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Prediction of scour depth around bridge piers in tandem arrangement using M5 and ANN regression models

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ABSTRACT

Purpose: Due to an increase in a number of bridges being constructed, scour depth around bridge piers is gradually being recognized as one of the possible reasons for bridge failure. According to [1] about 53% of bridge failures in the US were caused due to floods and corresponding scour in the rivers. Lots of work has been carried out around the single pier but in the case of group piers, the work is very less. Hence, it becomes necessary to calculate the actual scour depth around the bridge piers considering the close location of bridges as well.

Design/methodology/approach: Recognizing the need for research in this direction, an experimental study was planned and conducted in the Hydraulics Laboratory of Civil Engineering Department of National Institute of Technology Kurukshetra, India. Experiments were conducted in a standard recirculating tilting bed water flume 15 m long, 0.4 m wide, and 0.60 m deep. The orientation of more than one pier, namely Tandem pattern was employed for the work. Two pier models, 62 mm and 42 mm diameter were used for the experimental study. The mobile bed used in the experiments had an average mean size, d_{50} = 0.23 mm, 0.30 mm and 0.50 mm.

Findings: The outcomes of the ANN function and M5 model analysis have been used to compare with experimental results. From the earlier studies, it was concluded that, when the clear spacing between the pier models was greater than 0D the scour depth around the piers increase with a rapid rate. However, in the case of modelling techniques, M5 models show higher predictive accuracy than ANN models.

Research limitations/implications: It is a significant area of research. However, the present study has been a time and facility- constrained study. Therefore, there is a large scope to conduct further studies on the subject, Different pattern i.e. Side by Side; Staggered and Group of piers can be adopted for further investigations.

Originality/value: Sufficient work has been done by number of researchers around the single bridge pier. But due to rapid urbanization a number of bridges constructed in close proximity to each other which affects the scour depth of each other. Modelling techniques used in hydraulic engineering are not always effective in practice. The present study discusses the effect of spacing on scouring around piers in a tandem arrangement using experimental as well as modelling techniques. To predict the scour depth of the Tandem arrangement 89 laboratory data sets have been used.

Keywords: Tandem arrangement, Scour depth, Sediment, Pier, ANN, M5 model

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction and literature review

In hydraulic engineering bridges play an important role in the emergency care and convenience of public transportation. Pier scour is a challenging problem for hydraulic, civil and bridge engineers. Failure of bridges because of scour at pier foundations is a cause of scouring. Therefore, it becomes a topic of continued interest, especially for hydraulic engineers. Many researchers conducted experiments and put forward different relations for the estimation of the equilibrium scour depth at various-shaped piers, e.g. [2-12].

C.R. Hannah [13] had studied about local scour around a group of cylindrical piers of size 33 mm with a steady uniform flow with a velocity of flow 0.285 m/s under clear water conditions. The average mean size of the sediment used in all tests had, $d_{50} = 0.75$ mm and standard deviation, $\sigma = 1.32$. Tests showed that scour depths were 80% of equilibrium scours depth after seven hours. Further, after seven hours, only minor changes occurred in scour and deposition patterns. The author explained the results and the behavioural pattern with the help of four observed phenomena i.e. reinforcing, sheltering, shedding of vortices and compressed “horseshoe vortex”.

N. Vittal, et. al. [14] replaced a solid cylindrical pier of diameter *b* by a group of small piers of diameter 0.302 *b* each thus forming a staggered pattern of three piers. The piers were placed at angular spacing of 120˚ and the size of the smaller pier was such that anyone of them could pass through the gap between the other two using sediment of four different sizes ranging from 0.775 mm to 1.844 mm median diameter, the scour at the pier group and pier of diameter of the circumscribing circle was determined under identical flow conditions. It was found that the relative scour ratio was about 0.61.

M. Beg [15] studied the effect of mutual interference due to the close proximity of piers on local scour, a group of clear water experiments under steady uniform flow conditions had been performed considering different types of pier arrangements and equations have been developed for the estimation of scour depth at a group of piers. The author concludes that when pier spacing equal to zero, in tandem arrangement (when two piers touching with each other in the direction of flow), the scour depth at front pier was found to be equal to that at an isolated pier and at downstream pier it was eighty three percent of the scour depth at isolated pier.

A. Keshavarzi, et. al. [12] had studied the estimation of scour depths at the upstream and downstream pier for two piers in a line. They found that the scour depth at the upstream pier always greater than that at the downstream pier and the maximum scour depths for two in line piers occurred at a spacing of L equal to 2.5 times the diameter of pier and it was 22% greater than that of the single pier. For L/D > 2.5, the maximum scour depths at the upstream pier decreased and become equal to that at a single pier at L approximately equal to 10 times the diameter of bridge pier. At L/D = 1, i.e. the two piers in a line touching each other, the maximum scour depth at the upstream pier was equal to that for the single pier. For 1 < L/D > 2.5, the maximum scour depth at upstream pier was increased rapidly as the spacing between the two in line piers increased.

To fill up the gaps present in the past studies a detailed study has been conducted to achieve the following objectives:

a) To predicted the effect of spacing on the scour depth around pier models in tandem arrangement;
b) Compare the results with those of previous researchers;
c) Analyze the results with different modeling techniques.

2. Experimental setup

A group of experiments were carried out in the Hydraulics Laboratory of National Institute of Technology Kurukshetra India. An experimental study was conducted on a mobile bed with a recirculating flume 15 m long, 0.40 m wide and 0.60 m high was used. The working section provided at 5 m downstream from the inlet section in the flume. The working section assuming it to be a section where the flow was fully established. A tailgate has been provided at the end of the flume to control the depth flow and velocity of flow. Two different size circular piers 62 mm and 42 mm external diameter were used for the experimental study. Experiments were carried out with three different sizes of sediments 0.23 mm, 0.30 mm and 0.50 mm. Figure 1 shows a systematic line diagram of two circular piers in a line.

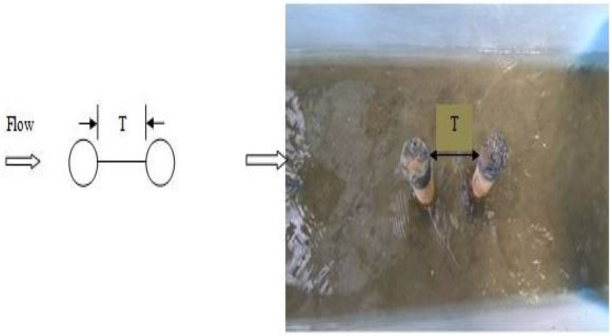

Fig. 1. Schematic line diagram of two piers at a clear spacing of 'T' in tandem arrangement

Table 1.
Detail and Specification of Experimental Setup

Content	Dimensions
Flume dimension L : B : H, m	15 : 0.4 : 0.6
Sediment size, mm	0.23, 0.30, 0.50
Size of the pier, cm	6.2, 4.2
Pier shape and material	Circular and Wooden
Velocity of flow, m/s	0.25 to 0.27
Depth of flow, cm	11.9 to 16.2

A set of 89 experiments using two piers in a line with different clear spacing was conducted for 6 hours duration. The duration of six hours was taken for the experimental study because, after 6 hours, the sediment movement around the piers was practically very slow [16]. All the experiments performed under clear water conditions. The measurement of scour depth on the upstream side of the piers was carried out by pointer gauge having a precision of ± 0.1 mm. Details of the experimental setup have been tabulated below in Table 1.

From eighty nine laboratory experimental results used in the tandem arrangement, 70%, i.e. 63 results were used for training the models and remaining 30% results randomly selected from the total data-set were used for testing purposes. Detailed reports about the data-set used for modelling have been tabulated in Table 2.

3. Result analysis and discussion

The experiments were conducted under uniform flow conditions in a tandem arrangement. When two piers arranged in a line the direction of flow was parallel to the pier arrangements, so that the angle of attack was zero. The two piers were placed in line with different non-dimensional clear spacing ranging from 0 to 18.

Figure 2 showed that the effect of clear spacing between the piers on the maximum scour depth at the upstream and downstream bridge piers when the mean size of sediment was, $d_{50} = 0.23$ mm. The data from the present study as well as the studied of C.R. Hannah [13] and M. Beg [15] have been included. Figure 2 showed that the maximum scour depth of the upstream pier and downstream pier occurs when the clear spacing between the piers was about four times the diameter of the pier (D = 6.2 cm) and it was 23% greater than that of the single pier. From the above results, it may be found that as the clear spacing between the piers increases the scour depth of the front pier also increased up to a spacing of 4D.

Table 2.
Characteristics of train and test data

Dimensional analysis						
	Training data-set			**Testing data-set**		
Parameter	Range	Mean	Std. deviation	Range	Mean	Std. deviation
Spacing	0-18	8.120967742	0.676109414	0-18	6.730769231	0.907757494
Depth of flow	11.9-16.2	13.95806452	1.439780082	12.1-16	13.78846154	1.464329723
Velocity	25-27.5	26.13548387	0.068761683	25-27	26.00769231	0.613790242
Mean sediment size	0.23-0.50	0.335322581	0.014501623	0.23-0.50	0.355384615	0.119907657
Non-dimensional parameters						
	Training data-set			**Testing data-set**		
Scour depth (H_{su}/D)	Range	Mean	Std. deviation	Range	Mean	Std. deviation
	0.4-1.58	0.97171	0.041881	0.4-1.61	1.033192308	0.316015951
Scour depth (H_{sd}/D)	0.4-1.9	0.913387	0.044279	0.4-1.24	0.885384615	0.275379458

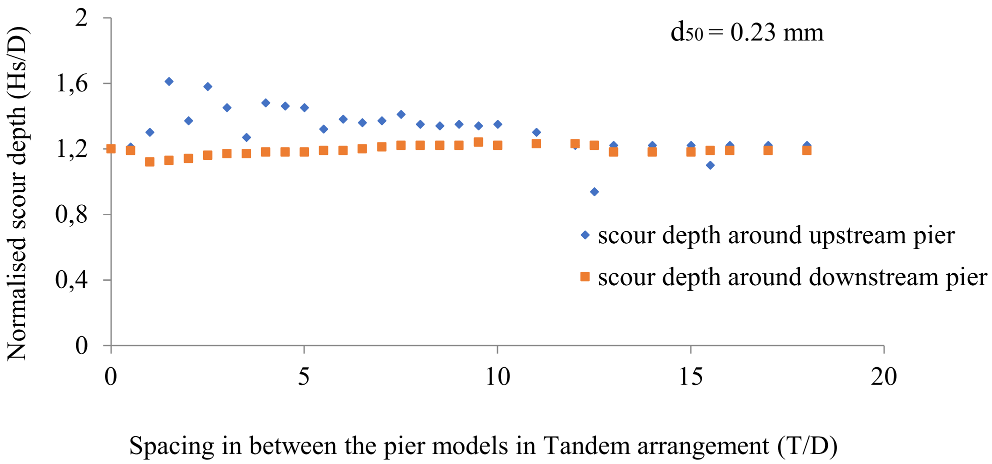

Fig. 2. Effect of clear spacing between the piers in tandem arrangement on scour depth

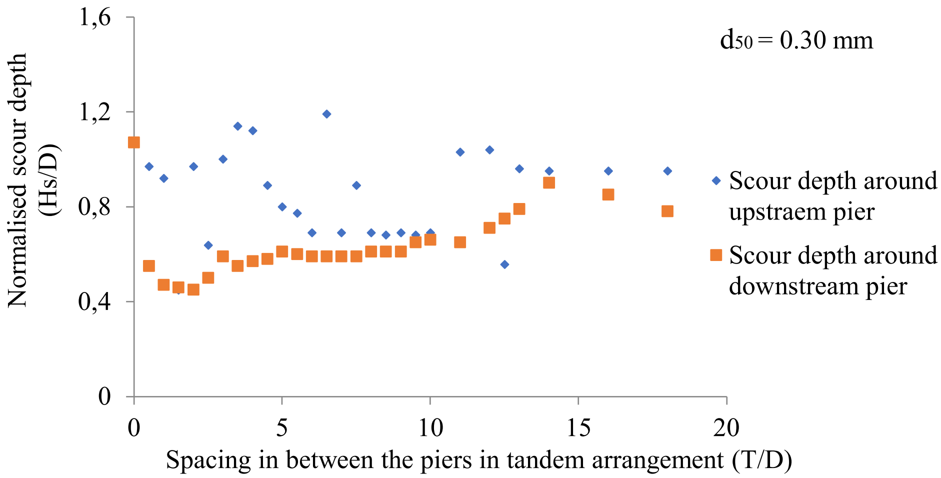

Fig. 3. Effect of clear spacing between the piers in tandem arrangement on scour depth

Figure 3 show that the effects of clear spacing between the piers on maximum scour depth at the upstream and downstream pier. Figure 3 shows that the maximum scour depth of the upstream pier and downstream pier occurred when the clear spacing between the piers were six times the diameter of pier and it was found that the maximum scour depth was found to be 9.1% greater than that for the isolated single pier. From the above observation, it may be found that as the clear spacing between the piers increased the scour depth of the upstream pier also increased up-to a spacing of six times the diameter of the isolated pier.

Figure 4 show that the effects of clear spacing on the maximum scour depth at the upstream and downstream bridge pier when the mean size of sediment was, d_{50}= 0.50 mm. Figure 4 showed that the maximum scour depth of the upstream pier and downstream pier occurs when the clear spacing between the piers was six times the diameter of the pier and it was found that the maximum scours depth was about 7.5% greater than that of the isolated pier. From the above observation, it may be concluded that as the clear spacing between the piers increased the scour depth of the upstream pier also increased up-to a spacing of six times the diameter of the isolated pier. The equation for the best fit line for the upstream and downstream piers was $y = -0.0001x^3 + 0.0036x^2 - 0.0738x + 1.1625$and $y = 0.001x^3 - 0.0275x^2 + 0.1515x + 0.9574$. The two equations gave a coefficient of determination values, R^2 of 0.6429 and 0.6476 respectively. Figure 5 shows a comparison between the results of the present study and the earlier studies of C.R. Hannah [13] and M. Beg [15].

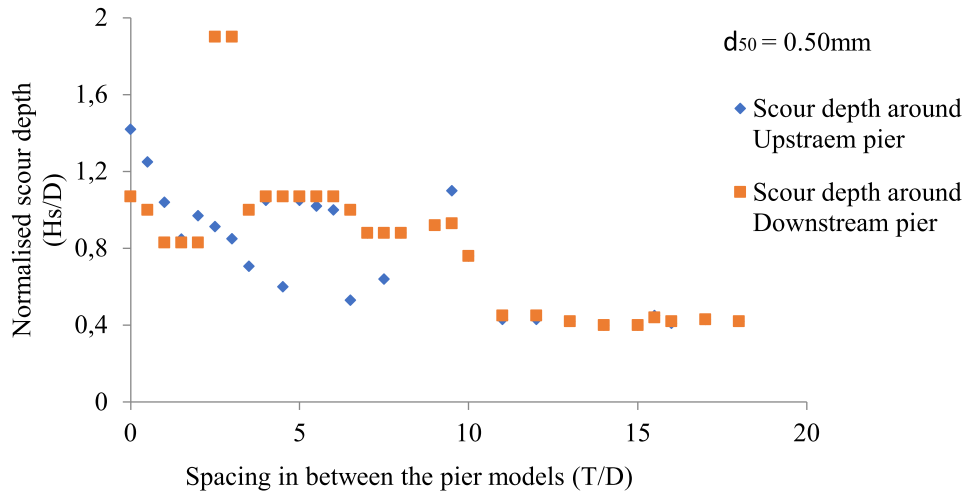

Fig. 4. Effect of clear spacing between the piers in tandem arrangement on scour depth

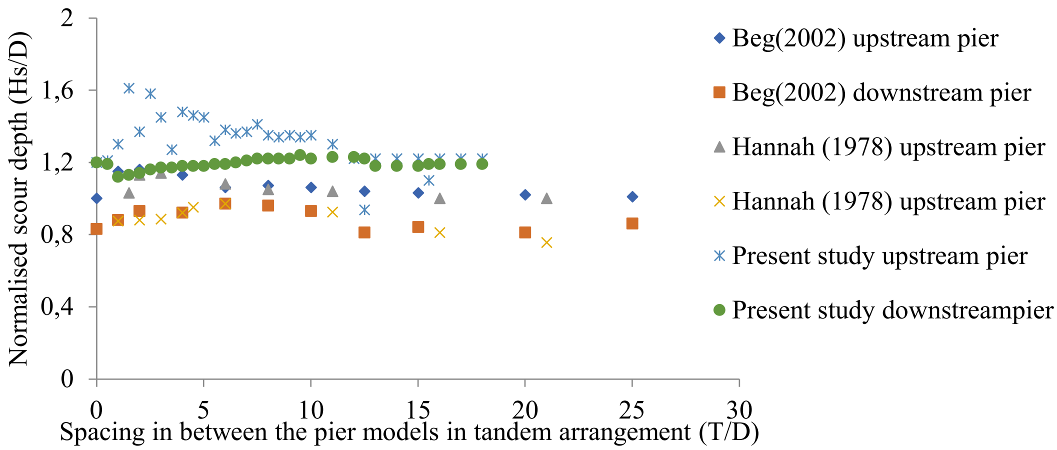

Fig. 5. Comparison between the [13,15] and present study (d_{50} = 0.23 mm) for the upstream and downstream pier

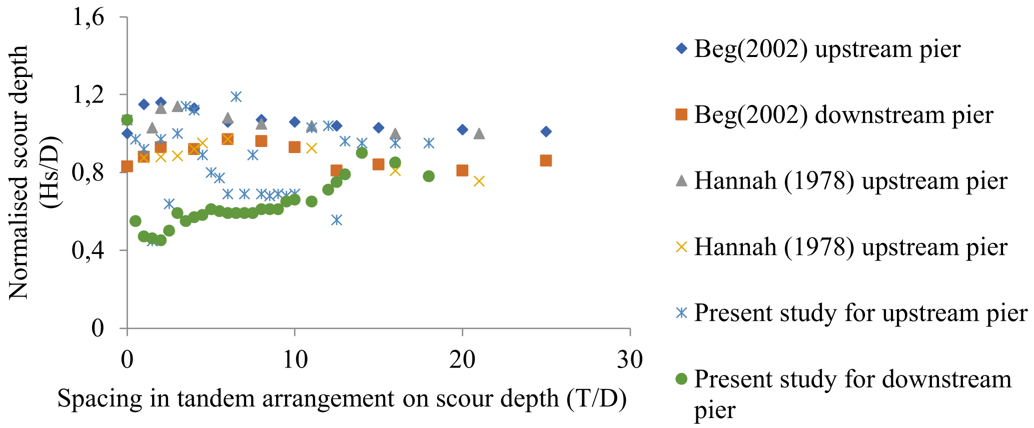

Fig. 6. Comparison between the [13,15] and present study (d_{50} = 0.30 mm) for the upstream and downstream pier

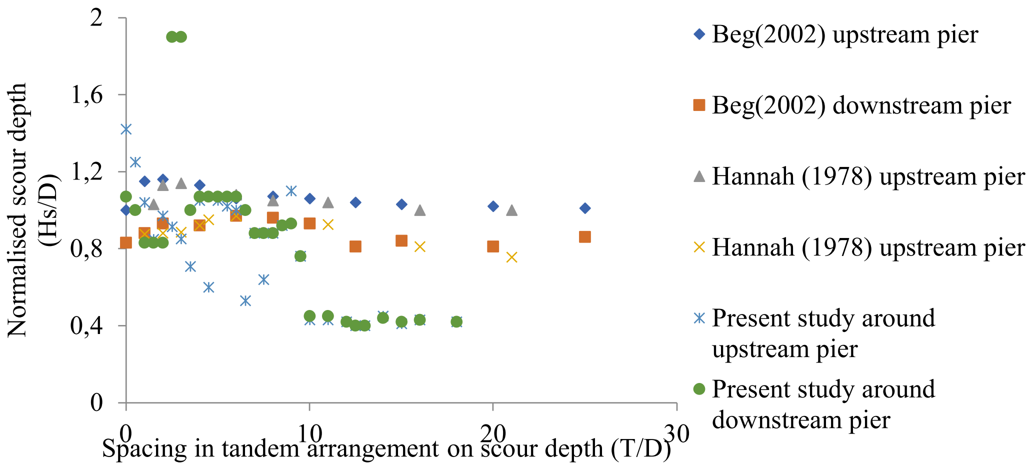

Fig. 7. Comparison between the [13,15] and present study (d_{50} = 0.50 mm) for the upstream and downstream pier

Figure 5 shows that the behaviour of the upstream and downstream piers was similar. But the difference in the magnitude could be possibly due to the different parameters like depth of flow and size of the sediment. While the average size of sediment and depth of flow used by M. Beg [15] was 1.0 mm and 14.0 cm, it was 0.23 mm and 12.5 cm in the present study. The front pier shows more scour depth as a comparison of the rear pier. Similarly, Figures 6 and 7 showed the comparison of past studies [13,15] with present studies when the size of sediment was 0.30 mm and 0.50 m.

4. Modelling techniques

4.1. M5 model technique

M5 version tree set of rules is a classifier of a choice tree that uses linear regression functions on the terminal nodes or leaves and classifies or divides the hassle into numerous sub-issues based totally at the inputs and employs a linear regression version to every categorized enter domain. The M5 choice tree method offers with non-stop magnificence issues in preference to discrete training and well-known shows piecewise facts of the character linear fashions constructed to approximately non-linear relationships of the information-set [17]. Building the M5 model tree involves separate steps. The first one consists of dividing statistics into subsets to construct a version tree. The dividing criterion is primarily based on the standard deviation of the subset values that reach a node as a measure of blunders price in that node and moreover calculating the expected reduction in the blunders because of trying out every characteristic at that node. A. Rahimikhoob [18] shows a comparison between conventional and M5 model tree methods for converting pan evaporation to reference evapotranspiration for semi-arid region. Some other researchers i.e. K.K. Singh et.al. [19] or M. Pal and S. Deshwal [20] also used M5 model in estimation of flood and evapotranspiration processes. Equation 1 of general deviation discount (SDR) is given as follows:

$$SDR = SD(T) - \sum \frac{Ti}{T} \times SD(T) \qquad (1)$$

where T is characterized as a set of examples that reaches the node; shows the subset of examples that have the i^{th} output of the potential set, and SD is the standard deviation.

4.2. Artificial Neural Network (ANN)

ANNs are called powerful gadget-gaining knowledge of techniques, which are substantially used for numerical prediction and class. ANN is also the operative gear to model nonlinear structures and wants a lesser quantity of information as inputs than normal mathematical strategies. A neural community is primarily based on processing fixed records that its analysis process has been stimulated through biological neural systems. Neural networks encompass a linked institution of neurons placed close to every different in layers that carry out data processing through neurons and their connections.

ANN protected one input layer, an output layer, and one or extra hidden layers; neurons of every layer are joined to all neurons of the earlier layer by means of weighted connections. The input vectors are obtained with the aid of neurons of the enter layer and the values can be conveyed to the next layer of processing factors across connections. The wide variety of neurons in each hidden layer is determined by the user. This manner continues up to the output layer. ANN modelling is done in two degrees consisting of training and checking out. In the schooling degree, after you have input information, the neural network makes attempts to convert the inputs into desired outputs. Connecting weights of network neurons are determined on this level; those weights inside the trying out stage have tested the use of numerous datasets. A. Rahimikhoob [21] estimated sunshine duration from other climatic data using artificial neural network for ET_0 estimation in an arid environment.

4.3. Implementation of the ANN and M5 modelling methods

Three widespread statistical measures, coefficient of dedication (R^2), Root Mean Square Error (RMSE) and Mean Absolute Error (MAE), have been used as criteria to decide the overall performance of modelling strategies. The optimization of consumer-described parameters was performed through sporting out several trials with those parameters at the training dataset and examining the validity of the shaped fashions at the testing dataset. Smaller values of RMSE and MAE infer closer estimation of measured values via the models. Larger R^2 values specify a stronger matching of tendencies in the measured information with the aid of the model outcomes. The first-rate model selected for output prediction becomes based totally at the least values of RMSE and MAE within the testing segment. The most advantageous values of identified person-described parameters acquired for the ANN and M5 version are given in Table 3.

Table3.
User defined function for ANN and M5model

Front Pier Scouring	
Modelling	User defined parameters
ANN	Learning rate=0.4, momentum=0.2, training time=500, neurons=7
M5	Instances=4
Rear Pier Scouring	
Modelling	User defined parameters
ANN	Learning rate=0.3, momentum=0.2, training time=250, neurons=7
M5	Instances=4

Performance assessment parameters

For the model performance assessment, three statistical parameters; root mean square error (RMSE) and Mean absolute error (MAE) were used.

The above mentioned assessments parameters have been calculated by using the equations (Eqs. 2 and 3) as follow:

$$\mathrm{RMSE} = \sqrt{\frac{1}{a}(\textstyle\sum_{i=1}^{a}(O - P)^2} \quad (2)$$

$$\mathrm{MAE} = \frac{1}{a}|O - P| \quad (3)$$

where O = Observed, P = Predicted, a = number of observations

4.4. Discussion of results using different modelling techniques

Figure 8 showed the actual and predicted values of scour depth for the upstream pier using the M5 tree and artificial neural network (ANN) for the given data-set. The predicted values of scour depth by M5 and ANN lie closer to the line of perfect agreement (Fig. 8). However, based on root mean square error and mean absolute error values (Tab. 4), the prediction accuracy of both the ANN (RMSE = 0.224735, MAE = 0.17519231) and M5 (RMSE = 0.224852, MAE = 0.1872308) is found almost comparable for the data set of upstream pier as the observed RMSE values are approximately equal. So, both of the models can be effective in approximating the actual scour depth for the upstream pier. The equation for the best fit line for the upstream pier using M5 and ANN regression technique was $T/D = 1.1454(T/D)^3 - 3.0178(T/D)^2 + 2.987(T/D) - 0.1818$ and $T/D = 0.7108(T/D)^3 - 1.6498(T/D)^2 + 1.8668T/D - 0.036$. The two equations gave a coefficient of determination value, R^2 of 0.5694 and 0.6475 respectively.

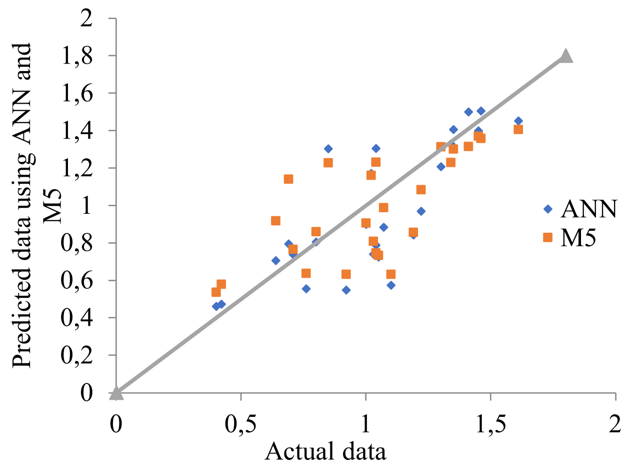

Fig. 8. Comparison between M5 model and ANN for testing data for upstream pier

Table 4.
RMSE and MAE statics for the training data using M5 and ANN model for front and rear pier

Training and testing analysis for the front pier			
Parameters	Method	Training	Testing
Correlation coefficient	ANN	0.923	0.786
Mean absolute error	ANN	0.108	0.175
RMSE	ANN	0.132	0.224
Correlation coefficient	M5	0.877	0.723
Mean absolute error	M5	0.126	0.187
RMSE	M5	0.156	0.224
Training and testing analysis for rear pier			
Parameters	Method	Training	Testing
Correlation coefficient	ANN	0.912	0.837
Mean absolute error	ANN	0.099	0.152
RMSE	ANN	0.143	0.188
Correlation coefficient	M5	0.850	0.851
Mean absolute error	M5	0.105	0.099
RMSE	M5	0.185	0.145

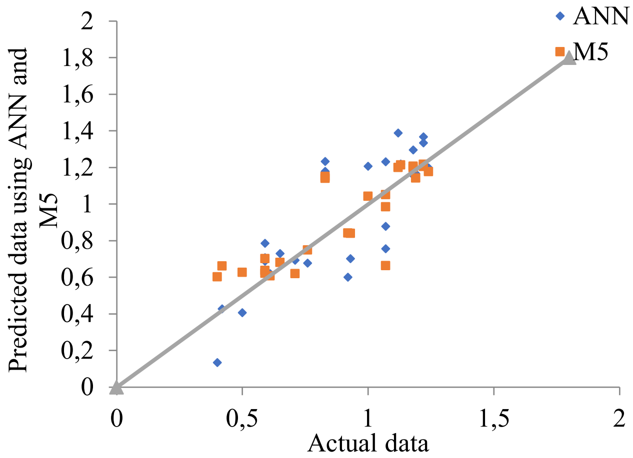

Fig. 9. Comparison between M5 model and ANN for testing data for downstream pier

Figure 9 showed the relationship of testing data between the actual values and predicted values of scour depth for the downstream pier using the M5 tree and artificial neural network (ANN) for the data-sets. The predicted values of scour depth for the downstream pier by M5 and ANN lie closer to the line of perfect agreement. However, based on root mean square error and mean absolute error (Tab. 4), the prediction accuracy of M5 (RMSE = 0.145299, MAE = 0.0994615) is found greater than ANN (RMSE = 0.188844, MAE = 0.15242308) for the downstream pier. The equation for the best fit line for the downstream pier using M5 and ANN regression technique was $T/D = -0.5098(T/D)^3 + 1.6202(T/D)^2 - 0.8121(T/D) + 0.695$ and $T/D = 4.2584(T/D)^3 - 11.026(T/D)^2 + 10.067T/D - 2.3076$. The two equations gave a coefficient of determination value, R^2 of 0.7357 and 0.733 respectively.

Figures 10 and 11 shows the difference between experimentally calculated and estimated scour depth values against observations of testing data. The residuals were found to be lower with M5 model as comparison to the ANN Model. The minimum and maximum calculated errors with M5 model were observed as 0.06 (negative) and 0.469 (positive) for the upstream pier and 0.003 (positive) and 0.407 (positive) for downstream piers, respectively.

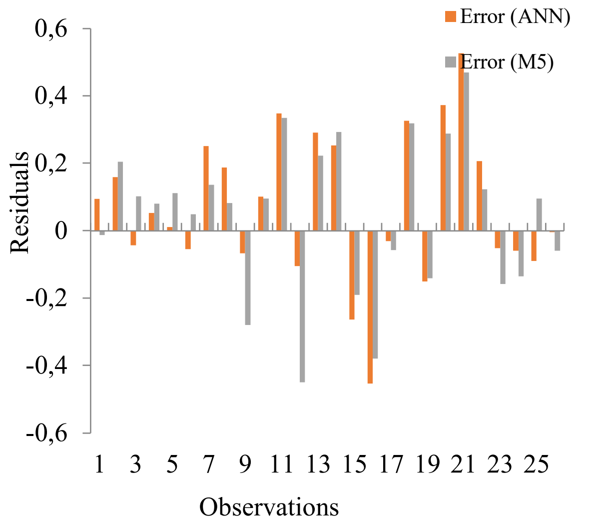

Fig. 10 Comparison errors between ANN and M5 model for testing data for front pier

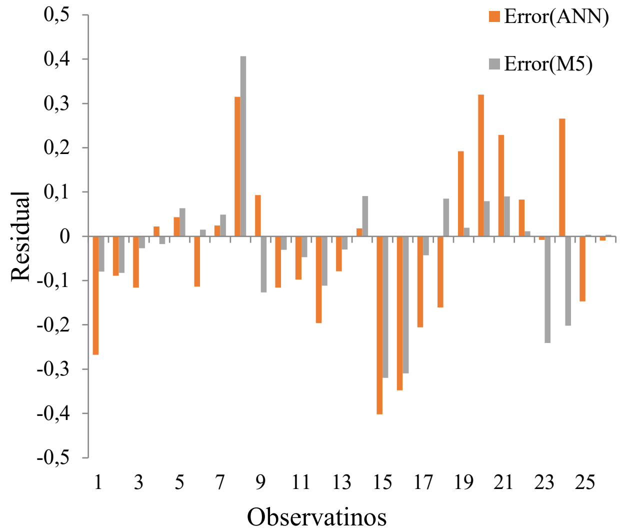

Fig. 11. Comparison errors between ANN and M5 model for testing data for rear stream pier

The minimum and maximum calculated errors with the ANN model was observed as 0.005 (positive) and 0.526 (positive) for the upstream pier and 0.008 (positive) and 0.402 (positive) for downstream pier. Most of the residuals were reside on the positive error zone which implies the dependency of estimation models in underestimation of the actual data. But in the case of downstream pier mostly values was lies on the negative error zone.

5. Conclusions

The following results are drawn about a tandem arrangement when two piers present in a line:

- The scour depth around the upstream pier always greater than the downstream pier;
- When the clear spacing between the piers was T/D = 0 the piers behaves as a single pier. But when the clear spacing between the piers increase, the upstream shows a rapid increase in scour depth;
- The scour depth around the piers depends upon the size of the sediment if the mean size of the sediment large the scour depth found to be the minimum;
- In estimating the scour depth around the piers, M5 models show higher predictive accuracy than ANN models. In addition to this, the comparative study of input data-set concludes better performance by the output data-set. Modelling and experimental results suggest that the influence of sediment size, velocity of flow and diameter is considerably higher than other parameters in affecting the scour depth around the bridge piers. The modelling in the present experimental study is useful in arriving at a combination of flow conditions corresponding to scour depth within the experimental range of this study.

6. Symbols

d – Depth of flow,
V – Flow velocity,
d_{50} – Mean size of the sediment,
L – Length of the flume,
B – Breadth of the flume,
H – Height of the flume,
D – Diameter of pier,
H_{sf} – Depth of scour around upstream pier,
H_{sr} – Depth of Scour around downstream pier,
T – Spacing in between the piers in tandem arrangement,
H_{sd}/D – Normalised scour depth around rear pier,
H_{su}/D – Normalised scour depth around front pier,
σ – Standard deviation.

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References

[1] K. Wardhana, F.C. Hadipriono, Analysis of Recent Bridge Failures in the United States, Journal of Performance of Constructed Facilities 17/3 (2003). DOI: https://doi.org/10.1061/(ASCE)0887-3828(2003)17:3(144)
[2] E.M. Laursen, A. Toch, Scour around bridge piers and abutments, Iowa Highway Research Board Bulletin No. 4 (1956) 1-60.
[3] B.W. Melville, Local scour at bridge sites, Ph.D. Thesis, School of Engineering, University of Auckland, Auckland, 1975.
[4] H.N.C. Breusers, A.J. Raudkivi, Scouring: Hydraulic Structure Design Manual Series, vol. 2, CRC Press, Balkema Rotterdam-Brookefield, 1991.
[5] U.C. Kothyari, R.C.J. Garde, K.G. Ranga Raju, Temporal variation of scour around circular bridge piers, Journal of Hydraulic Engineering 118/8 (1992) 1091-1106. DOI: https://doi.org/10.1061/(ASCE)0733-9429(1992)118:8(1091)
[6] B.W. Melville, Pier and abutment scour: integrated approach, Journal of Hydraulic Engineering 123/2 (1997) 125-136. DOI: https://doi.org/10.1061/(ASCE)0733-9429(1997)123:2(125)
[7] B. Setia, Scour around bridge piers: mechanism and protection, PhD. Thesis, Department of Civil Engineering, Indian Institute of Technology, Kanpur, India, 1997.
[8] A.H. Cardoso, R. Bettess, Effects of Time and Channel Geometry on Scour at Bridge Abutments, Journal of Hydraulic Engineering 125/4 (1999) 388-399. DOI: https://doi.org/10.1061/(ASCE)0733-9429(1999)125:4(388)
[9] B.W. Melville, S.E. Coleman, Bridge scour, Water Resources Publications, Colorado, USA, 2000.
[10] S. Dey, S.K. Bose, G.L.N. Sastry, Clear water scour at circular piers: a model, Journal of Hydraulic Engineering 121/12 (1995) 869-876. DOI: https://doi.org/10.1061/(ASCE)0733-9429(1995)121:12(869)

[11] R. Malik, B. Setia, Experimental study on behaviour of closely placed bridge pier models, Proceedings of the National Conference in Department of Civil Engineering, National Institute of Technology Kurukshetra, Kurukshetra, India, 2014.

[12] A. Keshavarzi, C.K. Shrestha, B. Melville, H. Khabbaz, M. Ranjbar-Zahedani, J. Ball, Estimation of maximum scour depths at upstream of front and rear piers for two in-line circular columns, Environmental Fluid Mechanics 18 (2018) 537-550. DOI: https://doi.org/10.1007/s10652-017-9572-6

[13] C.R. Hannah, Scour at pile groups, Research Report No. 78-3, Civil Engineering Department, University of Canterbury, New Zeland, 1978.

[14] N. Vittal, U.C. Kothyari, M. Haghighat, Clear water scour around bridge pier group, Journal of Hydraulic Engineering 120/11 (1994) 1309-1318. DOI: https://doi.org/10.1061/(ASCE)0733-9429(1994)120:11(1309)

[15] M. Beg, Mutual interference around bridge piers on local scour, Proceedings of the 2nd International Conference on Scour and Erosion „ICSE2", Singapore, 2004, 111–118.

[16] B. Setia, Equilibrium scour depth time, Proceedings of 3rd IASME/WSEAS International Conference on Water Resources, Hydraulics & Hydrology "WHH'08", University of Cambridge, 2008, 114-117.

[17] J.R. Quinlan, Learning with continuous classes, Proceedings of Australian Joint Conference on Artificial Intelligence, Hobart, 1992, 343–348.

[18] A. Rahimikhoob, M. Asadi, M. Mashal, A comparison between conventional and M5 model tree methods for converting pan evaporation to reference evapotranspiration for semi-arid region, Water Resources Management 27/14 (2013) 4815-4826. DOI: https://doi.org/10.1007/s11269-013-0440-y

[19] K.K. Singh, M. Pal, V.P. Singh, Estimation of mean annual flood in Indian catchments using back propagation neural network and M5 model tree, Water Resources Management 24/10 (2010) 2007-2019. DOI: https://doi.org/10.1007/s11269-009-9535-x

[20] M. Pal, S. Deswal, M5 model tree based modelling of reference evapotranspiration, Hydrological Processes 23/10 (2009) 1437-1443. DOI: https://doi.org/10.1002/hyp.7266

[21] A. Rahimikhoob, Estimating sunshine duration from other climatic data by artificial neural network for ET_0 estimation in an arid environment, Theoretical and Applied Climatology 118/1-2 (2014) 1-8. DOI: https://doi.org/10.1007/s00704-013-1047-1