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Reflection of surface spin waves from the interface of uniaxial and biaxial ferromagnets in a planar magnetic field

S. O. Reshetnyak, M. Y. Nastenko, A. S. Berezhinsky*

National Technical University of Ukraine "Kyiv Polytechnic Institute",
37 Peremohy av., Kiev, 03056, Ukraine
*Phone: +380976036806

*E-mail address: berejinskiy@gmail.com

ABSTRACT

The article investigates the process of reflection of surface spin waves passing through the interface of uniaxial and biaxial ferromagnets in a planar external magnetic field directed along the hard axis of ferromagnet. The problem is solved using the spin density formalism and the Landau-Lifshitz equations for the case of the absence of dissipation in the system. Geometrical optics formalism is used to describe the processes of refraction of surface spin waves propagating in the ferromagnetic medium with non uniform distribution of magnetic parameters. Quantum mechanical approach is used for calculation of the amplitudes of reflected and transmitted waves. It is shown that spin wave birefringence phenomenon appears at the interface of two uniform ferromagnetic components. Frequency and field dependencies of reflection coefficients for different branches of spin waves are obtained in the study. It is shown that it is possible to change the "optical" parameters of the system by only changing a magnitude of the external homogeneous magnetic field. It is also shown that reflection amplitude depends heavily on the angle of incidence.

***Keywords*:** Surface spin waves; ferromagnetic medium; spin pinning; reflection

1. INTRODUCTION

Recent advances in nanotechnology and nanoelectronics may enable one in employing high-frequency spin waves in various applications. In particular, the use of spin waves in practical applications is of interest since spin waves exhibit several peculiar characteristics. The wave approach is the most popular approach for describing behavior of propagating spin waves. This approach is successfully used, for instance, for determining various spectral characteristics of magnetic materials [1-4,].

Refraction index of bulk spin waves was calculated in paper [5]. Also the following array of information was discussed in this paper: usage of JWKB approximation for describing behavior of propagating bulk spin waves in uniaxial ferromagnets, behavior of bulk spin waves at the interface between two uniaxial ferromagnets with different magnetic parameters, influence of material parameters on reflection characteristics of bulk spin waves, calculation of parameters of devices that could use discussed phenomena. Refraction and reflection of bulk spin waves in a biaxial ferromagnet was studied in [6]. The most interesting results were obtained for the case of propagation of surface spin waves in biaxial ferromagnets. The phenomenon of spin wave birefringence occurs in such systems. As demonstrated in [6] direction of magnetic field affects the birefringence phenomenon. E.g. birefringence occurs in planar magnetic field only for the case of surface spin waves.

The growing interest in spintronic devices [9,10] and dramatic progress in the field of nanotechnologies suggests the need of theoretical investigation of behavior of high frequency spin waves in the inhomogeneous media of different configurations. In particular, a series of experimental works of authors A. Serga, M. Kostylev, B. Hillebrands, A. Chumak [10-14] allowed to get the picture of possibilities of management, filtration of spin waves, and also creation of the modules for spin waves generation, switching, etc. in devices based on exchange spin waves [16,17].

This paper is devoted to the problem of reflection of spin waves in the structure consisting of two homogeneous media, one of which is an uniaxial ferromagnet, the second one is a biaxial ferromagnet. The structure is placed into magnetic field directed along the heavy axis of biaxial ferromagnet.

2. EXPERIMENT

Let's consider unbounded ferromagnet which consists of two half-infinite parts contacting along a plane *x*O*z*. The part situated in the area that corresponds to negative values of *y* possesses uniaxial anisotropy and has the value of saturation magnetization M_{01}, value of parameter of the exchange interaction α_1, value of uniaxial magnetic anisotropy β_1. The second part is characterized by the corresponding parameters M_{02}, α_2, β_2 and by the parameter of rhombic anisotropy ρ_2. The easy magnetic axis is directed along the axis O*z*. The heavy axis of biaxial ferromagnet and external magnetic field are directed along the axis O*y*. Let's calculate refraction indices and reflection coefficients of spin waves in such structure.

2. 1. Basic equations for surface spin waves in biaxial ferromagnet in a planar magnetic field

The energy density magnetic of the described configuration in the exchange (high-frequency) approximation under the assumption $\mathbf{M}_j^2 = const$ has the following form [7,8]:

$$w = \sum_{j=1}^{2} \theta\left[\left(-1\right)^j y\right] w_j + A\delta\left(y\right)\mathbf{M}_1\mathbf{M}_2, \tag{1}$$

where:

$$w_1 = \frac{\alpha_1}{2}\left(\frac{\partial m_1}{\partial x_k}\right)^2 - \frac{\beta_1}{2} m_{1z}^2 - H_0 M_{1y}, \tag{2}$$

$$w_2 = \frac{\alpha_2}{2}\left(\frac{\partial m_2}{\partial x_k}\right)^2 - \frac{\beta_2}{2} m_{2z}^2 - \frac{\rho_2}{2}\left(m_{2x}^2 + m_{2z}^2\right) - H_0 M_{2y}, \tag{3}$$

where $\theta(x)$ – Heaviside step function; A – parameter, that characterizes the exchange interaction in the interface between the half-space at y = 0; $\mathbf{M}_j = M_{0j}\mathbf{e}_y + \mathbf{m}_j$, $\mathbf{m}_j$ – small deviations of the magnetization from the ground state, j = 1,2.

We use the formalism of spin density [7], according to which the magnetization can be written as:

$$\mathbf{M}_j(\mathbf{r},t) = M_{0j}\Psi_j^+(\mathbf{r},t)\boldsymbol{\sigma}\Psi_j(\mathbf{r},t), \quad j = 1,2. \tag{4}$$

where $\Psi_j(\mathbf{r},t)$ – quasiclassical wave functions which play the role of the order parameter of spin density, $\mathbf{r}$ – radius vector in the Cartesian coordinates, $\boldsymbol{\sigma}$ – vector of Pauli matrices.

The principle of least action leads to the following equations for the Lagrangian Ψ_j in the case of absence of damping in the system [7]:

$$i\hbar\frac{\partial\Psi_j(\mathbf{r},t)}{\partial t} = -\mu_0\mathbf{H}_{ej}(\mathbf{r},t)\boldsymbol{\sigma}\Psi_j(\mathbf{r},t), \tag{5}$$

where μ_0 – Bohr magneton, $\mathbf{H}_{ej} = -\dfrac{\partial w_j}{\partial \mathbf{M}_j} + \dfrac{\partial}{\partial x_k}\dfrac{\partial w_j}{\partial\left(\partial\mathbf{M}_j/\partial x_k\right)}$.

Using linear perturbation theory and the fact that $\mathbf{M}_j^2 = const$ it is possible to show that the solution of equation (4) can be written as following:

$$\Psi_j(\mathbf{r},t) = \exp\left(i\,\mu_0 H_0 t/\hbar\right)\cdot\begin{pmatrix}1\\0\end{pmatrix} + \exp\left(i\,\mu_0 H_0 t/\hbar\right)\cdot\begin{pmatrix}\xi_j(\mathbf{r},t)\\ \chi_j(\mathbf{r},t)\end{pmatrix}. \tag{6}$$

Here $\xi_j(\mathbf{r},t)$, $\chi_j(\mathbf{r},t)$ – small additions that describe the deviation of the magnetic moment from the ground state. Taking into account that $\mathbf{M}_j^2 = const$ we obtain $\chi = i\xi$. Linearization of equation (5) leads to the following equations.

In uniaxial medium we obtain:

$$-\frac{\hbar^2}{\left(2\mu_0 M_{0j}\right)^2}\frac{\partial^2\xi_j(\mathbf{r},t)}{\partial t^2}=\left[\alpha^2\Delta^2-2\alpha\left(\tilde{H}_{0j}-\beta/2\right)\Delta+\tilde{H}_{0j}\left(\tilde{H}_{0j}-\beta\right)\right]\xi_j(\mathbf{r},t),$$

in biaxial one:

$$-\frac{\hbar^2}{\left(2\mu_0 M_{0j}\right)^2}\frac{\partial^2\xi_j(\mathbf{r},t)}{\partial t^2}=\left[\alpha^2\Delta^2-2\alpha\left(\tilde{H}_{0j}-\beta/2-\rho\right)\Delta+\right.$$
$$\left.+\left(\tilde{H}_{0j}-\rho\right)\left(\tilde{H}_{0j}-\beta-\rho\right)\right]\xi_j(\mathbf{r},t), \tag{7}$$

where $\tilde{H}_{0j}=\dfrac{H_0}{M_{0j}}$.

On the surface z = 0 boundary conditions must be fulfilled [8]:

$$\frac{\partial\xi_j}{\partial z}(x,y,0,t)-L_j\xi_j(x,y,0,t)=0,$$

where L_j is spin pinning parameter at the surface of the magnet. Then, using this condition and the Fourier transform ($\chi_j(\mathbf{r},t)\sim e^{k_z z}e^{i(\mathbf{k}_\perp\mathbf{r}_\perp-\omega t)}$) one can obtain from equation (7) the dispersion relation for the surface spin wave in uniaxial medium which decays exponentially along the axis *Oz*:

$$\Omega_1^2=\left[\alpha_1(\mathbf{r}_\perp)k_\perp^2(\mathbf{r}_\perp)+\tilde{H}_{01}-\alpha_1(\mathbf{r}_\perp)L_1^2\right]\times$$
$$\times\left[\alpha_1(\mathbf{r}_\perp)k_{1\perp}^2(\mathbf{r}_\perp)-\beta_1(\mathbf{r}_\perp)+H_{01}-\alpha_1(\mathbf{r}_\perp)L_1^2\right]. \tag{8}$$

Similarly, expression for spin wave spectrum in the biaxial medium is given by:

$$\Omega_2^2=\left[\alpha_2(\mathbf{r}_\perp)k_\perp^2(\mathbf{r}_\perp)-\rho_2(\mathbf{r}_\perp)+\tilde{H}_{02}-\alpha_2(\mathbf{r}_\perp)L_2^2\right]\times$$
$$\times\left[\alpha_2(\mathbf{r}_\perp)k_{2\perp}^2(\mathbf{r}_\perp)-\rho_2(\mathbf{r}_\perp)-\beta_2(\mathbf{r}_\perp)+H_{02}-\alpha_2(\mathbf{r}_\perp)L_2^2\right], \tag{9}$$

where $\Omega_j=\dfrac{\omega\hbar}{2\mu_0 M_{0j}}$, ω –frequency, $\mathbf{k}=(\mathbf{k}_\perp,k_z)$ –wave vector, $\mathbf{r}_\perp=(x,y)$.

2. 2. Refraction of spin waves

Basing on equations (8) and (9), we can find:

$$\alpha\left(\mathbf{r}_{\perp}\right)k_{1\perp}^{2}\left(\mathbf{r}_{\perp}\right)=\alpha\left(\mathbf{r}_{\perp}\right)L_{1}^{2}+\frac{\beta_{1}\left(\mathbf{r}_{\perp}\right)}{2}-\tilde{H}_{01}\pm\sqrt{\Omega_{1}^{2}+\frac{\beta_{1}^{2}\left(\mathbf{r}_{\perp}\right)}{4}},$$

$$\alpha_{2}(\mathbf{r}_{\perp})k_{2\perp}^{2}(\mathbf{r}_{\perp})=\alpha_{2}(\mathbf{r}_{\perp})L_{2}^{2}+\frac{\beta_{2}(\mathbf{r}_{\perp})}{2}+\rho_{2}(\mathbf{r}_{\perp})-H_{02}\pm\sqrt{\Omega_{2}^{2}+\beta_{2}^{2}(\mathbf{r}_{\perp})/4}.$$

If a spin wave wavelength λ satisfies the condition of geometrical optics:

$$\lambda \ll a, \tag{10}$$

where a – characteristic size of an inhomogeneity, then an analogue of classical Hamilton-Jacobi equation can be used:

$$\left(\nabla_{\perp}s_{j}\left(\mathbf{r}_{\perp}\right)\right)^{2}=n_{j}^{2}\left(\mathbf{r}_{\perp}\right), \tag{11}$$

where $\nabla_{\perp}=\mathbf{e}_{x}\frac{\partial}{\partial x}+\mathbf{e}_{y}\frac{\partial}{\partial y}, n_{j}^{2}\left(\mathbf{r}_{\perp}\right)=\frac{k_{j}^{2}\left(\mathbf{r}_{\perp}\right)}{k_{0}^{2}}$, k_0 is wave number at the infinity from the side of incident wave. On the border between uniform uniaxial and biaxial ferromagnets we obtain refractive indices:

$$n^{\pm}=\frac{\sin\theta_{1}^{\pm}}{\sin\theta_{2}^{\pm}}=\frac{k_{2}^{\pm}}{k_{1}^{\pm}}=\sqrt{\frac{\alpha_{1}}{\alpha_{2}}\frac{\alpha_{2}L_{2}^{2}+\beta_{2}/2+\rho_{2}-\tilde{H}_{02}\pm\sqrt{\Omega_{2}^{2}+\beta_{2}^{2}/4}}{\alpha_{1}L_{1}^{2}+\beta_{1}/2-\tilde{H}_{01}\pm\sqrt{\Omega_{1}^{2}+\beta_{1}^{2}/4}}}, \tag{12}$$

θ_1 is angle of incidence, θ_2 is angle of refraction.

Boundary conditions for ferromagnetic materials are determined by integrating the equations of motion of the magnetic moment in a small neighborhood of the interface and equating the result to zero with decreasing radius of the integration domain to zero.

So, we get the following system:

$$\begin{aligned}&\left[A\gamma\left(\xi_{2}-\xi_{1}\right)+\alpha_{1}\xi_{1}'\right]_{y=0}=0,\\&\left[A\left(\xi_{1}-\xi_{2}\right)-\gamma\alpha_{2}\xi_{2}'\right]_{y=0}=0.\end{aligned} \tag{13}$$

Note that in the case of "ideal" exchange, that is the absence of defects at the interface (this corresponds to the values $A\to\infty$),and uniform saturation magnetization obtained expressions transform into standard exchange boundary conditions: $\xi_2=\xi_1$, $\alpha_1\xi_1'=\alpha_2\xi_2'$.

2. 3. Reflection of surface spin waves at the interface of two inhomogeneous biaxial media

Suppose, the incident wave function is given by $\xi_I = \exp\left(i\left(\mathbf{k}_0\mathbf{r} - \omega t\right)\right)$,

reflected wave – by $\xi_R = R\exp\left(i\left(\mathbf{k}_1\mathbf{r} - \omega t\right)\right)$,

transmitted wave – by $\xi_D = D\exp\left(i\left(\mathbf{k}_2\mathbf{r} - \omega t\right)\right)$, where R is complex amplitude of the reflection of spin waves from the interface, D – transmission amplitude, $\mathbf{k}_0$, $\mathbf{k}_1$ – wave vectors of the incident and reflected waves respectively, $\mathbf{k}_2$ – wave vector of transmitted wave.

The next expressions for the amplitudes of reflection and transmission can be obtained from (13) using this wave function exponential notation:

$$R = \frac{k_0\alpha_1\alpha_2\gamma\cos\theta_1\sqrt{n^2-\sin^2\theta_1} - iA\left(\alpha_1\cos\theta_1 - \alpha_2\gamma^2\sqrt{n^2-\sin^2\theta_1}\right)}{k_0\alpha_1\alpha_2\gamma\cos\theta_1\sqrt{n^2-\sin^2\theta_1} - iA\left(\alpha_1\cos\theta_1 + \alpha_2\gamma^2\sqrt{n^2-\sin^2\theta_1}\right)},$$

$$D = \frac{-2iA\alpha_1\cos\theta_1}{k_0\alpha_1\alpha_2\gamma\cos\theta_1\sqrt{n^2-\sin^2\theta_1} - iA\left(\alpha_1\cos\theta_1 + \alpha_2\gamma^2\sqrt{n^2-\sin^2\theta_1}\right)} \qquad (14)$$

3. DISCUSSION OF RESULTS

Fig. 1 shows the dependency of the intensity of reflected wave $I_{R^+} = \left|R^+\right|^2$ on the frequency of surface spin wave for typical values of the parameters of ferrite-garnets [15]. It is shown that it is possible to achieve the required ratio of intensities of reflected and transmitted waves by choosing appropriate set of parameters for the given spin wave frequency.

Furthermore, as follows from Fig. 2, the intensity of the reflection depends strongly on the value of the external uniform magnetic field, which enables one to change the intensity of the reflected wave in a wide range by only changing the value of the external magnetic field and keeping other parameters of the material constant.

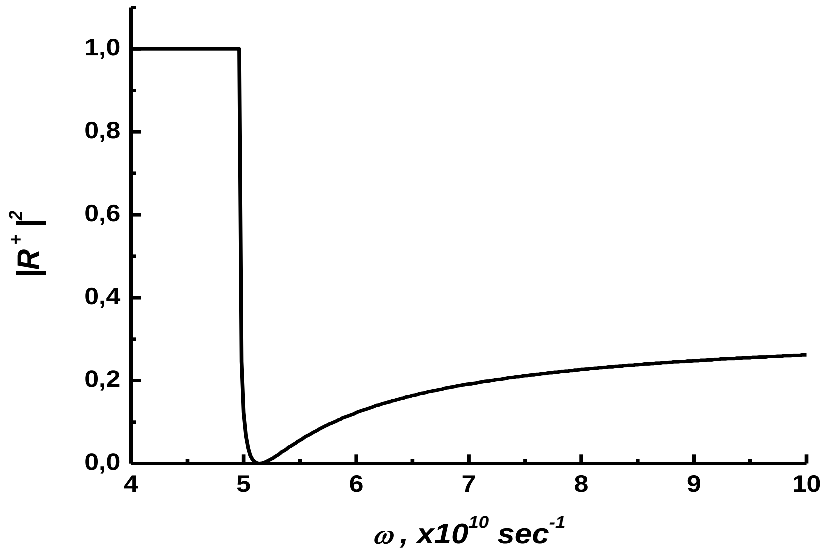

Figure 1. Dependency of the reflection coefficient $\left|R^{+}\right|^{2}$ on the frequency of spin waves ω at $\alpha_1 = 10^{-11}$ cm^2, $\alpha_2 = 3 \cdot 10^{-11}$ cm^2, $\beta_1 = 10$, $\beta_2 = 15$, $\rho_2 = 2$, $L_1 = 10^6$cm^{-1}, $L_2 = 10^5$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm, $\theta = \pi/80$, $H_0 = 5000$ Oe

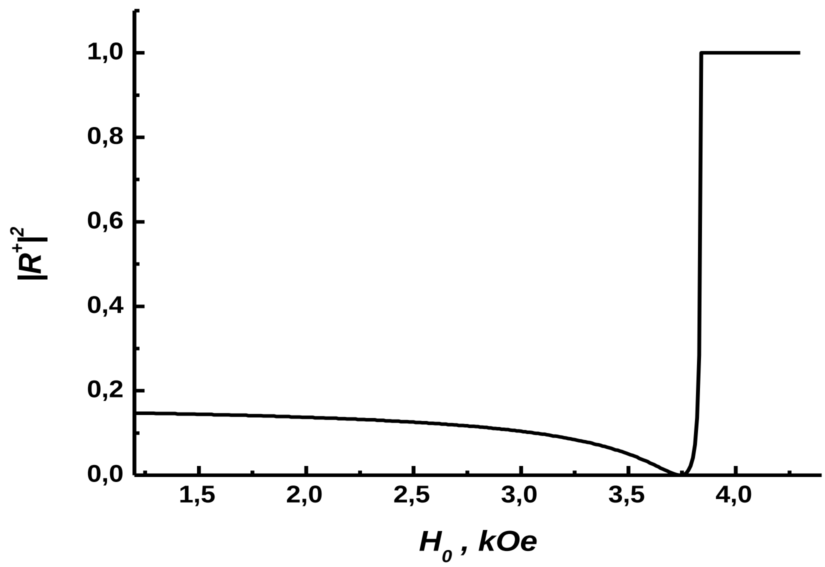

Figure 2. Dependency of the reflection coefficient $\left|R^{+}\right|^{2}$ on of external magnetic field H_0 at $\alpha_1 = 10^{-11}$ cm^2, $\alpha_2 = 1{,}8 \cdot 10^{-11}$ cm^2, $\beta_1 = 10$, $\beta_2 = 15$, $\rho_2 = 2$, $L_1 = 10^6$cm^{-1}, $L_2 = 5 \cdot 10^5$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm, $\theta = \pi/80$, $\omega = 3.6 \cdot 10^{10} sec^{-1}$.

Fig. 3 and 4 show the dependency of the intensity of reflected wave $I_{R^-} = \left|R^-\right|^2$ on the frequency of surface spin waves and magnitude of the external magnetic field at values of parameters of the material that allow existence of this spin wave branch.

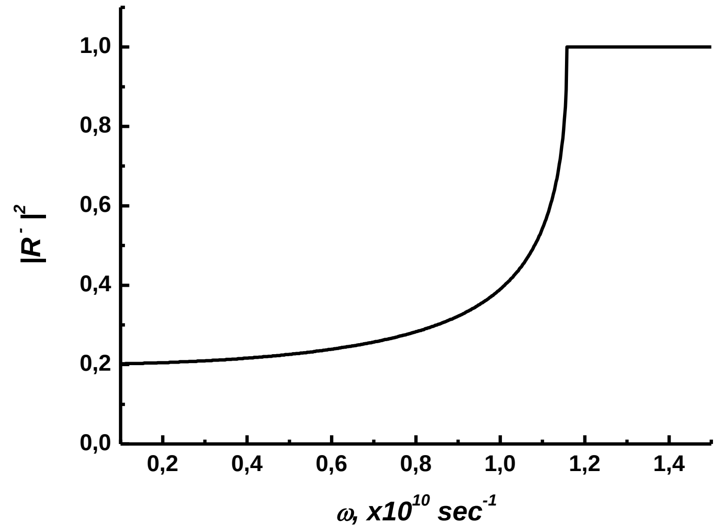

Figure 3. Dependency of the reflection coefficient $\left|R^-\right|^2$ on the frequency of spin waves ω at

$\alpha_1 = 10^{-11}$ cm^2, $\alpha_2 = 1{,}5 \cdot 10^{-11}$ cm^2, $\beta_1 = 10$, $\beta_2 = 15$, $\rho_2 = 2$, $L_1 = 10^6$cm^{-1}, $L_2 = 10^6$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm, $\theta = \pi/80$, $H_0 = 1200$ Oe.

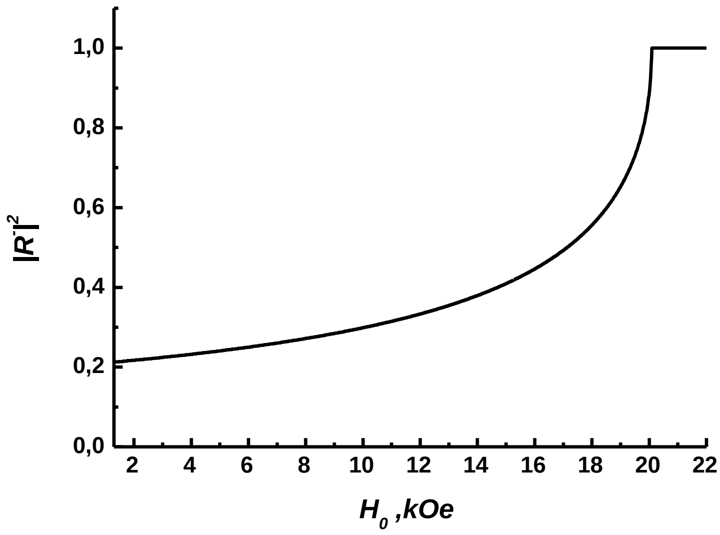

Figure 4. Dependency of the reflection coefficient $\left|R^-\right|^2$ on the value of external magnetic field H_0 at $\alpha_1 = 10^{-11}$ cm^2, $\alpha_2 = 10^{-11}$ cm^2, $\beta_1 = 10$, $\beta_2 = 15$, $\rho_2 = 2$, $L_1 = 5 \cdot 10^6$cm^{-1}, $L_2 = 10^7$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm, $\theta = \pi/80$, $\omega = 10^{11} sec^{-1}$.

Figs. 5-10 show dependencies of the reflection intensities $I_{R+} = |R^{+}|^2$ and $I_{R-} = |R^{-}|^2$ on the parameter of exchange interaction α, parameter of uniaxial magnetic anisotropy β and the angle of incidence θ.

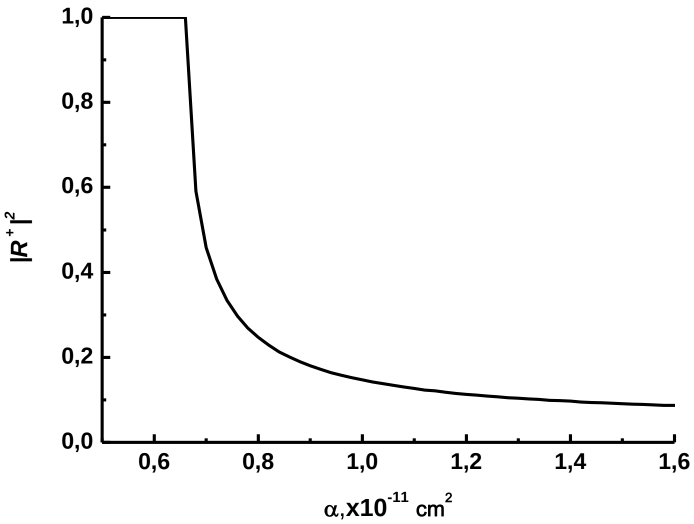

Figure 5. Dependency of the reflection coefficient $|R^{+}|^2$ on the parameter of exchange interaction α at $\beta_1 = 10$, $\beta_2 = 15$, $\rho_2 = 2$, $L_1 = 10^6$cm^{-1}, $L_2 = 10^6$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm, $\theta = \pi/80$, $\omega = 10^{10} sec^{-1}$, $H_0 = 1900$ Oe.

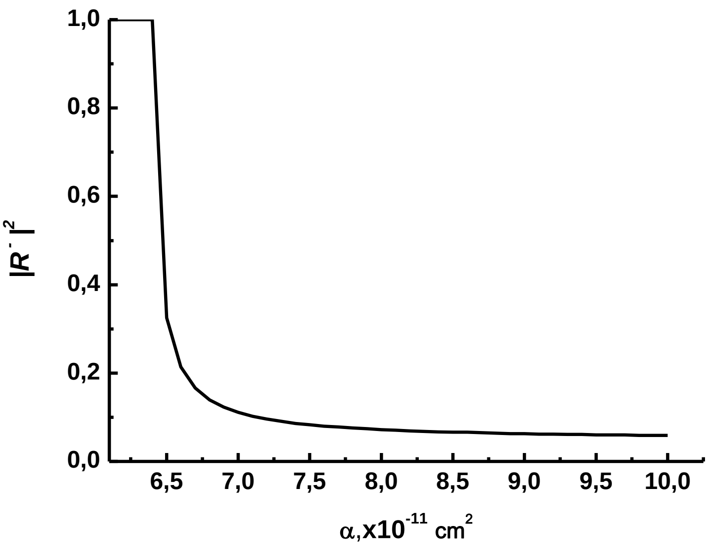

Figure 6. Dependency of the reflection coefficient $|R^{-}|^2$ on the parameter of exchange interaction α at $\beta_1 = 10$, $\beta_2 = 15$, $\rho_2 = 2$, $L_1 = 10^6$cm^{-1}, $L_2 = 10^6$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm, $\theta = \pi/80$, $\omega = 10^{11} sec^{-1}$, $H_0 = 1500$ Oe

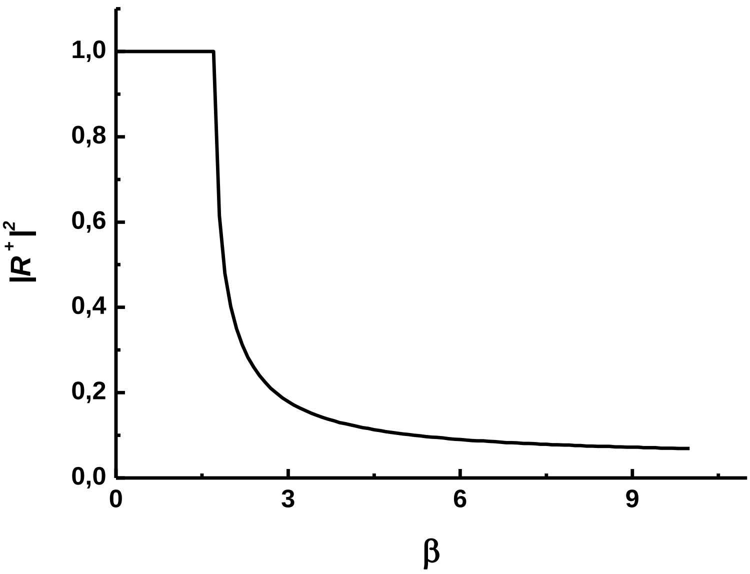

Figure 7. Dependency of the reflection coefficient $\left|R^{+}\right|^{2}$ on the parameter of uniaxial magnetic anisotropy β at $\rho_2 = 2$, $L_1 = 10^6$cm^{-1}, $L_2 = 10^6$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm, $\theta = \pi/80$, $\omega = 1{,}5 \cdot 10^{10} sec^{-1}$, $H_0 = 1200$ Oe, $\alpha_1 = 3 \cdot 10^{-12}$ cm^2, $\alpha_2 = 3 \cdot 10^{-12}$ cm^2

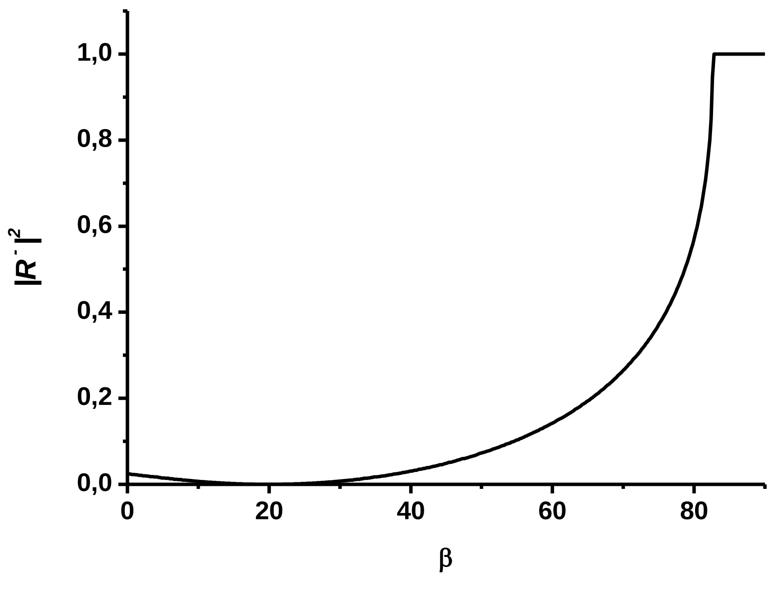

Figure 8. Dependency of the reflection coefficient $\left|R^{-}\right|^{2}$ on the parameter of uniaxial magnetic anisotropy β at $\rho_2 = 2$, $L_1 = 10^6$cm^{-1}, $L_2 = 10^6$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm, $\theta = \pi/80$, $\omega = 10^{10} sec^{-1}$, $H_0 = 1400$ Oe, $\alpha_1 = 1{,}3 \cdot 10^{-11}$ cm^2, $\alpha_2 = 1{,}3 \cdot 10^{-11}$ cm^2

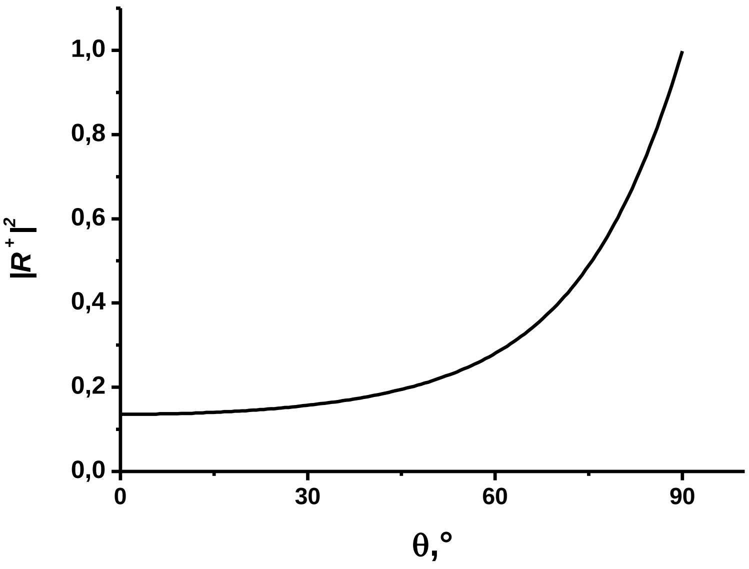

Figure 9. Dependency of the reflection coefficient $\left|R^{+}\right|^{2}$ on the incidence angle θ at $\rho_2 = 2$, $L_1 = 10^6$cm^{-1}, $L_2 = 10^6$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm,$\omega = 10^{11} sec^{-1}$, $H_0 =$ 1500 Oe, $\alpha_1 = 10^{-11}$ cm^2, $\alpha_2 = 1{,}5 \cdot 10^{-11}$ cm^2, $\beta_1 = 10$, $\beta_2 = 15$.

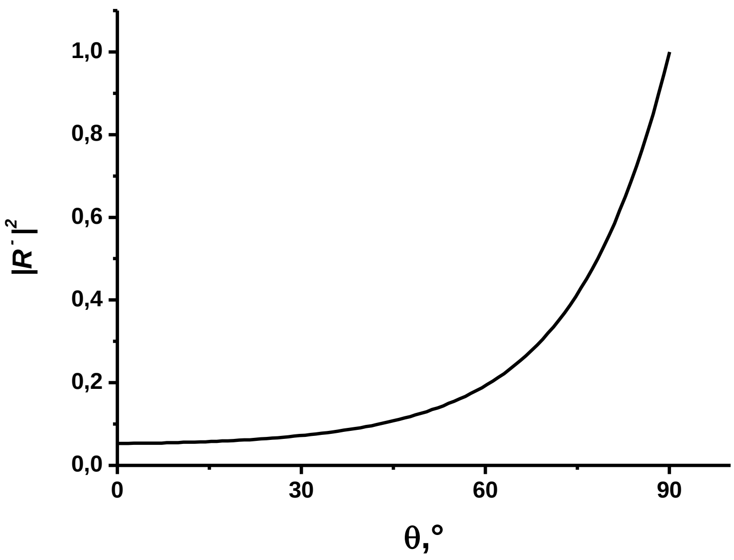

Figure 10. Dependency of the reflection coefficient $\left|R^{-}\right|^{2}$ on the incidence angle θ $at \rho_2 = 2$, $L_1 = 10^6$cm^{-1}, $L_2 = 0{,}5 \cdot 10^6$cm^{-1}, $M_{01} = 100$ G, $M_{02} = 105$ G, $A = 10^7$cm,$\omega = 3{,}6 \cdot 10^{10} sec^{-1}$, $H_0 = 1500$ Oe, $\alpha_1 = 10^{-11}$ cm^2, $\alpha_2 = 10^{-11}$ cm^2, $\beta_1 = 10$, $\beta_2 = 15$.

4. CONCLUSIONS

The JWKB approximation applied to the case of surface spin waves propagation in ferromagnets is used to show that spin wave birefringence phenomenon occurs at the interface between uniaxial and biaxial homogeneous ferromagnetic media. Each branch of refracted spin wave has its own shape of reflection intensity dependency. This fact enables one to establish required ratio of the "negative" branch reflection intensity to "positive" branch reflection intensity. Due to the fact that branches have different allowed zones one can completely get rid of one of the branches by changing magnetic parameters of the media. It's worth noting that it is possible to control reflection coefficient by changing value of the external magnetic field and keeping other parameters constant only if the exchange parameter is big enough. Value of the maximum transmission amplitude tends to zero as $A \to 0$.

The results of the paper can be successfully used for the development of spin wave microelectronics devices (spin wave filters, spin wave analogues of optical devices, etc.). In particular, these calculations enable one to build a spin wave lens or mirror that has manageable focal distance and reflectiveness.

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