

SAT-BASED BOUNDED MODEL CHECKING FOR TIMED INTERPRETED SYSTEMS AND THE RTECTLK PROPERTIES

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ABSTRACT

We define an SAT-based bounded model checking (BMC) method for RTECTLK (the existential fragment of the real-time computation tree logic with knowledge) that is interpreted over timed models generated by timed interpreted systems. Specifically, we translate the model checking problem for RTECTLK to the model checking problem for a variant of branching temporal logic (called E_y CTLK) interpreted over an abstract model, and we redefine an SAT-based BMC technique for E_y CTLK.

1. INTRODUCTION

The *Interpreted system* (IS) [5] is the formalism, which was designed to model multi-agent systems (MASs) [11], and to reason about the agents' epistemic and temporal properties. The *timed interpreted system* (TIS) [14] is the formalism that extends ISs to make feasible reasoning about real-time aspects of MASs. The TIS gives a computationally grounded semantics on which it is feasible to interpret both the time-bounded temporal modalities and the conventional epistemic modalities.

The fundamental thought of the SAT-based bounded model checking (BMC) systems [2, 10] comprises in translating the existential model checking problem for a modal logic and for a Kripke structure to the SAT problem [6], furthermore, exploiting the sophistication of present day SAT-solvers, i.e., programs (tools) that automatically decide whether a propositional formula is satisfiable.

To express the specifications of MASs different extensions of classic temporal logics [3] with epistemic [5], doxastic [7], and deontic (to represent the correct functioning behaviour) [9] modalities have been proposed. In this paper we consider RTECTLK, i.e., an epistemic extension of the existential fragment of the soft real-time CTL (RTECTL) [4], which is a propositional

branching-time temporal logic with bounded operators, and which was introduced to permit specification and reasoning about time-critical correctness properties. We interpret RTECTLK over *timed models* generated by timed interpreted systems.

A version of the SAT-based BMC method for specifications expressed in RTECTLK has been published in [12, 13]. However, the underlying model for RTECTLK was the interpreted system [5] with the asynchronous semantics (interleaving semantics). Here we use, as the underlying model for RTECTLK, the timed interpreted systems with the synchronous semantics, thus the agents over this semantics perform a joint action at a given time in a global state. Moreover, the RTECTLK properties cannot be expressed using nested applications of the next state operators.

In the paper we make the following contribution. We define the SAT-based BMC method for RTECTLK interpreted over timed models generated by timed interpreted systems. Specifically, we translate the model checking problem for RTECTLK to the model checking problem for a variant of branching temporal logic (called E_y CTLK) interpreted over an abstract model, and we redefine and improve the SAT-based BMC technique for E_y CTLK of [8]. The improvement of the SAT-based BMC [8] consists in utilizing the SAT-based BMC method for ECTL [15]. Its main idea is to translate every subformula ψ of the formula φ using only $f_k(\psi)$ paths of length k . So, our new BMC algorithm uses a reduced number of paths, what results in significantly smaller and less complicated propositional formulae that encode the RTECTLK properties.

The rest of the paper is organised as follows. In Section 2 we introduce the TIS and the RTECTLK logic. In Section 3 we show how to translate the model checking problem for RTECTLK to the model checking problem for E_y CTLK. In Section 4 we provide a BMC method for E_y CTLK and for ATIS. Finally in Section 5 we conclude the paper.

2. PRELIMINARIES

Let us start by fixing some notation used through the paper. \mathbb{N} is the set of non-negative integers, $\mathbb{N}_+ = \mathbb{N} \setminus \{0\}$, \mathcal{PV} is a set of propositional variables, and X is a finite set of non-negative integers variables, called *clocks*. A *clock valuation* is a function $v : X \rightarrow \mathbb{N}$ that assigns to each clock $x \in X$ a non-negative integer value $v(x)$. $\mathbb{N}^{|X|}$ is the set of all the clock valuations. For $X' \subseteq X$, the valuation $v' = v[X' := 0]$ is defined as: $\forall x \in X', v'(x) = 0$ and $\forall x \in X \setminus X', v'(x) = v(x)$. For $\delta \in \mathbb{N}$, $v + \delta$ denotes the valuation v' such that $\forall x \in X, v'(x) = v(x) + \delta$.

Let $x \in X$, $c \in \mathbb{N}$, and $\sim \in \{\leq, <, =, >, \geq\}$. The set $\mathcal{C}(X)$ of *clock constraints* over X is defined by the following grammar:

$$\phi := true \mid x \sim c \mid \phi \wedge \phi$$

Let v be a clock valuation, and $\phi \in \mathcal{C}(X)$. The satisfaction relation $v \models \phi$ is defined inductively with the following rules:

- $v \models true$,
- $v \models x \sim c$ iff $v(x) \sim c$,
- $v \models \phi \wedge \phi'$ iff $v \models \phi$ and $v \models \phi'$.

Finally, by the *time successor* of v (written $succ(v)$) we denote the clock valuation v' such that $\forall x \in X, v'(x) = v(x) + 1$.

Timed Interpreted Systems. Let $\mathcal{A} = \{1, \dots, n\}$ be the non-empty and finite set of agents, \mathcal{E} be a special agent that is used to model the environment in which the agents operate, and $\mathcal{PV} = \bigcup_{\mathbf{c} \in \mathcal{A}} \mathcal{PV}_{\mathbf{c}} \cup \mathcal{PV}_{\mathcal{E}}$ be a set of propositional variables such that $\mathcal{PV}_{\mathbf{c}_1} \cap \mathcal{PV}_{\mathbf{c}_2} = \emptyset$ for all $\mathbf{c}_1, \mathbf{c}_2 \in \mathcal{A} \cup \{\mathcal{E}\}$. The set of agents \mathcal{A} together with the environment constitute a multi-agent system (MAS), to model which we utilize the formalism of *timed interpreted system* (TIS).

In TIS, each agent $\mathbf{c} \in \mathcal{A}$ is modelled by:

- $L_{\mathbf{c}}$ - a non-empty and finite set of *local states*,
- $Act_{\mathbf{c}}$ - a non-empty and finite set of *possible actions* such that the special *null* action $\epsilon_{\mathbf{c}}$ belongs to $Act_{\mathbf{c}}$; it is assumed that actions are "public",
- $X_{\mathbf{c}}$ - a non-empty and finite set of *clocks*,
- $P_{\mathbf{c}} : L_{\mathbf{c}} \rightarrow 2^{Act_{\mathbf{c}}}$ - a *protocol function* that characterizes rules according to which actions may be performed in every local state,
- $t_{\mathbf{c}} : L_{\mathbf{c}} \times L_{\mathcal{E}} \times \mathcal{C}(X_{\mathbf{c}}) \times 2^{X_{\mathbf{c}}} \times Act \rightarrow L_{\mathbf{c}}$ with $Act = \prod_{\mathbf{c} \in \mathcal{A}} Act_{\mathbf{c}} \times Act_{\mathcal{E}}$ - a (partial) *evolution function* which defines local transitions; each element of Act and $\mathcal{C}(X_{\mathbf{c}})$ is called a *joint action* and an *enabling condition*, respectively,
- $\mathcal{V}_{\mathbf{c}} : L_{\mathbf{c}} \rightarrow 2^{\mathcal{PV}}$ - a *valuation function* which assigns to every local state a set of propositional variables that are assumed to be true at that state,
- $\mathcal{I}_{\mathbf{c}} : L_{\mathbf{c}} \rightarrow \mathcal{C}(X_{\mathbf{c}})$ - an *invariant function* which specifies the amount of time agent \mathbf{c} may spend in its local states.

We assume that if $\epsilon_{\mathbf{c}} \in P_{\mathbf{c}}(\ell_{\mathbf{c}})$, then $t_{\mathbf{c}}(\ell_{\mathbf{c}}, \ell_{\mathcal{E}}, \phi_{\mathbf{c}}, X, (a_1, \dots, a_n, a_{\mathcal{E}})) = \ell_{\mathbf{c}}$ for $a_{\mathbf{c}} = \epsilon_{\mathbf{c}}$, any $\phi_{\mathbf{c}} \in \mathcal{C}(X_{\mathbf{c}})$, and any $X \in 2^{X_{\mathbf{c}}}$. Finally, we assume that the sets of clocks are pairwise disjoint.

Correspondingly to the other agents, the environment \mathcal{E} is modelled by

- $L_{\mathcal{E}}$ - a non-empty and finite set of *local states*,
- $Act_{\mathcal{E}}$ - a non-empty and finite set of *possible actions*,
- $X_{\mathcal{E}}$ - a non-empty and finite set of *clocks*,

- $P_{\mathcal{E}} : L_{\mathcal{E}} \rightarrow 2^{Act_{\mathcal{E}}}$ - a protocol function,
- $t_{\mathcal{E}} : L_{\mathcal{E}} \times \mathcal{C}(X_{\mathcal{E}}) \times 2^{X_{\mathcal{E}}} \times Act \rightarrow L_{\mathcal{E}}$ - a (partial) *evolution function*,
- $\mathcal{V}_{\mathcal{E}} : L_{\mathcal{E}} \rightarrow 2^{\mathcal{P}\mathcal{V}_{\mathcal{E}}}$ - a *valuation function*,
- $\mathcal{I}_{\mathcal{E}} : L_{\mathcal{E}} \rightarrow \mathcal{C}(X_{\mathcal{E}})$ - and an *invariant function* which specifies the amount of time agent \mathcal{E} may spend in its local states.

It is assumed that local states, actions and clocks for \mathcal{E} are "public".

Let the symbol $S = \prod_{\mathbf{c} \in \mathcal{A} \cup \mathcal{E}} L_{\mathbf{c}} \times \mathbb{N}^{|X_{\mathbf{c}}|}$ denote the non-empty set of all *global states*, and $s = ((\ell_1, v_1), \dots, (\ell_n, v_n), (\ell_{\mathcal{E}}, v_{\mathcal{E}})) \in S$. Then, the symbols $l_{\mathbf{c}}(s) = \ell_{\mathbf{c}}$ and $v_{\mathbf{c}}(s) = v_{\mathbf{c}}$ denote, respectively, the local component and the clocks valuation of agent $\mathbf{c} \in \mathcal{A} \cup \{\mathcal{E}\}$ in the global state s . Finally, given a set of agents \mathcal{A} , the environment \mathcal{E} , and a set of initial global states $\iota \subseteq S$ such that for all $\mathbf{c} \in \mathcal{A} \cup \{\mathcal{E}\}$ and for all $x \in X_{\mathbf{c}}$ it holds $v_{\mathbf{c}}(x) = 0$, a *timed interpreted system* (TIS) is a tuple

$$\mathbb{I} = (\{L_{\mathbf{c}}, Act_{\mathbf{c}}, X_{\mathbf{c}}, P_{\mathbf{c}}, t_{\mathbf{c}}, \mathcal{V}_{\mathbf{c}}, \mathcal{I}_{\mathbf{c}}\}_{\mathbf{c} \in \mathcal{A} \cup \{\mathcal{E}\}}, \iota)$$

For a given time interpreted system \mathbb{I} we define a *timed model* as a tuple

$$M = (\Sigma, \iota, S, T, \mathcal{V}) :$$

- $\Sigma = Act \cup \mathbb{N}$ is the set of labels (i.e., joint actions and natural numbers),
- S and $\iota \in S$ are defined as above,
- $\mathcal{V} : S \rightarrow 2^{\mathcal{P}\mathcal{V}}$ is the valuation function defined as $\mathcal{V}(s) = \bigcup_{\mathbf{c} \in \mathcal{A}} \mathcal{V}_{\mathbf{c}}(l_{\mathbf{c}}(s))$,
- $T \subseteq S \times (Act \cup \mathbb{N}) \times S$ is a transition relation defined by action and time transitions:
 - (1) Action transition: for any $\bar{a} \in Act$, $(s, \bar{a}, s') \in T$ iff for all $\mathbf{c} \in \mathcal{A}$, there exists a local transition $t_{\mathbf{c}}(l_{\mathbf{c}}(s), l_{\mathcal{E}}(s), \phi_{\mathbf{c}}, X', \bar{a}) = l_{\mathbf{c}}(s')$ such that $v_{\mathbf{c}}(s) \models \phi_{\mathbf{c}} \wedge \mathcal{I}(l_{\mathbf{c}}(s))$ and $v'_{\mathbf{c}}(s') = v_{\mathbf{c}}(s)[X' := 0]$ and $v'_{\mathbf{c}}(s') \models \mathcal{I}(l_{\mathbf{c}}(s'))$, and there exists a local transition $t_{\mathcal{E}}(l_{\mathcal{E}}(s), \phi_{\mathcal{E}}, X', \bar{a}) = l_{\mathcal{E}}(s')$ such that $v_{\mathcal{E}}(s) \models \phi_{\mathcal{E}} \wedge \mathcal{I}(l_{\mathcal{E}}(s))$ and $v'_{\mathcal{E}}(s') = v_{\mathcal{E}}(s)[X' := 0]$ and $v'_{\mathcal{E}}(s') \models \mathcal{I}(l_{\mathcal{E}}(s'))$.
 - (2) Time transition: let $\delta \in \mathbb{N}$, $(s, \delta, s') \in T$ iff for all $\mathbf{c} \in \mathcal{A} \cup \{\mathcal{E}\}$, $l_{\mathbf{c}}(s) = l_{\mathbf{c}}(s')$ and $v_{\mathbf{c}}(s) \models \mathcal{I}(l_{\mathbf{c}}(s))$ and $v'_{\mathbf{c}}(s') = v_{\mathbf{c}}(s) + \delta$ and $v'_{\mathbf{c}}(s') \models \mathcal{I}(l_{\mathbf{c}}(s))$.

We assume that the relation T is total, i.e. for any $s \in S$ there exists $s' \in S$ and there exist either a non-empty joint action $\bar{a} \in Act$ or natural number $\delta \in \mathbb{N}$ such that it holds $T(s, \bar{a}, s')$ or $T(s, \delta, s')$.

Given a time interpreted system \mathbb{I} one can define the indistinguishability relation $\sim_{\mathbf{c}} \subseteq S \times S$ for agent \mathbf{c} as follows: $s \sim_{\mathbf{c}} s'$ iff $l_{\mathbf{c}}(s') = l_{\mathbf{c}}(s)$ and $v_{\mathbf{c}}(s') = v_{\mathbf{c}}(s)$.

Let M be a timed model generated by a TIS \mathbb{I} . A *run* of \mathbb{I} is an infinite sequence $\rho = s_0 \xrightarrow{\delta_0, \bar{a}_0} s_1 \xrightarrow{\delta_1, \bar{a}_1} s_2 \xrightarrow{\delta_2, \bar{a}_2} \dots$ of global states such that the following conditions hold for all $i \in \mathbb{N}$: $s_i \in S$, $\bar{a}_i \in Act$, $\delta_i \in \mathbb{N}_+$, and there exists $s'_i \in S$ such that $(s_i, \delta_i, s'_i) \in T$ and $(s'_i, \bar{a}_i, s_{i+1}) \in T$. Note

that the definition of the run does not allow two consecutive joint actions to be performed one after the other, i.e., between each two joint actions some time must pass.

The symbol $\Pi_{\mathbb{I}}(s)$ denotes the set of all the runs in \mathbb{I} that start at the state s . $\Pi = \bigcup_{s^0 \in \iota} \Pi_{\mathbb{I}}(s^0)$.

RTECTLK. Let $p \in \mathcal{PV}$, $\mathbf{c} \in \mathcal{A}$, $\Gamma \subseteq \mathcal{A}$, and I be an interval in \mathbb{N} of the form: $[a, b)$ or $[a, \infty)$, for $a, b \in \mathbb{N}$ and $a \neq b$. The existential fragment of RTCTL with knowledge (RTECTLK) is defined by the following grammar:

$$\begin{aligned} \varphi := & \top \mid \perp \mid p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \\ & \text{E}(\varphi \text{U}_I \varphi) \mid \text{E} \text{G}_I \varphi \mid \overline{\text{K}}_{\mathbf{c}} \varphi \mid \overline{\text{E}}_{\Gamma} \varphi \mid \overline{\text{D}}_{\Gamma} \varphi \mid \overline{\text{C}}_{\Gamma} \varphi \end{aligned}$$

The symbol E (*for some path*) is the path quantifier. The symbols U_I (*bounded until*) and G_I (*bounded globally*) are temporal modalities. The derived basic temporal modalities for *bounded eventually* and *bounded release* are defined as follows: $\text{EF}_I \varphi \stackrel{\text{def}}{=} \text{E}(\top \text{U}_I \varphi)$, $\text{E}(\varphi \text{R}_I \psi) \stackrel{\text{def}}{=} \text{E}(\psi \text{U}_I (\psi \wedge \varphi)) \vee \text{EG}_I \psi$. Hereafter, if the interval I is of the form $[0, \infty)$, then we omit it for the simplicity of the presentation. The symbols $\overline{\text{K}}_{\mathbf{c}}$ (agent \mathbf{c} considers possible), $\overline{\text{E}}_{\Gamma}$ (possibly everyone in Γ knows), $\overline{\text{D}}_{\Gamma}$ (possible distributed knowledge in the group Γ), and $\overline{\text{C}}_{\Gamma}$ (possible common knowledge among agents in Γ) are the dualities to standard epistemic modalities.

To define the satisfiability relation for RTECTLK, we define the notion of a *discrete path* λ_{ρ} *corresponding to run* ρ (this can be done in a unique way because of the assumption that the runs are strongly monotonic), and we assume the following definitions of epistemic relations: $\sim_{\Gamma}^{E \text{def}} \stackrel{\text{def}}{=} \bigcup_{\mathbf{c} \in \Gamma} \sim_{\mathbf{c}}$,

$\sim_{\Gamma}^{C \text{def}} \stackrel{\text{def}}{=} (\sim_{\Gamma}^E)^+$ (the transitive closure of \sim_{Γ}^E), $\sim_{\Gamma}^{D \text{def}} \stackrel{\text{def}}{=} \bigcap_{\mathbf{c} \in \Gamma} \sim_{\mathbf{c}}$, where $\Gamma \subseteq \mathcal{A}$.

Let $\Delta_0 = [b_0, b_1)$, $\Delta_1 = [b_1, b_2)$, \dots be the sequence of pairwise disjoint intervals, where: $b_0 = 0$ and $b_i = b_{i-1} + \delta_{i-1}$ if $i > 0$. For each $t \in \mathbb{N}$, let $\text{id}x_{\rho}(t)$ denote the unique index i such that $t \in \Delta_i$. A *path* λ_{ρ} *corresponding to* ρ is a mapping $\lambda_{\rho} : \mathbb{N} \rightarrow \mathcal{S}$ such that $\lambda_{\rho}(t) = ((\ell_1^i, v_1^i + t - b_i), \dots, (\ell_n^i, v_n^i + t - b_i))$, $(\ell_{\mathcal{E}}^i, v_{\mathcal{E}}^i + t - b_i) = s_i + t - b_i$, where $i = \text{id}x_{\rho}(t)$.

Let $Y \in \{\text{D}, \text{E}, \text{C}\}$. The *satisfiability* relation \models , which indicates truth of a RTECTLK formula in the timed model M at state s , is defined inductively with the classical rules for propositional operators and with the following rules for the temporal and epistemic modalities:

$$\begin{aligned} M, s \models \text{E}(\alpha \text{U}_I \beta) & \text{ iff } (\exists \rho \in \Pi_{\mathbb{I}}(s)) (\exists i \in I) (M, \lambda_{\rho}(i) \models \beta \text{ and} \\ & \quad (\forall 0 \leq j < i) M, \lambda_{\rho}(j) \models \alpha) \\ M, s \models \text{EG}_I \alpha & \text{ iff } (\exists \rho \in \Pi_{\mathbb{I}}(s)) (\forall i \in I) (M, \lambda_{\rho}(i) \models \alpha) \\ M, s \models \overline{\text{K}}_{\mathbf{c}} \alpha & \text{ iff } (\exists \rho \in \Pi_{\mathbb{I}}) (\exists i \geq 0) (s \sim_{\mathbf{c}} \lambda_{\rho}(i) \text{ and } M, \lambda_{\rho}(i) \models \alpha) \\ M, s \models \overline{Y}_{\Gamma} \alpha & \text{ iff } (\exists \rho \in \Pi_{\mathbb{I}}) (\exists i \geq 0) (s \sim_{\Gamma}^Y \lambda_{\rho}(i) \text{ and } M, \lambda_{\rho}(i) \models \alpha) \end{aligned}$$

An RTECTLK formula φ *holds* in the model M (denoted $M \models \varphi$) iff $M, s^0 \models \varphi$ for some state $s^0 \in \iota$. The *model checking problem* asks whether $M \models \varphi$.

3. FROM RTECTLK TO E_yCTLK

The translation of the model checking problem for RTECTLK to the model checking problem for E_yCTLK, a language defined below and interpreted over an *abstract model* for an *augmented timed interpreted system* is based on [8], where the translation of the model checking problem for the existential part of TCTL [1] augmented with knowledge (TECTLK) with a dense-time semantics defined over timed automata to the model checking problem for E_yCTLK with a semantics defined over the region graph has been introduced.

We start by defining *augmented timed interpreted systems* (ATIS) for a given timed interpreted system $\mathbb{I} = (\{L_{\mathbf{c}}, Act_{\mathbf{c}}, X_{\mathbf{c}}, P_{\mathbf{c}}, t_{\mathbf{c}}, \mathcal{V}_{\mathbf{c}}, \mathcal{I}_{\mathbf{c}}\}_{\mathbf{c} \in \mathcal{AU}\{\mathcal{E}\}}, \iota)$, and an RTECTLK formula φ .

Let m be the number of intervals appearing in φ . Then, an ATIS \mathbb{I}_{φ} is defined as the following tuple

$$(\{L_{\mathbf{c}}, \mathcal{V}_{\mathbf{c}}, \mathcal{I}_{\mathbf{c}}\}_{\mathbf{c} \in \mathcal{AU}\{\mathcal{E}\}}, \{Act_{\mathbf{c}}, X_{\mathbf{c}}, P_{\mathbf{c}}, t_{\mathbf{c}}\}_{\mathbf{c} \in \mathcal{A}}, Act'_{\mathcal{E}}, X'_{\mathcal{E}}, P'_{\mathcal{E}}, t'_{\mathcal{E}}) :$$

- $Act'_{\mathcal{E}} = Act_{\mathcal{E}} \cup \{a_y\}$, where a_y is a new action corresponding to setting to zero a new clock y .
- $X'_{\mathcal{E}} = X_{\mathcal{E}} \cup \{y\}$, where the new clock y corresponds to all the intervals appearing in φ ; one clock is sufficient to perform the BMC algorithm that is presented in the next section.
- $P'_{\mathcal{E}} : L_{\mathcal{E}} \rightarrow 2^{Act'_{\mathcal{E}}}$ is an extension of the protocol function $P_{\mathcal{E}} : L_{\mathcal{E}} \rightarrow 2^{Act_{\mathcal{E}}}$ such that $\{a_y\} \subseteq P'_{\mathcal{E}}(\ell)$ for all $\ell \in L_{\mathcal{E}}$.
- $t'_{\mathcal{E}} : L_{\mathcal{E}} \times \mathcal{C}(X'_{\mathcal{E}}) \times 2^{X'_{\mathcal{E}}} \times Act' \rightarrow L_{\mathcal{E}}$ is an extension of $t_{\mathcal{E}}$ such that $Act' = \prod_{i=1}^n Act_i \times Act'_{\mathcal{E}}$ and $t'_{\mathcal{E}}(\ell_{\mathcal{E}}, true, \{a_y\}, (\epsilon_1, \dots, \epsilon_n, a_y)) = \ell_{\mathcal{E}}$.

An abstract model for ATIS. Let φ be an RTECTLK formula, $\mathcal{PV}' = \mathcal{PV} \cup \{p_{y \in I} \mid I \text{ is an interval in } \varphi\}$, and $\mathbb{I}_{\varphi} = (\{L_{\mathbf{c}}, Act_{\mathbf{c}}, X_{\mathbf{c}}, P_{\mathbf{c}}, t_{\mathbf{c}}, \mathcal{V}_{\mathbf{c}}, \mathcal{I}_{\mathbf{c}}\}_{\mathbf{c} \in \mathcal{AU}\{\mathcal{E}\}}, \iota)$ be an ATIS. The *abstract model* for \mathbb{I}_{φ} is a tuple $M_{\varphi} = (\Sigma_{\varphi}, \iota, S_{\varphi}, T_{\varphi}, \mathcal{V}_{\varphi})$, where

- $\Sigma_{\varphi} = Act \cup \{\tau\}$, where $Act = \prod_{\mathbf{c} \in \mathcal{AU}\{\mathcal{E}\}} Act_{\mathbf{c}}$,
- $S_{\varphi} = \prod_{\mathbf{c} \in \mathcal{AU}\{\mathcal{E}\}} L_{\mathbf{c}} \times \mathbb{N}^{|X_{\mathbf{c}}|}$ is the set of all possible global states,
- $\mathcal{V}_{\varphi} : S_{\varphi} \rightarrow 2^{\mathcal{PV}'}$ is the valuation function such that:
 - (1) $p \in \mathcal{V}_{\varphi}(s)$ iff $p \in \bigcup_{\mathbf{c} \in \mathcal{AU}\{\mathcal{E}\}} \mathcal{V}_{\mathbf{c}}(l_{\mathbf{c}}(s))$ for all $p \in \mathcal{PV}$,
 - (2) $p_{y \in I} \in \mathcal{V}_{\varphi}(((\ell_1, v_1), \dots, (\ell_n, v_n), (\ell_{\mathcal{E}}, v_{\mathcal{E}})))$ iff $v_{\mathcal{E}}(y) \in I$,
- $T_{\varphi} \subseteq S_{\varphi} \times \Sigma_{\varphi} \times S_{\varphi}$ is a transition relation defined by action and time transitions. Let $\bar{a} \in Act$:

1. Action transition: $(s, \bar{a}, s') \in T_\varphi$ iff $(\forall \mathbf{c} \in \mathcal{A}) (\exists \phi_{\mathbf{c}} \in \mathcal{C}(X_{\mathbf{c}}))$
 $(\exists X'_{\mathbf{c}} \subseteq X_{\mathbf{c}}) (t_{\mathbf{c}}(l_{\mathbf{c}}(s), l_{\mathcal{E}}(s), \phi_{\mathbf{c}}, X'_{\mathbf{c}}, \bar{a}) = l_{\mathbf{c}}(s')$ and $v_{\mathbf{c}}(s) \models \phi_{\mathbf{c}} \wedge$
 $\mathcal{I}(l_{\mathbf{c}}(s))$ and $v'_{\mathbf{c}}(s') = v_{\mathbf{c}}(s)[X'_{\mathbf{c}} := 0]$ and $v'_{\mathbf{c}}(s') \models \mathcal{I}(l_{\mathbf{c}}(s'))$)
and $(\exists \phi_{\mathcal{E}} \in \mathcal{C}(X_{\mathcal{E}})) (\exists X'_{\mathcal{E}} \subseteq X_{\mathcal{E}}) (t'_{\mathcal{E}}(l_{\mathcal{E}}(s), \phi_{\mathcal{E}}, X'_{\mathcal{E}}, \bar{a}) = l_{\mathcal{E}}(s')$
and $v_{\mathcal{E}}(s) \models \phi_{\mathcal{E}} \wedge \mathcal{I}(l_{\mathcal{E}}(s))$ and $v'_{\mathcal{E}}(s') = v_{\mathcal{E}}(s)[X'_{\mathcal{E}} := 0]$ and
 $v'_{\mathcal{E}}(s') \models \mathcal{I}(l_{\mathcal{E}}(s'))$)
2. Time transition: $(s, \tau, s') \in T_\varphi$ iff $(\forall \mathbf{c} \in \mathcal{A} \cup \{\mathcal{E}\}) (l_{\mathbf{c}}(s) = l_{\mathbf{c}}(s')$
and $v_{\mathbf{c}}(s) \models \mathcal{I}(l_{\mathbf{c}}(s))$ and $v'_{\mathbf{c}}(s') = \text{succ}(v_{\mathbf{c}}(s))$ and $v'_{\mathbf{c}}(s') \models$
 $\mathcal{I}(l_{\mathbf{c}}(s))$).

Note that each transition is followed by a possible reset of new clocks. This is to ensure that the new clocks can be reset along the evolution of the system any time it is needed.

Given an augmented time interpreted system \mathbb{I}_φ one can define the indistinguishability relation $\sim_{\mathbf{c}} \subseteq S_\varphi \times S_\varphi$ for agent \mathbf{c} as follows: $s \sim_{\mathbf{c}} s'$ iff $l_{\mathbf{c}}(s) = l_{\mathbf{c}}(s')$ and $v_{\mathbf{c}}(s) = v_{\mathbf{c}}(s')$.

The E_y CTLK language. In order to translate a RTECTLK formula φ into the corresponding E_y CTLK formula ψ we map the RTECTLK language into E_y CTLK by reinterpreting the temporal operators, denoted by $E_y U$ and $E_y G$. Formally, for $p \in \mathcal{PV}'$, $\mathbf{c} \in \mathcal{A}$, and $\Gamma \subseteq \mathcal{A}$, the E_y CTLK formulae are defined by the following grammar:

$$\varphi := \top \mid \perp \mid p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid E_y(\varphi U \varphi) \mid E_y G \varphi \mid \bar{K}_{\mathbf{c}} \varphi \mid \bar{E}_\Gamma \varphi \mid \bar{D}_\Gamma \varphi \mid \bar{C}_\Gamma \varphi$$

In addition, we introduce some useful derived temporal modalities:

- $E_y(\varphi R \psi) \stackrel{\text{def}}{=} E_y(\psi U(\varphi \wedge \psi)) \vee E_y G \psi$ (*release*),
- $E_y F \varphi \stackrel{\text{def}}{=} E_y(\top U \varphi)$ (*eventually*).

The E_y CTLK formulae are interpreted over the abstract model M_φ . Let $T_{\bar{\Gamma}}$ denote the part of T_φ , where transitions are labelled with elements of $\text{Act} \cup \{\tau\}$, and T_y denotes the transitions that reset the clock y .

Definition 1. A path π in M_φ is a sequence $\pi = (s_0, s_1, \dots)$ of states such that $(s_0, \tau, s_1) \in T_{\bar{\Gamma}}$, and for each $i > 0$, either $(s_i, \bar{a}_i, s_{i+1}) \in T_{\bar{\Gamma}}$ or $(s_i, \tau, s_{i+1}) \in T_{\bar{\Gamma}}$, and if $(s_i, \bar{a}_i, s_{i+1}) \in T_{\bar{\Gamma}}$ holds, then $(s_{i+1}, \tau, s_{i+2}) \in T_{\bar{\Gamma}}$ holds, and $\bar{a}_i \in \text{Act}$ for each $i \geq 0$.

Observe that the above definition of the path ensures that the first transition is the time one, and between each two action transitions at least one time transition appears.

For a path π , $\pi(i)$ denotes the i -th state s_i of π . $\Pi_\varphi(s)$ denotes the set of all the paths starting at $s \in S_\varphi$, and $\Pi_\varphi = \bigcup_{s^0 \in S_\varphi} \Pi_\varphi(s^0)$.

The satisfiability relation \models , which indicates truth of ψ in M_φ at state s (in symbols $M_\varphi, s \models \psi$), is defined inductively with the classical rules for

propositional operators and with the following rules for the temporal and epistemic modalities:

- $M_\varphi, s \models E_y(\alpha U \beta)$ iff $(\exists s' \in S_\varphi)((s, (\epsilon_1, \dots, \epsilon_n, a_y), s') \in T_y$ and $(\exists \pi \in \Pi_\varphi(s'))(\exists m \geq 0) [M_\varphi, \pi(m) \models \beta$ and $(\forall j < m) M_\varphi, \pi(j) \models \alpha])$,
- $M_\varphi, s \models E_y G \alpha$ iff $(\exists s' \in S_\varphi)((s, (\epsilon_1, \dots, \epsilon_n, a_y), s') \in T_y$ and $(\exists \pi \in \Pi_\varphi(s'))(\forall m \geq 0) M_\varphi, \pi(m) \models \alpha)$,
- $M_\varphi, s \models \overline{K}_c \alpha$ iff $(\exists \pi \in \Pi_\varphi)(\exists m \geq 0)(s \sim_c \pi(m)$ and $M_\varphi, \pi(m) \models \alpha)$,
- $M_\varphi, s \models \overline{Y}_\Gamma \alpha$ iff $(\exists \pi \in \Pi_\varphi)(\exists m \geq 0)(s \sim_\Gamma^Y \pi(m)$ and $M_\varphi, \pi(m) \models \alpha)$.

An E_y CTLK formula φ is *valid on* M_φ (denoted $M_\varphi \models \varphi$) iff $M_\varphi, s^0 \models \varphi$ for some $s^0 \in \iota$, i.e., φ is true at some initial state of the model M_φ .

Having defined syntax and semantics of the E_y CTLK logic, we can now introduce the translation mentioned above. An RTECTLK formula φ is translated inductively into the E_y CTLK formula $\mathcal{H}(\varphi)$ as follows:

- $\mathcal{H}(p) = p$ if $p \in \mathcal{PV}'$, $\mathcal{H}(\neg p) = \neg p$ if $p \in \mathcal{PV}'$,
- $\mathcal{H}(\alpha \vee \beta) = \mathcal{H}(\alpha) \vee \mathcal{H}(\beta)$, $\mathcal{H}(\alpha \wedge \beta) = \mathcal{H}(\alpha) \wedge \mathcal{H}(\beta)$,
- $\mathcal{H}(EG_I \alpha) = E_y G(\neg p_{y \in I} \vee \mathcal{H}(\alpha))$,
- $\mathcal{H}(E(\alpha U_I \beta)) = E_y(\mathcal{H}(\alpha) U(\mathcal{H}(\beta) \wedge p_{y \in I}))$,
- $\mathcal{H}(\overline{K}_c \alpha) = \overline{K}_c \mathcal{H}(\alpha)$, $\mathcal{H}(\overline{Y}_\Gamma \alpha) = \overline{Y}_\Gamma \mathcal{H}(\alpha)$, where $Y \in \{D, E, C\}$.

The main theorem of the section states that the validity of the RTECTLK formula φ over the timed model is equivalent to the validity of the corresponding E_y CTLK formula $\mathcal{H}(\varphi)$ over the abstract model. The proof of the theorem can be completed by an induction of the formula φ .

Theorem 1. *Let M be the timed model, φ an RTECTLK formula, and M_φ the abstract model. Then, $M \models \varphi$ iff $M_\varphi \models \mathcal{H}(\varphi)$.*

4. AN SAT-BASED BMC METHOD FOR E_y CTLK

Bounded semantics. Let $M_\varphi = (\Sigma_\varphi, \iota, S_\varphi, T_\varphi, \mathcal{V}_\varphi)$ be an abstract model, $k \in \mathbb{N}$, and $0 \leq l \leq k$. As before, we denote by $T_{\mathbb{I}}$ the subset of T_φ , where transitions are labelled with elements of $Act \cup \{\tau\}$, and by T_y the set of transitions resetting the clock y .

Definition 2. *A k -path π is a finite sequence $\pi = (s_0, \dots, s_k)$ of states such that $(s_0, \tau, s_1) \in T_{\mathbb{I}}$, and for each $0 < i < k$, either $(s_i, \bar{a}_i, s_{i+1}) \in T_{\mathbb{I}}$ or $(s_i, \tau, s_{i+1}) \in T_{\mathbb{I}}$, and if $(s_i, \bar{a}_i, s_{i+1}) \in T_{\mathbb{I}}$ holds, then $(s_{i+1}, \tau, s_{i+2}) \in T_{\mathbb{I}}$ holds, and $\bar{a}_i \in Act$ for each $0 \leq i < k$.*

The symbol $\Pi_k(s)$ denotes the set of all the k -paths starting at s in M_φ , and $\Pi_k = \bigcup_{s^0 \in \iota_\varphi} \Pi_k(s^0)$.

Definition 3. *Let $\pi(i) = ((\ell_1^i, v_1^i), \dots, (\ell_n^i, v_n^i), (\ell_\mathcal{E}^i, v_\mathcal{E}^i))$ for all $i \leq k$. A k -path $\pi = (\pi(0), \dots, \pi(k))$ is a loop if there exists $0 \leq l < k$ and $(\forall \mathbf{c} \in \mathcal{A} \cup \{\mathcal{E}\})(\ell_\mathbf{c}^k = \ell_\mathbf{c}^l)$ and $(\forall \mathbf{c} \in \mathcal{A})(v_\mathbf{c}^k = v_\mathbf{c}^l)$ and $v_{\mathcal{E} \downarrow X_\mathcal{E}}^k = v_{\mathcal{E} \downarrow X_\mathcal{E}}^l$, where $\downarrow X_\mathcal{E}$*

denoted the projection of the clock valuation $v_{\mathcal{E}} : X_{\mathcal{E}} \cup \{y\} \rightarrow \mathbb{N}$ on the clock valuation $v'_{\mathcal{E}} : X_{\mathcal{E}} \rightarrow \mathbb{N}$.

Satisfaction of the temporal operator $E_y G$ on a k -path π in the bounded case depends on whether or not π is a loop. Therefore, we assume a function $loop : \Pi_k \mapsto 2^{\mathbb{N}}$ which returns the set of all the indices of the states for which there is a transition from the last state of a k -path π . Note that if a k -path is a loop, then it represents an *infinite* path.

The *bounded satisfiability* relation \models_k , which indicates truth of ψ in M_{φ} at state s (denoted $M_{\varphi}, s \models_k \psi$) is defined inductively with the classical rules for propositional operators and with the following rules for the temporal and epistemic modalities:

- $M_{\varphi}, s \models_k E_y(\alpha U \beta)$ iff $(\exists s' \in S_{\varphi})((s, (\epsilon_1, \dots, \epsilon_n, a_y), s') \in T_y$ and $(\exists \pi \in \Pi_k(s'))(\exists 0 \leq m \leq k) (M_{\varphi}, \pi(m) \models_k \beta$ and $(\forall j < m) M_{\varphi}, \pi(j) \models \alpha)$),
- $M_{\varphi}, s \models_k E_y G \alpha$ iff $(\exists s' \in S_{\varphi})((s, (\epsilon_1, \dots, \epsilon_n, a_y), s') \in T_y$ and $(\exists \pi \in \Pi_k(s'))(\forall 0 \leq j \leq k) (M_{\varphi}, \pi(j) \models \alpha$ and $loop(\pi) \neq \emptyset)$),
- $M_{\varphi}, s \models \bar{K}_{\mathbf{c}} \alpha$ iff $(\exists \pi \in \Pi_k)(\exists 0 \leq m \leq k)(s \sim_{\mathbf{c}} \pi(m)$ and $M_{\varphi}, \pi(m) \models \alpha)$,
- $M_{\varphi}, s \models \bar{Y}_{\Gamma} \alpha$ iff $(\exists \pi \in \Pi_k)(\exists 0 \leq m \leq k)(s \sim_{\Gamma}^Y \pi(m)$ and $M_{\varphi}, \pi(m) \models \alpha)$, where $Y \in \{D, E, C\}$.

We use the following notation $M_{\varphi} \models_k \psi$ iff $M_{\varphi}, s^0 \models_k \psi$ for some $s^0 \in \iota_{\varphi}$. The *bounded model checking problem* consists in finding out whether there exists $k \in \mathbb{N}$ such that $M_{\varphi} \models_k \psi$.

The following theorem shows that for some particular bound the bounded and unbounded semantics are equivalent.

Theorem 2. *Let φ be an RTECTLK formula, M_{φ} an abstract model, and $\psi = \mathcal{H}(\varphi)$ an E_y CTLK formula. The following equivalence holds: $M_{\varphi} \models \varphi$ iff there exists $k \geq 0$ such that $M_{\varphi} \models_k \psi$.*

Translation to SAT. Let M_{φ} be an abstract model, ψ an E_y CTLK formula, and $k \geq 0$ a bound. The presented propositional encoding of the BMC problem for E_y CTLK improves the BMC encoding of [8] and it is based on the BMC encoding of [16]. It relies on defining the propositional formula $[M_{\varphi}, \psi]_k := [M_{\varphi}^{\psi, \iota}]_k \wedge [\psi]_{M_{\varphi}, k}$, which is satisfiable if and only if $M_{\varphi} \models_k \psi$ holds.

The definition of $[M_{\varphi}, \psi]_k$ assumes that both the states and the joint actions of M_{φ} are encoded symbolically. This is possible, since both the set of states and the set of joint actions are finite. Also, since we work with a set of k -paths, we can bound the clocks valuation to the set $\mathbb{D} = \{0, \dots, c + 1\}$ with c being the largest constant appearing in any enabling condition or state invariants of all the agents and in intervals appearing in φ . Moreover, this definition assumes knowledge of the number of k -paths of M_{φ} that are sufficient to validate ψ . To this aim, as usually, we define the auxiliary

function $f_k : E_y\text{CTLK} \rightarrow \mathbb{N}$: $f_k(\top) = f_k(\perp) = f_k(p) = f_k(\neg p) = 0$, where $p \in \mathcal{PV}'$; $f_k(\alpha \wedge \beta) = f_k(\alpha) + f_k(\beta)$; $f_k(\alpha \vee \beta) = \max\{f_k(\alpha), f_k(\beta)\}$; $f_k(E_y(\alpha \cup \beta)) = k \cdot f_k(\alpha) + f_k(\beta) + 1$; $f_k(E_y G \alpha) = (k + 1) \cdot f_k(\alpha) + 1$; $f_k(\overline{C}_\Gamma \alpha) = f_k(\alpha) + k$; $f_k(Y \alpha) = f_k(\alpha) + 1$ for $Y \in \{\overline{K}_c, \overline{D}_\Gamma, \overline{E}_\Gamma\}$.

Formally, let $\mathbf{c} \in \mathcal{A}$. We assume that each state $s \in S_\varphi$ is represented by a *symbolic state* $\mathbf{w} = ((\mathbf{w}_1, \mathbf{v}_1) \dots, (\mathbf{w}_n, \mathbf{v}_n), (\mathbf{w}_\mathcal{E}, \mathbf{v}_\mathcal{E}))$, where each symbolic local state $(\mathbf{w}_c, \mathbf{v}_c)$ is a pair of vectors of propositional variables; the first element encodes local states of L_c and the second element encodes the clock valuations over \mathbb{D} . Next, we assume that each join action $\bar{a} \in \text{Act}$ is represented by a *symbolic action* $\mathbf{a} = (\mathbf{a}_1, \dots, \mathbf{a}_n, \mathbf{a}_\mathcal{E})$, where each symbolic local action \mathbf{a}_c is a vector of propositional variables. Moreover, we assume that the time action τ is represented by a proposition variable \wp_τ . Finally, we assume a symbolic representation of a k -path π , the number of which is j , and we call it the j -th symbolic k -path $\pi_j = (\mathbf{w}_{0,j}, \dots, \mathbf{w}_{k,j})$, where $0 \leq j < f_k(\psi)$, $0 \leq i \leq k$, and $\mathbf{w}_{i,j}$ is a symbolic state.

Let \mathbf{w} and \mathbf{w}' be two different symbolic states, and \mathbf{a} a symbolic action. We assume definitions of the following auxiliary propositional formulae:

- $p(\mathbf{w})$ - encodes the set of states of M_φ in which $p \in \mathcal{PV}'$ holds.
- $I_s(\mathbf{w})$ - encodes the state s of M_φ .
- $H_c(\mathbf{w}, \mathbf{w}')$ - encodes the equality of two local states and two local clock valuations of agent $\mathbf{c} \in \mathcal{A}$.
- $H(\mathbf{w}, \mathbf{w}') := \bigwedge_{\mathbf{c} \in \mathcal{A} \cup \{\mathcal{E}\}} H_c(\mathbf{w}, \mathbf{w}')$ - encodes equality of two global states.
- $\mathcal{T}(\mathbf{w}, \mathbf{a}, \mathbf{w}')$ is a formula over \mathbf{w} , \mathbf{w}' , and \mathbf{a} , which is true for valuations $s_{\mathbf{w}}$ of \mathbf{w} , $s_{\mathbf{w}'}$ of \mathbf{w}' , and $s_{\mathbf{a}}$ of \mathbf{a} iff either $(s_{\mathbf{w}}, s_{\mathbf{a}}, s_{\mathbf{w}'}) \in T_{\mathbb{I}}$ or $(s_{\mathbf{w}}, \wp_\tau, s_{\mathbf{w}'}) \in T_{\mathbb{I}}$ (encodes non-resetting transitions of M_φ).
- $\mathcal{T}_y(\mathbf{w}, \mathbf{w}')$ is a formula over \mathbf{w} and \mathbf{w}' , which is true for two valuations $s_{\mathbf{w}}$ of \mathbf{w} and $s_{\mathbf{w}'}$ of \mathbf{w}' iff $(s_{\mathbf{w}}, (\epsilon_1, \dots, \epsilon_n, a_y), s_{\mathbf{w}'}) \in T_y$ (encodes transitions resetting the clock y).

Let $F_k(\psi) = \{j \in \mathbb{N} \mid 1 \leq j \leq f_k(\psi)\}$, $\mathbf{w}_{i,j}$ and $\mathbf{a}_{i,j}$ be, respectively, symbolic states and symbolic actions, for $0 \leq i \leq k$ and $j \in F_k(\psi)$. The formula $[M_\varphi^{\psi, \iota}]_k$, which encodes the unfolding of the transition relation of M_φ $f_k(\psi)$ -times to the depth k , is defined as follows:

$$[M_\varphi^{\psi, \iota}]_k := \bigvee_{s \in \iota} I_s(\mathbf{w}_{0,0}) \wedge \bigwedge_{n=1}^{f_k(\psi)} \bigwedge_{m=0}^{k-1} \mathcal{T}(\mathbf{w}_{m,n}, \mathbf{a}_{m,n}, \mathbf{w}_{m+1,n}).$$

The next step is a translation of a $E_y\text{CTLK}$ formula ψ to a propositional formula $[\psi]_{M_\varphi, k} := [\psi]_k^{[0,0, F_k(\psi)]}$, where $[\alpha]_k^{[m,n,A]}$ denotes the translation of α at the symbolic state $\mathbf{w}_{m,n}$ by using the set $A \subseteq F_k(\psi)$. To define $[\psi]_k^{[0,0, F_k(\psi)]}$, we have to know how to divide the set $F_k(\psi)$ into subsets

needed for translating the subformulae of ψ . To accomplish this goal we use some auxiliary functions ($g_l, g_r, g_s, h_k^U, h_k^G$) that were defined in [16].

Let M_φ be a model, ψ a E_yCTLK formula, and $k \geq 0$ a bound. The formula $[\psi]_k^{[0,0,F_k(\psi)]}$ that encodes the bounded semantics for E_yCTLK is inductively defined as shown below.

Namely, let $0 \leq n < f_k(\psi)$, $m \leq k$, $n' = \min(A)$, $h_k^U = h_k^U(g_s(A), f_k(\beta))$, and $h_k^G = h_k^G(g_s(A), f_k(\alpha))$.

$$\begin{aligned}
[\top]_k^{[m,n,A]} &:= \top, [\perp]_k^{[m,n,A]} := \perp, \\
[p]_k^{[m,n,A]} &:= p(w_{m,n}), [\neg p]_k^{[m,n,A]} := \neg p(w_{m,n}), \\
[\alpha \wedge \beta]_k^{[m,n,A]} &:= [\alpha]_k^{[m,n,g_l(A,f_k(\alpha))]} \wedge [\beta]_k^{[m,n,g_r(A,f_k(\beta))]}, \\
[\alpha \vee \beta]_k^{[m,n,A]} &:= [\alpha]_k^{[m,n,g_l(A,f_k(\alpha))]} \vee [\beta]_k^{[m,n,g_l(A,f_k(\beta))]}, \\
[E_y(\alpha U \beta)]_k^{[m,n,A]} &:= \mathcal{T}_y(\mathbf{w}_{m,n}, \mathbf{w}_{0,n'}) \wedge \bigvee_{i=0}^k ([\beta]_k^{[i,n',h_k^U(k)]} \wedge \bigwedge_{j=0}^{i-1} [\alpha]_k^{[j,n',h_k^U(j)]}), \\
[E_y G \alpha]_k^{[m,n,A]} &:= \mathcal{T}_y(\mathbf{w}_{m,n}, \mathbf{w}_{0,n'}) \wedge \bigwedge_{j=0}^k [\alpha]_k^{[j,n',h_k^G(k)]} \wedge \bigvee_{l=0}^k H(\mathbf{w}_{k,n'}, \mathbf{w}_{l,n'}), \\
[\overline{K} \alpha]_k^{[m,n,A]} &:= (\bigvee_{s \in \iota} I_s(\mathbf{w}_{0,n'})) \wedge \bigvee_{j=0}^k ([\alpha]_k^{[j,n',g_\mu(A)]} \wedge H_{\mathbf{c}}(\mathbf{w}_{m,n}, \mathbf{w}_{j,n'})), \\
[\overline{D} \Gamma \alpha]_k^{[m,n,A]} &:= (\bigvee_{s \in \iota} I_s(\mathbf{w}_{0,n'})) \wedge \bigvee_{j=0}^k ([\alpha]_k^{[j,n',g_\mu(A)]} \wedge \bigwedge_{\mathbf{c} \in \Gamma} H_{\mathbf{c}}(\mathbf{w}_{m,n}, \mathbf{w}_{j,n'})), \\
[\overline{E} \Gamma \alpha]_k^{[m,n,A]} &:= (\bigvee_{s \in \iota} I_s(\mathbf{w}_{0,n'})) \wedge \bigvee_{j=0}^k ([\alpha]_k^{[j,n',g_\mu(A)]} \wedge \bigvee_{\mathbf{c} \in \Gamma} H_{\mathbf{c}}(\mathbf{w}_{m,n}, \mathbf{w}_{j,n'})), \\
[\overline{C} \Gamma \alpha]_k^{[m,n,A]} &:= [\bigvee_{j=1}^k (\overline{E} \Gamma)^j \alpha]_k^{[m,n,A]}.
\end{aligned}$$

The following theorem guarantees that the BMC problem for E_yCTLK and for an augmented timed interpreted system can be reduced to the SAT-problem. The theorem can be proven by induction on the length of the formula ψ .

Theorem 3. *Let M_φ be an abstract model, and ψ an E_yCTLK formula. For every $k \in \mathbb{N}$, $M_\varphi \models_k \psi$ if, and only if, the propositional formula $[M_\varphi, \psi]_k$ is satisfiable.*

5. CONCLUSIONS

We have defined an SAT-based BMC for timed interpreted system and for properties expressed in RTECTLK. The method is based on a translation of the model checking problem for RTECTLK to the model checking problem

for $E_y\text{CTLK}$, and then on the translation of the model checking problem for $E_y\text{CTLK}$ to the SAT-problem.

In [8] a formalism of real time interpreted systems (RTIS) has been defined to model MASs with hard real-time deadlines and an SAT-based BMC for the existential version of the timed CTLK (TECTLK) has been defined. However, in contrast to the semantics adopted in this work, the semantics of the RTIS model is asynchronous, the agents are just pure timed automata, and the $E_y\text{CTLK}$ logic is interpreted on the region graph for timed automata.

In the future, we plan to implement and experimentally evaluate the proposed SAT-based BMC. Next, we plan to define SMT-based BMC for TIS and for RTECTLK, and compare it with the SAT-based one.

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