ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2016
Seria: ORGANIZACJA I ZARZĄDZANIE z. 96 Nr kol. 1963

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RESOURCE ALLOCATION UNDER COMPLETE UNCERTAINTY – CASE OF ASYMMETRIC PAYOFFS[1]

Summary. The resource allocation problem has been investigated in many contributions both for the deterministic (known parameters) and stochastic case (scenarios with known probability distribution). In this paper we propose a decision rule enabling one to find a proper solution under complete uncertainty, i.e. when possible scenarios are known, but the decision maker has no information (or does not intend to use it) about their likelihood. The procedure takes into account the decision maker's nature and the specificity of particular sets of possible cumulative payoffs (range, average, asymmetry, dispersion).

Keywords: resource allocation, complete uncertainty, optimism, pessimism, decision rule, optimization model, scenarios, payoff matrix.

ROZDZIAŁ ZASOBÓW W WARUNKACH CAŁKOWITEJ NIEPEWNOŚCI – PRZYPADEK ASYMETRYCZNYCH WYPŁAT

Streszczenie. Problem rozdziału zasobów jest analizowany w wielu pracach zarówno dla przypadku deterministycznego (znane parametry), jak i stochastycznego (scenariusze ze znanym prawdopodobieństwem). W tym opracowaniu proponowana jest reguła decyzyjna umożliwiająca wyznaczenie odpowiedniego rozwiązania w warunkach całkowitej niepewności (znane są scenariusze, lecz decydent nie dysponuje wiedzą o prawdopodobieństwie bądź nie zamierza z tej wiedzy skorzystać). Podejście uwzględnia naturę decydenta oraz charakterystyczne cechy poszczególnych zbiorów możliwych skumulowanych wypłat (przedział, średnia, asymetria, rozproszenie).

Słowa kluczowe: rozdział zasobu, całkowita niepewność, optymizm, pesymizm, reguła decyzyjna, model optymalizacyjny, scenariusze, macierz wypłat.

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[1] This research is financed by the National Science Center in Poland (project registration number: 2014/15/D/HS4/00771).

1. Introduction

The resource allocation problem (RAP) concerns the assignment of available resources (materials, money, manpower, machinery, merchandise etc.) to various uses (activities). Deterministic (known parameters) [33, 34, 35], stochastic (parameters are random variables with known probability distributions) [29, 37] and strategic (parameters are random variables with unknown probability distributions) [9, 20] versions of the RAP have been already discussed in the literature. The goal of this contribution is to analyze uncertain RAPs for different types of decision maker's preferences and cases where profits resulting from the assignment of a quantity of resource to a given activity, are distributed asymmetrically and the payoff dispersions for particular resource quantities are quite diverse. We recommend a procedure based on the Hurwicz and Bayes rules and we formulate an optimization model allowing one to find a suitable solution. The paper is organized as follows. Section 2 deals with models and procedures applied to RAP. Section 3 defines RAP under complete uncertainty (i.e. with unknown probabilities/frequencies) and focuses on the case where the decision maker is able to declare his/her coefficients of optimism/pessimism. Section 4 presents a new rule and model. Section 5 provides a case study. Conclusions are gathered in the last part.

2. Resource allocation problem – models and procedures

In the resource allocation problem it is assumed that the decision maker (DM) has a given resource quantity (b), which may be allocated to n activities (j=1, …, n). The total outcome depends on the quantity of resource (x_j) allocated to particular activities. The outcome function $f_j(x_j)$ is different for each activity. The goal of the DM is to find such a resource allocation which maximizes the total profit. The optimization model describing the aforementioned problem may be formulated in the following way:

$$\sum_{j=1}^{n} f_j(x_j) \rightarrow \max \tag{1}$$

$$\sum_{j=1}^{n} x_j \le b \tag{2}$$

$$0 = d_j \le x_j \le g_j \qquad j = 1, ..., n \tag{3}$$

where $\{d_j,\dots,g_j\}$ denotes the set of possible values of the investment outlay x_j.

There are three main variants of the classic RAP. In the first one, the DM intends to allocate at most b units of resource (constraint 2 is an inequality). In the second one, the DM is willing to allocate exactly b units (constraint 2 becomes an equation). In the third one, the total resource quantity is not limited – formula (2) may be omitted. RAP is analyzed on the following assumptions:

1. The resource allocation is performed in integer units.
2. Benefits from particular activities may be measured by means of the same unit.
3. The profit from a given activity does not depend on the quantity of resource allocated to another activity.
4. The overall profit resulting form the resource allocation is the sum of benefits connected with particular activities.

The problem can be solved by means of the dynamic programming (DP) and Bellman's principle of optimality [2, 3, 4]. There also exist simplified procedures applied to RAP and described e.g. in [9, 10], such as the method of local extrema (MLE) and the method of marginal profits (MMP). Problem (1)-(3) can be solved on the basis of DP or MMP. On the other hand, problem (1), (3) may be solved using DP, MLE or MMP. Note that when applying MMP marginal profit functions for each activity ought to be non-increasing. The aforementioned resource allocation problem is explained in detail e.g. in [9, 23, 34]. RAP may be also analyzed in the context of total cost minimization [9]. A review of other resource allocation problems, not investigated in this contribution, can be found for instance in [1, 5, 6, 8, 19, 25, 30, 32, 39].

In the deterministic version of RAP we assume that profits resulting from allocating a given quantity of resource to particular activities are perfectly known. Nevertheless, in real-world situations, those parameters are not completely known. Therefore, instead of using exact values, it is desirable to refer to random variables with a known probability distribution (stochastic version, see: decision making under risk or stochastic uncertainty) or unknown probability distribution (strategic version, see: decision making under uncertainty or complete uncertainty). The random variable may be discrete or continuous. The stochastic approach has been discussed in many contributions, e.g. in [21, 26, 28, 29, 37]. Note that sometimes the estimation of the likelihood is quite hampered due to the existence of many discrepant definitions of probability [27], the lack of historical data (for totally new decisions and events), the lack of sufficient knowledge about particular states or the fact that the set of possible scenarios forecasted by experts in the scenario planning stage does not satisfy probability axioms (the sum of state probabilities should be equal to 1, the whole sample space must be precisely defined), see [22]. Additionally, according to von Mises [36], the theory of probability can never lead to a definite statement concerning a single event (the probability of a single event cannot be presented numerically). Furthermore, De Finetti [7] argues that objective probabilities do not exist ("No matter how much information you have, there is no scientific method to assign a probability to an event", there are only subjective probabilities – different for particular DMs). In connection with all these facts, we dare to state that the analysis of RAP with probability-like quantities [20] or under complete uncertainty (without applying probability-like quantities) is extremely desired .

3. Resource allocation problem under complete uncertainty (RAP-CU)

Within the framework of resource allocation under complete uncertainty we will analyze only the case which can be presented by means of Table 1. Hence, there are n possible activities. Cumulative outcomes $O^j_{k,j}$ depend on the quantity of resource (Q_k) allocated to particular activities (A_j) and on the scenario which will occur S^j_i, where $k=1,\ldots,r$; $j=1,\ldots,n$; $i=1,\ldots,m(j)$ (there are $m(j)$ possible states of nature and the number of scenarios may be different for each activity). Quantity of resource Q_k belongs to set $\{d_j,\ldots,g_j\}$, where d_j and g_j denote the minimal and maximal possible quantity of resource allocated to activity A_j. Cumulative profits are estimated by experts during the scenario planning stage.

Table 1

Cumulative payoff matrix for resource allocation problem with scenarios (general case)

Resource quantity	Activities								
	A_1			A_j			A_n		
	S^1_1	S^1_i	$S^1_{m(1)}$	S^j_1	S^j_i	$S^j_{m(j)}$	S^n_1	S^n_i	$S^n_{m(n)}$
Q_1	$O^1_{1,1}$	$O^1_{1,i}$	$O^1_{1,m(1)}$	$O^j_{1,1}$	$O^j_{1,i}$	$O^j_{1,m(j)}$	$O^n_{1,1}$	$O^n_{1,i}$	$O^n_{1,m(n)}$
Q_k	$O^1_{k,1}$	$O^1_{k,i}$	$O^1_{k,m(1)}$	$O^j_{k,1}$	$O^j_{k,i}$	$O^j_{k,m(j)}$	$O^n_{k,1}$	$O^n_{k,i}$	$O^n_{k,m(n)}$
Q_r	$O^1_{r,1}$	$O^1_{r,i}$	$O^1_{r,m(1)}$	$O^j_{r,1}$	$O^j_{r,i}$	$O^j_{r,m(j)}$	$O^n_{r,1}$	$O^n_{r,i}$	$O^n_{r,m(n)}$

Source: Prepared by the author.

Gaspars [9] has been already analyzed that case and has described diverse procedures and optimization models based on Wald, Savage, Hurwicz and Bayes (Laplace) decision rules on the assumption that: a) the total quantity (b) of resource is limited, b) the total quantity of resource is not limited. The aforementioned rules are well-known approaches [18, 24, 31, 38]. Thus, we just briefly remind the main features of particular methods. Wald rule (max-min) is designed for pessimists (maximization of minimal payoffs). Savage rule (min-max) consists in minimizing maximal relative losses and should be also applied by cautious DMs. Hurwicz rule enables one to take into consideration the levels of optimism and pessimism (and leads to the maximization of a weighted average of the highest and lowest payoff). Bayes rule, in contrast to the first three procedures (applicable to one-shot decisions), can be used when the selected decision is going to be performed many times (the arithmetical average of all profits is maximized).

In this contribution, we would like to focus only on the uncertain resource allocation method referring to Hurwicz rule, i.e. on the situation where the DM makes the decision on the basis of scenario planning and his/her coefficients of pessimism/optimism. In [9] the optimization model based on that rule consists of the following equations:

$$\sum_{j=1}^{n}\sum_{k=d_j}^{g_j}((\alpha\cdot\min_{i=1,\dots,m(j)}\{O_{k,i}^{j}\}+\beta\cdot\max_{i=1,\dots,m(j)}\{O_{k,i}^{j}\})x_{k,j})=\sum_{j=1}^{n}\sum_{k=d_j}^{g_j}H_{k,j}\cdot x_{k,j}\rightarrow\max \tag{4}$$

$$\sum_{k=d_j}^{g_j}x_{k,j}=1 \qquad j=1,\dots,n \tag{5}$$

$$x_{k,j}\in\{0,1\} \qquad k=d_j,\dots,g_j;\ j=1,\dots,n \tag{6}$$

$$\sum_{k=d_j}^{g_j}(k\cdot\sum_{j=1}^{n}x_{k,j})\leq b \tag{7}$$

Equation (4) maximizes the sum of Hurwicz indices ($H_{k,j}$) multiplied by respective binary variables $x_{k,j}$. Symbol α and β signify coefficients of pessimism and optimism, where $\alpha+\beta=1$ and α, $\beta\geq0$. Those parameters may be estimated intuitively by the DM or calculated on the basis of some psychological tests. Variable $x_{k,j}$ equals 1 when the quantity of resource allocated to activity A_j should amount to Q_k, otherwise that variable equals 0. Formula (5) signifies that the quantity of resource allocated to activity A_j may be equal to d_j, d_j+1, …, g_j-1 or g_j (i.e. to Q_1, Q_2, …, Q_{r-1} or Q_r), where usually $d_j=0$. Constraint (7) guarantees that the total amount of resource allocated to all activities will not exceed b (that formula is optional).

4. 2-criteria (H+B)-rule for RAP under complete uncertainty (RAP-UC)

On the whole, model (4)-(7), based on the Hurwicz approach, leads to rational results. Nevertheless, recent papers [11, 12, 13, 14, 15, 16, 17] contain several remarks concerning the recommendations offered by the classic Hurwicz decision rule. As a matter of fact, that procedure suggests logic solutions if and only if the discrete distribution of payoffs is (almost) symmetric and the profit dispersions for particular alternatives are quite similar. In other cases, the aforementioned decision rule should not be used since it does not reflect DM’s preferences (asymmetric payoffs may be also problematic in the case of Wald, Savage or Bayes rules, but that topic is not investigated in this article). Therefore, we would like to present an optimization model for RAP-UC which takes into consideration:

- DM’s preferences measured by the coefficient of optimism (pessimism) – feature characteristic of Hurwicz rule,
- all payoffs connected with a given alternative – feature characteristic of Bayes rule,

- the range and asymmetry (the frequency of the highest and lowest outcomes) of the payoff distributions.

Hence, the new model, as is the case of model (4)-(7), gives the opportunity to adjust the choice of a solution to the DM's nature (pessimist, moderate, optimist), and what's more, it takes into account the specificity of payoff distributions. In the novel approach, function (8) is subjected to constraints (5)-(7) and (9). If there is no upper bound *b*, Equation (7) may be omitted. As we can notice, this time a modified Hurwicz index (i.e. $HB_{k,j}$) is applied. The way how to compute that index will be explained in the next paragraph. Thanks to constraint (9) the model recommends only such quantities of resource for which the range of possible payoffs is not too large. The more optimist the DM is, the larger the range of possible payoffs can be. Symbol R_{max} denotes the maximal range in the set of all pairs "quantity Q_k – activity A_j" <*k*,*j*>. Range R'_{min} is defined as the minimal range in the set of pairs <*k*,*j*> with positive ranges. $\overline{R}'$ signifies the average of all positive ranges. Of course, instead of using formula (9), one may apply other constraints to limit payoff dispersions (e.g. set the upper bound arbitrarily or replace ranges with standard deviations), see for instance [17]. Constraint (9) is just a suggestion.

$$\sum_{j=1}^{n}\sum_{k=d_j}^{g_j} HB_{k,j}\cdot x_{k,j} \rightarrow \max \tag{8}$$

$$R_{k,j}\cdot x_{k,j} \le \beta(R_{\max}-\overline{R}') + R'_{\min} \qquad j=1,...,n; k=d_j,...,g_j \tag{9}$$

The procedure enabling to compute index $HB_{k,j}$ consists of the following steps:

1. Determine α and β. If $\alpha \in [0,0.5)$, then $\alpha=\alpha_o, \beta=\beta_o$ (α_o and β_o are optimist's coefficients). If $\alpha \in (0.5,1]$, then $\alpha=\alpha_p, \beta=\beta_p$ (α_p and β_p are pessimist's coefficients).
2. Find non-increasing sequences of outcomes $Sq_{k,j}=(a_{k,j}^1,...,a_{k,j}^t,...,a_{k,j}^z)$ for each pair <*k*,*j*>: $a_{k,j}^t \ge a_{k,j}^{t+1}$ (t=1,…,z-1), z – number of terms in the sequence (i.e. $m(j)$), t – number of the term in the sequence.
3. Calculate, for each pair <*k*,*j*>, index $HB_{k,j}$ ($HB_{k,j}^p$, $HB_{k,j}^o$ or $HB_{k,j}^{0.5}$). If $\alpha \in (0.5,1]$, calculate $HB_{k,j}^p$ (index for pessimists) according to Equation (10). If $\alpha \in [0,0.5)$, compute $HB_{k,j}^o$ (index for optimists) following formula (11). If α = 0.5, calculate $HB_{k,j}^{05}$ using Equation (12), where $b_{k,j}$ denotes the Bayes criterion, i.e. the average of all payoffs.

$$HB_{k,j}^p = \frac{\alpha_p\cdot a_{k,j}^z + \beta_p\cdot\sum_{t=1}^{z-1} a_{k,j}^t}{\alpha_p+(z-1)\cdot\beta_p} \qquad j=1,...,n; k=d_j,...,g_j \tag{10}$$

$$HB^{o}_{k,j}=\frac{\alpha_o\cdot\sum_{t=2}^{z}a^{t}_{k,j}+\beta_o\cdot a^{1}_{k,j}}{(z-1)\cdot\alpha_o+\beta_o}\qquad j=1,...,n;k=d_j,...,g_j \tag{11}$$

$$HB^{0.5}_{k,j}=HB^{p}_{k,j}=HB^{o}_{k,j}=b_{k,j}=\frac{1}{m(j)}\cdot\sum_{i=1}^{m(j)}O^{j}_{k,i}\qquad j=1,...,n;k=d_j,...,g_j \tag{12}$$

The denominators in Equations (10)-(11) are introduced so that the final value of particular indices belongs to interval $[a^{z}_{k,j}, a^{1}_{k,j}]$. Denominators are not crucial – they can be omitted when formulating objective function (8).

The hybrid of the Hurwicz and Bayes decision rules has been already described in previous contributions [13, 16] and was originally designed for problems with n decisions, m scenarios and $n\cdot m$ payoffs. The general idea of H+B was to assign, for a pessimist, α to the last term of the non-increasing sequence of all payoffs related to a given decision and β to the remaining terms of that sequence. For an optimist, weights are set in a different way: β is connected with the first term of the sequence and α with the remaining ones (optimists expect the occurrence of the best scenario, so it should have the biggest weight). The goal of the hybrid presented in [13] is to recommend for a strong pessimist an alternative with a relatively high payoff $a_{j,min}$ (i.e. the lowest one for decision D_j) or with quite frequent payoffs (almost) equal to $a_{j,max}$ (the highest one). On the other hand, that rule suggests for a strong optimist an alternative with the highest (or almost the highest) payoff $a_{j,max}$, but its highest payoffs do not have to be frequent. For extreme optimists (β=1) and extreme pessimists (α=1) solutions recommended by that procedure are the same as those suggested by the classic Hurwicz rule. Due to possible diverse payoff dispersions in the case of RAP, we suggest to combine the original (H+B) with an additional criterion (range, standard deviation or other similar measures).

5. Illustrative example

In this Section we are going to solve an uncertain resource allocation problem presented in Table 2. There are 3 possible activities (e.g. stores) with 3, 5 and 4 scenarios, respectively. Potential resource quantities (e.g. sellers) for each activity belong to interval $[d_j, g_j]$=[0,5]. We assume that the payoff matrix was estimated (objectively) by experts and that a DM (with α=0.7) intends to allocate at most 8 units of resource. His (her) goal is to maximize the cumulative profit. The DM makes the decision on the basis of data gathered in Table 2.

Table 2

Cumulative payoff matrix for resource allocation problem with scenarios

Resource quantity	Activities											
	A_1			A_2					A_3			
	S^1_1	S^1_2	S^1_3	S^2_1	S^2_2	S^2_3	S^2_4	S^2_5	S^3_1	S^3_2	S^3_3	S^3_4
0	0	0	0	0	0	0	0	0	0	0	0	0
1	4	6	0	1	1	1	1	5	5	5	5	1
2	8	11	5	6	6	7	7	8	7	7	7	2
3	12	15	10	13	9	13	11	11	10	8	8	8
4	15	17	13	20	17	19	11	12	20	10	11	12
5	18	11	15	30	20	16	6	10	40	14	15	16

Source: Prepared by the author.

At the beginning, the 2-criteria (H+B) rule for RAP under uncertainty requires the computation of indices $HB_{k,j}$. The DM is a moderate pessimist since α=0.7. Hence, $\alpha=\alpha_p$, $\beta=\beta_p$. Non-increasing sequences are as follows: $Sq_{0,1}$=(0,0,0), $Sq_{1,1}$=(6,4,0), …, $Sq_{5,3}$=(40,16,15,14). There are 18 indices (see Table 3). Two of them are calculated below: $H_{1,2}=\frac{0.7\cdot 1+0.3\cdot(5+1+1+1)}{0.7+4\cdot 0.3}=1.63$, $H_{1,3}=\frac{0.7\cdot 1+0.3\cdot(5+5+5)}{0.7+3\cdot 0.3}=3.25$.

Table 3

Indices $H_{k,j}$ and ranges $R_{k,j}$

Resource quantity	Activities		
	A_1	A_2	A_3
0	0.00/ 0	0.00/ 0	0.00/ 0
1	2.31 / 6	1.63/ 4	3.25/ 4
2	7.08 / 6	6.63/ 2	4.81/ 5
3	11.62/ 5	10.89/ 4	8.37/ 2
4	14.38/ 4	14.79/ 9	12.44/ 10
5	13.54/ 7	14.21/ 24	19.44/ 26

Source: Prepared by the author.

Now, let us formulate and solve the following optimization model:

$$0x_{0,1}+2.31x_{1,1}+7.08x_{2,1}+\ldots+0x_{0,2}+1.63x_{1,2}+6.63x_{2,2}+\ldots+19.44x_{5,3}\rightarrow \max, \tag{13}$$

$$x_{0,1}+x_{1,1}+x_{2,1}+x_{3,1}+x_{4,1}+x_{5,1}=1, \quad \sum_{k=0}^{5}x_{k,2}=1, \quad \sum_{k=0}^{5}x_{k,3}=1, \tag{14}$$

$$x_{k,j}=\{0,1\} \qquad k=0,\ldots,5;\ j=1,2,3 \tag{15}$$

$$0 \cdot (x_{0,1} + x_{0,2} + x_{0,3}) + 1 \cdot \sum_{j=1}^{3} x_{1,j} + 2 \cdot \sum_{j=1}^{3} x_{2,j} + 3 \cdot \sum_{j=1}^{3} x_{3,j} + 4 \cdot \sum_{j=1}^{3} x_{4,j} + 5 \cdot \sum_{j=1}^{3} x_{5,j} \leq 8 \qquad (16)$$

$$0x_{0,1}, 6x_{1,1}, ..., 0x_{0,2}, 4x_{1,2}, ..., 26x_{5,3} \leq 0.3 \cdot (26 - 7.87) + 2 = 7.44 \qquad (17)$$

This model cannot be solved by means of such methods as dynamic programming (DP) or method of marginal profits (MMP) since, due to uncertain future cumulative outcomes and their diverse ranges, the suggested procedure assumes that the DM takes into account two criteria (weighted payoff and range of original payoffs). Therefore, we are going to apply a computer optimization tool. The model contains 18 binary variables, 1 linear objective function and 22 linear constraints. We use here SAS/OR (optmodel procedure), but it is not the only possible computer tool (see for instance CPLEX, R, minizinc). The optimal solution is $x_{4,1}$=1; $x_{3,2}$=1; $x_{1,3}$=1 (the remaining variables are equal to 0), which means that 4 units should be allocated to A_1, 3 units to activity A_2 and 1 unit to activity A_3: (4,3,1). For values lower than *b*=8 optimal results are as follows: *b*=1: (0,0,1), *b*=2: (2,0,0), *b*=3: (3,0,0), *b*=4: (3,0,1), *b*=5: (3,2,0), *b*=6: (3,3,0), *b*=7: (3,3,1).

6. Conclusions

The contribution contains a description of a decision rule applied to resource allocation problem under complete uncertainty on the assumption that payoffs for particular quantities of resource are asymmetric. Its goal is to maximize the profit resulting from allocating resources to different activities within limited resources and to take into account possible scenarios and the decision maker's nature. The use of that procedure may support the decision making process and improve the performance of an organization (project). Note that we assume in the paper that the cumulative payoff matrix is estimated by experts. If it is estimated by the decision maker himself, then the information concerning his nature is included in the matrix and the use of the suggested approach is not rational. The method presented in the article may be also preceded by an additional stage consisting in transforming original payoffs to utilities.

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Omówienie

Zagadnienie optymalnego rozdziału zasobu jest omawiane w wielu pracach zarówno dla przypadku deterministycznego (znane parametry), jak i stochastycznego (znane scenariusze oraz ich rozkład prawdopodobieństwa). W artykule została zaproponowana reguła decyzyjna umożliwiająca znalezienie odpowiedniej strategii w warunkach całkowitej niepewności, czyli sytuacji, w której znane są poszczególne stany natury, lecz decydent nie dysponuje informacją o prawdopodobieństwach. Procedura uwzględnia naturę decydenta, a także specyfikę poszczególnych zbiorów możliwych skumulowanych zysków.