

## DYNAMICALLY POSITIONED SHIP STEERING MAKING USE OF BACKSTEPPING METHOD AND ARTIFICIAL NEURAL NETWORKS

**Anna Witkowska**

Gdańsk University of Technology

**Tacjana Niksa Rynkiewicz**

Gdańsk University of Technology

### ABSTRACT

*The article discusses the issue of designing a dynamic ship positioning system making use of the adaptive vectorial backstepping method and RBF type artificial neural networks. In the article, the backstepping controller is used to determine control laws and neural network weight adaptation laws. The artificial neural network is applied at each time instant to approximate nonlinear functions containing parametric uncertainties. The proposed control system does not require precise knowledge of the model of ship dynamics and external disturbances, it also eliminates the problem of analytical determination of the regression matrix when designing the control law with the aid of the adaptive backstepping procedure.*

**Keywords:** backstepping, neural networks, RBF, dynamic ship positioning

### INTRODUCTION

Issues related with automatic control of ship motion at sea still remain the area of active research, due to new and more complicated operational tasks to be undertaken by ships. Ship control is usually executed using a multi-layer control structure. The highest layer in this structure is the supervisory control system, which is responsible for planning the desired ship trajectory [10] from the initial point to the set destination point, based on the data achieved from navigation devices [13]. Here, algorithms are proposed in the literature which make use of the game theory [12], or artificial intelligence methods, including evolutionary algorithms [19,20] or fuzzy neural systems [15].

The next control layers are composed of algorithms responsible for ship's motion along the set trajectory, manoeuvring, and keeping the set heading and speed parameters, or position and heading parameters. These tasks are executed depending on the speed of the moving ship. Currently, a large number of watercrafts are equipped with dynamic positioning (DP) systems [17]. The list of those watercrafts includes drilling platforms, floating cranes, cable ships, store ships, fire boats, research and passenger ships, underwater work assistance ships used, for instance, for laying underwater pipelines, warships, reloading terminals, etc. The basic task of the DP systems is maintaining the set ship position and heading, or assistance in ship manoeuvring at low speed (up to 2 m/s), in the presence of environmental

disturbances acting on the ship hull. This task is executed by controlling ship's movements in three degrees of freedom (DOF's): surge, sway, and yaw, with the aid of propellers and rudders. An overview of essential research and technological development in designing DP controllers is presented in [17].

The majority of the presently used DP control methods is based on equations describing the mathematical model of dynamics and kinematics of the object. When maintaining the set position and heading of the DP ship, nonlinear damping forces, Coriolis forces, and centripetal forces can be neglected due to their small effect on ship dynamics, and the nonlinearity can be only taken into account in the kinematic model. On the other hand, tracking the set trajectory at low ship speed consists in simultaneous control of ship heading and its longitudinal and transverse position. Changes of the operating point of the system and hydrodynamic phenomena are the sources of nonlinearities and varying coefficients in equations of ship dynamics and kinematics. Controlling nonlinear systems with uncertainties is the area which still needs further research. At present, two basic approaches, referred to as robust and adaptive control, are applied to deal with system uncertainties. Robust control methods, such as sliding mode control [21] or  $H^\infty$  control [8], consist in designing a controller with fixed structure which ensures proper performance in the entire range of process changes. Adaptive control methods, in turn, such as backstepping and its modifications: Dynamic Surface Control (DSC) [18], and Active Direct Surface Control (ADSC) [14] provide opportunities for designing a dynamically changing feedback loop.

The basic idea of designing an adaptive control law consists firstly in assessing the value of the unknown parameter and determining its estimate. Then, the static part of the controller, which contains the estimated parameters, is continuously updated to reflect changing conditions of system operation.

Currently, backstepping is one of basic methods used to design nonlinear control systems with uncertainties. It belongs to the group of recursive methods based on the theory of Lyapunov functions [9]. The structure of both the control law with feedback, and the accompanying Lyapunov function for systems with unknown parameters is systematised.

A complex system is firstly divided into lower-dimension subsystems. Then, the Lyapunov function and the intermediate virtual control inputs are determined for each subsystem. Designing is done recursively until the entire system reaches the real control input. This way the goal of control can be achieved at reduced control effort [22]

The basic limitation in the use of the adaptive backstepping method is the need for analytical calculation of the time derivative of the "virtual control signal" [9] at each procedure step, which leads to a complicated algorithm requiring much computational effort. What is more, the level of complicity of controller's structure increases with the increase of the system order. To eliminate the need for calculating complex derivatives, first- and second-order filters are frequently used [24]. The other limitation of the standard adaptive backstepping method is the assumption that the

functions with uncertainties are to be linear with respect to unknown parameters and, consequently, are able to be presented in the form of the regression model. Determining the regression matrix requires laborious analysis. Moreover, the complexity of the regression matrix and the number of unknown parameters increase in consecutive backstepping procedure steps, thus generating the so-called effect of overparametrization [9].

Some attempts to solve this problem which can be found in the literature make use of fuzzy systems or artificial neural networks [1, 6]. The latter approach can be applied at each step of the backstepping procedure. Neural networks are used for assessing nonlinear functions, the form of which depends on the values of the estimated parameters. This method makes it possible to design the control law in which the analytical form of the regression matrix and the assumption of linearity with respect to parameters are not required.

This paper presents a multidimensional nonlinear DP controller designed using the adaptive vectorial backstepping method and Radial Basic Function (RBF) type artificial neural networks in the feedback loop. When designing the control law, the presence of parametric uncertainties was assumed in the matrices of damping, Coriolis forces, and environmental disturbances. The artificial RBF type neural network was used at each time instant to approximate nonlinear functions with parametric uncertainties. The network weight adaptation laws were determined based on the Lyapunov's theory of stability, depending on the operating point of the system. This way, the neural network does not require preliminary offline weight tuning.

The applied radial network is the structure consisting of three layers: input, hidden, and output. The architecture of the network is relatively simple. The input signals are given to the input layer, while the radial neurons are accumulated in the hidden layer. The neurons play a special role, as they map radially the space surrounding the set points. The output layer usually comprises only one neuron, the role of which is to combine the weighted signals coming from the hidden layer. This approach makes it possible to map the entire space of points.

Unlike the already existing DP systems, the proposed control system does not require detailed knowledge of the model of object's dynamics and external disturbances, thus eliminating the problem of analytical calculation of derivatives and the regression matrix.

The use of the theory of Lyapunov functions and RBF networks makes that the designed feedback loops ensure the convergence of ship position and heading to the set values, and the boundedness of signals in the closed control system loop [11].

## STRUCTURE OF CONTROL SYSTEM

A structurally simplified general working scheme of the DP system is shown in Fig. 1. The set values of ship position and heading, composing the ship position vector, are introduced

to the system via user's interface, while the estimated vectors of current ship position and speed are calculated based on the navigation reference system and the state observer. The position error vector, being the difference between the set vector and the estimated vector, is passed to the DP controller, which then calculates forces and torque required to minimise deviations from the set values.

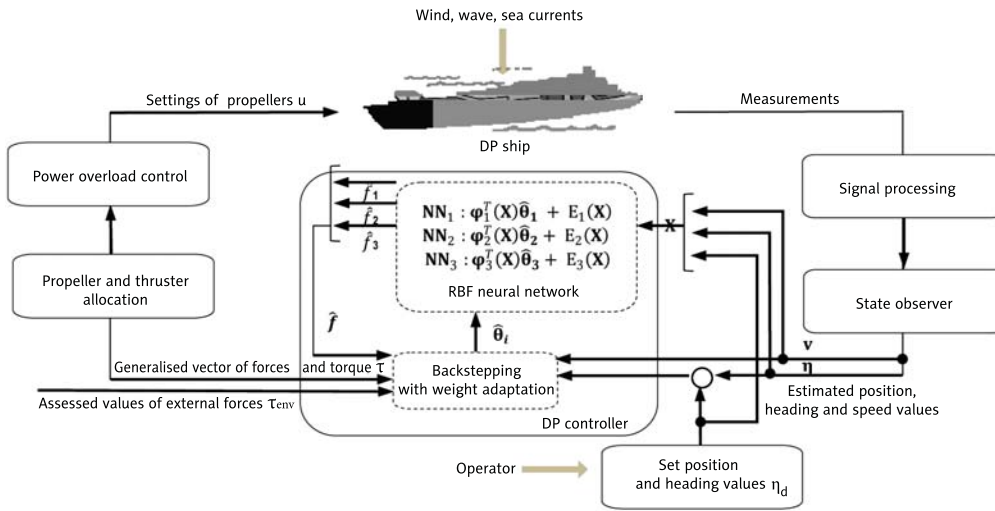


Fig. 1. Structure of DP control system with backstepping controller and RBF type neural network

The resultant vector of forces and torque is passed to the allocation control system, where the set values of forces and torque are converted to control signals for actuator settings, at the same time minimising the energy needed to execute the control task. The actuator settings refer to rotational speeds of main propellers and azimuth and tunnel thrusters, and rudder angles.

Bearing in mind the required precision and safety of control executed by a DP system, the number of actuators is, as a rule, larger than the number of the controlled degrees of freedom of ship motion. This over-actuation is the reason why the conversion of forces and torque to control signals and their allocation into individual actuators is not always unambiguous. In those cases, it can be approximated in the square optimisation process with limitations placed on maximal amplitude and rate of changes of actuator settings, at simultaneous minimisation of economic losses resulting from excessive activity of propellers and rudders.

## PROBLEM FORMULATION

This research aimed at designing a controller which would perform basic tasks of dynamic ship positioning: manoeuvres of ship position and heading change or ship stabilisation at a point, in the case of inaccurate data on the mathematical model of the object used for designing the control law. This means that the designed control system meets the condition of control error convergence to zero after change

of the operating point of the system and/or environmental conditions.

The DP controller was synthesised using the adaptive vectorial backstepping method and artificial RBF type neural networks. The artificial neural networks were applied to approximate nonlinear functions with uncertainties, while the backstepping method was used to determine the control laws and the RBD network weight adaptation mechanism.

## MATHEMATICAL MODEL OF DP SHIP

When designing the control law based on the backstepping method and neural networks, the mathematical model of a ship dynamically positioned in the horizontal plane was adopted. This model is given by the following system of differential equations (1)-(2).

$$\dot{\eta} = J(\eta)v \quad (1)$$

$$M(v)\dot{v} = \tau + J(\eta)^T b - D(v)v - C(v)v \quad (2)$$

where  $\tau = [\tau_x, \tau_y, \tau_z]^T$  is the generalised vector of forces and torque acting on the ship,  $\eta = [x, y, \psi]^T$  is the vector of ship position and heading, and  $v = [u, v, r]^T$  is the vector of longitudinal, lateral and angular ship velocity components. The matrices  $M \in \mathbb{R}^{3 \times 3}$ ,  $D \in \mathbb{R}^{3 \times 3}$ ,  $C \in \mathbb{R}^{3 \times 3}$ , and  $J(\eta) \in \mathbb{R}^{3 \times 3}$  represent, respectively, the matrices of inertia, damping, and Coriolis forces, and the state dependent matrix which converts coordinates from the system fixed to the ship's centre of gravity to the Earth fixed system. The vector  $b = [b_1, b_2, b_3]^T$  represents unmodelled slowly varying environmental disturbances.

The ship model takes into account three degrees of freedom of ship motion: longitudinal motion (surge), transverse motion (sway), and change of ship heading angle.

The model equations have the following properties [7]:

$$M^T = M \Rightarrow x^T M x > 0, x \neq 0, \quad (3)$$

$$J^{-1}(\eta) = J^T(\eta), \|J(\eta)\| = 1, \quad (4)$$

$$\frac{d}{dt} \mathbf{J}(\boldsymbol{\eta}) = -r \mathbf{S} \mathbf{J}(\boldsymbol{\eta})^T, \text{ where } \mathbf{S} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{s} = -\mathbf{S}^T \quad (5)$$

$$\mathbf{z}_1^T(t) \mathbf{S} \mathbf{z}_1(t) = \mathbf{0}, \mathbf{S} = -\mathbf{S}^T. \quad (6)$$

The system (1)-(2) has a cascaded structure. The control input given to the system is the vector  $\boldsymbol{\tau}$ , while the “virtual control output” is the vector  $\mathbf{v}$ , being simultaneously the “virtual control input” for the first subsystem (1). The control output from the entire system is the ship position and heading vector  $\boldsymbol{\eta}$ .

When designing the control law, it was assumed that the model of ship dynamics contains parametric uncertainties in matrices  $\mathbf{D}$ ,  $\mathbf{C}$ , and  $\mathbf{M}$ , and that the components of the vector  $\mathbf{b}$  are unknown but slowly varying. Moreover, it was assumed that all state variables are bounded and measurable (or estimable). The set position and heading trajectories  $\boldsymbol{\eta}_d = [x_d, y_d, \psi_d]^T$  and their first- and second-order derivatives are smooth and bounded in time.

## STRUCTURE OF RBF TYPE NETWORK

A special variation in the family of artificial neural networks (NN) is the group of networks with radial basic functions (RBF). The hidden layer in this network consists of neurons bearing the name of basic or radial neurons [2]. A radial neuron represents a hypersphere in which circular division around the central point  $x_i$ , where  $i = 1 \dots l$  [4,16], takes place. The vector of radial functions  $\boldsymbol{\varphi}(\mathbf{x}) = [\varphi_1(\mathbf{x}), \varphi_2(\mathbf{x}), \dots, \varphi_l(\mathbf{x})]$  is determined in the space of input signals. It is assumed that there exists the vector  $\boldsymbol{\theta}^T \boldsymbol{\varphi}(\mathbf{x})$ , which represents the border between two classes, and its value indicates belongingness to a given class, as  $\boldsymbol{\theta}^T \boldsymbol{\varphi}(\mathbf{x}) < 0$  or  $\boldsymbol{\theta}^T \boldsymbol{\varphi}(\mathbf{x}) > 0$ . That means that the space division is nonlinearly  $\varphi$ -separable.

In [4], the authors have proved that each set of patterns randomly distributed in the multidimensional space is  $\varphi$ -separable with probability equal to 1 if only a sufficiently large dimension  $l$  of the projection space is assumed.

It is stressed in the literature that assuming a sufficiently large number  $l$  of radial neurons in the hidden layer ensures correct solution when using three network layers: the input layer, the hidden layer in which the vector  $\boldsymbol{\varphi}(\mathbf{x})$  is executed, and the output layer consisting of one linear neuron described by the weight vector  $\boldsymbol{\theta}$ . The operation of the network can be described by formula (7)

$$\bar{F}(x) = \sum_{i=1}^K \boldsymbol{\theta}_i \mathbf{G}(\|\mathbf{x} - \mathbf{x}_i\|) \quad (7)$$

Selecting the type of norm can be arbitrary. In the proposed approach, the Euclidean norm was used together with Green's functions of Gauss type [2,3,16] (8).

$$\mathbf{G}(x; \mathbf{x}_i) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}_i\|^2}{2\sigma_i^2}\right) = \exp\left(-\frac{1}{2\sigma_i^2} \sum_{k=1}^N (x_k - x_{ki})\right) \quad (8)$$

Here,  $\mathbf{x}_i$  is the vector of mean values (centres) and  $\sigma_i^2$  is the variance. When creating the radial neural network to the presented problem, the number  $K$  of basic functions had to be assumed. The initial values of the centres  $\mathbf{x}_i$  of the radial functions were selected using the Fuzzy C-Means (FCM) algorithm, which is used and described in the literature [5].

A simplified scheme of RBF network structure is shown in Fig 2.

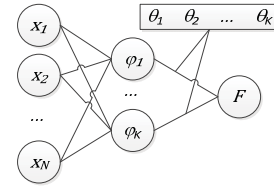


Fig. 2. Simplified scheme of RBF network structure

The approximation task consists in selecting appropriate Green's functions  $\mathbf{G}(\mathbf{x}, \mathbf{x}_i)$  and weights  $\boldsymbol{\theta}_i$ . The nonlinear radial function for each hidden neuron has different parameters  $\mathbf{x}_i$  and  $\sigma_i$ . The argument of the radial function is the distance of the given sample  $\mathbf{x}$  from the centre  $\mathbf{x}_i$ .

## DP CONTROLLER

In accordance with the backstepping methodology, for the system (1)-(2), new state variables were defined in the form of control errors  $\mathbf{z}_1(t) \in \mathbb{R}^{3 \times 1}$  and  $\mathbf{z}_2(t) \in \mathbb{R}^{3 \times 1}$ , and the vector  $\boldsymbol{\alpha} \in \mathbb{R}^{3 \times 1}$  of functions stabilising the first subsystem. In the coordinate system fixed to the moving ship, the control errors take the following form:

$$\mathbf{z}_1 = \mathbf{J}(\boldsymbol{\eta})^T (\boldsymbol{\eta} - \boldsymbol{\eta}_d) \quad (9)$$

$$\mathbf{z}_2 = \mathbf{v} - \boldsymbol{\alpha} \quad (10)$$

The vector of stabilising functions will be determined when designing the control law.

Based on the kinematics and dynamics equations (1)-(2) and taking into account the model property (5), the control error derivatives were determined as:

$$\dot{\mathbf{z}}_1 = -r \mathbf{S} \mathbf{z}_1 + \mathbf{z}_2 + \boldsymbol{\alpha} - \mathbf{J}(\boldsymbol{\eta})^T \dot{\boldsymbol{\eta}}_d \quad (11)$$

$$\mathbf{M} \dot{\mathbf{z}}_2 = \boldsymbol{\tau} + \mathbf{f}(\boldsymbol{\eta}, \mathbf{v}, \boldsymbol{\eta}_d, \mathbf{v}_d) \quad (12)$$

Denoting the matrix dependent on state variables by  $\mathbf{X} = [\boldsymbol{\eta}, \mathbf{v}, \boldsymbol{\eta}_d, \mathbf{v}_d]$ , the function  $f \in \mathbb{R}^{3 \times 1}$  takes the form:

$$f(\mathbf{X}) = -\mathbf{C}\mathbf{v} - \mathbf{D}\mathbf{v} + \mathbf{J}(\boldsymbol{\eta})^T \mathbf{b} - \mathbf{M}\dot{\boldsymbol{\alpha}} \quad (13)$$

This function contains unknown model parameters.

To determine the adaptation laws for parameters of matrices  $\mathbf{M}$ ,  $\mathbf{D}$ ,  $\mathbf{C}$  and vector  $\mathbf{b}$ , the function  $f$  is to be presented in the form of the regression model, after which the standard backstepping procedure can be applied. This approach is labour-intensive, requires huge computational effort, and leads to an excessively large number of estimated parameters, as stated in [23]. Instead, the components of the function  $f = [f_1, f_2, f_3]^T$  can be approximated using three artificial RBF type neural networks  $NN_i$  with the number of neurons  $l > 1$ . The outputs  $\hat{f} = [\hat{f}_1, \hat{f}_2, \hat{f}_3]^T$  from these networks have the form of the following regression model:

$$\hat{f}_i(\mathbf{X}_i) = \boldsymbol{\varphi}_i^T(\mathbf{X}_i) \boldsymbol{\theta}_i; \quad i = 1..3 \quad (14)$$

In Equation (14),  $\mathbf{X}_i$  is the input vector to network  $NN_i$ ,  $\mathbf{X}_i = [\boldsymbol{\eta}(i), \mathbf{v}(i), \boldsymbol{\eta}_d(i), \mathbf{v}_d(i)]^T \in \mathbb{R}^{4 \times 1}$ , while  $\boldsymbol{\theta}_i$  is the determined vector of weights between the second and third layer in network  $NN_i$ ,  $\boldsymbol{\theta}_i = [\theta_{i1}, \theta_{i2}, \dots, \theta_{il}]^T \in \mathbb{R}^{l \times 1}$ . The vector  $\boldsymbol{\varphi}_i$  represents the set of basic function values in network  $NN_i$ ,  $\boldsymbol{\varphi}_i = [\varphi_{i1}, \varphi_{i2}, \dots, \varphi_{il}]^T \in \mathbb{R}^{l \times 1}$ .

Each RBF network has a predefined number  $l$  of radial neurons. The selection of their weights is made in the process of control system adaptation to changing operating conditions. In the proposed system, the number of radial neurons was determined experimentally as equal to  $l = 4$ .

After complementing with the RBF network equations, Equation (6) of control error dynamics takes the form:

$$\mathbf{M}\dot{\mathbf{z}}_2 = \boldsymbol{\tau} + \boldsymbol{\varphi}^T(\mathbf{X})\boldsymbol{\theta} \quad (15)$$

The regression vector, i.e. the vector of RBF network weights, is defined as  $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T, \boldsymbol{\theta}_3^T]^T \in \mathbb{R}^{3 \times l}$ , while the regression matrix  $\boldsymbol{\varphi}^T \in \mathbb{R}^{3 \times 3l}$  has the form:

$$\boldsymbol{\varphi}^T = \begin{bmatrix} \boldsymbol{\varphi}_1^T(\mathbf{X}) & \mathbf{0}_{1 \times l} & \mathbf{0}_{1 \times l} \\ \mathbf{0}_{1 \times l} & \boldsymbol{\varphi}_2^T(\mathbf{X}) & \mathbf{0}_{1 \times l} \\ \mathbf{0}_{1 \times l} & \mathbf{0}_{1 \times l} & \boldsymbol{\varphi}_3^T(\mathbf{X}) \end{bmatrix} \quad (16)$$

The task of the backstepping controller is to determine: the indirect control law  $\boldsymbol{\alpha}$  which stabilises the first subsystem (1), the control law  $\boldsymbol{\tau}$  which stabilises the entire system, and the adaptation law for the weight vector  $\boldsymbol{\theta}$  with respect to the Lyapunov function of the system.

Applying the certain equivalence principle [9], the vector  $\boldsymbol{\theta}$  in Equation (15) was replaced by the sum of the vectors of estimates and estimation errors,  $\hat{\boldsymbol{\theta}} + \tilde{\boldsymbol{\theta}}$ .

$$\mathbf{M}\dot{\mathbf{z}}_2 = \boldsymbol{\tau} + \boldsymbol{\varphi}^T(\mathbf{X})\hat{\boldsymbol{\theta}} + \boldsymbol{\varphi}^T(\mathbf{X})\tilde{\boldsymbol{\theta}} \quad (17)$$

The control law was determined with respect to the Lyapunov function  $V_a$  being the sum of squares of control errors and the term related with the error of estimation of the unknown weight vector  $\boldsymbol{\theta}$ .

$$V_a = \frac{1}{2} \mathbf{z}_1^T \mathbf{z}_1 + \frac{1}{2} \mathbf{z}_2^T \mathbf{M} \mathbf{z}_2 + \frac{1}{2} \tilde{\boldsymbol{\theta}}^T \boldsymbol{\Gamma}^{-1} \tilde{\boldsymbol{\theta}} \quad (18)$$

where:  $\boldsymbol{\Gamma} > 0$  is the diagonal matrix of controller gains,  $\dim \boldsymbol{\Gamma} = 3l \times 3l$ .

Assuming that the estimated parameters are slowly varying, i.e. the equation  $\dot{\tilde{\boldsymbol{\theta}}} = -\tilde{\boldsymbol{\theta}}$  is fulfilled, the derivative of the Lyapunov function  $V_a$  (18) takes the form:

$$\dot{V}_a = \mathbf{z}_1^T \dot{\mathbf{z}}_1 + \mathbf{z}_2^T \mathbf{M} \dot{\mathbf{z}}_2 - \tilde{\boldsymbol{\theta}}^T \boldsymbol{\Gamma}^{-1} \dot{\tilde{\boldsymbol{\theta}}} \quad (19)$$

Substituting the error dynamics equations (11) and (17) into equation (19) and eliminating the term  $\mathbf{z}_1^T(t) \mathbf{S} \mathbf{z}_1(t) = 0$  (6) gives:

$$\begin{aligned} \dot{V}_a &= \mathbf{z}_1^T [\boldsymbol{\alpha}_1 - \mathbf{J}(\boldsymbol{\eta})^T \dot{\boldsymbol{\eta}}_d] \\ &+ \mathbf{z}_2^T [\mathbf{z}_1 + \boldsymbol{\tau} + \boldsymbol{\varphi}^T(\mathbf{X})\hat{\boldsymbol{\theta}}] \\ &+ \tilde{\boldsymbol{\theta}}_1^T (\boldsymbol{\varphi}(\mathbf{X})\mathbf{z}_2 - \boldsymbol{\Gamma}^{-1} \dot{\tilde{\boldsymbol{\theta}}}) \end{aligned} \quad (20)$$

The control laws  $\boldsymbol{\alpha}$ ,  $\boldsymbol{\tau}$  were selected such that the Lyapunov function (20) in the system of new variables was negative semidefinite:

$$\dot{V}_a = -\mathbf{z}_1^T \mathbf{K}_1 \mathbf{z}_1 - \mathbf{z}_2^T \mathbf{K}_2 \mathbf{z}_2 \leq 0 \quad (21)$$

Here,  $\mathbf{K}_j \in \mathbb{R}^{3 \times 3}$ ,  $j = \{1, 2\}$  is the diagonal and positive definite matrix of controller gains. Comparing (20) and (21), the following relationships can be determined:

- weight adaptation mechanism:

$$\dot{\tilde{\boldsymbol{\theta}}} = \boldsymbol{\Gamma} \boldsymbol{\varphi}(\mathbf{X}) \mathbf{z}_2 \quad (22)$$

- vector of stabilising functions,  $\boldsymbol{\alpha}$ , independent of the vector of estimates  $\hat{\boldsymbol{\theta}}$ :

$$\boldsymbol{\alpha} = -\mathbf{K}_1 \mathbf{z}_1 + \mathbf{J}(\boldsymbol{\eta})^T \dot{\boldsymbol{\eta}}_d \quad (23)$$

- vector of controls,  $\boldsymbol{\tau}$ , dependent of the vector of estimates  $\hat{\boldsymbol{\theta}}$  (RBF network weights) calculated in accordance with (19), under the assumption that the network has a sufficient number of neurons,



$$\boldsymbol{\tau} = -\mathbf{K}_2 \mathbf{z}_2 - \mathbf{z}_1 - \boldsymbol{\varphi}^T(\mathbf{X}) \hat{\boldsymbol{\theta}} \quad (24)$$

If rapidly varying disturbances do not occur in the system, then the control law (24) together with the adaptation law (22) ensure asymptotical convergence of ship position and heading to their set values,  $\boldsymbol{\eta}(t) \rightarrow \boldsymbol{\eta}_d(t)$ , when  $\mathbf{v}(t) \approx 0$ . They also ensure the boundedness of changes of signals  $\boldsymbol{\eta}(t)$  and  $\mathbf{v}(t)$ ,  $t \rightarrow \infty$ , at bounded changes of estimated parameter values.

## SIMULATION TESTS

The computer simulations were performed on the control system with the structure shown in Fig.1. In these simulations, the issues of power overload and state observer control were not analysed, assuming that all state variables are bounded and measurable (estimable).

The simulation tests were performed using the mathematical model of a store ship with length of  $L=76.2$  m and mass of 4591 [t]. The ship motion equation was analysed by controlling the ship motion in 3 degrees of freedom with the aid of two main propellers with rudders, bow tunnel thruster, and rotating azimuth bow thruster.

The following dimensionless parameters of the model (1)-(2), determined in the Bis scaling system, were assumed [7]:

$$\begin{aligned} \mathbf{C}''(\mathbf{v}) &= [0 \ 0 \ -1.8902v'' + 0.0744r''; \\ &\quad 0 \ 0 \ 1.1274 u''; \\ &\quad 1.8902v'' - 0.0744r'' \ -1.1274 u'' \ 0], \\ \mathbf{D}'' &= [0.0414 \ 0 \ 0; \ 0 \ 0.1775 \ -0.0141; \\ &\quad 0 \ -0.1073 \ 0.0568], \\ \mathbf{M}'' &= [1.1274 \ 0 \ 0; \ 0 \ 1.8902 \ -0.0744; \\ &\quad 0 \ -0.0744 \ 0.1278]. \end{aligned}$$

During manoeuvring operations, the set ship position and heading trajectories  $\boldsymbol{\eta}_d$  and their derivatives  $\dot{\boldsymbol{\eta}}_d$  were generated in accordance with the reference model (25) to determine smooth and bounded set signals for the DP controller. Assuming that  $\xi = 0.8$  and  $\omega_n = 0.05$  rad/s, the value of  $G_f(s)$  was calculated from formula (22):

$$G_f(s) = \frac{\omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)}. \quad (25)$$

The DP controller was defined by the control law (24), the neural network with radial basic functions (7), and the weight adaptation law (22). In the simulations, the initial ship position and heading, and the initial estimated weight values were assumed equal to zero.

The nonlinear function of the backstepping controller was approximated using the RBF network, due to its properties described in Section 5 and the possibility to present the output in the form of regression model (7).

The proposed system was designed and constructed in MatLab environment. Three RBF networks were created. For each network, 4 sets of input signals were given, and each time one output signal was obtained as a result of its operation. The number of basic neurons was chosen experimentally. The centres of basic functions were set using the FCM algorithm. The RBF network structure underwent preliminary verification by using it as an ordinary function approximator (13). After selecting its parameters, the network was implemented in the control structure with adaptive controller, shown in Fig. 1.

The tests with adaptive controller included programmable inertial changes of set position and heading values in the presence of slowly varying environmental disturbances, modelled using the Markov process [7].

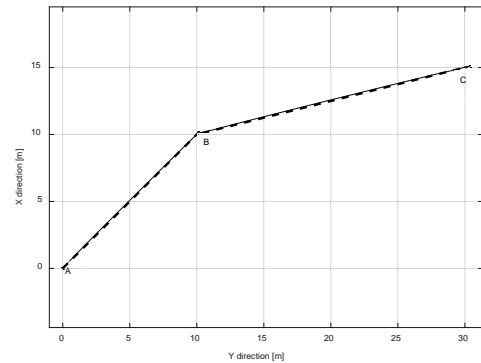


Fig. 3. Set (--) and real (-) trajectory (x, y) of DP ship

Fig. 3 shows the set and real trajectory of the ship. At time zero, the ship is at point A. Then it begins the manoeuvre of position and heading change to the set position B. After time  $t_1 = 1115.4$ s, the next change of the set ship position and heading takes place (towards point C). The robustness of the controller with radial neural network was analysed by introducing a disturbance signal after time  $t_2 = 836.55$ s. This signal was added to the forces acting on the ship in yaw direction. Based on the results of the performed simulations and the obtained time-histories, it can be concluded that the ship's position and heading track the set trajectory with good accuracy (Fig. 4).

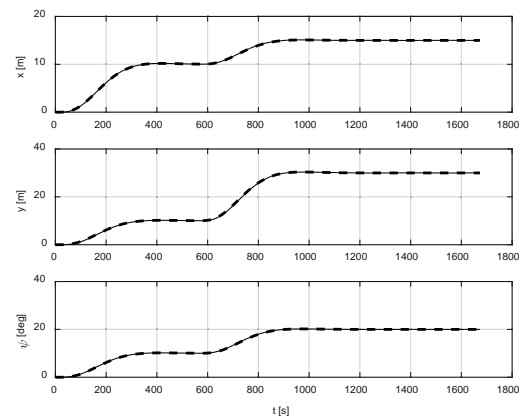


Fig. 4. Set (--) and real (-) time-histories of ship position and heading

The control errors tend asymptotically to zero, without over-regulation in three degrees of freedom of ship motion. This result has been obtained for the system without a priori knowledge of ship model parameters and slowly varying environmental disturbances. The normalised control inputs are shown in Figure 5.

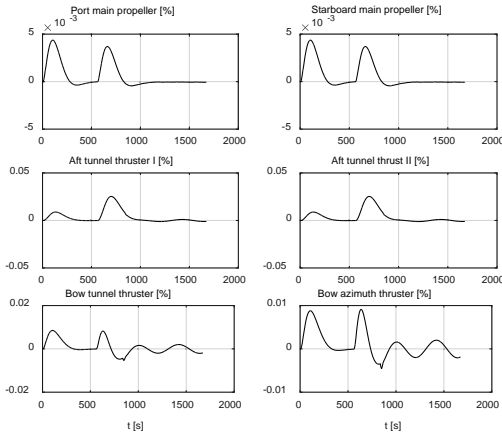


Fig. 5. Control inputs normalised in Bis system

These values become saturated after exceeding 1 and compensate the effect of environmental disturbances. As shown in Fig. 4, small tracking errors were recorded, but they did not exceed acceptable limits.

The present study did not aim at analysing properties of the neural network. Better performance of the DP system can be achieved by further tuning the updating gains and/or by increasing the number of neurons. The present study only demonstrates the possibility of using neural networks for approximating the dynamics of a DP ship with the aid of the theory of Lyapunov functions when designing the control law and when estimating online the RBF network weights.

In a general case, the backstepping method ensures that the values of estimates change in a limited manner and are approximately constant in steady-state conditions (Fig. 6).

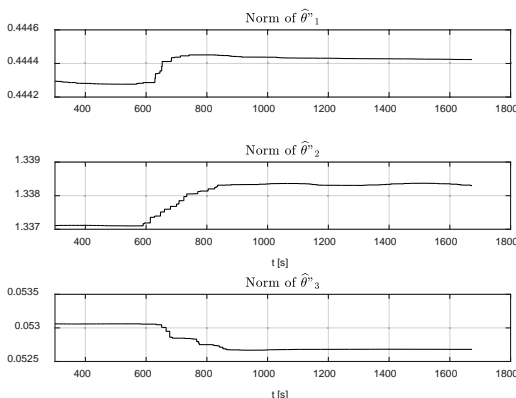


Fig. 6. Norm of RBF network weight vector ( $\hat{\theta}_i$  - weight vector for NNi network,  $i=1..3$  in Bis system)

The obtained results confirm correct operation of the DB system with model and disturbance uncertainties at the analysed operating points.

## CONCLUSIONS

The control system with adaptive backstepping controller was designed for a DP ship. In this system, the RBF neural network was used to estimate the nonlinear function of ship model. The performance of the system was checked in simulation tests. The proposed system does not require a priori knowledge of parameters of matrices of ship damping, Coriolis forces, and/or inertia. It neither requires precise modelling of slowly varying environmental disturbances. The use of the RBF network significantly simplifies designing a backstepping controller, as it does not require analytical description of the regression matrix. Determining the weight adaptation law makes preliminary network tuning unnecessary, as a consequence of which the network can be used online. The performed simulation tests have proved that the adaptive controller tracks the set position and heading trajectory with acceptably small error, at the same time ensuring the boundedness of signals in the closed loop of the control system. Thus, the results of computer simulations illustrate high operating effectiveness of the proposed control method making use of RBF neural networks.

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#### CONTACT WITH THE AUTHORS

**Anna Witkowska**

*e-mail: anna.witkowska@pg.edu.pl*  
 Gdańsk University of Technology  
 Gabriela Narutowicza 11/12  
 80-233 Gdańsk  
**POLAND**

**Tacjana Niksa Rynkiewicz**

*e-mail: tacniksa@pg.edu.pl*  
 Gdańsk University of Technology  
 Gabriela Narutowicza 11/12  
 80-233 Gdańsk  
**POLAND**