

## MATHEMATICAL MODEL OF ATTITUDE CONTROL BUCKET-WHEEL EXCAVATOR

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### Abstract:

This lecture deals with the application problems of convertibility GPS system at paddle excavator K 800. The claims of the modern operating surface mining of the excavators requires a lot of information for monitoring of mining process, capacity mining, selective extraction etc. The utilization of monitoring the excavator setting by GPS system proved to be the only one proper because the receivers are resistant to the vibration, dust, temperature divergence and weather changeable. Only the direct contact with communications satellite is required. It means that they can't be located in a metal construction space (shadow caused by construction elements, influence of electrical high voltage cables) even they can't be located close to the paddle wheel on the paddle boom (shadow possibility caused by cutting edge created during lower gangplanks mining). This is the reason that GPS receivers are set uppermost on the metal construction excavator and the mathematical formulation is required for determination of paddle wheel petting. The relations for calculation of the paddle wheel coordinate were defined mathematically and after that the mathematical model was composed.

**Key words:** GPS system, excavator K 800, GPS receivers, mathematical model

### INTRODUCTION

We need exactly determine the location of paddle axes possibly the lower edge of paddle to get objective control of mining process. The global convertible systems allow accomplish the requirements that are set for the control of paddle wheel excavator. This systems work with precision  $\pm 3$  cm even at vertical axis nowadays.

The problems of the application GPS systems at paddle excavators for surface mining are put on technology and technique itself. The machines usually work deep under the horizon terrain which markedly makes worse the signal receiving. When we place GPS receiver at the paddle boom

near the paddle wheel, the quality of signal is affected by near high voltage electrical cables and significant vibration of paddle boom. The receivers must be placed uppermost on the metal construction but because of that the mathematical formulation for determination of paddle wheel is considerably complicated. The geometrical parameters of machine must be considered which most of them are variable. The inclination of machine has impact for that significantly which change during every machine transposition.

This thesis deals with GPS application at paddle excavator K800/N1/103 (Fig. 1) that is fixed at coal cutting DNT Tušimice from the point of view mathematical relation between GPS data and paddle wheel position [3, 4, 5].



Fig. 1 Bucket-wheel excavator K800/N1/103



Fig. 2 The point cloud

## MEASUREMENT

### Data processing

By registering, we combined point clouds from individual scanning positions and placed them in the chosen coordinate system. To connect individual frames, control points were used, focussing on classical geodetic methods. Control points are scanned during measurement

with higher density, automatically calculating the exact position in space. Levelling and error analysis can be done from redundant control points. The average results of the analysis give the limits of identifying control elements after transformation to a value of 3 mm in space.

The maximum correction to control elements then has the value of 6 mm in the area.

Series data taken from individual positions were combined into one unit and at the same time, unwanted objects and surrounding terrain were cut out (Fig. 2). Point clouds, which formed the structure of the excavator bucket wheel contained 600 million points.

Subject to subsequent evaluation, there was determined the spatial relationship between the turntable axis telescopic arm, the turntable axis of the wheel and the front and rear GPS antenna devices located on the excavator.

Using the modular system Cyclone, there were modeled, the individual details and intended intersections of the wheel axis with the axis of the telescopic arm, intersections of the shoulder rotating axis with the axis of the retractable shoulder and the GPS antenna reference points. The wheel circumference was also determined. One of the outputs is a 3D drawing of the relationship between the rotating axes and GPS devices (Fig. 3).

The data series gathered from different scanner stations were combined (Fig. 2) and the following step was data filtering and data processing. The CloudWorx for AutoCAD application allows simple tools to be used for the processing of large point clouds, such as selecting sections of the point cloud. Time consuming and demanding for computer hardware as the processing of large data volumes is, it is crucial to select appropriate point field density in each point cloud section. In this application, the main structural elements of the excavators were gradually vectorized, until a generalized 3D model was created (Fig. 3).

Selected geometric parameters obtained from laser scanning are checked against the parameters obtained on the basis of Geodesy, GPS and inclinometer measurements (Table 1).

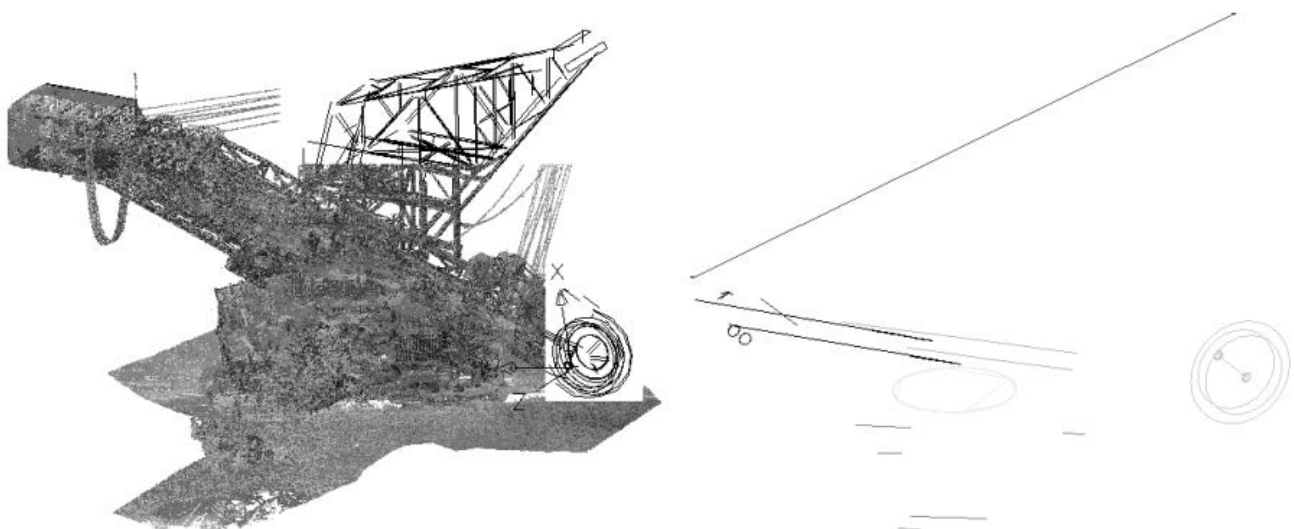


Fig. 3 Bucket-wheel excavator vector model

Table 1

K800/N1/103 bucket-wheel excavator geometric parameters, in [m]

Dimension	Symbol	Geodesic surveying, GPS, inclinometers	Laser scanning	Difference
Bucket-wheel boom length	$l_v$	35.966	35.95	-0.02
Distance from IRC centre, having a point of intersection on GPS1 vertical, to a parallel with the boom running through IRC centre	$l_{po}$	7.557	7.57	+0.01
Vertical distance of GPS1 sensor from the joint of bucket-wheel boom in upper position	$h_{l_0}$		4.29	
Vertical distance between bucket-wheel centre and IRC centre	$h_2$		10.2	
GPS1 head over the notional point of intersection on GPS1 vertical determining I	$h_a$	1.770	1.8	+0.03
Horizontal distance of GPS1 sensor from the joint of bucket-wheel boom in upper position	$l_{l_0}$		7.16	
Horizontal distance between GPS sensors	$h_{GPS}$		41.18	
Vertical distance between GPS sensors with excavator in horizontal position	$Z_{GPS_0}$			
Vertical distance between GPS sensors	$Z_{GPS}$		12.4	
Bucket-wheel axis distance from GPS1	$L_0$		41.64	
Bucket-wheel boom travel hoist angle	$\alpha$	19.648°	19.2°	-0.4°
Vertical distance between GPS1 sensor and ball bearing slewing ring	$Z_{IRC}$		12.43	
Bucket-wheel max. diameter to teeth edge	$D_k$		7.52	
GPS sensor positions relative to excavator vertical axis – bucket-wheel boom joint distance from excavator axis in upper position	$l_3$		13.29	
GPS sensor lengthwise positions – distance from excavator lengthwise plane				0.01
Ball bearing slewing ring head above undercarriage bottom edge	$Z_{KD}$		6.8	-0.0

**ESTABLISHING KEY BUCKET-WHEEL EXCAVATOR GEOMETRIC PARAMETERS**

Key bucket-wheel excavator geometric parameters were derived from the 3D vector model by means of the Microstation V8 software and with the help of auxiliary dimension-giving elements added to the vector model. The abbreviation IRC is a marked sensor of the Incremental Rotary Encoder. Movement of the wheel boom causes movement of its joint (IRC) on the beam, which records the number of encoder pulses. The number of pulses can be subsequently translated along the length of the extended boom using an impulse conversion constant.

Geometric parameters were referred to by the following machine features:

- PS receiver locations;
- Bucket-wheel;
- Centre of the ball-bearing slewing ring;
- Bucket-wheel boom hoisting direction;
- Boom travel rails;
- IRC (incremental sensor positions);
- Undercarriage bottom edge.

GPS sensor vertical distance  $Z_{GPS}$  indicates excavator lengthwise tilt. What is needed is the GPS sensor vertical distance  $Z_{GPS_0}$  relative to the excavator in absolutely horizontal position.

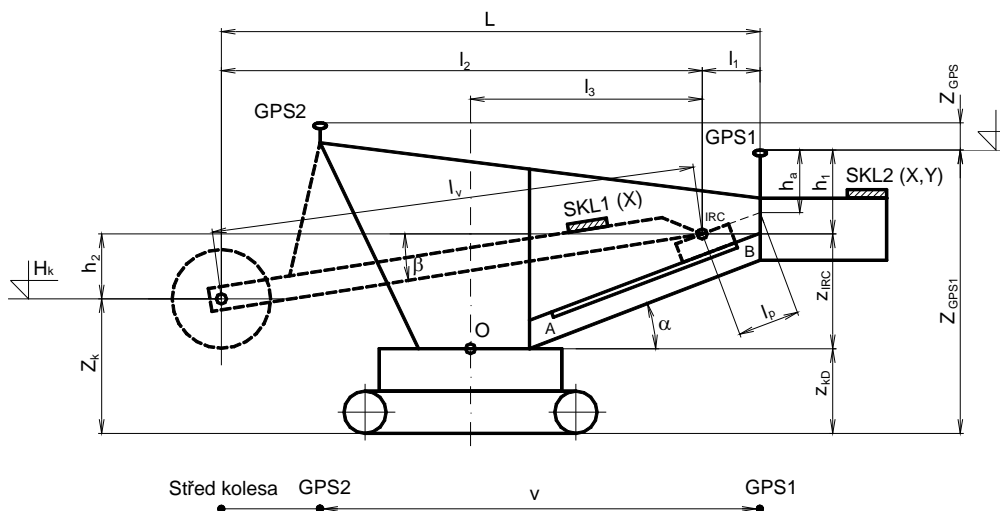


Fig. 4 Bucket-wheel excavator sketch

**DATA EVALUATION [1, 2, 4, 6]**

A suitable mathematical model is required to calculate the 3D position of the centre of the bucket wheel from the data described in the previous section. The definition of the bucket-wheel position is based on GPS sensor data. The formula definitions are based on Figure 4.

The parameters to calculate include:

- Bucket-wheel geodesic head;
- Bucket wheel-to-machine axis horizontal distance;
- Bucket-wheel horizontal incline from vertical plane running through GPS1;
- 3D bucket wheel position.

All variables and some constant parameters will change with the slewing motion while excavator superstructure is off horizontal, i.e. almost every time. This is why the impact of excavator tilt on length projections in the horizontal and vertical planes must be accounted for. Let us now carry out a rough calculation of what the tilt impact on the lengths is. Different excavators feature different max values of lengthwise and crosswise working tilt determined by individual design. For max tilt values in selected excavators see Table 2

**Table 2**  
**Geometric parameters**

Type of Excavator	Lengthwise tilt	Cross tilt
K 10000	1:14.3 = 7%	
KU 800	7%	
K 2000	5.6%	
K 800 N	3%	
KU 300	14%	5%

The 3% tilt in case of K 800/N1/103 corresponds to an angle of 1.72°. The excavator superstructure design does not allow bigger tilts without compromising machine stability. Let us calculate the maximum inclinations for the superstructure axis and for its perpendicular plane for maximum allowable tilt:

**a) Horizontal inclination:**

Difference between maximum distance of bucket wheel from GPS1 sensor in horizontal plane and at maximum tilt of superstructure axis. The result is 0.025 m.

**b) Vertical inclination:**

GPS1 sensor shift from the horizontal plane at maximum tilt perpendicular to superstructure axis relative to the plane of travel. The result is 0.011 m. The resulting differences are not very big, but the horizontal inclination will be taken account for in the following four scenarios of relative positions of the bucket-wheel boom and the excavator:

- I. Bucket-wheel excavator in horizontal position
- II. Bucket-wheel excavator inclined with bucket wheel below horizontal plane intersecting IRC centre ( $|\gamma| < |\beta|$ )
- III. a) Bucket wheel below horizontal plane (sign  $\beta =$  sign  $\gamma$ )  
b) Bucket wheel above horizontal plane (sign  $\beta = -$  sign  $\gamma$ )

Regarding calculation formulas, the present article will only show formulas with input data and after derivation of formulas. The input data descriptions are based on Table 1.

*Option I – Bucket wheel excavator in horizontal position (Fig. 4)*

The following calculation is based on Figure 4 and Table 1.

$\gamma$  – excavator tilt in horizontal plane:

$$\gamma = SKL_2 = 0$$

$\beta$  – SKL<sub>1</sub> inclinometer angle reading; absolute value will be used:

$$\beta = SKL_1$$

$\alpha$  – bucket-wheel boom travel tilt angle:

$$\alpha = 19.151^\circ$$

$l_1$  – horizontal distance between GPS<sub>1</sub> and bucket-wheel boom joint in deliberate position:

$$l_1 = l_p \cdot \cos \alpha$$

$l_p$  – distance between IRS centre with point of intersection on GPS<sub>1</sub> vertical and between a parallel line to the beam running through IRC centre:

$$l_p = l_{p0} + IRC \cdot \frac{12,03}{40423}$$

$l_2$  – horizontal distance between IRC and bucket-wheel axis:

$$l_2 = l_v \cdot \cos \beta$$

$L$  – bucket-wheel axis distance from GPS<sub>1</sub>:

$$L = l_1 + l_2$$

$$L = l_p \cdot \cos \alpha + l_v \cdot \cos \beta$$

$$L = \left( l_{p0} + IRC \cdot \frac{12,03}{40423} \right) \cdot \cos \alpha + l_v \cdot \cos \beta$$

The Z coordinate of bucket-wheel centre comes from:

$$ZK = Z1 - (h_1 + h_2)$$

$$Z1 = Z_{GPS1}$$

$$h_1 = h_a + l_p \cdot \sin \alpha$$

$$h_1 = 1,804 + \left( 7,575 + IRC \cdot \frac{12,03}{40423} \right) \cdot \sin \alpha$$

$$h_2 = l_v \cdot \sin \beta$$

$$h_2 = 35,952 \cdot \sin \beta$$

By substituting we obtain a general formula for the Z coordinate of the bucket-wheel centre as follows:

ZK – geodesic gead of bucket-wheel axis

Z – coordinate of bucket-wheel centre

$$ZK = Z1 - h_a - \left( l_{p0} + IRC \cdot \frac{12,03}{40423} \right) \cdot \sin \alpha - l_v \cdot \sin \beta$$

*Option II – Bucket-wheel excavator not in horizontal position with bucket wheel below horizontal plane intersecting IRC centre ( $|\gamma| < |\beta|$ ) (Fig. 5)*

Now the L value is required to calculate the X and Y coordinates.

$L$  – bucket-wheel axis distance from GPS<sub>1</sub>:

$$L = l_p \cos(\alpha + \gamma) - h_a \sin \gamma + l_v \cos \beta$$

$$ZK = Z1 - h_a \cos \gamma - l_p \sin(\alpha + \gamma) - l_v \sin \beta$$

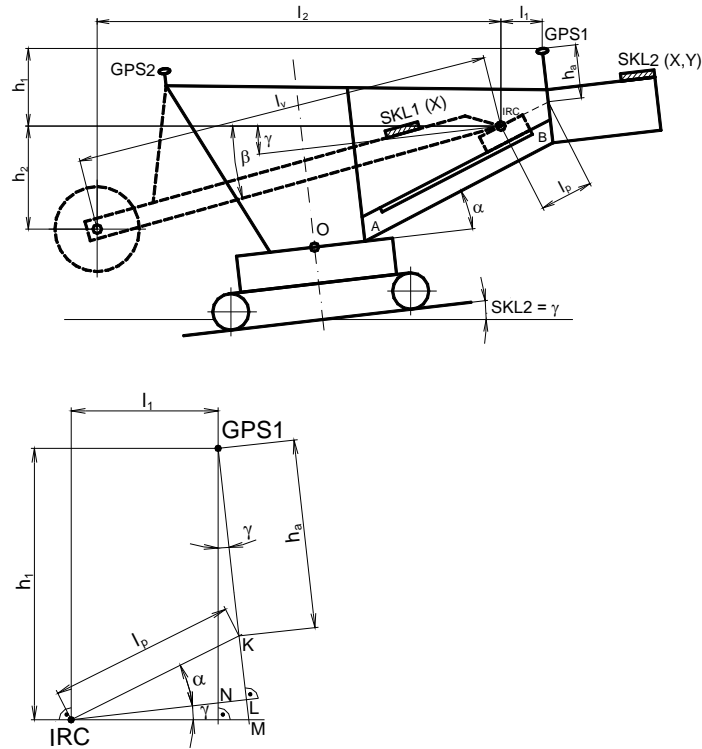


Fig. 5 Nclined bucket-wheel excavator with bucket-wheel below IRC plane ( $|\gamma| < |\beta|$ ) and IRC joint detail image

Option III Bucket-wheel excavator in inclined positions, III. a), III. b)

$$L = h_a \cdot \sin \gamma + l_p \cdot \cos(\alpha - \gamma) + l_v \cdot \cos \beta$$

III. a) Bucket wheel below horizontal plane ( $\text{sign} \beta = \text{sign} \gamma$ ) (Fig. 6).

The Z coordinate calculation will be different for position 1 with the bucket wheel below the horizontal plane and  $\text{sgn} \beta = -\text{sgn} \gamma$ .

Thus:

$$ZK = Z1 - h_1 - h_2$$

$$ZK = Z1 - h_a \cdot \cos \gamma - \left( l_{p0} + IRC \cdot \frac{12,03}{40423} \right) \cdot \sin(\alpha - \gamma) - l_v \cdot \sin \beta$$

III. b) Bucket wheel above horizontal plane ( $\text{sign} \beta = -\text{sign} \gamma$ ) (Fig. 7).

In position 2, bucket wheel above horizontal plane and  $\text{sgn} \beta = -\text{sgn} \gamma$ , in absolute values  $|\beta| < |\gamma|$ , we get:

$$ZK = Z1 - h_1 - h_2$$

$$ZK = Z1 - h_a \cdot \cos \gamma - \left( l_{p0} + IRC \cdot \frac{12,03}{40423} \right) \cdot \sin(\alpha - \gamma) - l_v \cdot \sin \beta$$

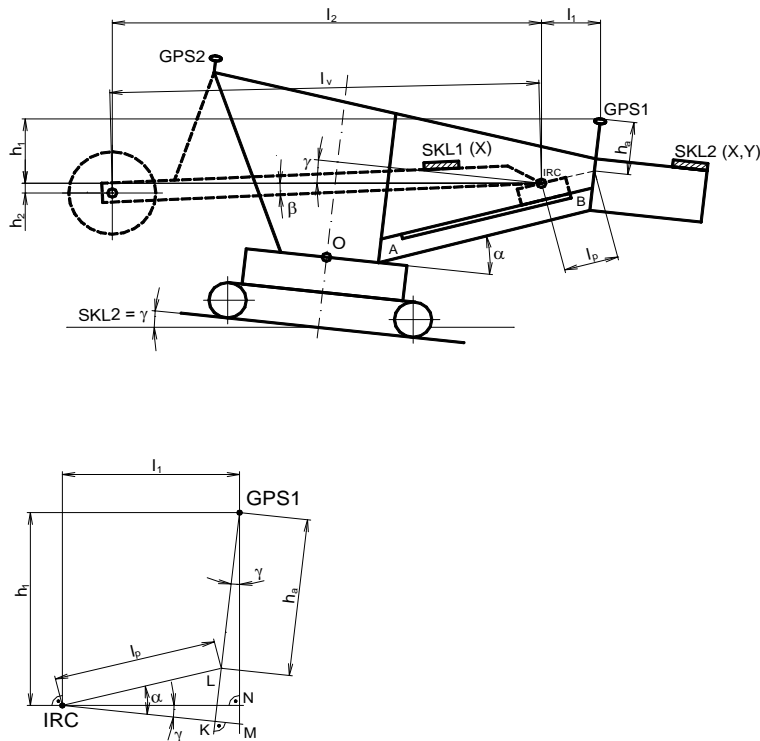


Fig. 6 Bucket wheel below horizontal plane  $\text{sign} \beta = \text{sign} \gamma$

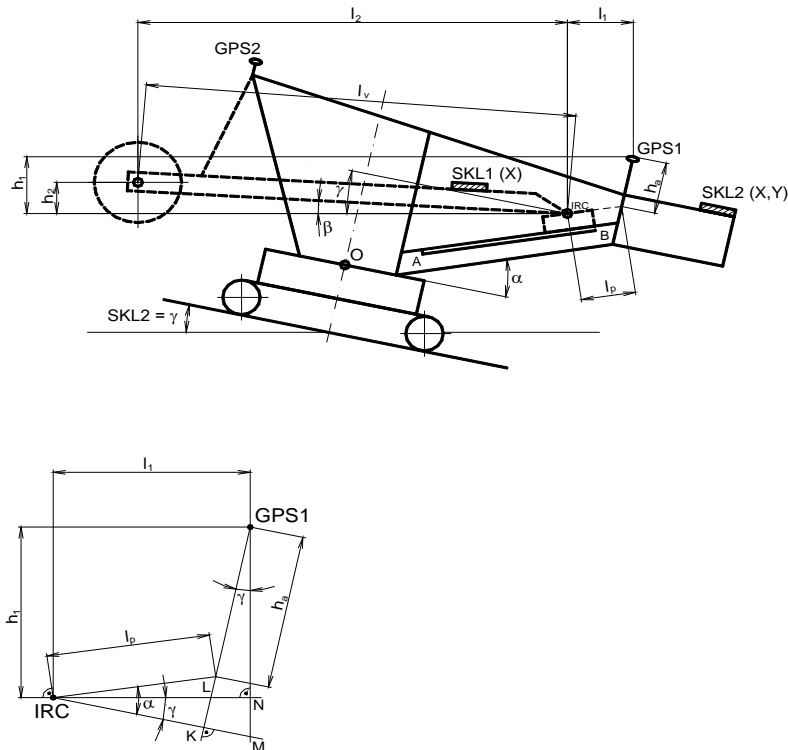


Fig. 7 Bucket wheel below horizontal plane sign  $\theta = \text{sign } \gamma$

Bucket-wheel centre X and Y coordinate calculation (Fig. 8)

The two sensors, GPS1 and GPS2, form a straight line represented by the following formula:

$$p: \quad x = X1 + (X2 - X1) \cdot t$$

$$y = Y1 + (Y2 - Y1) \cdot t$$

for GPS1:

$$t_1 = 0 : \quad x = X1 \quad , \quad y = Y1$$

for GPS2:

$$t_2 = 1 : \quad x = X2 \quad , \quad y = Y2$$

Distance GPS1 to GPS2:

$$v = |GPS1, GPS2| = \sqrt{(X2 - X1)^2 + (Y2 - Y1)^2}$$

K is the bucket-wheel centre.

Bucket-wheel axis distance from GPS1:

$$L = |GPS1, K|$$

Bucket-wheel centre parameter  $t_k$  comes from the following rule of proportion:

$$v \dots \dots \dots t = t_2 - t_1 = 1$$

$$L \dots \dots \dots t_k = ?$$

$$t_k = \frac{L}{v}$$

Substituting the result in the p-line parametric formula, we get the X and Y coordinates of bucket-wheel centre K as follows:

$$XK = X1 + (X2 - X1) \cdot t_k$$

$$YK = Y1 + (Y2 - Y1) \cdot t_k$$

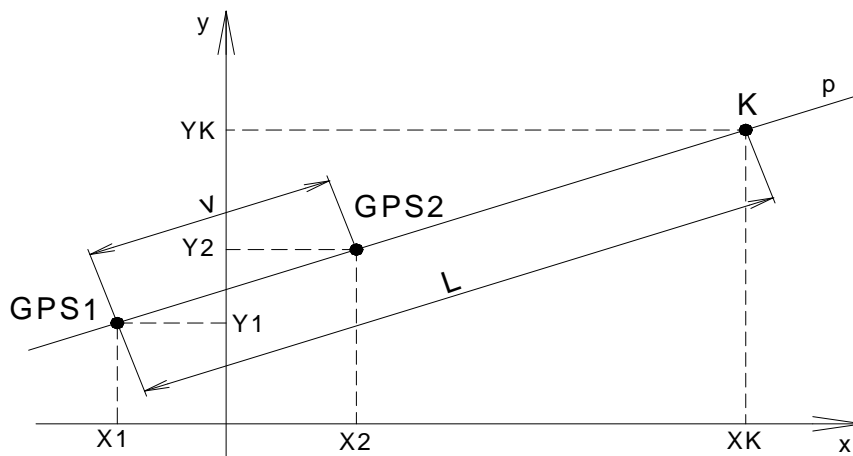


Fig. 8 GPS1, GPS2 and bucket-wheel K centre horizontal plan

*Proposed mathematical model*

The mathematical model is based on geometric dimensions and on mathematical formulas shown in the preceding sections using the following input data:

- GPS1 receiver data;
- GPS2 receiver data;
- IRC incremental sensor data;
- SKL1 bucket-wheel boom mounted inclinometer;
- SKL2 support-frame mounted inclinometer;
- Excavator geometric data.

The output consists of the following bucket-wheel centre coordinates:

$$XK = X1 + (X2 - X1) \cdot t_k$$

$$YK = Y1 + (Y2 - Y1) \cdot t_k$$

$$ZK = Z1 - h_1 + h_2$$

**CONCLUSIONS**

A mathematical model describing bucket-wheel excavator movement in 3D space was built on the basis of 3D laser scanning and on additional data measurements. The mathematical model was processed by means of the MATLAB software. The exercise also aims at creating a useful technique for the surveying of bucket-wheel excavators. Such data will enable creating 3D visualisations

of bucket-wheel excavator positions required to monitor the quality of extracted coal in real-time control of the excavation process.

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