, \tag{7}$$

where $v_p(t)$ is the control voltage (parametric function), α is the gain factor of the multiplier.

The comparison of Equations (6), (7) shows that:

$$C(t) = \left(1 - \alpha v_p(t)\right) C_x, \tag{8}$$

$$\frac{\mathrm{d}C(t)}{\mathrm{d}t} = -C_x\alpha\frac{\mathrm{d}v_p(t)}{\mathrm{d}t}. \tag{9}$$

It is proved that the proposed system realizes the function of a grounded parametric capacitor with time variable capacitance $C(t)$ controlled by the voltage $v_p(t)$. In this article the exponential variability of parametric capacitance $C(t)$ is assumed:

$$C(t) = C_0 + \beta e^{-\gamma t}, \qquad \beta\gamma \in R^+, \quad \beta > -C_0\,. \tag{10}$$

Coefficient C_0 in (10) is the capacitance of a stationary prototype of an LTV capacitor, parameter γ determines the rate of reaching set value C_0, coefficient β determines the value of the capacitance at time $t = 0$.

The obtained results were illustrated with examples, assuming that variabilities of the parameters are exponential functions because the several factors mentioned below influenced the selection of the exponential functions as varying parameters: The usage of LTV systems with exponentially variable parameters, i.e. [1, 9, 17], is known from the literature. Moreover, the works e.g. [4, 5], show that for such a selection of exponentially varying coefficients it is possible to determine parametric solutions of differential equations describing the LTV systems and to present an impulse response in an analytically closed form as well as a response to any excitation of such a system. There are some papers e.g. [3], related to the time domain analysis of the LTV systems, which show that the non-periodic function describing parameter variability can be represented by a generalized Fourier series with an exponential basis.

The stability examination, e.g. [29, 31], proves the stability of the LTV systems described by the first and the second order differential equations for strictly positive parametric functions. However, it is worth to emphasize that it is only the sufficient condition of the LTV system stability and limits the class of parametric functions. Extending the parametric function class is still a problem to research and will be considered in further works on the synthesis of parametric elements. Summarizing, coefficients of the parametric function were selected in order to ensure stability of the system [29]. The waveforms of parametric functions $C(t)$ and $v_p(t)$ are shown in Fig. 6.

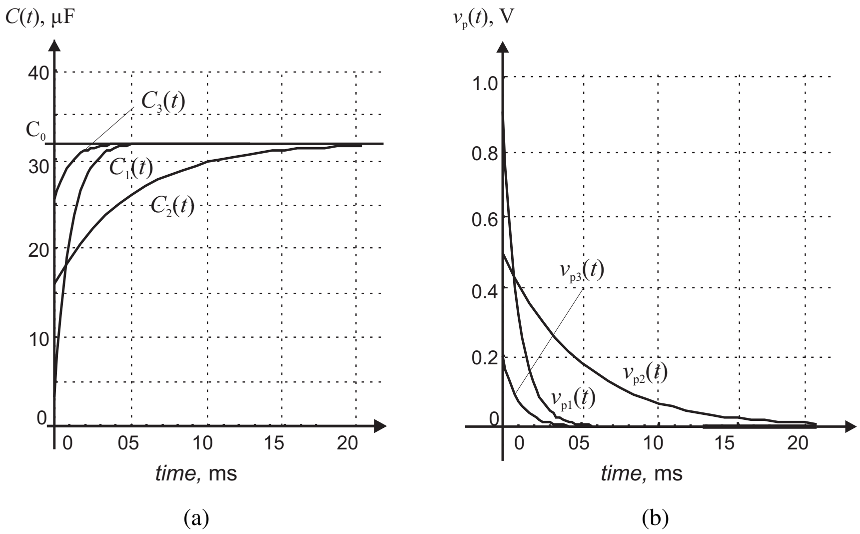

Fig. 6. Parameters variability of LTV capacitor: varying capaciance $C(t)$ (a); control voltage $v_p(t)$ (b)

The proposed parametric capacitor has been used to compose allow pass LTV filter. The prototype of the LTV filter was the classical LTI (linear time invariant) RC filter with the parameters chosen in such a way that cut-off frequency was set at 50 Hz. ($R = 100\,\Omega$, $C_0 = 32\,\mu$F). The scheme of the LTV filter is shown in Fig. 7 and the principle of operation is described by the first order parametric differential Equation (11):

$$\frac{\mathrm{d}v_C(t)}{\mathrm{d}t} + \frac{RC_0\alpha\dfrac{\mathrm{d}v_p(t)}{\mathrm{d}t} + 1}{RC_0\left(1 - \alpha v_p(t)\right)} v_2(t) = \frac{v_1(t)}{RC_0\left(1 - \alpha v_p(t)\right)}, \tag{11}$$

thus:

$$\frac{\mathrm{d}v_C(t)}{\mathrm{d}t} + \omega(t)v_C(t) = k(t)v_1(t). \tag{12}$$

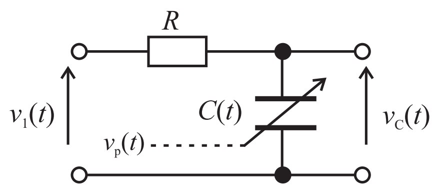

Fig. 7. Example realization of the low-pass LTV filter

One can notice that the function $\omega(t)$ can be treated as a variable cut off angular frequency of the LTV filter and is the function of controlled voltage. The waveform of the step response of the LTV filter is shown in Fig. 8.

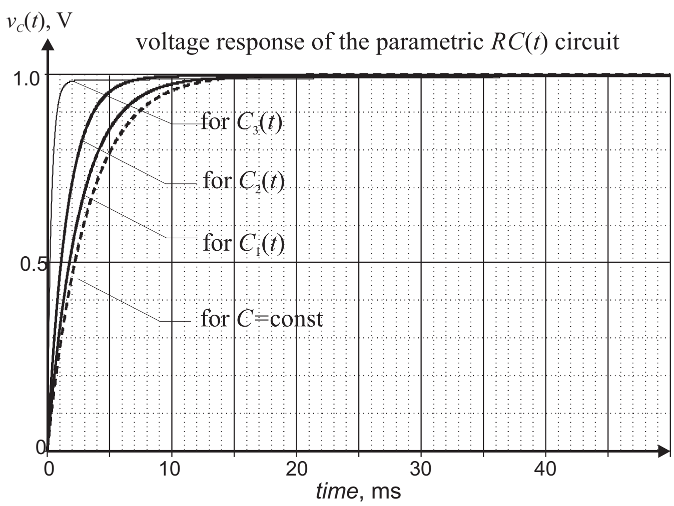

Fig. 8. The LTV filter step responses with different variation of capacitance $C(t)$

It is worth to notice that if parametric function $C(t)$, (11), reaches fixed value C_0, the considered filter will became an equivalent to the classical LTV low pass filter with parameters R, C and angular cut-off frequency ω_0 ($f_0 = 50$ Hz).

2.3. Parametric coil

A parametric coil belongs to elements of the LTV (*linear time varying*) class and its characteristic feature is that inductance $L(t)$ changes with time, Fig. 9. The current-voltage relation for a parametric coil can be expressed as:

$$v_L(t) = L(t)\frac{\mathrm{d}i_L(t)}{\mathrm{d}t}. \tag{13}$$

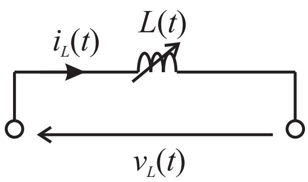

Fig. 9. Model of a coil of LTV class

Equation (13) describes the dependence between voltage and current for the coil of the LTV class which inductance changes with time and is described by function $L(t)$, called a parametric function of the system.

Fig. 2 with inductor L_x in the feedback-loop presents a system which is a realization of the dependence, (13). From the point of view of terminals AB, the system is considered as a coil of which variable inductance $L(t)$ is controlled by voltage $v_p(t)$. From the analysis of the performance of the system in 0, with the switch in position 1.3 and with $i_{\mathrm{in}}(t) = i_L(t)$ and $v_{\mathrm{in}}(t) = v_L(t)$, it can be inferred that the voltage at input terminals AB of the system equals:

$$v_{AB}(t) = v_L(t) = \frac{L_x}{\left(1 - \alpha v_p(t)\right)}\frac{\mathrm{d}i_L(t)}{\mathrm{d}t}, \tag{14}$$

where the $v_p(t)$ is the control voltage of a parametric function, and α is the multiplier gain coefficient. Comparing Formula (14) to the current-voltage relation (13) of the LTV coil, the following equation is obtained:

$$L(t) = \frac{L_x}{1 - \alpha v_p(t)}, \tag{15}$$

which can be interpreted as inductance varying in time. Thus, the proposed system performs the function of a grounded parametric coil with parametric function $L(t)$ tuned by voltage $v_p(t)$. The waveform of parametric function $L(t)$ (interpreted as time-varying inductance) is given as:

$$L(t) = L_0 + \beta e^{-\gamma t}, \qquad \gamma > 0, \quad \beta \geq -L_0. \tag{16}$$

Coefficients L_0 in Equation (16) is equal to the inductance of the stationary prototype of the LTV coil, parameter γ determines the pace of reaching set value L_0, and coefficient β determines the value of inductance at time $t = 0$. Coefficients of the parametric function were selected in such a way which ensures stable work of the system [29]. Details concerning the analysis of the parametric first order differential equation can be found in previous publications, e.g. [1, 3]. Examples of waveforms of the $L(t)$ function are presented in 0. If parametric function $L(t)$ is given, (16), control voltage waveform $v_p(t)$ can be determined by solving the equations:

$$\frac{L_x}{(1 - \alpha v_p)} = L + \beta e^{-\gamma t} \Rightarrow v_p(t) = \frac{L - L_x + \beta e^{-\gamma t}}{\alpha(L_x + \beta e^{-\gamma t})}. \tag{17}$$

The waveforms of control function $v_p(t)$ are shown in Fig. 10(b).

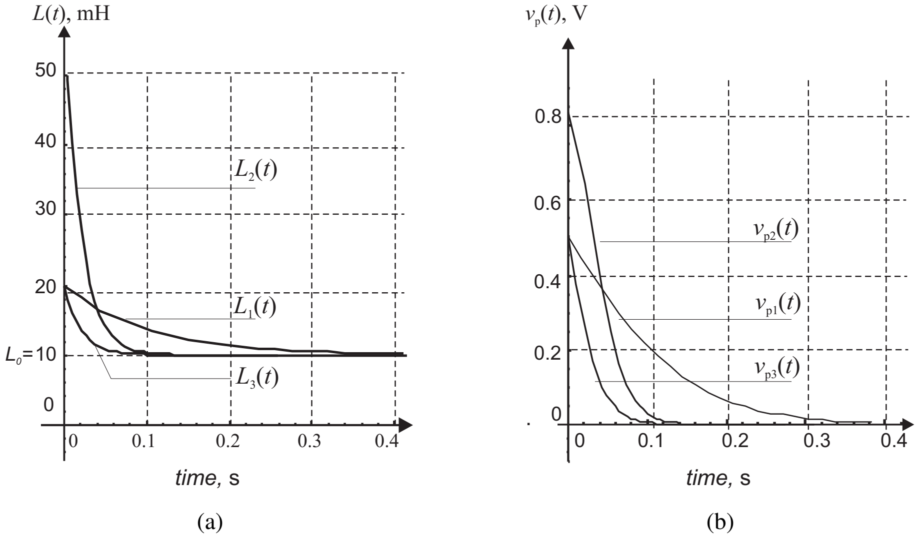

Fig. 10. Parameters variability of LTV coil: varying inductance: $L(t)$ (a); control voltage $v_p(t)$ (b)

The performance of the parametric coil was presented based on the *RL* system of the LTV class. Its prototype was a classical, stationary *RL* system with parameters: $R = 100\,\Omega$, $L = 10$ mH.

The scheme of the LTV system under scrutiny in which a proposed model of a parametric inductor was used was presented in Fig. 11. The performance of the $RL(t)$ system is determined by the first order parametric differential Equation (18):

$$L(t)\frac{\mathrm{d}i_L(t)}{\mathrm{d}t} + Ri(t) = v_1(t). \tag{18}$$

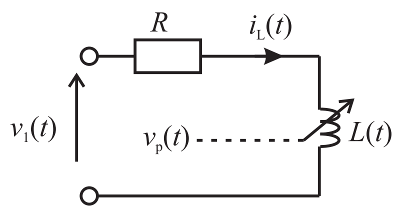

Fig. 11. $RL(t)$ system

Fig. 12 shows waveforms of current of an LTV coil in the case of unit step excitation and for various time-varying inductances $L(t)$. The figure shows clearly that the selection of the waveform of a parametric function has a significant influence on the current waveform of the system.

It should be noticed that when parametric function $L(t)$ reaches the set value L_0, the considered parametric system becomes equal to the stationary system with parameters R, L_0 and the benefit of its application is the possibility to regulate the dynamics of the circuit. The authors emphasize that they have not optimized the variability of the parametric function due to the dynamic properties of the system, however this issue will be considered in the future.

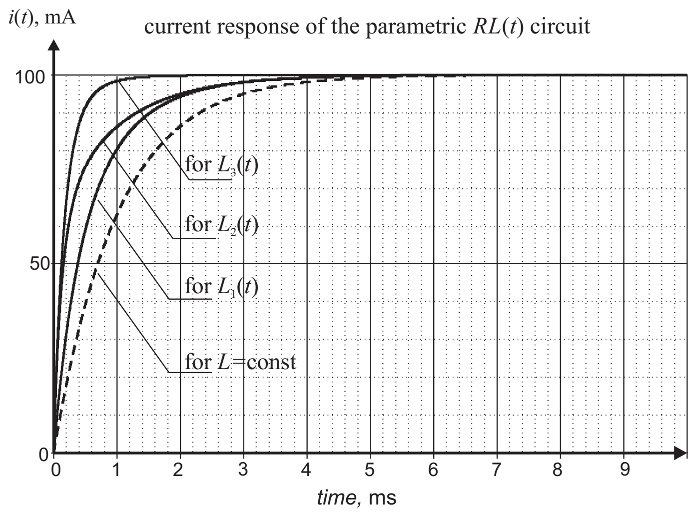

Fig. 12. Current response of $RL(t)$ system of an LTV filter to a unit step in the case of various alterations of parameter $L(t)$

3. Conclusions

The synthesis method described in this article enables one to obtain elements $R(t)$, $L(t)$, $C(t)$ in the form of three poles, or in the form of one-ports assuming forced voltage. The range and variability of parameters $R(t)$, $L(t)$, $C(t)$ can be very wide and might depend on the waveforms of control voltages. Such elements can be used to construct various, much more complex systems with parameters varying in time (LTV) and an appropriate selection of the waveform of the parametric function provides possibility to improve the dynamic properties of the system and to shorten the duration of a transient state.

There is a possibility to generalize the presented method for the system with single parametric elements and stationary or parametric two-ports in a feedback branch. However, the mentioned two-ports must be described using classic or parametric convolution. This issue has not been considered in this article and requires further research. The stability of such generalized elements requires further analysis as well.

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