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Identification of complex technical systems operation processes

Keywords

system operation, semi-markov process, statistical identification, transportation systems

Abstract

There is presented the contents of the training course addressed to industry. The curriculum of the course includes the methods, algorithms and procedures for identification of the unknown parameters of the operation processes of the complex technical systems and their application in practice. It is based on theoretical backgrounds concerned with the semi-markov modelling of the complex technical systems operation processes, the statistical methods of identification of the complex technical systems operation processes. The illustrations of the proposed methods and procedures practical application in port, shipyard and maritime transport sector are included.

1. Introduction

The training course is concerned with the methods, algorithms and procedures of identification of the operation processes of the complex technical systems and their application in practice and it is based on the results given in [5] and [1]. The participants of the course are provided training materials and a disk with the computer program included in [2]. Presented at the training course examples of practical applications are coming from [3]-[4] and [6]-[7].
The training course includes the following items:

- Theoretical backgrounds based on [5]: mathematical model of the complex technical system operation process and its basic parameters and characteristics;
- Methodology of description of the complex technical systems: fixing the system designation and operation conditions, fixing the system subsystems and components;
- Methodology of defining parameters of the system operation process based on [1]: fixing the number of disjoin operation states of the system, defining the operation states of the system, fixing the possible transitions between the system operation states, fixing the set of the unknown parameters of the system operation process model;
- Procedure of the system operation process data collection based on [1]: fixing the experiment duration time, defining the system operation process single realization, fixing the number of the observed realizations of the system operation process, fixing the numbers of staying of the system operation process in the particular operation states at the initial moments of all observed realizations of the system operation process, fixing the numbers of the transitions between the system operation process states during all observed realizations of the system operation process, fixing the numbers of departures of the system operation process from the particular operation states during all observed realizations of the system operation process, fixing the realizations of the conditional sojourn times of the system operations process at the particular operation states in all observed realizations of the system operation process;
- Procedure of evaluating the unknown parameters of the system operation process

based on [1]: fixing the realizations of the probabilities of the initial states of the system operation process, fixing the realizations of the probabilities of the system operation process transitions between the operation states during the experiment time, evaluating the unknown parameters of distributions of the conditional sojourn time of the system operation process in the particular operation states for typical distributions;

- Procedure of identifying the distributions of the system conditional sojourn times in the operation states for the fixed in [5] distributions based on [1]: constructing and plotting the realization of the histogram of the system conditional sojourn times at the particular operation states, analyzing the realization of the histograms, comparing them with the graphs of the density functions of the previously distinguished typical distributions and formulating the hypotheses concerning the unknown form of the distribution functions of the conditional sojourn times in the particular operation states, verifying (testing) the hypotheses concerning the unknown form of the distribution functions of the conditional sojourn times in the particular operation states;
- Procedure of identifying the mean values of the system conditional sojourn times in the operation states for the fixed in [5] distributions based on [1]: determining the mean values of the system conditional sojourn times in the operation states for the fixed in [5] distributions, determining approximate empirical values of the system conditional sojourn times in the operation states;
- Procedure of applying the computer program for identification of the system operation process based on [2];
- Application of the procedures and computer program for identification of the operations processes of real complex technical systems: identification of the operation process of the port oil piping transportation system based on [6], identification of the operation process of the shipyard ship-rope elevator based on [3], identification of the operation process of the shipyard ground ship-rope transportation system based on [4], identification of the operation process of the ferry technical system based on [7].

2. Theoretical backgrounds

Training material is given in [5].

3. Procedures of identification of the operation process of the complex technical system

3.1. Methodology of description of the complex technical system

The description of the complex technical systems should include at least the following items:
- the system designation,
- the system operation conditions,
- the system subsystem and components.

3.2. Methodology of defining the parameters of the system operation process

To make the estimation of the unknown parameters of the system operations process the experiment delivering the necessary statistical data should be precisely planned.
Firstly, before the experiment, we should perform the following preliminary steps:

i) to analyze the system operation process;

ii) to fix or to define its following general parameters:

- the number of the operation states of the system operation process ν,

- the operation states of the system operation process $z_1, z_2, \ldots, z_\nu$;

iii) to fix the possible transitions between the system operation states;

iv) to fix the set of the unknown parameters of the system operation process semi-markov model.

3.3. Procedure of the system operation process data collection

To estimate the unknown parameters of the system operations process, during the experiment, we should collect necessary statistical data performing the following steps:

i) to fix and to collect the following statistical data necessary to evaluating the probabilities of the initial states of the system operations process:

- the duration time of the experiment Θ,

- the number of the investigated (observed) realizations of the system operation process $n(0)$,

- the numbers of staying of the operation process respectively in the operations states z_1, z_2, …, z_v, at the initial moment $t = 0$ of all $n(0)$ observed realizations of the system operation process

$$n_1(0),\ n_2(0),\ \ldots,\ n_v(0),$$

where

$$n_1(0) + n_2(0) + n_v(0) = n(0),$$

- the vector of the realizations of the numbers of staying of the operation process in the operation states at the initial moment

$$[n_b(0)] = [n_1(0), n_2(0), ..., n_v(0)];$$

ii) to fix and to collect the following statistical data necessary to evaluating the transient probabilities between the system operation states:

- the numbers n_{bl}, b, l = 1,2,...,v, $b \neq l$, of the transitions of the system operation process from the operation state z_b to the operation state z_l during all observed realizations of the system operation process

$$n_{11} = 0,\ n_{12},\ n_{13},\ \ldots,\ n_{1v},$$
$$n_{21},\ n_{22} = 0,\ n_{23},\ ...,\ n_{2v},$$
$$\ldots$$
$$n_{v1},\ n_{v2},\ n_{v3},\ \ldots,\ n_{vv} = 0,$$

- the matrix of the realizations of the transitions' numbers of the system operation process between the operation states

$$[n_{bl}] = \begin{bmatrix} n_{11}\ n_{12} \ldots n_{1v} \\ n_{21}\ n_{22} \ldots n_{2v} \\ \ldots \\ n_{v1}\ n_{v2} \ldots n_{vv} \end{bmatrix},$$

- the numbers n_b, b = 1,2,...,v, of departures of the system operation process from the operation states z_b (the sum of the numbers of the b-th row of the matrix $[n_{bl}]$)

$$n_1 = n_{11} + n_{12} + ... + n_{1v},$$
$$n_2 = n_{21} + n_{22} + ... + n_{2v},$$
$$\ldots$$
$$n_v = n_{v1} + n_{v2} + ... + n_{vv};$$

iii) to fix and to collect the following statistical data necessary to evaluating the unknown parameters of the distributions of the conditional sojourn times of the system operation process in the particular operation states:

- the numbers n_{bl}, b, l = 1,2,...,v, $b \neq l$, of realizations of the conditional sojourn times θ_{bl} b, l = 1,2,...,v, $b \neq l$, of the system operations process at the operation state z_b when the next transition is to the operation state z_l during the observation time,

- the realizations θ_{bl}^k, k = 1,2, …, n_{bl}, of the conditional sojourn times θ_{bl} of the system operations process at the operation state z_b when the next transition is to the operation state z_l during the observation time for each b, l = 1,2,...,v, $b \neq l$.

3.4. Procedure of evaluating the unknown parameters of the system operation process

After collecting the statistical data, it is possible to estimate the unknown parameters of the system operation process performing the following steps:

i) to determine the vector

$$[p(0)] = [p_1(0),\ p_2(0), \ldots,\ p_v(0)], \tag{1}$$

of the realizations of the probabilities $p_b(0)$, $b = 1,2,...,v$, of the initial states of the system operation process, according to the formula

$$p_b(0) = \frac{n_b(0)}{n(0)} \text{ for } b = 1,2,...,v, \tag{2}$$

where

$$n(0) = \sum_{b=1}^{v} n_b(0), \tag{3}$$

is the number of the realizations of the system operation process starting at the initial moment t = 0;

ii) to determine the matrix

$$[p_{bl}] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1v} \\ p_{21} & p_{22} & \cdots & p_{2v} \\ \cdots \\ p_{v1} & p_{v2} & \cdots & p_{vv} \end{bmatrix}, \quad (4)$$

of the realizations of the probabilities p_{bl}, $b,l = 1,2,...,\nu$, of the system operation process transitions from the operation state z_b to the operation state z_l during the experiment time Θ, according to the formula

$$p_{bl} = \frac{n_{bl}}{n_b} \text{ for } b,l = 1,2,...,\nu,\ b \neq l,\ p_{bb} = 0 \quad (5)$$

for $b = 1,2,...,\nu$,

where

$$n_b = \sum_{b \neq l}^{\nu} n_{bl},\ b = 1,2,...,\nu, \quad (6)$$

is the realization of the total number of the system operation process departures from the operation state z_b during the experiment time Θ;

iii) to determine the following empirical characteristics of the realizations of the conditional sojourn time of the system operation process in the particular operation states:

- the realizations of the mean values $\bar{\theta}_{bl}$ of the conditional sojourn times θ_{bl} of the system operation process at the operation state z_b when the next transition is to the operation state z_l, according to the formula

$$\bar{\theta}_{bl} = \frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} \theta_{bl}^k,\ b,l = 1,2,...,\nu,\ b \neq l, \quad (7)$$

- the number $\bar{r}$ of the disjoint intervals $I_j =< a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}$, that include the realizations θ_{bl}^k, $k = 1,2,...,n_{bl}$, of the conditional sojourn times θ_{bl} at the operation state z_b when the next transition is to the operation state z_l, according to the formula

$$\bar{r}_{bl} \cong \sqrt{n_{bl}},$$

- the length d_{bl} of the intervals $I_j =< a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, according to the formula

$$d_{bl} = \frac{\bar{R}_{bl}}{\bar{r}_{bl} - 1},$$

where

$$\bar{R}_{bl} = \max_{1 \le k \le n_{bl}} \theta_{bl}^k - \min_{1 \le k \le n_{bl}} \theta_{bl}^k,$$

- the ends a_{bl}^j, b_{bl}^j, of the intervals $I_j =< a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{bl}$, according to the formulae

$$a_{bl}^1 = \max\{\min_{1 \le k \le n_{bl}} \theta_{bl}^k - \frac{d_{bl}}{2}, 0\},\ b_{bl}^j = a_{bl}^1 + j d_{bl},$$

$$j = 1,2,...,\bar{r},$$

$$a_{bl}^j = b_{bl}^{j-1},\quad j = 2,3...,\bar{r}_{bl},$$

in the way such that

$$n_{bl}^j = \#\{k : \theta_{bl}^k \in I_j,\ k \in \{1,2,...,n_{bl}\}\},$$

$$j = 1,2,...,\bar{r}_{bl},$$

where

$$\sum_{j=1}^{\bar{r}} n_{bl}^j = n_{bl},$$

whereas the symbol # means the number of elements of the set;

iv) to estimate the parameters of the distributions of the conditional sojourn times of the system operation process in the particular operation states for the following distinguished distributions respectively in the following way:

- the uniform distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{1}{y_{bl} - x_{bl}}, & x_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where

$0 \le x_{bl} < y_{bl} < +\infty,$

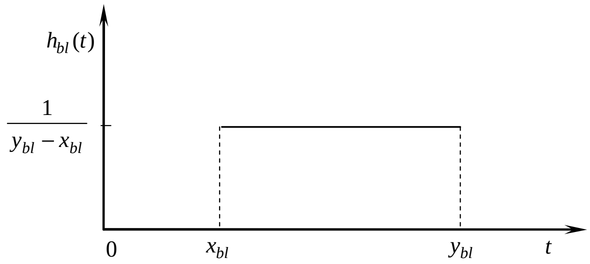


Figure 1. The graph of the uniform distribution's density function

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^{1}, \; y_{bl} = x_{bl} + \bar{r} d_{bl};$$

- the triangular distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \frac{2}{y_{bl}-x_{bl}} \frac{t-x_{bl}}{z_{bl}-x_{bl}}, & x_{bl} \le t \le z_{bl} \\ \frac{2}{y_{bl}-x_{bl}} \frac{y_{bl}-t}{y_{bl}-z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where

$0 \le x_{bl} < z_{bl} < y_{bl} < +\infty,$

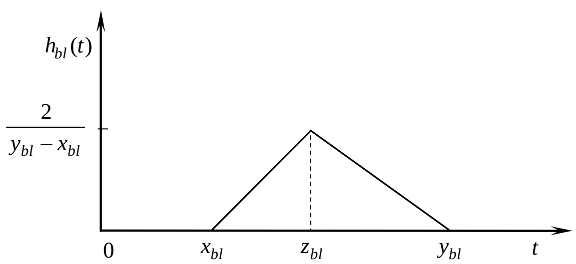


Figure 2. The graph of the triangular distribution's density function

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^{1}, \; y_{bl} = x_{bl} + \bar{r} d_{bl}, \; z_{bl} = \bar{\theta}_{bl};$$

- the double trapezium distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + \frac{C_{bl}-q_{bl}}{z_{bl}-x_{bl}}(t-x_{bl}), & x_{bl} \le t \le z_{bl} \\ w_{bl} + \frac{C_{bl}-w_{bl}}{y_{bl}-z_{bl}}(y_{bl}-t), & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where

$$C_{bl} = \frac{2 - q_{bl}(z_{bl}-x_{bl}) - w_{bl}(y_{bl}-z_{bl})}{y_{bl}-x_{bl}},$$

$0 \le x_{bl} < z_{bl} < y_{bl} < +\infty, \; 0 \le q_{bl} < +\infty,$
$0 \le w_{bl} < +\infty, \; 0 \le q_{bl}(z_{bl}-x_{bl}) + w_{bl}(y_{bl}-z_{bl}) \le 2,$

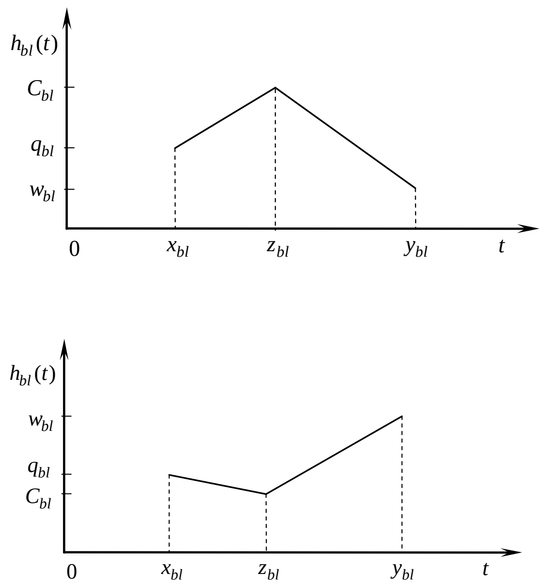


Figure 3. The graph of the double trapezium distribution's density function

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^{1}, \; y_{bl} = x_{bl} + \bar{r} d, \; q_{bl} = \frac{n_{bl}^{1}}{n_{bl} d_{bl}},$$

$$w_{bl} = \frac{n_{bl}^{\bar{r}}}{n_{bl} d_{bl}}, \; z_{bl} = \bar{\theta}_{bl};$$

- the quasi-trapezium distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + \dfrac{A_{bl} - q_{bl}}{z_{bl}^1 - x_{bl}}(t - x_{bl}), & x_{bl} \le t \le z_{bl}^1 \\ A_{bl}, & z_{bl}^1 \le t \le z_{bl}^2 \\ w_{bl} + \dfrac{A_{bl} - w_{bl}}{y_{bl} - z_{bl}^2}(y_{bl} - t), & z_{bl}^2 \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

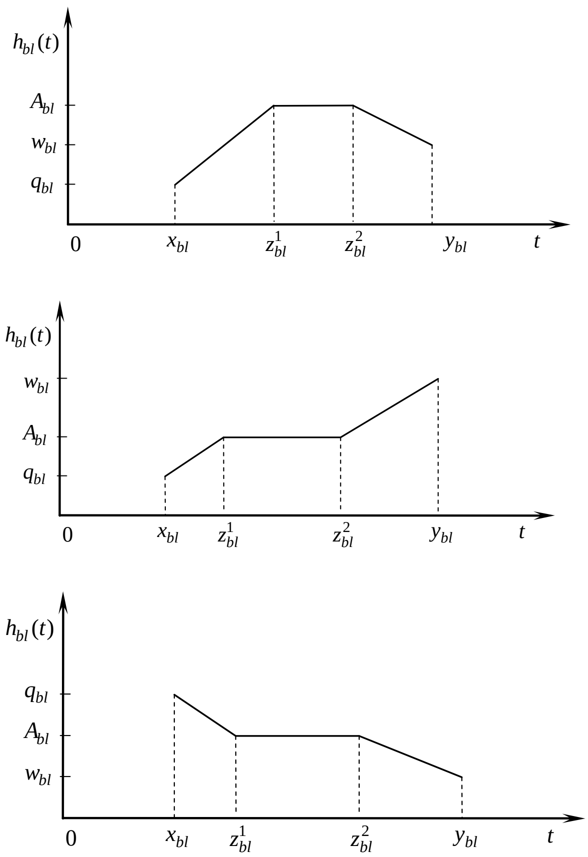
where

$$A_{bl} = \frac{2 - q_{bl}(z_{bl}^1 - x_{bl}) - w_{bl}(y_{bl} - z_{bl}^2)}{z_{bl}^2 - z_{bl}^1 + y_{bl} - x_{bl}},$$

$$0 \le x_{bl} < z_{bl}^1 \le z_{bl}^2 < y_{bl} < +\infty,\ \ 0 \le q_{bl} < +\infty,$$

$$0 \le w_{bl} < +\infty,$$

$$0 \le q_{bl}(z_{bl}^1 - x_{bl}) + w_{bl}(y_{bl} - z_{bl}^2) \le 2,$$



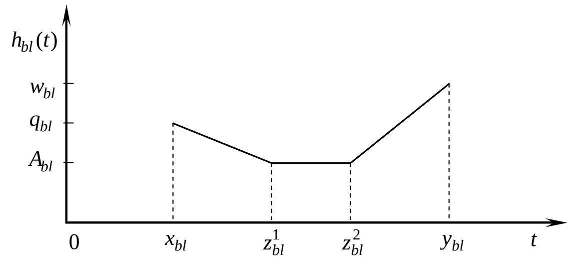


Figure 4. The graph of the quasi-trapezium distribution's density function

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^1,\ \ y_{bl} = x_{bl} + \bar{r} d_{bl},\ \ q_{bl} = \frac{n_{bl}^1}{n_{bl} d_{bl}},$$

$$w_{bl} = \frac{n_{bl}^{\bar{r}}}{n_{bl} d_{bl}},\ \ z_{bl}^1 = \bar{\theta}_{bl}^1,\ \ z_{bl}^2 = \bar{\theta}_{bl}^2,$$

where

$$\bar{\theta}_{bl}^1 = \frac{1}{n_{(me)}} \sum_{j=1}^{n(me)} \theta_{bl}^j,\ \ \bar{\theta}_{bl}^2 = \frac{1}{n_{bl} - n_{(me)}} \sum_{j=n(me)+1}^{n_{bl}} \theta_{bl}^j,$$

$$n_{(me)} = [\frac{n_{bl} + 1}{2}],$$

and $[x]$ denotes the entire part of *x*;

- the exponential distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl},\ x_{bl} \ge 0 \\ \alpha_{bl} \exp[-\alpha_{bl}(t - x_{bl})], & t \ge x_{bl}, \end{cases}$$

where

$$0 \le \alpha_{bl} < +\infty,\ \ 0 \le x_{bl} = a_{bl}^1,$$

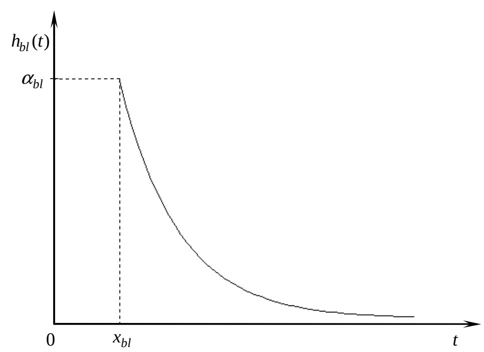


Figure 5. The graph of the exponential distribution's density function

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^1, \ \alpha_{bl} = \frac{1}{\bar{\theta}_{bl} - x_{bl}};$$

- the Weibull's distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl}, x_{bl} \geq 0 \\ \alpha_{bl}\beta_{bl}(t - x_{bl})^{\beta_{bl}-1}\exp[-\alpha_{bl}(t - x_{bl})^{\beta_{bl}}], & t \geq x_{bl}, \end{cases}$$

where

$$0 \leq \alpha_{bl} < +\infty, \ 0 \leq \beta_{bl} < +\infty, \ 0 \leq x_{bl} = a_{bl}^1,$$

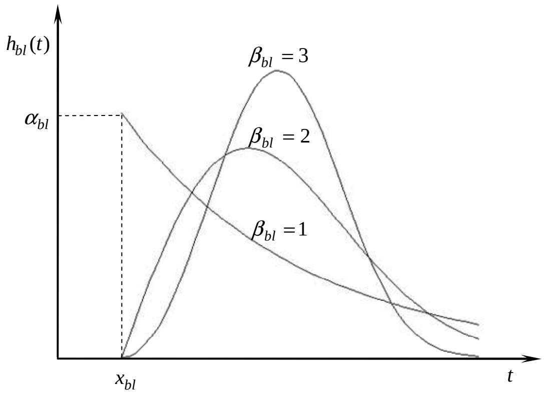


Figure 6. The graph of the Weibull distribution's density function

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^1, \ \alpha_{bl} = \frac{n_{bl}}{\sum_{j=1}^{n_{bl}} (\theta_{bl}^j)^{\beta_{bl}}},$$

$$\alpha_{bl} = \frac{\frac{n_{bl}}{\beta_{bl}} + \sum_{j=1}^{n_{bl}} \ln(\theta_{bl}^j - x_{bl})}{\sum_{j=1}^{n_{bl}} (\theta_{bl}^j)^{\beta_{bl}} \ln(\theta_{bl}^j - x_{bl})};$$

- the normal distribution with a density function

$$h_{bl}(t) = \frac{1}{\sigma_{bl}\sqrt{2\pi}} \exp[-\frac{(t - m_{bl})^2}{2\sigma_{bl}^2}], \quad t \in (-\infty, \infty),$$

where

$$-\infty < m_{bl} < +\infty, \ 0 \leq \sigma_{bl} < +\infty,$$

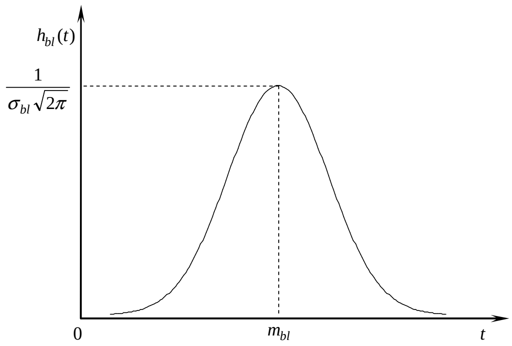


Figure 7. The graph of the normal distribution's density function

the estimates of the unknown parameters of this distribution are:

$$m_{bl} = \bar{\theta}_{bl}, \ \sigma_{bl}^{\ 2} = \bar{\sigma}_{bl}^2 = \frac{1}{n_{bl}} \sum_{j=1}^{n_{bl}} (\theta_{bl}^j - m_{bl})^2;$$

- the chimney distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \frac{a_{bl}}{z_{bl}^1 - x_{bl}}, & x_{bl} \leq t \leq z_{bl}^1 \\ \frac{c_{bl}}{z_{bl}^2 - z_{bl}^1}, & z_{bl}^1 \leq t \leq z_{bl}^2 \\ \frac{d_{bl}}{y_{bl} - z_{bl}^2}, & z_{bl}^2 \leq t \leq y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where

$$0 \leq x_{bl} < z_{bl}^1 < z_{bl}^2 < y_{bl} < +\infty,$$

$$0 \leq q_{bl} < +\infty, \ 0 \leq w_{bl} < +\infty,$$

$$a_{bl} \geq 0, \ c_{bl} \geq 0, \ d_{bl} \geq 0, \ a_{bl} + c_{bl} + d_{bl} = 1.$$

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^1, \ y_{bl} = x_{bl} + \bar{r}d_{bl},$$

and moreover, if

$$\hat{n}_{bl} = \max_{1 \leq j \leq \bar{r}} \{n_{bl}^j\}, \ i = j,$$

where $j \in \{1,2,...,\bar{r}\}$, is the number of the interval having the largest number of realizations i.e. such as $n_{bl}^{j} = \hat{n}_{bl}$,
then:

- for $i = 1$

$$z_{bl}^{1} = x_{bl} + (i-1)d, \quad z_{bl}^{2} = x_{bl} + id_{bl},$$

$$a_{bl} = 0, \quad c_{bl} = \frac{n_{bl}^{i}}{n_{bl}}, \; d_{bl} = \frac{n_{bl}^{i+1} + ... + n_{bl}^{\bar{r}}}{n_{bl}},$$

while $n_{bl}^{i+1} = 0$ or $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} \geq 3$,

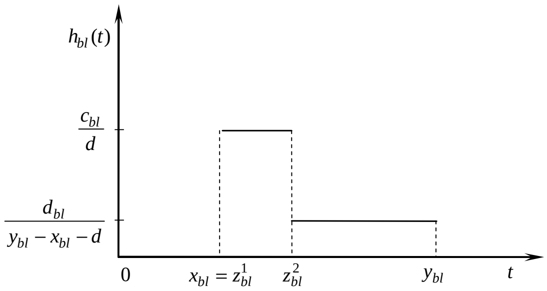


Figure 8. The graph of the chimney distribution's density function for $i = 1$

$$z_{bl}^{1} = x_{bl} + (i-1)d_{bl}, \quad z_{bl}^{2} = x_{bl} + (i+1)d_{bl},$$

$$a_{bl} = 0, \quad c_{bl} = \frac{n_{bl}^{i} + n_{bl}^{i+1}}{n_{bl}}, \; d_{bl} = \frac{n_{bl}^{i+2} + ... + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} < 3$,

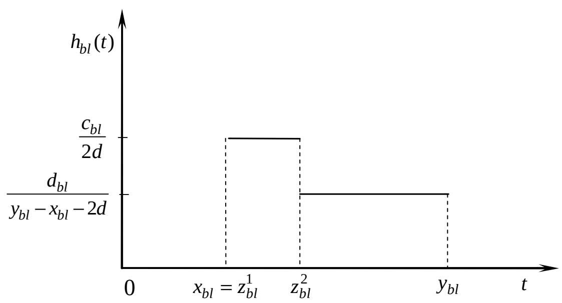


Figure 9. The graph of the chimney distribution's density function for $i = 1$

- for $i = 2,3,...,\bar{r} - 1$

$$z_{bl}^{1} = x_{bl} + (i-1)d_{bl}, \quad z_{bl}^{2} = x_{bl} + id_{bl},$$

$$a_{bl} = \frac{n_{bl}^{1} + ... + n_{bl}^{i-1}}{n_{bl}},$$

$$c_{bl} = \frac{n_{bl}^{i}}{n_{bl}}, \; d_{bl} = \frac{n_{bl}^{i+1} + ... + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i-1} = 0$ or $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} \geq 3$ and when $n_{bl}^{i+1} = 0$ or $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} \geq 3$,

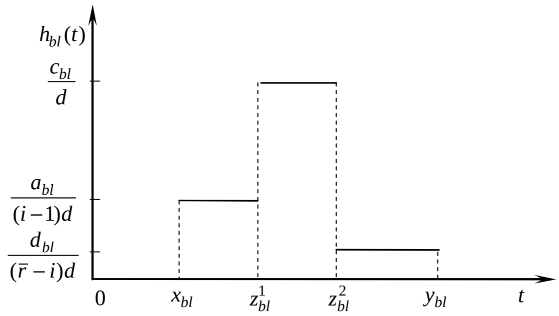


Figure 10. The graph of the chimney distribution's density function for $i = 2,3,...,\bar{r} - 1$

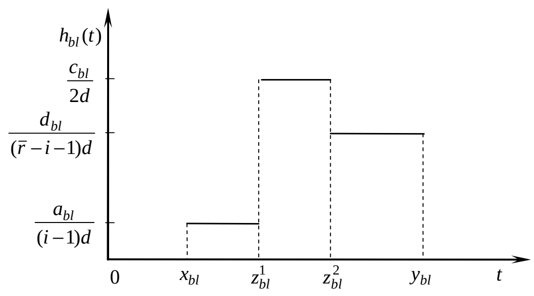
$$z_{bl}^{1} = x_{bl} + (i-1)d_{bl}, \quad z_{bl}^{2} = x_{bl} + (i+1)d,$$

$$a_{bl} = \frac{n_{bl}^{1} + ... + n_{bl}^{i-1}}{n_{bl}},$$

$$c_{bl} = \frac{n_{bl}^{i} + n_{bl}^{i+1}}{n_{bl}}, \; d_{bl} = \frac{n_{bl}^{i+2} + ... + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i-1} = 0$ or $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} \geq 3$ and

when $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} < 3$,



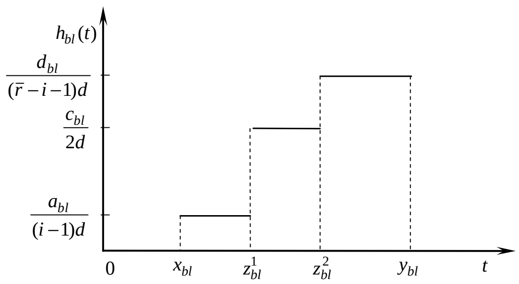


Figure 11. The graph of the chimney distribution's density function for $i = 2,3,...,\bar{r}-1$

$$z_{bl}^1 = x_{bl} + (i-2)d_{bl}, \; z_{bl}^2 = x_{bl} + id_{bl},$$

$$a_{bl} = \frac{n_{bl}^1 + ... + n_{bl}^{i-2}}{n_{bl}}, \; c_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^{i}}{n_{bl}},$$

$$d_{bl} = \frac{n_{bl}^{i+1} + ... + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} < 3$ and when $n_{bl}^{i+1} = 0$ or $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} \geq 3$,

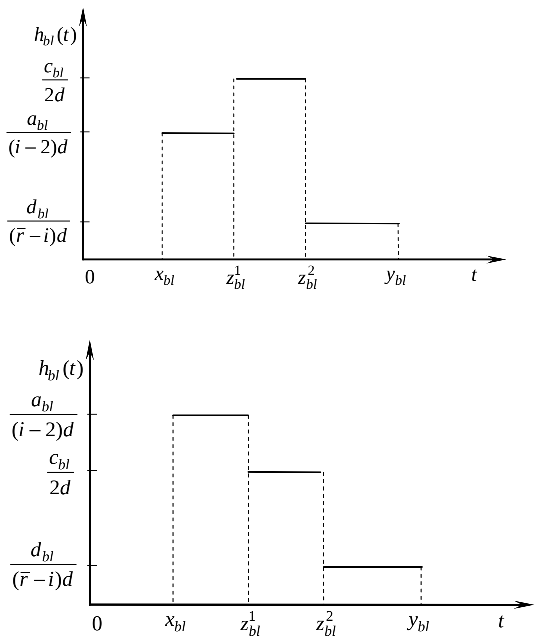


Figure 12. The graph of the chimney distribution's density function for $i = 2,3,...,\bar{r}-1$

$$z_{bl}^1 = x_{bl} + (i-2)d_{bl}, \; z_{bl}^2 = x_{bl} + (i+1)d_{bl},$$

$$a_{bl} = \frac{n_{bl}^1 + ... + n_{bl}^{i-2}}{n_{bl}},$$

$$c_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^{i} + n_{bl}^{i+1}}{n_{bl}}, \; d_{bl} = \frac{n_{bl}^{i+2} + ... + n_{bl}^{\bar{r}}}{n_{bl}},$$

while $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} < 3$ and while $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} < 3$,

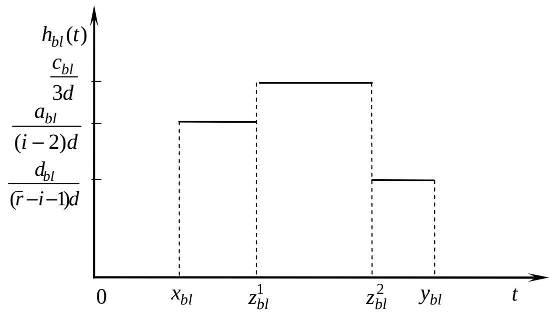


Figure 13. The graph of the chimney distribution's density function for $i = 2,3,...,\bar{r}-1$

- for $i = \bar{r}$

$$z_{bl}^1 = x_{bl} + (i-1)d_{bl}, \; z_{bl}^2 = x_{bl} + id_{bl},$$

$$a_{bl} = \frac{n_{bl}^1 + ... + n_{bl}^{i-1}}{n_{bl}}, \; c_{bl} = \frac{n_{bl}^{i}}{n_{bl}}, \; d_{bl} = 0,$$

while $n_{bl}^{i-1} = 0$ or $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} \geq 3$,

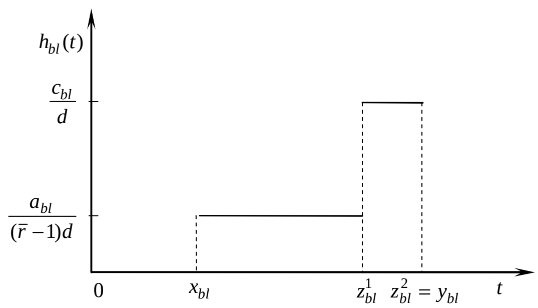


Figure 14. The graph of the chimney distribution's density function for $i = \bar{r}$

$$z_{bl}^1 = x_{bl} + (i-2)d_{bl}, \; z_{bl}^2 = x_{bl} + id_{bl},$$

$$a_{bl} = \frac{n_{bl}^1 + ... + n_{bl}^{i-2}}{n_{bl}}, \; c_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^{i}}{n_{bl}}, \; d_{bl} = 0,$$

when $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} < 3$.

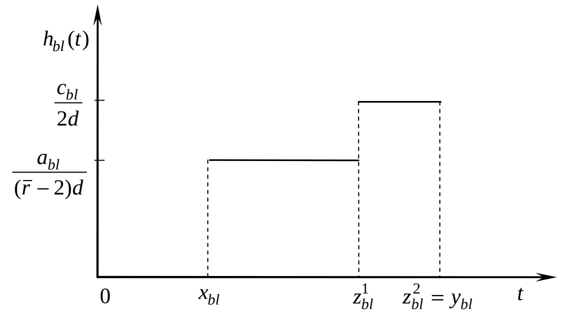


Figure 15. The graph of the chimney distribution's density function for $i = \bar{r}$

3.5. Procedure of identifying the distributions of the system conditional sojourn times in operation states

To formulate and next to verify the non-parametric hypothesis concerning the form of the distribution function $H_{bl}(t)$ of the system conditional sojourn time θ_{bl} at the operation state z_b when the next transition is to the operation state z_l, on the basis of its realizations θ_{bl}^k, $k = 1,2,...,n_{bl}$, it is necessary to proceed according to the following scheme:

- to construct and to plot the realization of the histogram of the system conditional sojourn time θ_{bl} at the operation state, defined by the following formula

$$\bar{h}_{n_{bl}}(t) = \frac{n_{bl}^j}{n_{bl}} \text{ for } t \in I_j,$$

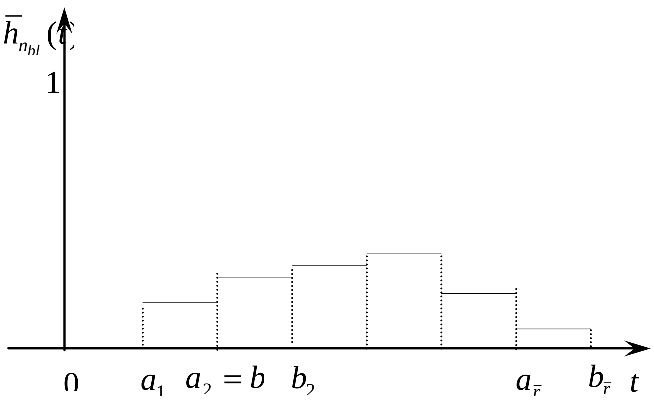


Figure 16. The graph of the realization of the histogram of the system conditional sojourn time θ_{bl} at the operation state

- to analyze the realization of the histogram, comparing it with the graphs of the density functions $h_{bl}(t)$ of the previously distinguished distributions, to select one of them and to formulate the null hypothesis H_0 and the alternative hypothesis H_A, concerning the unknown form of the distribution function $H_{bl}(t)$ of the conditional sojourn time θ_{bl} in the following form:

H_0 : The system conditional sojourn time θ_{bl} at the operation state z_b when the next transition is to the operation state z_l, has the distribution function $H_{bl}(t)$,

H_A : The system conditional sojourn time θ_{bl} at the operation state z_b when the next transition is to the operation state z_l, has the distribution function different from $H_{bl}(t)$;

- to join each of the intervals I_j that has the number n_{bl}^j of realizations less than 4 either with the neighbor interval I_{j+1} or with the neighbor interval I_{j-1} this way that the numbers of realizations in all intervals are not less than 4;

- to fix a new number of intervals $\bar{\bar{r}}$;

- to determine new intervals $\bar{I}_j = <\bar{a}_{bl}^j, \bar{b}_{bl}^j)$, $j = 1,2,..,\bar{\bar{r}}$;

- to fix the numbers $\bar{n}_{bl}^j$ of realizations in new intervals $\bar{I}_j$, $j = 1,2,..,\bar{\bar{r}}$;

- to calculate the hypothetical probabilities that the variable θ_{bl} takes values from the interval $\bar{I}_j$, under the assumption that the hypothesis H_0 is true, i.e. the probabilities

$$p_j = P(\theta_{bl} \in \bar{I}_j) = P(\bar{a}_{bl}^j \le \theta_{bl} < \bar{b}_{bl}^j)$$

$$= H_{bl}(\bar{b}_{bl}^j) - H_{bl}(\bar{a}_{bl}^j), \; j = 1,2,...,\bar{\bar{r}},$$

where $H_{bl}(\bar{b}_{bl}^j)$ and $H_{bl}(\bar{a}_{bl}^j)$ are the values of the distribution function $H_{bl}(t)$ of the random variable θ_{bl} defined in the null hypothesis H_0;

- to calculate the realization of the χ^2 (chi-square)-Pearson's statistics $U_{n_{bl}}$, according to the formula

$$u_{n_{bl}} = \sum_{j=1}^{\bar{\bar{r}}} \frac{(\bar{n}_{bl}^{j} - n_{bl} p_j)^2}{n_{bl} p_j};$$

- to assume the significance level α ($\alpha = 0.01$, $\alpha = 0.02$, $\alpha = 0.05$ or $\alpha = 0.10$) of the test;

- to fix the number $\bar{\bar{r}} - l - 1$ of degrees of freedom, substituting for l for the distinguished distributions respectively the following values: $l = 0$ for the uniform, triangular, double trapezium, quasi-trapezium and chimney distributions, $l = 1$ for the exponential distribution, $l = 2$ for the Weibull's and normal distributions;

- to read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level α and the number of degrees of freedom $\bar{\bar{r}} - l - 1$ such that the following equality holds

$$P(U_{n_{bl}} > u_\alpha) = 1 - \alpha,$$

and next to determine the critical domain in the form of the interval $(u_\alpha, +\infty)$ and the acceptance domain in the form of the interval $< 0, u_\alpha >$.

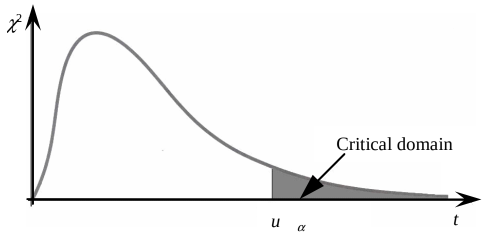


Figure 17. The graphical interpretation of the critical interval and the acceptance interval for the chi-square goodness-of-fit test

- to compare the obtained value $u_{n_{bl}}$ of the realization of the statistics $U_{n_{bl}}$ with the read from the Tables critical value u_α of the chi-square random variable and to verify previously formulated the null hypothesis H_0 in the following way: if the value $u_{n_{bl}}$ does not belong to the critical domain, i.e. when $u_{n_{bl}} \le u_\alpha$, then we do not reject the hypothesis H_0, otherwise if the value $u_{n_{bl}}$ belongs to the critical domain, i.e. when $u_{n_{bl}} > u_\alpha$, then we reject the hypothesis H_0 in favor of the hypothesis H_A.

3.6. Procedure of identifying the mean values of the system conditional sojourn times in operation states

After identifying the matrix of the conditional density functions of the system conditional sojourn times θ_{bl}, $b, l = 1,2,...,v,\ b \ne l,$ in operation states

$$[h_{bl}(t)]_{v \times v} = \begin{bmatrix} h_{11}(t)\ h_{12}(t) \dots h_{1v}(t) \\ h_{21}(t)\ h_{22}(t) \dots h_{2v}(t) \\ \dots \\ h_{v1}(t)\ h_{v2}(t) \dots h_{vv}(t) \end{bmatrix}$$

it is possible to determine the mean values of the system conditional sojourn times in the operation states using the following formula

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t h_{bl}(t) dt, \quad b, l = 1,2,...,v,\ b \ne l. \qquad (8)$$

The results, for the distinguished in [5] distributions, are as follows:

- for the uniform distribution

$$M_{bl} = E[\theta_{bl}] = \frac{x_{bl} + y_{bl}}{2}; \qquad (9)$$

- for the triangle distribution

$$M_{bl} = E[\theta_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{3}; \qquad (10)$$

- for the double trapezium distribution

$$M_{bl} = E[\theta_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{3} + \frac{w_{bl}(y_{bl})^2 - q_{bl}(x_{bl})^2}{2}$$

$$+ \frac{w_{bl} + q_{bl}}{6}\left[(x_{bl} z_{bl} - y_{bl} z_{bl}) + \frac{x_{bl} y_{bl}(x_{bl} + y_{bl})}{y_{bl} - x_{bl}}\right]$$

$$- \frac{(x_{bl})^3 q_{bl} + (y_{bl})^3 w_{bl}}{3(y_{bl} - x_{bl})}; \qquad (11)$$

- for the quasi-trapezium distribution

$$M_{bl} = E[\theta_{bl}] = \frac{q_{bl}}{2}[(z_{bl}^1)^2 - (x_{bl})^2]$$

$$- \frac{A_{bl} - q_{bl}}{6}[2(z_{bl}^1)^2 - 5x_{bl} z_{bl}^1 - (x_{bl})^2]$$

$$+\frac{A_{bl}}{2}[(z_{bl}^2)^2-(z_{bl}^1)^2]+\frac{w_{bl}}{2}[(y_{bl})^2-(z_{bl}^2)^2]$$

$$-\frac{w_{bl}-A_{bl}}{6}[2(z_{bl}^2)^2-5y_{bl}z_{bl}^2-(y_{bl})^2]; \qquad (12)$$

- for the exponential distribution

$$M_{bl}=E[\theta_{bl}]=x_{bl}+\frac{1}{\alpha_{bl}}; \qquad (13)$$

- for the Weibull distribution

$$M_{bl}=E[\theta_{bl}]=x_{bl}+\alpha_{bl}^{-\frac{1}{\beta_{bl}}}\Gamma(1+\frac{1}{\beta_{bl}}), \qquad (14)$$

where

$$\Gamma(u)=\int_0^{+\infty}t^{u-1}e^{-t}dt,\ \ u>0,$$

is the gamma function;

- for the normal distribution

$$M_{bl}=E[\theta_{bl}]=m_{bl}; \qquad (15)$$

- for the steep (chimney) distribution

$$M_{bl}=E[\theta_{bl}]=\frac{1}{2}[a_{bl}(x_{bl}+z_{bl}^1)$$

$$+c_{bl}(z_{bl}^1+z_{bl}^2)+d_{bl}(z_{bl}^2+x_{bl})]. \qquad (16)$$

In the case when the identification of the conditional density functions of the system conditional sojourn times θ_{bl}, $b,l=1,2,...,v,\ b\neq l,$ in operation states is not possible we may determine the approximate empirical values of the system conditional sojourn times in the operation states according to the formula (7) or use their approximate values coming from experts.

4. Procedure of applying the computer program for identification of the system operation process

Training material is given in [2].

5. Identification of the operation processes of real complex technical systems – using procedures

5.1. Statistical identification of the port oil piping transportation system operation process

5.1.1. The port oil piping transportation system description

The considered port oil piping transportation system is the main part of the Oil Terminal in Dębogórze that is designated for the reception from ships, the storage and sending by carriages or by cars the oil products such like petrol and oil. It is also designated for receiving from carriages or cars, the storage and loading the tankers with oil products.
The considered terminal is composed of three parts *A*, *B* and *C*, linked by the piping transportation systems with the pier. The scheme of this terminal is presented in *Figure 18*.

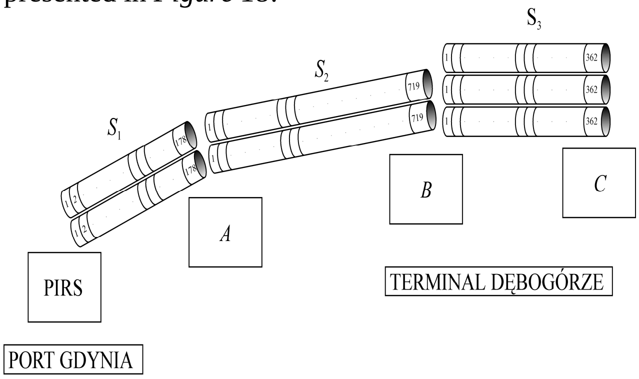


Figure 18. The scheme of the port oil piping transportation system.

The unloading of tankers is performed at the pier placed in the Port of Gdynia. The pier is connected with terminal part *A* through the transportation subsystem S_1 built of two piping lines composed of steel pipe segments with diameter of 600 mm. In the part *A* there is a supporting station fortifying tankers pumps and making possible further transport of oil by the subsystem S_2 to the terminal part *B*. The subsystem S_2 is built of two piping lines composed of steel pipe segments of the diameter 600 mm. The terminal part *B* is connected with the terminal part *C* by the subsystem S_3. The subsystem S_3 is built of one piping line composed of steel pipe segments of the diameter 500 mm and two piping lines composed of steel pipe segments of diameter 350 mm. The terminal part *C* is designated for the loading the rail cisterns with oil products and for the wagon sending to the railway station of the Port of Gdynia and further to the interior of the country.
The oil pipeline system consists three subsystems S_1, S_2, S_3:

- the subsystem S_1 composed of two identical pipelines, each composed of 178 pipe segments of length 12m and two valves,
- the subsystem S_2 composed of two identical pipelines, each composed of 717 pipe segments of length 12m and two valves,
- the subsystem S_3 composed of three different pipelines, each composed of 360 pipe segments of either 10 m or 7,5 m length and two valves.

5.1.2. Defining the parameters of the port oil piping transportation system operation process

Taking into account the expert opinion on the operation process of the considered port oil pipeline transportation system we fix:

- the number of the pipeline system operation process states $\nu = 7$
and we distinguish the following as its seven operation states:

- an operation state $z_1 -$ transport of one kind of medium from the terminal part B to part C using two out of three pipelines in subsystem S_3,
- an operation state $z_2 -$ transport of one kind of medium from the terminal part C (from carriages) to part B using one out of three pipelines in subsystem S_3,
- an operation state $z_3 -$ transport of one kind of medium from the terminal part B through part A to pier using one out of two pipelines in subsystem S_2 and one out of two pipelines in subsystem S_1,
- an operation state $z_4 -$ transport of two kinds of medium from the pier through parts A and B to part C using one out of two pipelines in subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines in subsystem S_3,
- an operation state $z_5 -$ transport of one kind of medium from the pier through part A to B using one out of two pipelines in subsystem S_1 and one out of two pipelines in subsystem S_2,
- an operation state $z_6 -$ transport of one kind of medium from the terminal part B to C using two out of three pipelines in subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines in parts S_1 and one out of two pipelines in subsystem S_2,
- an operation state $z_7 -$ transport of one kind of medium from the terminal part B to C using one out of three pipelines in part S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines in part S_3.

Moreover, we fix that there are possible the transitions between all system operation states. Thus, the unknown parameters of the system operation process semi-markov model are:

- the initial probabilities $p_b(0)$, $b = 1,2,...,7,$ $b \neq l,$ of the pipeline system operation process transients in the particular states z_b at the moment $t = 0$,
- the transition probabilities p_{bl}, $b, l = 1,2,...,7,$ of the pipeline system operation process from the operation state z_b into the operation state z_l,
- the distributions of the conditional sojourn times θ_{bl}, $b, l = 1,2,...,7,$ $b \neq l,$ in the particular operation states and their mean values.

To identify all these parameters of the pipeline system operation process the statistical data about this process is needed. The statistical data that has been collected is given in the Appendix 1A in *Tables 1-41* [6]. From data given in these Tables, on the basis of methods and procedures given in the previous sections, in further sections the piping system operation process statistical data are fixed and its unknown parameters are estimated.

5.1.3. The port oil piping transportation system operation process data collection

The collected statistical data necessary to evaluating the initial transient probabilities of the piping system operation process in the particular states are:

- the pipeline system operation process observation/experiment time $\Theta =$ 329 days = 47 weeks,

- the number of the pipeline system operation process realizations $n(0) =$ 41,

- the realization $n_b(0)$ of the number of the pipeline system operation process transients in the particular operation states z_b at the initial moment $t = 0$

$$n_1(0) = 14,\ n_2(0) = 2,\ n_3(0) = 0,\ n_4(0) = 0,$$

$$n_5(0) = 9,\ n_6(0) = 8,\ n_7(0) = 8,$$

where

$$n_1(0)+\ n_2(0)+\ n_3(0)+\ n_4(0)$$

$$+\,n_5(0)+\ n_6(0)+\ n_7(0)=41,$$

- the vector of realizations of the numbers of the pipeline system operation process transitions in the particular operation states z_b at the initial moment t = 0

$$[n_b(0)]=[n_1(0),n_2(0),n_3(0),n_4(0),$$

$$n_5(0),n_6(0),n_7(0)]=[14,2,0,0,9,8,8]\,.$$

The collected statistical data necessary to evaluating the transition probabilities of the pipeline system operation process between the operation states are:

- the realization n_{bl} of the numbers of pipeline system operation process transitions from the state z_b into the state z_l during the experiment time $\Theta = 329$ days

$n_{11} = 0,\ n_{12} = 1,\ n_{13} = 1,\ n_{14} = 0\,,\ n_{15} = 24\,,$
$n_{16} = 5\,,\ n_{17} = 14\,,$
$n_{21} = 1,\ n_{22} = 0\,,\ n_{23} = 0,\ n_{24} = 0\,,\ n_{25} = 0\,,$
$n_{26} = 0\,,\ n_{27} = 4\,,$
$n_{31} = 1,\ n_{32} = 0\,,\ n_{33} = 0,\ n_{34} = 0\,,\ n_{35} = 0\,,$
$n_{36} = 0\,,\ n_{37} = 0\,,$
$n_{41} = 0,\ n_{42} = 0\,,\ n_{43} = 0,\ n_{44} = 0\,,\ n_{45} = 0\,,$
$n_{46} = 0\,,\ n_{47} = 1\,,$
$n_{51} = 21,\ n_{52} = 1\,,\ n_{53} = 0,\ n_{54} = 1\,,\ n_{55} = 0\,,$
$n_{56} = 10\,,\ n_{57} = 10\,,$
$n_{61} = 2,\ n_{62} = 0\,,\ n_{63} = 0,\ n_{64} = 0\,,\ n_{65} = 14\,,$
$n_{66} = 0\,,\ n_{67} = 5\,,$
$n_{71} = 17,\ n_{72} = 2\,,\ n_{73} = 0,\ n_{74} = 0\,,\ n_{75} = 7\,,$
$n_{76} = 7\,,\ n_{77} = 0\,,$

- the matrix of realizations n_{bl} of the numbers of the pipeline system operation process transitions from the state z_b into the state z_l during the experiment time $\Theta = 329$ days

$$[\,n_{bl}\,] = \begin{bmatrix} 0 & 1 & 1 & 0 & 24 & 5 & 14 \\ 1 & 0 & 0 & 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 21 & 1 & 0 & 1 & 0 & 10 & 10 \\ 2 & 0 & 0 & 0 & 14 & 0 & 5 \\ 17 & 2 & 0 & 0 & 7 & 7 & 0 \end{bmatrix},$$

- the realization n_b of the total numbers of the pipeline system operation process transitions from the operation state z_b during the experiment time $\Theta = 329$ days (the sums of the numbers of the matrix $[n_{bl}]$)

$$n_1 = n_{11} + n_{12} + n_{13} + n_{14} + n_{15} + n_{16} + n_{17} = 45,$$
$$n_2 = n_{21} + n_{22} + n_{23} + n_{24} + n_{25} + n_{26} + n_{27} = 5,$$
$$n_3 = n_{31} + n_{32} + n_{33} + n_{34} + n_{35} + n_{36} + n_{37} = 1,$$
$$n_4 = n_{41} + n_{42} + n_{43} + n_{44} + n_{45} + n_{46} + n_{47} = 1,$$
$$n_5 = n_{51} + n_{52} + n_{53} + n_{54} + n_{55} + n_{56} + n_{57} = 43,$$
$$n_6 = n_{61} + n_{62} + n_{63} + n_{64} + n_{65} + n_{66} + n_{67} = 21,$$
$$n_7 = n_{71} + n_{72} + n_{73} + n_{74} + n_{75} + n_{76} + n_{77} = 33,$$

- the matrix of realizations of the total numbers of the pipeline system operation process transitions from the operation state z_b during the experiment time $\Theta = 329$ days

$$[n_b] = [n_1, n_2, n_3, n_4, n_5, n_6, n_7] = [45,5,1,1,43,21,33]\,.$$

The collected statistical data necessary to evaluating the unknown parameters of the distributions of the conditional sojourn times of the port oil pipeline transportation system operation process in the particular operation states are as follows:

- the realizations θ_{bl}^k, k = 1,2, …, n_{bl}, of the conditional sojourn times θ_{bl} of the port oil pipeline transportation system operation process at the operation state z_b when the next transition is to the operation state z_l during the observation time:

- the variable θ_{11}, the number of realizations n_{11} = 0, realizations:

θ_{11}^k - these realizations are not possible,

- the variable θ_{12}, the number of realizations $n_{12} = 1$, realizations:

$\theta_{12}^1 = 1920,$

- the variable θ_{13}, the number of realizations $n_{13} = 1$, realizations:

$\theta_{13}^{1} = 480$,

- the variable θ_{14}, the number of realizations $n_{14} = 0$, realizations:

there are no realizations,

- the variable θ_{15}, the number of realizations $n_{15} = 24$,
realizations:

$\theta_{15}^{1} = 930$, $\theta_{15}^{2} = 3840$, $\theta_{15}^{3} = 1290$, $\theta_{15}^{4} = 480$,
$\theta_{15}^{5} = 5575$, $\theta_{15}^{6} = 4680$, $\theta_{15}^{7} = 4350$, $\theta_{15}^{8} = 2100$,
$\theta_{15}^{9} = 840$, $\theta_{15}^{10} = 2460$, $\theta_{15}^{11} = 1560$, $\theta_{15}^{12} = 1020$,
$\theta_{15}^{13} = 1860$, $\theta_{15}^{14} = 960$, $\theta_{15}^{15} = 930$, $\theta_{15}^{16} = 910$,
$\theta_{15}^{17} = 480$, $\theta_{15}^{18} = 410$, $\theta_{15}^{19} = 960$, $\theta_{15}^{20} = 480$,
$\theta_{15}^{21} = 1440$, $\theta_{15}^{22} = 4710$, $\theta_{15}^{23} = 540$, $\theta_{15}^{24} = 5180$,

- the variable θ_{16}, the number of realizations $n_{16} = 5$, realizations:

$\theta_{16}^{1} = 1380$, $\theta_{16}^{2} = 840$, $\theta_{16}^{3} = 540$, $\theta_{16}^{4} = 1930$,
$\theta_{16}^{5} = 1560$,

- the variable θ_{17}, the number of realizations $n_{17} = 14$,
realizations:

$\theta_{17}^{1} = 2400$, $\theta_{17}^{2} = 1020$, $\theta_{17}^{3} = 255$, $\theta_{17}^{4} = 890$,
$\theta_{17}^{5} = 435$, $\theta_{17}^{6} = 470$, $\theta_{17}^{7} = 435$, $\theta_{17}^{8} = 470$,
$\theta_{17}^{9} = 490$, $\theta_{17}^{10} = 480$, $\theta_{17}^{11} = 1350$, $\theta_{17}^{12} = 1770$,
$\theta_{17}^{13} = 4570$, $\theta_{17}^{14} = 780$,

- the variable θ_{21}, the number of realizations $n_{21} = 1$, realizations:

$\theta_{21}^{1} = 9960$,

- the variable θ_{22}, the number of realizations $n_{22} = 0$, realizations:

θ_{22}^{k} - these realizations are not possible,

- the variable θ_{23}, the number of realizations $n_{23} = 0$, realizations:

there are no realizations,

- the variable θ_{24}, the number of realizations $n_{24} = 0$, realizations:

there are no realizations,

- the variable θ_{25}, the number of realizations $n_{25} = 0$, realizations:
there are no realizations,

- the variable θ_{26}, the number of realizations $n_{26} = 0$, realizations:

there are no realizations,

- the variable θ_{27}, the number of realizations $n_{27} = 4$,
realizations:

$\theta_{27}^{1} = 900$, $\theta_{27}^{2} = 780$, $\theta_{27}^{3} = 840$, $\theta_{27}^{4} = 720$,

- the variable θ_{31}, the number of realizations $n_{31} = 1$, realizations:

$\theta_{31}^{1} = 757$,

- the variable θ_{32}, the number of realizations $n_{32} = 0$, realizations:

there are no realizations,

- the variable θ_{33}, the number of realizations $n_{33} = 0$, realizations:

θ_{33}^{k} - these realizations are not possible,

- the variable θ_{34}, the number of realizations $n_{34} = 0$, realizations:

there are no realizations,

- the variable θ_{35}, the number of realizations $n_{35} = 0$, realizations:

there are no realizations,

- the variable θ_{36}, the number of realizations $n_{36} = 0$, realizations:

there are no realizations,

- the variable θ_{37}, the number of realizations $n_{37} = 0$,

realizations:
there are no realizations,

- the variable θ_{41}, the number of realizations $n_{41} = 0$, realizations:

there are no realizations,

- the variable θ_{42}, the number of realizations $n_{42} = 0$, realizations:

there are no realizations,

- the variable θ_{43}, the number of realizations $n_{43} = 0$, realizations:

there are no realizations,

- the variable θ_{44}, the number of realizations $n_{44} = 0$, realizations:

θ_{44}^{k} - these realizations are not possible,

- the variable θ_{45}, the number of realizations $n_{45} = 0$, realizations:

there are no realizations,

- the variable θ_{46}, the number of realizations $n_{46} = 0$, realizations:

there are no realizations,

- the variable θ_{47}, the number of realizations $n_{47} = 1$, realizations:

$\theta_{47}^{1} = 380,$

- the variable θ_{51}, the number of realizations $n_{51} = 21$,
realizations:

$\theta_{51}^{1} = 1320,\ \theta_{51}^{2} = 930,\ \theta_{51}^{3} = 811,\ \theta_{51}^{4} = 450,$
$\theta_{51}^{5} = 1270,\ \theta_{51}^{6} = 960,\ \theta_{51}^{7} = 1200,\ \theta_{51}^{8} = 540,$
$\theta_{51}^{9} = 1020,\ \theta_{51}^{10} = 900,\ \theta_{51}^{11} = 360,\ \theta_{51}^{12} = 955,$
$\theta_{51}^{13} = 430,\ \theta_{51}^{14} = 42,\ \theta_{51}^{15} = 140,\ \theta_{51}^{16} = 3855,$
$\theta_{51}^{17} = 780,\ \theta_{51}^{18} = 300,\ \theta_{51}^{19} = 840,\ \theta_{51}^{20} = 510,$
$\theta_{51}^{21} = 755,$

- the variable θ_{52}, the number of realizations $n_{52} = 1$, realizations:

$\theta_{52}^{1} = 480,$

- the variable θ_{53}, the number of realizations $n_{53} = 0$, realizations:

there are no realizations,

- the variable θ_{54}, the number of realizations $n_{54} = 0$, realizations:

$\theta_{54}^{1} = 300,$

- the variable θ_{55}, the number of realizations $n_{55} = 0$, realizations:

θ_{55}^{k} - these realizations are not possible,

- the variable θ_{56}, the number of realizations $n_{56} = 10$,
realizations:

$\theta_{56}^{1} = 250,\ \theta_{56}^{2} = 918,\ \theta_{56}^{3} = 360,\ \theta_{56}^{4} = 140,$
$\theta_{56}^{5} = 390,\ \theta_{56}^{6} = 290,\ \theta_{56}^{7} = 360,\ \theta_{56}^{8} = 405,$
$\theta_{56}^{9} = 870,\ \theta_{56}^{10} = 380,$

- the variable θ_{57}, the number of realizations $n_{57} = 10$,
realizations:

$\theta_{57}^{1} = 540,\ \theta_{57}^{2} = 5760,\ \theta_{57}^{3} = 930,\ \theta_{57}^{4} = 235,$
$\theta_{57}^{5} = 345,\ \theta_{57}^{6} = 670,\ \theta_{57}^{7} = 350,\ \theta_{57}^{8} = 430,$
$\theta_{57}^{9} = 1000,\ \theta_{57}^{10} = 165,$

- the variable θ_{61}, the number of realizations $n_{61} = 2$, realizations:

$\theta_{61}^{1} = 480,\ \theta_{61}^{2} = 170,$

- the variable θ_{62}, the number of realizations $n_{62} = 0$, realizations:
there are no realizations,

- the variable θ_{63}, the number of realizations $n_{63} = 0$,
realizations:

there are no realizations,

- the variable θ_{64}, the number of realizations $n_{64} = 0$,

realizations:

there are no realizations,

- the variable θ_{65}, the number of realizations $n_{65} = 14$,
realizations:

$\theta_{65}^1 = 960,\ \theta_{65}^2 = 60,\ \theta_{65}^3 = 560,\ \theta_{65}^4 = 425,\ \theta_{65}^5 = 435,$
$\theta_{65}^6 = 480,\ \theta_{65}^7 = 450,\ \theta_{65}^8 = 480,\ \theta_{65}^9 = 480,$
$\theta_{65}^{10} = 480,\ \theta_{65}^{11} = 480,\ \theta_{65}^{12} = 780,\ \theta_{65}^{13} = 540,$
$\theta_{65}^{14} = 540,$

- the variable θ_{66}, the number of realizations $n_{66} = 0$, realizations:

θ_{66}^k - these realizations are not possible,

- the variable θ_{67}, the number of realizations $n_{67} = 5$, realizations:

$\theta_{67}^1 = 370,\ \theta_{67}^2 = 445,\ \theta_{67}^3 = 165,\ \theta_{67}^4 = 430,$
$\theta_{67}^5 = 780,$

- the variable θ_{71}, the number of realizations $n_{71} = 17$,
realizations:

$\theta_{71}^1 = 840,\ \theta_{71}^2 = 1380,\ \theta_{71}^3 = 2460,\ \theta_{71}^4 = 2370,$
$\theta_{71}^5 = 480,\ \theta_{71}^6 = 480,\ \theta_{71}^7 = 480,\ \theta_{71}^8 = 420,$
$\theta_{71}^9 = 405,\ \theta_{71}^{10} = 850,\ \theta_{71}^{11} = 480,\ \theta_{71}^{12} = 480,$
$\theta_{71}^{13} = 480,\ \theta_{71}^{14} = 420,\ \theta_{71}^{15} = 900,\ \theta_{71}^{16} = 720,$
$\theta_{71}^{17} = 820,$

- the variable θ_{72}, the number of realizations $n_{72} = 2$,
realizations:

$\theta_{72}^1 = 360,\ \theta_{72}^2 = 660,$

- the variable θ_{73}, the number of realizations $n_{73} = 0$,
realizations:

there are no realizations,

- the variable θ_{74}, the number of realizations $n_{74} = 0$, realizations:

there are no realizations,

- the variable θ_{75}, the number of realizations $n_{75} = 7$, realizations:

$\theta_{75}^1 = 3840,\ \theta_{75}^2 = 960,\ \theta_{75}^3 = 960,\ \theta_{75}^4 = 10090,$
$\theta_{75}^5 = 390,\ \theta_{75}^6 = 980,\ \theta_{75}^7 = 880,$

- the variable θ_{76}, the number of realizations $n_{76} = 7$,
realizations:

$\theta_{76}^1 = 3970,\ \theta_{76}^2 = 3705,\ \theta_{76}^3 = 2565,\ \theta_{76}^4 = 1320,$
$\theta_{76}^5 = 2600,\ \theta_{76}^7 = 120,$

- the variable θ_{77}, the number of realizations $n_{77} = 0$, realizations:

θ_{77}^k - these realizations are not possible.

5.1.4. Evaluating the unknown parameters of the port oil piping transportation system operation process

On the basis of the statistical data from Section 5.1.3, using the formulae given in Section 3.4, it is possible to evaluate

- the vector of realizations

$$[p(0)] = [0.34, 0.05, 0, 0, 0.23, 0.19, 0.19]$$

of the initial probabilities $p_b(0)$, $b = 1,2,...7$, (1)-(3) of the pipeline system operation process transients in the particular states z_b at the moment $t = 0$,

- the matrix of realizations

$$[p_{bl}] = \begin{bmatrix} 0 & 0.022 & 0.022 & 0 & 0.534 & 0.111 & 0.311 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0.8 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.488 & 0.023 & 0 & 0.023 & 0 & 0.233 & 0.233 \\ 0.095 & 0 & 0 & 0 & 0.667 & 0 & 0238 \\ 0.516 & 0.064 & 0 & 0 & 0.226 & 0.194 & 0 \end{bmatrix}$$

of the transition probabilities p_{bl}, $b, l = 1,2,...,7$, (4)-(6) of the pipeline system operation process from the operation state z_b into the operation state z_l during the experiment time $\Theta = 329$ days.

On the basis of the statistical data from Section 4.1.3, using the formulae given in Section 3.4, it is possible to determine the following empirical characteristics of the realizations of the conditional sojourn time of the pipeline system operation process in the particular operation states:

- the realizations of the mean values $\bar{\theta}_{15}$ of the conditional sojourn times θ_{15} of the pipeline system operation process at the operation state z_1 when the next transition is to the operation state z_5

$$\bar{\theta}_{15} = \frac{1}{24}\sum_{k=1}^{24}\theta_{15}^k = 1999.4 \quad b,l = 12,...,7 \quad b \neq l,$$

- the number $\bar{r}_{15}$ of the disjoint intervals $I_j =< a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{15}$, that include the realizations θ_{15}^k, $k = 1,2,...,24$, of the conditional sojourn times θ_{15} at the operation state z_1 when the next transition is to the operation state z_5

$$\bar{r}_{15} \cong \sqrt{24} \cong 5,$$

- the length d_{15} of the intervals $I_j =< a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,5$,

$$d_{15} = \frac{\bar{R}_{15}}{\bar{r}_{15} - 1} = \frac{5165}{4} = 1291,$$

where

$$\bar{R}_{15} = \max_{1\le k\le 24}\theta_{15}^k - \min_{1\le k\le 24}\theta_{15}^k = 5575 - 410 = 5165,$$

- the ends a_{bl}^j, b_{bl}^j, of the intervals $I_j =< a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,5$, according to the formulae

$$\min_{1\le k\le 24}\theta_{15}^k - \frac{d_{15}}{2} = 410 - \frac{1291}{2} = -235.5,$$

$$a_{15}^1 = \max\{-235.5, 0\} = 0,$$

$$b_{15}^1 = a_{15}^1 + 1291 = 0 + 1291 = 1291,$$

$$a_{15}^2 = b_{15}^1 = 1291,$$

$$b_{15}^2 = a_{15}^1 + 2\cdot 1291 = 0 + 2582 = 2582,$$

$$a_{15}^3 = b_{15}^2 = 2582,$$

$$b_{15}^3 = a_{15}^1 + 3\cdot 1291 = 0 + 3873 = 3873,$$

$$a_{15}^4 = b_{15}^3 = 3873,$$

$$b_{15}^4 = a_{15}^1 + 4\cdot 1291 = 0 + 5164 = 5164,$$

$$a_{15}^5 = b_{15}^4 = 5164,$$

$$b_{15}^5 = a_{15}^1 + 5\cdot 1291 = 0 + 6455 = 6455,$$

- the numbers n_{15}^j of the realizations θ_{15}^k in particular intervals I_j, $j = 1,2,...,5$,

$$n_{15}^1 = 13,\ n_{15}^2 = 5,\ n_{15}^3 = 1,\ n_{15}^4 = 4,\ n_{15}^5 = 1,$$

Histogram of the conditional sojourn time θ_{15}					
$I_j =< a_{bl}^j, b_{bl}^j)$	0 – 1291	1291 – 2582	2582 – 3873	3873 – 5164	5164 – 6455
n_{15}^j	13	5	1	4	1
$\bar{h}_{15}(t) = n_{15}^j / n_{15}$	13/24	5/24	1/24	4/24	1/24

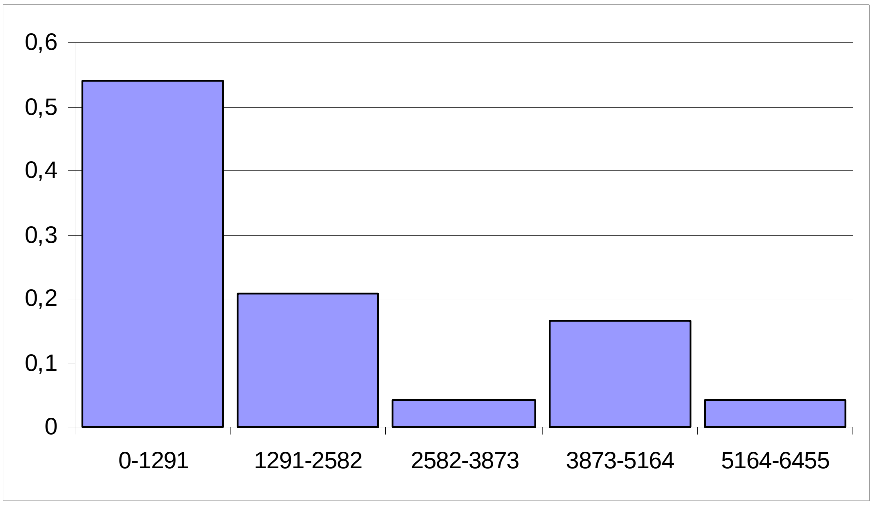


Figure 19. Histogram of the conditional sojourn time θ_{15}

- the realizations of the mean values $\bar{\theta}_{71}$ of the conditional sojourn times θ_{71} of the pipeline system operation process at the operation state z_7 when the next transition is to the operation state z_1

$$\bar{\theta}_{71} = \frac{1}{17}\sum_{k=1}^{17}\theta_{71}^{k} = 851 \;\; b,l = 12,...,7 \,,\; b \neq l,$$

- the number $\bar{r}_{71}$ of the disjoint intervals $I_j =< a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,\bar{r}_{71}$, that include the realizations θ_{71}^k, $k = 1,2,...,17$, of the conditional sojourn times θ_{71} at the operation state z_7 when the next transition is to the operation state z_1

$$\bar{r}_{71} \cong \sqrt{17} \cong 4\,,$$

- the length d_{71} of the intervals $I_j =< a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,4$,

$$d_{71} = \frac{\bar{R}_{71}}{\bar{r}_{71} - 1} = \frac{2055}{3} = 685\,,$$

where

$$\bar{R}_{71} = \max_{1\le k\le 17}\theta_{71}^k - \min_{1\le k\le 17}\theta_{71}^k = 2460 - 405 = 2055\,,$$

- the ends a_{bl}^j, b_{bl}^j, of the intervals $I_j =< a_{bl}^j, b_{bl}^j)$, $j = 1,2,...,4$, according to the formulae

$$a_{71}^1 = \min_{1\le k\le 17}\theta_{71}^k - \frac{d_{71}}{2} = 405 - \frac{685}{2} = 62.5\,,$$

$$b_{71}^1 = a_{71}^1 + 685 = 62.5 + 685 = 747.5,$$

$$a_{71}^2 = b_{71}^1 = 747.5\,,$$

$$b_{71}^2 = a_{71}^1 + 2\cdot 685 = 62.5 + 1370 = 1432.5,$$

$$a_{71}^3 = b_{71}^2 = 1432.5\,,$$

$$b_{71}^3 = a_{71}^1 + 3\cdot 685 = 62.5 + 2055 = 2117.5,$$

$$a_{71}^4 = b_{71}^3 = 2117.5\,,$$

$$b_{71}^4 = a_{71}^1 + 4\cdot 685 = 62.5 + 2740 = 2802.5,$$

- the numbers n_{71}^j of the realizations θ_{71}^k in particular intervals I_j, $j = 1,2,...,4$,

$$n_{71}^1 = 10,\;\; n_{71}^2 = 5,\;\; n_{71}^3 = 0,\;\; n_{71}^4 = 2,$$

Histogram of the conditional sojourn time θ_{71}				
$I_j =< a_{bl}^j, b_{bl}^j)$	62.5 – 747.5	747.5 – 1432.5	1432.5 – 2117.5	2117.5 – 2802.5
n_{71}^j	10	5	0	2
$\bar{h}_{71}(t) = n_{71}^j / n_{71}$	10/17	5/17	0	2/17

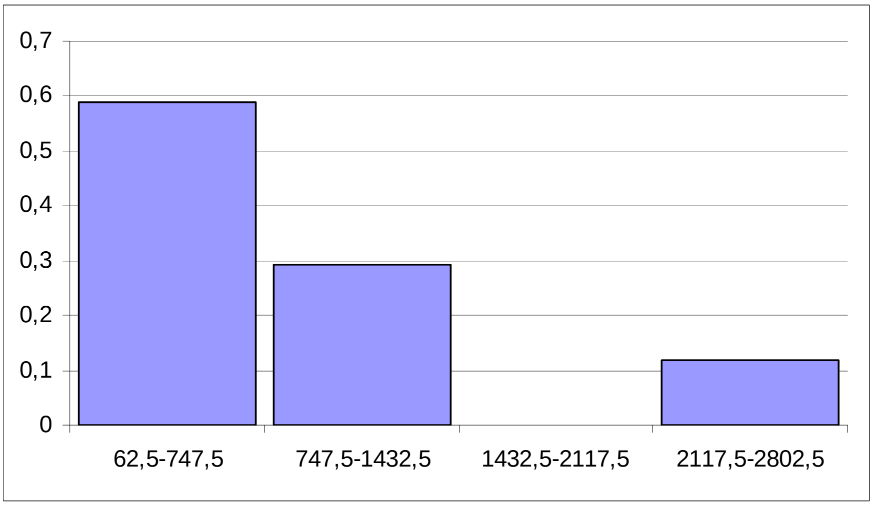


Figure 20. Histogram of the conditional sojourn time θ_{71}

5.1.5. Identifying the distributions of the conditional sojourn times in the operation states of the port oil piping transportation system

Using the procedure given in Section 3.5 we may verify the hypotheses on the distributions of the conditional sojourn times θ_{bl}, $b, l = 1,2,...,7$, $b \neq l$, in the particular operation states. At the moment, because of the lack of statistical data coming from experiment it was possible to verify only two hypotheses on the distributions of the sojourn times. Namely, we have the following results:

- the conditional sojourn time θ_{15} has a chimney distribution with the density function

$$h_{15}(t) = \begin{cases} 0, & t < 0, \\ 0.0003, & 0 \le t < 1721.67, \\ 0.00007, & 1721.67 \le t < 6886.67, \\ 0, & t \ge 6886.67; \end{cases}$$

- the conditional sojourn time θ_{71} has a chimney distribution with the density function

$$h_{71}(t) = \begin{cases} 0, & t < 62.5, \\ 0.0006, & 62.5 \le t < 1432.5, \\ 0.00009, & 1432.5 \le t < 2802.5, \\ 0, & t \ge 2802.5. \end{cases}$$

5.1.6. Identifying the mean values of the system conditional sojourn times in operation states of the port oil piping transportation system

For the distributions identified in Section 5.1.5, using the formulae (8) or (16) , we can find the following mean values of the conditional sojourn times in the particular operation states:

$$M_{15} = 1999.4, \; M_{71} = 874.1.$$

In the remaining cases of not identified in Section 5.1.5 distributions, using formula (7), it is possible to find only the empirical values of the mean values $M_{bl} = E[\theta_{bl}]$ of the conditional sojourn times in the particular operation states that are as follow:

$$M_{12} = 1920, \; M_{13} = 480,$$

$M_{16} = 1250,\ M_{17} = 1129.6,$
$M_{21} = 9960,\ M_{27} = 810,$
$M_{31} = 575,\ M_{47} = 380,$
$M_{51} = 874.7,\ M_{52} = 480,$
$M_{54} = 300,\ M_{56} = 436.3,\ M_{57} = 1042.5,$
$M_{61} = 325,\ M_{65} = 510.7,\ M_{67} = 438,$
$M_{72} = 510,\ M_{75} = 2585.7,\ M_{76} = 2380.$

As there are no realizations of conditional sojourn times θ_{14}, θ_{23}, θ_{24}, θ_{25}, θ_{26}, θ_{32}, θ_{34}, θ_{35}, θ_{36}, θ_{37}, θ_{41}, θ_{42}, θ_{43}, θ_{45}, θ_{46}, θ_{53}, θ_{62}, θ_{63}, θ_{64}, θ_{73} and θ_{74}, then it is impossible to estimate their empirical conditional mean values M_{14}, M_{23}, M_{24}, M_{25}, M_{26}, M_{32}, M_{34}, M_{35}, M_{36}, M_{37}, M_{41}, M_{42}, M_{43}, M_{45}, M_{46}, M_{53}, M_{62}, M_{63}, M_{64}, M_{73}, M_{74}.

5.2. Statistical identification of the shipyard ship-rope elevator operation process

5.2.1. The shipyard ship-rope elevator description

Ship-rope elevators are used to dock and undock ships coming to shipyards for repairs. The elevator utilized in the Naval Shipyard in Gdynia, with the scheme presented in *Figure 21*, is composed of a steel platform-carriage placed in its syncline (hutch). The platform is moved vertically with 10 rope-hoisting winches fed by separate electric motors. Motors are equipped in ropes "Bridon" with the diameter 47 mm each rope having a maximum load of 300 tones. During ship docking the platform, with the ship settled in special supporting carriages on the platform, is raised to the wharf level (upper position). During undocking, the operation is reversed. While the ship is moving into or out of the syncline and while stopped in the upper position the platform is held on hooks and the loads in the ropes are relieved. Since the platform-carriage and electric motors are highly reliable in comparison to the ropes, which work in extremely aggressive conditions, in our further analysis we will discuss the reliability of the rope system only.

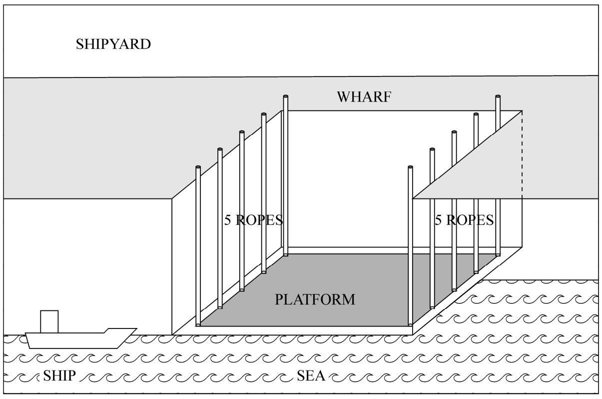


Figure 21. The scheme of the ship-rope transportation system

5.2.2. Defining the parameters of the shipyard ship-rope elevator operation process

Taking into account the expert opinion on the operation process of the considered ship-rope elevator we fix:
- the number of the ship-rope elevator operation process states $\nu = 5$
and we distinguish the following as its five operation states:

- an operation state z_1 – loading over 0 up to 500 tones,
- an operation state z_2 – loading over 500 up to 1000 tones,
- an operation state z_3 – loading over 1000 up to 1500 tones,
- an operation state z_4 – loading over 1500 up to 2000 tones,
- an operation state z_5 – loading over 2000 up to 2500 tones.

Moreover, we fix that there are possible the transitions between all system operation states. Thus, the unknown parameters of the system operation process semi-markov model are:

- the initial probabilities $p_b(0)$, $b = 1,2,...,5,$ $b \neq l,$ of the ship-rope elevator operation process transients in the particular states z_b at the moment *t* = 0,
- the transition probabilities p_{bl}, $b, l = 1,2,...,5,$ of the ship-rope elevator operation process from the operation state z_b into the operation state z_l,
- the distributions of the conditional sojourn times θ_{bl}, $b, l = 1,2,...,5,\ b \neq l,$ in the particular operation states and their mean values.

To identify all these parameters of the pipeline system operation process the statistical data about this process is needed. The statistical data that has been collected is given in the Appendix 2A in *Tables 2-21* [3]. From data given in these Tables, on the basis of methods and procedures given in the previous sections, in further sections the ship-rope elevator operation process statistical data are fixed and its unknown parameters are estimated.

5.2.3. The shipyard ship-rope elevator operation process data collection

The collected statistical data necessary to evaluating the initial transient probabilities of the ship-rope elevator operation process in the particular states are:

- the ship-rope elevator operation process observation/experiment time $\Theta = 616$ days,

- the number of the ship-rope elevator operation process realizations $n(0) = 19$,

- the realization $n_b(0)$ of the number of the ship-rope elevator operation process transients in the particular operation states z_b at the initial moment $t = 0$

$$n_1(0) = 8,\ \ n_2(0) = 7,\ \ n_3(0) = 4,$$
$$n_4(0) = 0,\ \ n_5(0) = 0,$$

- the vector of realizations of the numbers of the ship-rope elevator operation process transitions in the particular operation states z_b at the initial moment $t = 0$

$$[n_b(0)] = [n_1(0), n_2(0), n_3(0), n_4(0), n_5(0)]$$

$$= [8,7,4,0,0].$$

The collected statistical data necessary to evaluating the transition probabilities of the ship-rope elevator operation process between the operation states are:

- the realization n_{bl} of the numbers of ship-rope elevator operation process transitions from the state z_b into the state z_l during the experiment time $\Theta = 616$ days

$$n_{12} = 6,\ \ n_{13} = 11,\ \ n_{14} = 2,\ \ n_{15} = 0,$$
$$n_{21} = 6,\ \ n_{23} = 8,\ \ n_{24} = 3,\ \ n_{25} = 1,$$
$$n_{31} = 8,\ \ n_{32} = 10,\ \ n_{34} = 2,\ \ n_{35} = 2,$$
$$n_{41} = 2,\ \ n_{42} = 2,\ \ n_{43} = 2,\ \ n_{45} = 1,$$
$$n_{51} = 2,\ \ n_{52} = 0,\ \ n_{53} = 2,\ \ n_{54} = 0,$$

- the matrix of realizations n_{bl} of the numbers of the ship-rope elevator operation process transitions from the state z_b into the state z_l during the experiment time $\Theta = 616$ days

$$[n_{bl}] = \begin{bmatrix} 0 & 6 & 11 & 2 & 0 \\ 6 & 0 & 8 & 3 & 1 \\ 8 & 10 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 1 \\ 2 & 0 & 2 & 0 & 0 \end{bmatrix},$$

- the realization n_b of the total numbers of the ship-rope elevator operation process transitions from the operation state z_b during the experiment time $\Theta = 616$ days (the sums of the numbers of the matrix $[n_{bl}]$)

$$n_1 = 19,\ \ n_2 = 18,\ \ n_3 = 22,\ \ n_4 = 7,\ \ n_5 = 4,$$

- the matrix of realizations of the total numbers of the ship-rope elevator operation process transitions from the operation state z_b during the experiment time $\Theta = 616$ days

$$[n_b] = [n_1, n_2, n_3, n_4, n_5, n_6] = [19,18,22,7,4].$$

The collected statistical data necessary to evaluating the unknown parameters of the distributions of the conditional sojourn times of the ship-rope elevator operation process in the particular operation states are as follows:

- the realizations θ_{bl}^k, k = 1,2, …, n_{bl}, of the conditional sojourn times θ_{bl} of the ship-rope elevator operation process at the operation state z_b when the next transition is to the operation state z_l during the observation time:

- the variable θ_{11}, the number of realizations n_{11} = 0,
realizations:

θ_{11}^k - these realizations are not possible,

- the variable θ_{12}, the number of realizations $n_{12} = 6$,
realizations: