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Unified design method of time delayed PI controller for first order plus dead-time process models with different dead-time to time constant ratio

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The time delay element present in the PI controller brings dead-time compensation capability and shows improved performance for dead-time processes. However, design of robust time delayed PI controller needs much responsiveness for uncertainty in dead-time processes. Hence in this paper, robustness of time delayed PI controller has been analyzed for First Order plus Dead-Time (FOPDT) process model. The process having dead-time greater than three times of time constant is very sensitive to dead-time variation. A first order filter is introduced to ensure robustness. Furthermore, integral time constant of time delayed PI controller is modified to attain better regulatory performance for the lag-dominant processes. The FOPDT process models are classified into dead-time/lag dominated on the basis of dead-time to time constant ratio. A unified tuning method is developed for processes with a number of dead-time to time constant ratio. Several simulation examples and experimental evaluation are exhibited to show the efficiency of the proposed unified tuning technique. The applicability to the process models other than FOPDT such as high-order, integrating, right half plane zero systems are also demonstrated via simulation examples.

Key words: PI controller, time delayed PI controller, dead-time compensation

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1. Introduction

The Proportional Integral (PI) and Proportional Integral Derivative (PID) controllers are commonly employed controllers in the industrial control applications because of its easiness, satisfactory performance, and availability of simple tuning relations [18]. The turning off derivative action with respect to noise makes a PI controller simple and less sensitive [9]. Hence, PI controllers are more desirable than the PID controllers owing to the availability of measurement noise. Widespread numbers of tuning procedures have been developed in the literature to tune PI controllers for different objectives. These techniques are discussed by O'Dwyer, 2006 [2] and Astrom and Hagglund, 2006 [17].

However, the well-known longstanding design methodology proposed by Ziegler and Nichols (ZN) shows better regulatory operation compared to model-based methods. Moreover, PI controller is appropriate for short dead-time systems, while if a process has long dead-time, a PI controller may not attain desired performance. The design of a robust dead-time control scheme is a challenging problem and has been drawing great attention [7]. The first Dead-time Compensation (DTC) scheme is a Smith predictor (SP) developed by Smith which effectively eliminates the exponential terms from closed-loop characteristic equation. Considerable Dead-time Compensation (DTC) schemes based on Smith's Predictor have been suggested in the literature [1,3,15,16]. Hagglund, 1996 [21] introduced a Predictive Proportional Integral (PPI) controller for a FOPDT process model that includes all the properties of Smith predictor. To improve robustness of PPI controller, Normy-Rico et al., 1997 [13] proposed a Filtered PPI controller. Also, detailed performance analysis and robustness in particular with a comparison of predictive and PI(D) controllers is reported in [3].

Many control schemes such as Smith predictor [21], Time delay control [14,23,24], Dahlin's controller, Internal Model Control [6,19], Repetitive control [12], Sliding mode control [5], PID dead-time controller [8] and Proportional Delayed Integral (PDI) controller [11] explicitly uses time delay in the structure of controller. In PID dead-time controller, time delay part is introduced into the integral feedback circuit of a PID controller and settings are provided for the distributed process. In a PDI controller, a time delay unit added to the integral term increases gain and phase margins. But, insertion of time delay element creates stability issues due to model uncertainty related to process dead-time. This issue should be addressed during the design of dead-time based control schemes [20].

In a nutshell, it can be stated that the IMC-PI tuning methods require modified integral time constant for processes with a small dead-time to time-constant ratio. The PI scheme requires additional compensation for processes with a large dead-time to time-constant ratio to overcome the effect of dead-time. To the best of our knowledge, there is no general time delayed PI controller tuning method which is applicable for FOPDT processes with a wide range of dead-time to time-constant ratio. Hence, development of a unified control design method towards a wide range of processes is much needed. Therefore, this paper focuses on the development of a unified time delayed PI controller design method for the FOPDT process model with different dead-time to the time constant ratio.

The paper is organized as follows. The time delayed PI control technique is presented in Section 2. The unified design method is presented in Section 3. Simulation and comparative analysis are presented in Section 4. Extension to High-order, integral plus dead-time and non-minimum phase process is also presented in the same section, together with the experimental validation. Finally, the derived conclusions are reported in Section 5.

2. Time delayed PI control scheme

A control method with a PI controller is given by,

$$C(s) = k_c \left(1 + \frac{1}{\tau_i s}\right). \tag{1}$$

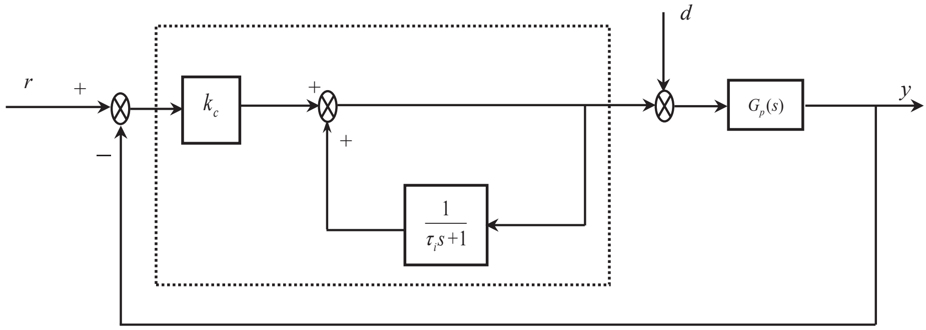
This has been analysed in reset configuration and presented in Fig. 1a. Here, $G_p(s)$ is the process transfer function, r, y, and d are the reference, process output, and input disturbance. This configuration is widely used in commercial controllers to realize the PI controller, and it has the advantage of integral windup protection by simple insertion of a saturation element. In reset configuration, a positive feedback loop around the first order filter with filter time constant τ_i generates the integral action and ensures the offset-free steady state control system for the step type reference and disturbance changes.

As stated in introduction, conventional PI controller needs additional compensation to overcome the effect of dead-time for processes with a large dead-time to time-constant ratio. Hence, a time delay element of is inserted with the first order filter in the positive feedback loop as presented in Fig. 1b. The time delay element delays the reset action of PI controller thereby provides capability of dead-time compensation.

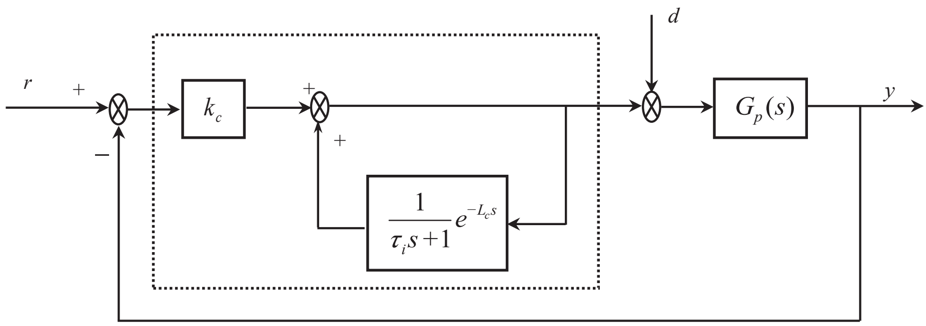
The time delayed PI controller transfer function is as follows,

$$C(s) = k_c \left(\frac{1}{1 - \dfrac{1}{\tau_i s + 1} e^{-L_c s}} \right). \tag{2}$$

Usually, the amount of time delay is taken as process time delay i.e., the reset action is performed only when the process starts response after the time delay.



(a)



(b)

Figure 1: a) PI controller in reset configuration, b) time delayed PI controller in reset configuration

The proportional controller initiates the controller action, and then the reset action is delayed. The delay period is made equivalent to estimated process dead-time. The plant model mismatch related to process dead-time leads to inappropriate delaying of reset action. The small positive incremental uncertainty in the process time delay will advance the controller output manipulation. This pre-computation may result in an unstable closed-loop response. Hence, the stability of time delayed PI controller has to be analyzed and suitable modifications have to be done to ensure the stability.

The rate of integration is taken as a process integration rate, i.e., the integral time constant is equivalent to process time constant. This choice may give good performance for the FOPDT process with large to medium dead-time to a time constant ratio processes. However, integral action should be modified to achieve better disturbance rejection response for small dead-time to time constant ratio processes. It is important to use a single controller structure for processes with different dead-time to time constant ratio. Due to different process characteristics, a mono tuning rule for a chosen controller may not be an optimal one. Hence, the tuning method has to be conceived based on different process characteristics such as dead-time dominant, lag dominant and normalized. Also, there must be a convenient method to classify the processes with respect to dead-time to time constant ratio.

Designing a unified controller design method will be much helpful for the plant operator to implement a single controller structure for processes with diverse characteristics. Thereby, in this research work, a novel method has been proposed to classify the FOPDT process model into dead-time dominant, lag dominant and normalized process based on the dead-time to time constant ratio. Further, an appropriate and improved tuning relation for each category has been suggested based on the existing well-known tuning relation.

3. Unified design method of time delayed PI controller

In the proposed work, FOPDT model classification based on the characteristics and its corresponding tuning relation of the time delayed PI controller has been presented in this section. FOPDT model can adequately represent most of the process dynamics, and it is given by,

$$G_m(s) = \frac{k_m}{\tau_m s + 1} e^{-L_m s}, \tag{3}$$

where, k_m, τ_m and L_m is the process gain, time constant and dead-time respectively. The above process model can be easily obtained by conducting a simple open-loop step response experiment. The time delayed PI controller has three tuning parameters, i.e., k_c, τ_i and time delay L_c. In order to analyze the robustness of the time delayed PI control scheme, the integral time constant and time delay is chosen as FOPDT model time constant and dead-time respectively.

$$\tau_i = \tau_m , \qquad L_c = L_m . \tag{4}$$

The above choice makes the controller dynamics same as that of dynamics process. To satisfy robustness requirements due to the presence of model uncertainty, the controller gain k_c can be designed for the user-specified gain margin specification. Gain margin specification is one of the well-known robustness indexes of closed-loop system.

The solution of following equations gives loop ultimate gain,

$$\arg\left[C(j\omega_{ph})\, G_m(j\omega_{ph})\right] = -\pi, \tag{5}$$

$$k_u = \frac{1}{\left|C(j\omega_{ph})\, G_p(j\omega_{ph})\right|} . \tag{6}$$

The frequency ω_p is phase crossover frequency at phase lag of $-180°$ in the Nyquist curve. The controller gain, k_c can be designed using the following relation,

$$\begin{aligned} k_u &= k_c\, GM, \\ k_c &= \frac{k_u}{GM}, \end{aligned} \tag{7}$$

where, k_u is ultimate gain and GM is user-specified gain margin.

3.1. Robustness of time delayed PI controller

The time delayed reset action may improve the robustness margins (gain and phase margins) compared to the conventional PI controller. The robustness improvement is analyzed in this section.

The PI controller time delayed transfer function 2 may be exhibited as by substituting $\tau_i = \tau_m$, $L_c = L_m$

$$C(s) = k_c \left(\frac{\tau_m s + 1}{\tau_m s + 1 - e^{-L_m s}} \right). \tag{8}$$

The loop transfer function is as follows,

$$C(s)G_p(s) = \frac{k_c k_m e^{-L_m s}}{\tau_m s + 1 - e^{-L_m s}}. \tag{9}$$

The above loop equation can be written as,

$$\begin{aligned} C(s)G_p(s) &= \frac{k_c k_m}{(\tau_m s + 1)e^{L_m s} - 1}, \\ C(j\omega)G_p(j\omega) &= \frac{k_c k_m}{(\cos(L_m\omega) - \tau_m\omega\sin(L_m\omega) - 1) + j(\sin(L_m\omega) + \tau_m\omega\cos(L_m\omega))}. \end{aligned} \tag{10}$$

Consider a FOPDT process model $\frac{1}{s+1}e^{-s}$; the time delayed PI controller parameters are chosen as $\tau_i = 1$; $L_c = 1$. Using Eqs. (5) and (6), the ultimate gain k_u is obtained as 3.25, the phase margin is calculated using the following relation,

$$\left| C(j\omega_g)G_p(j\omega_g) \right| = 1, \tag{11}$$

$$PM = \arg\left[C(j\omega_g)\, G_p(j\omega_g) \right] + \pi. \tag{12}$$

The obtained phase margin value is 68 deg.

To compute the margins of conventional PI controller let us substitute $L_m = 0$ in the controller Eq. (9), then the controller transfer function becomes,

$$C(s) = k_c \left(\frac{\tau_m s + 1}{\tau_m s} \right). \tag{13}$$

For the above PI controller transfer function, the calculated gain and phase margin values are *GM* – 1.57 and *PM* – 32.7 deg. The Nyquist plots are drawn for with and without delay element in the positive feedback loop and are displayed in Fig. 2. Comparing the robustness margins, the time delayed PI controller gives high gain and phase margin values that are reflected in the Nyquist plot. This analysis clearly shows that the delay in the positive feedback path increases the gain and phase margins. This high margin level of the time delayed PI controller allows higher controller gain for the improved regulatory closed-loop operation.

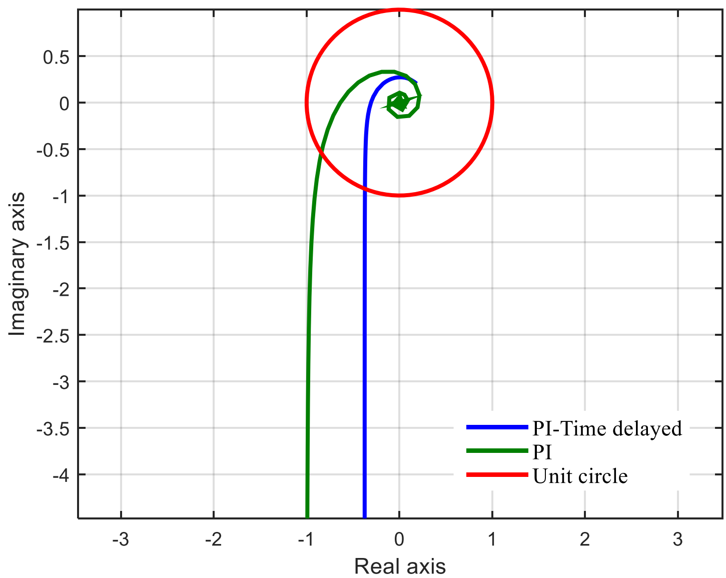


Figure 2: Nyquist plots of PI-Time delayed and PI controller

3.2. Stability analysis of time delayed PI control scheme

The existence of controller time delay element enhances robustness margins. In another part, control loop may be sensitive to dissimilar variations in dead-time process. Consider a process model $G_m(s) = \frac{1}{s+1}e^{-L_m s}$ with different values of dead-time. The loop transfer functions of Nyquist plots are presented in Fig. 3 for different dead-times. From Fig. 3, it is inferred that there is an occurrence of multiple crossover frequencies for higher values of dead-time. The existence of time delay process in control structure creates a multiple crossover phenomenon for processes with maximum values of dead-time compared to the process time constant. Additionally, it is inferred that the occurrence is related to ratio of dead-time to time constant. The number of crossovers maximizes as the ratio maximizes. This multiple crossovers start when the ratio greater than 3. Hence, the process can be classified as dead-time dominant when the ratio is greater than 3. Based on the occurrence of multiple crossovers the process is classified as dead-time dominant.

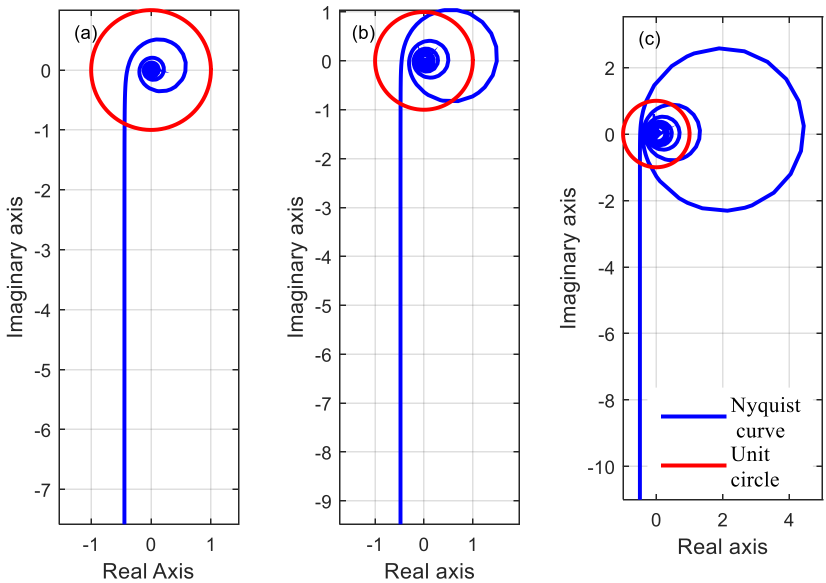


Figure 3: Nyquist plot of the control loop with dead-time values as $L = 2$ (a), 4 (b), and 8 (c)

In this work, the graphical analysis tool such as the Nyquist plot has been extensively used to investigate the stability of the system.

Using Eq. (11), the $\left|C(j\omega_g)G_p(j\omega_g)\right|$ is computed as follows,

$$\left|C(j\omega_g)G_p(j\omega_g)\right| = \\ = \frac{k_c k_m}{sqrt\left[2 + \left(\tau_m\omega_g\right)^2 + 2\left(\tau_m\omega_g \sin(L_m\omega_g) - \cos(L_m\omega_g)\right)\right]}. \tag{14}$$

The condition for gain crossover frequencies is,

$$\frac{k_c k_m}{sqrt\left[2 + \left(\tau_m\omega_g\right)^2 + 2\left(\tau_m\omega_g \sin(L_m\omega_g) - \cos(L_m\omega_g)\right)\right]} = 1. \tag{15}$$

The gain crossover frequencies satisfy,

$$\tau_m^2\omega_g^2 + 2\tau_m \sin(L_m\omega_g)\omega_g + 2\left(1 - \cos(L_m\omega_g)\right) - k_c^2 k_m^2 = 0. \tag{16}$$

Solving the above equation the gain crossover frequencies information can be obtained. The condition for non-multiple crossover frequencies, i.e., the single gain crossover frequency is,

$$\tau_m\omega_g \sin(L_m\omega_g) - \cos(L_m\omega_g) > 0 \tag{17}$$

implies that

$$\cos(L_m\omega_g) < 0 \quad \text{if} \quad \sin(L_m\omega_g) > 0$$

also

$$\sin(L_m\omega_g) < 0 \quad \text{if} \quad \cos(L_m\omega_g) > 0. \tag{18}$$

High values of dead-time L_m give multiple gain crossover frequencies at lower frequencies. The term $\tau_m\omega_g \sin(L_m\omega_g) - \cos(L_m\omega_g)$ becomes negative for high values of L_m and creates multiple gain crossover frequencies. So, the process long dead-time might affect the system closed-loop stability.

A simulation analysis has been performed to assess system stability under process dead-time uncertainty. A first-order process $G_m(s) = \dfrac{1}{s+1}e^{-10s}$ having a 25% of incremental dead-time uncertainty has been considered. The following time delayed PI controller is designed for the settings of $k_c = 1$, $\tau_i = 1$, $L_c = 10$

$$C(s) = \left(\frac{s+1}{s+1-e^{-10s}}\right). \tag{19}$$

For the above controller the loop transfer function with process dead-time uncertainty is as below,

$$C(s)G_p(s) = \frac{e^{-12.5s}}{s+1-e^{-10s}}. \tag{20}$$

The loop transfer functions of Nyquist plot with and without dead-time uncertainty is shown in Fig. 4. The Nyquist plot encircles the critical point −1, for system with +25% variations in dead-time process. Therefore closed-loop becomes unstable due to dead-time uncertainty. From this evaluation, it is inferred that systems having multiple crossover frequencies is sensitive to small positive changes in dead-time.

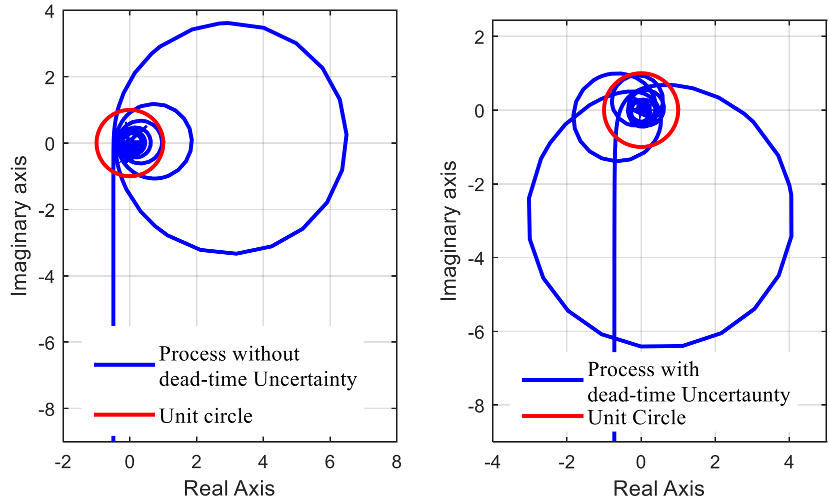


Figure 4: Nyquist plot of the control loop with and without dead-time uncertainty

3.3. Tuning of time delayed PI controller for dead-time dominant process

This occurrence of multiple crossovers makes the loop as sensitive to variations in dead-time. Hence, it should be eliminated by suitable compensation. The closed-loop system robustness against uncertainty in dead-time can be ensured by increasing the filtering properties. A first-order filter transfer function is added in both process feedback path and the controller feedback path to achieve a robust control system. The filtered time delayed PI controller is exhibited in Fig. 5. The purpose of filter is to eliminate the occurrence of multi-crossover phenomenon. The effective elimination can be examined with the help of the following example.

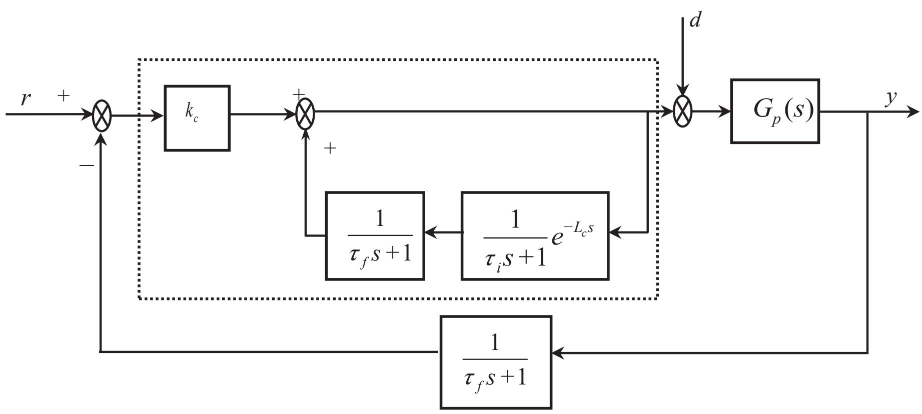


Figure 5: Filtered time delayed PI control scheme

Consider the same first-order process having a high value of dad-time $\left(G_m(s) = \dfrac{1}{s+1}e^{-10s}\right)$, the loop transfer function of Nyquist plot for different values of filter time constants $\tau_f = 2, 4, 6$ is shown in Fig. 6. It can be inferred that the addition of filter eliminates the occurrence of multiple gain crossover frequencies. Further, it is inferred that by selecting $\tau_f \geqslant \dfrac{L}{3}$, multiple gain crossover frequencies can be avoided. The addition of filter and its proper tuning eliminates the presence of number of crossovers in Nyquist plot with a unit circle, but it affects the regulatory performance. It is the conventional trade-off in-between robustness and operation. Owing to this analysis following tuning relation is proposed for dominant time-delay processes,

$$k_c = 1/k_m\,, \qquad \tau_i = \tau_m\,, \qquad L_c = L_m\,, \qquad \tau_f = \frac{L}{3}\,.$$

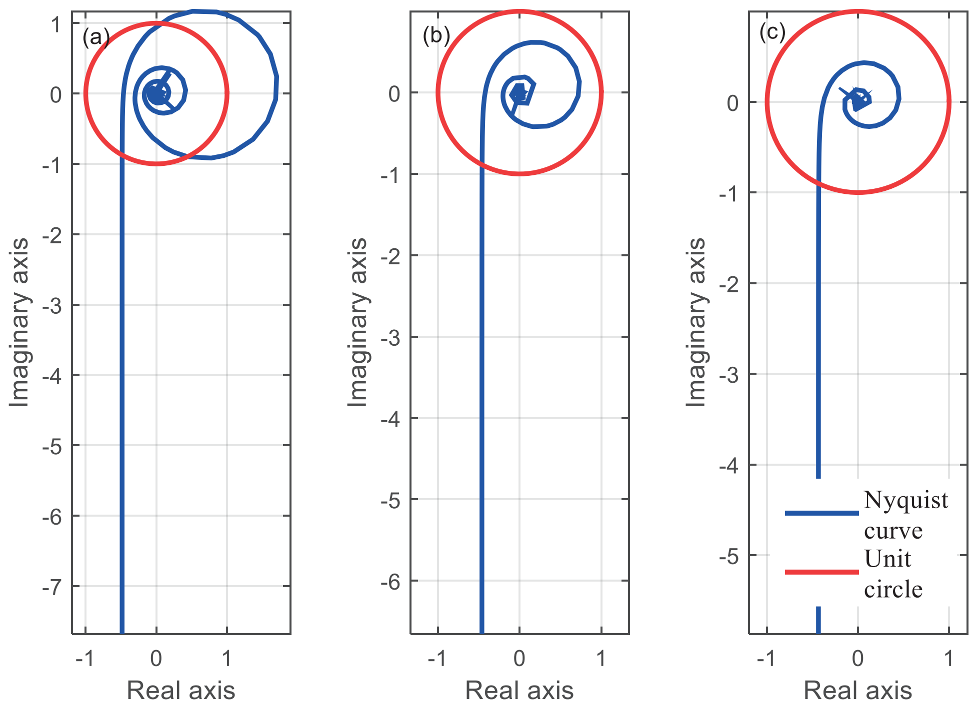


Figure 6: Nyquist plot of the control loops with different values of filter time constants $\tau_f = 2$; (a), 4; (b); and 6; (c)

The above controller tuning relation neglects the dead-time element from characteristics equation and brings the property of Smith predictor. The proposed controller tuning relations will give the following servo and regulatory responses.

Also, it provides better robustness against uncertainty in dead-time for dead-time dominant processes,

$$\begin{aligned} \frac{y(s)}{r(s)} &= \frac{1}{\tau_m s + 1} e^{-L_m s}, \\ \frac{y(s)}{d(s)} &= \frac{1}{(\tau_m s + 1)\left(\tau_f s + 1\right)} e^{-L_m s}. \end{aligned} \tag{21}$$

3.4. Tuning of time delayed PI controller for lag-dominated process

The time delayed PI control technique provides good response of servo and regulatory for the process with high dead-time or consistently large normalized dead-time processes. However, it is familiar that, for lag-dominant processes the pole-zero cancellation methods consequences in a longer settling time for the changes in the input disturbance. There is a necessity to modify the integral time constant value to maximize the input operation of disturbance rejection.

In literature, several tuning methods are proposed for the PI controller to maximize the function of disturbance rejection, and it is discussed in the introduction section. On the other hand, ZN closed loop method gives a good regulatory response to the lag dominated process. The ZN setting of integral time constant is based on the ultimate period.

The objective of controller design is to match the controller dynamics with the process dynamics and by tuning the controller gain to achieve satisfactory performance. For slowly changing process, the rate of controller integration can be set according to the value of the rate of ultimate input manipulation (ultimate period).

By decreasing the controller integral time constant, the load disturbance rejection performance can be improved. So, the controller integral time constant value can be selected as minimum value of either the process time constant and ultimate period. Hence, simulation study shows that this choice gives an improved disturbance rejection response. In order to find the point when the ultimate period is minimum when compared to process time constant and this can be seen as follows,

The phase angle equation for the FOPDT model is given by,

$$-\tan^{-1}(\tau_m \omega_p) - (L_m \omega_p) = -\pi\,. \tag{22}$$

Ultimate period is $pu = \dfrac{2\pi}{\omega_p}$.

For lag dominated process the time constant process is higher than the dead-time process. Hence, it is expressed as follows,

$$\tau_m > k \times L_m\,, \tag{23}$$

where, k is the multiplying factor and it should be greater than unity. Our objective is to investigate the value of k such that,

$$\tau_m > pu. \tag{24}$$

Substituting the above inequalities of Eqs. (23) and (24) in Eq. (22) and solving the following, k value is obtained as,

$$k = 3.6349 \tag{25}$$

when process having time constant greater than 3.6 times of process dead-time, the ultimate period is less than the process time constant. Hence, for processes having $\tau_m > 3.6349L_m$ is classified as the lag dominant process and controller integral time constant was taken as ultimate period.

As discussed above the controller integral time constant is set as the loop ultimate period,

$$\tau_i = pu. \tag{26}$$

3.4.1. Setpoint weighting

High integral gain gives minimum error for the regulatory response, but this provides more overshoot or oscillatory response for setpoint changes. It is considerably condensed by adding a setpoint filter in the setpoint as shown in Fig. 7. The setpoint filter has a weighting parameter b bounded between $0 \leqslant b \leqslant 1$. This filter provides two degrees of freedom controller design, and this filter design improves the setpoint response by reducing the overshoot.

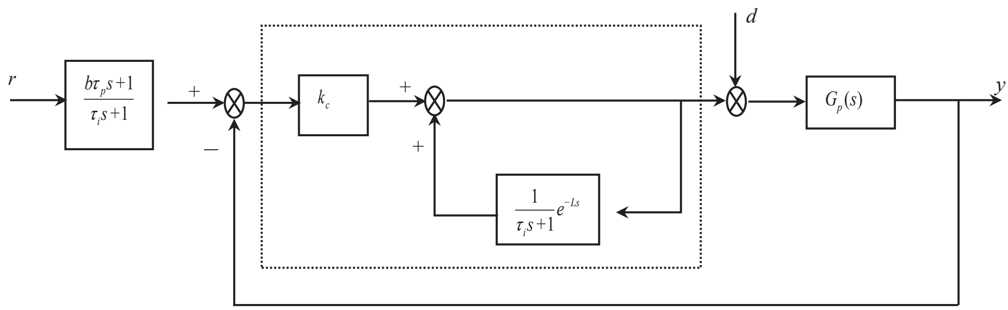


Figure 7: Setpoint weighted time delayed PI control scheme

3.5. Design flowchart of time delayed PI controller

A first order filter is added to ensure stability of the closed-loop system for time delay dominant processes. This addition may degrade the regulatory performance, but it is mandatory to attain a stable system in the existence of uncertainty in dead-time. The filter is also inserted in controller positive feedback path to match the process characteristics.

To achieve better regulatory performance, the integral time constant is modified for lag-dominant processes. The integral time constant is set as the loop ultimate period. As well, a setpoint filter is added to minimize the overshoot in reference tracking response. From extensive analysis, the process models are classified as dead-time dominant when $\tau_m > 3.6349L_m$ and lag dominant when $\tau_m > 3.6349L_m$.

The design flowchart of the time delayed PI controller is presented in Fig. 8 for FOPDT process models with different dead-time to time constant ratio. The controller dynamics is modelled with respect to the process dynamics, and further the controller gain is designed to achieve user-specified gain margin specification.

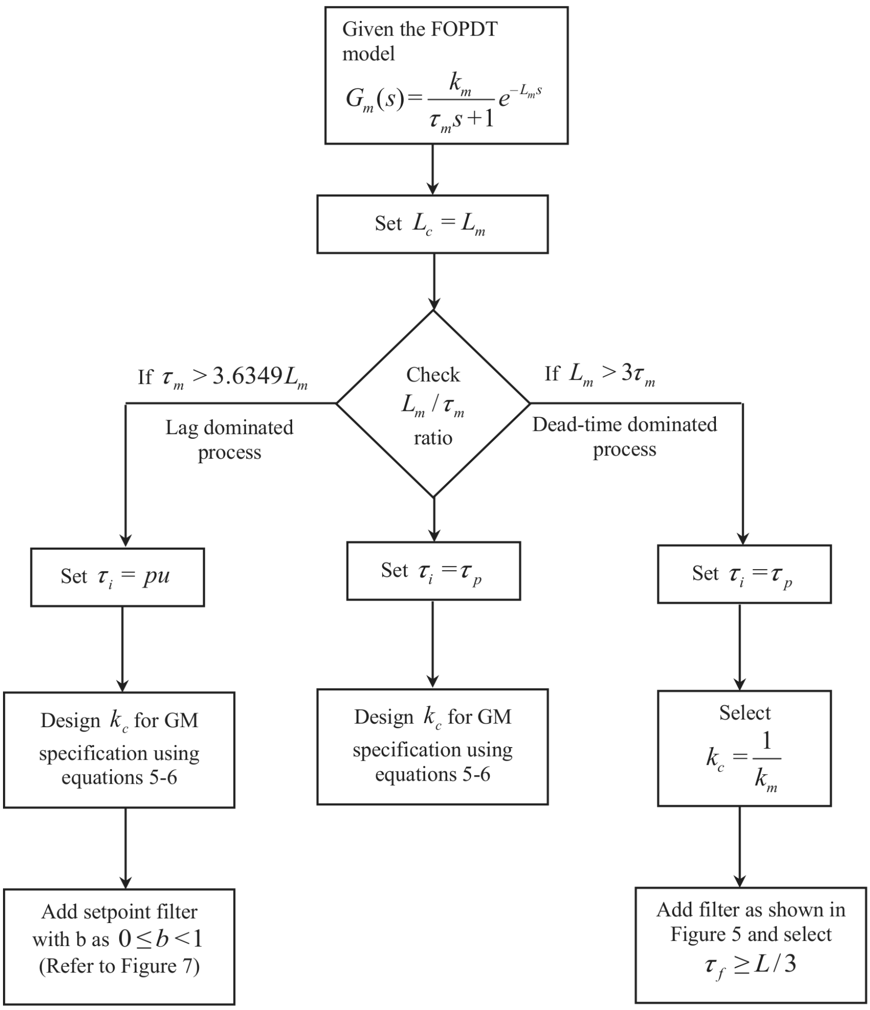


Figure 8: Design flowchart for time delayed PI controller

4. Simulation results and comparative analysis

The performance of the time delayed PI controller is illustrated through simulation examples. The performance measures such as Integral Absolute Error (IAE) and Total Variation (TV) of the computed input u are considered as performance criteria for comparison analysis.

The IAE is presented as follows,

$$\mathrm{IAE} = \int_0^{\infty} |r(t) - y(t)| \,\mathrm{d}t. \tag{27}$$

The TV is defined as follows,

$$\mathrm{TV} = \sum_{k=1}^{\infty} |u(k+1) - u(k)| \,. \tag{28}$$

Further, the robustness indexes such as gain margin, phase margin and Maximum sensitivity (M_s) of the designed control schemes are calculated for all simulation examples. It is noted that M_s is a convenient measure of both operation and stability robustness. Let the sensitivity function $S(s)$ shall be expressed as below,

$$S(s) = \frac{1}{1 + C(s)G_m(s)} = \frac{1}{1 + L(s)}, \tag{29}$$

where, $L(s) = C(s)G_m(s)$ is the loop transfer function.

The parameter M_s is defined as follows,

$$M_s = \max_{\omega} |S(j\omega)| \,. \tag{30}$$

Example 1

Consider a model $\frac{1}{s+1}e^{-0.5s}$ with L_m/τ_m the ratio of 0.5 and the time delayed PI controller is tuned for gain margin specification of 2. For comparison, PDI controller [11] and IMC-PI controller [10, 22] is considered. In Table 1, the controller parameters are presented. The simulated process for the reference and input disturbance changes are shown in Fig. 9. The computed performance and robustness indexes are presented in Table 1. From the results, it is observed that the time delayed PI controller gives lower IAE value for regulatory response when compared to PDI and IMC-PI (Chien) controllers. The time delayed PI controller IAE value is slightly higher when compared to PDI for setpoint, and this is due to high controller gain that gives good regulatory response but gives slight overshoot in the setpoint response (Refer Fig. 9).

Table 1: Performance and robustness index of time delayed PI controller, PDI and IMC-PI controller for FOPDT process with $L_m/\tau_m = 0.5$

Method	k_c	τ_i	GM	PM	Ms	Setpoint		Load disturbance	
						IAE	TV	IAE	TV
Time delayed PI – Proposed	2.4	1	2.0	50.4	2.12	0.93	4.56	0.63	1.55
PDI – Lee & Edgar ($\lambda = 0.3$)	2.14	1.7	2.12	64	1.94	0.87	3.9	0.8	1.27
IMC-PI-Chien ($\tau_c = 0.333$)	1.2	1	2.61	55.5	1.78	1.05	2.8	0.83	1.27

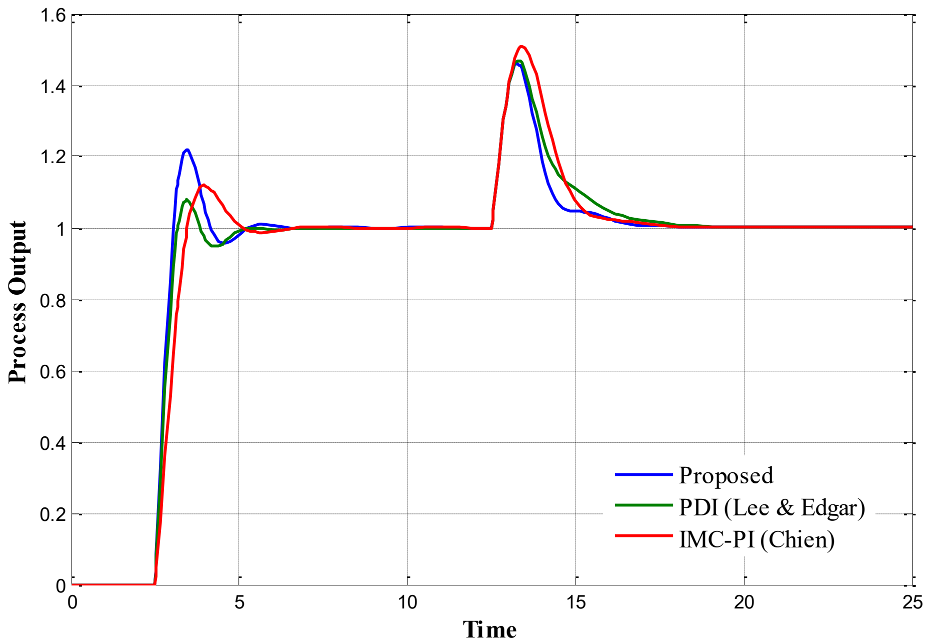


Figure 9: Process output responses for FOPDT process with $L_m/\tau_m = 0.5$

It is noted that the time delayed PI controller gives comparable phase margin and M_s value. The analytic tuning rules of PDI controller has been derived by approximating the IMC controller and has a single tuning parameter λ. Additionally, the PDI controller tuning relation requires process ultimate gain information. For this example, the tuning parameter λ is taken as 0.3 and controller parameters were considered that were being listed by Lee and Edgar, 2002. Compared to the PDI scheme the time delayed PI controller tuning is simple and it is tuned for a particular specification.

Example 2

Consider a lag dominated process model $\frac{100}{100s+1}e^{-s}$, the time delayed PI controller is modelled, and the controller gain k_c is determined to gain margin specification of 2.5. For comparison, direct synthesis of PI controller for disturbance response proposed by Chen and Seborg and SIMC-PI controller is considered and is presented in Table 2.

Table 2: Performance and robustness index of time delayed PI controller, Ds-d-PI and SIMC-PI controllers for FOPDT process with $L_m/\tau_m = 0.01$

Method	k_c	τ_i	GM	PM	Ms	Setpoint		Load disturbance	
						IAE	TV	IAE	TV
Proposed ($b = 1$)	0.72	4	2.5	36.48	1.95	3.35	1.68	6.99	1.9
Proposed ($b = 0.5$)	0.72	4	2.5	36.48	1.95	2.24	0.91	6.99	1.9
DS-d-PI ($\tau_c = 10$)	0.55	4.91	2.58	38.3	1.93	3.58	1.53	8.91	1.84
SIMC-PI	0.5	8	3.0	47.98	1.69	3.78	1.2	16.0	1.51

The process of the control techniques for the reference and input disturbance changes are shown in Fig. 10. The evaluated operation and robustness indexes

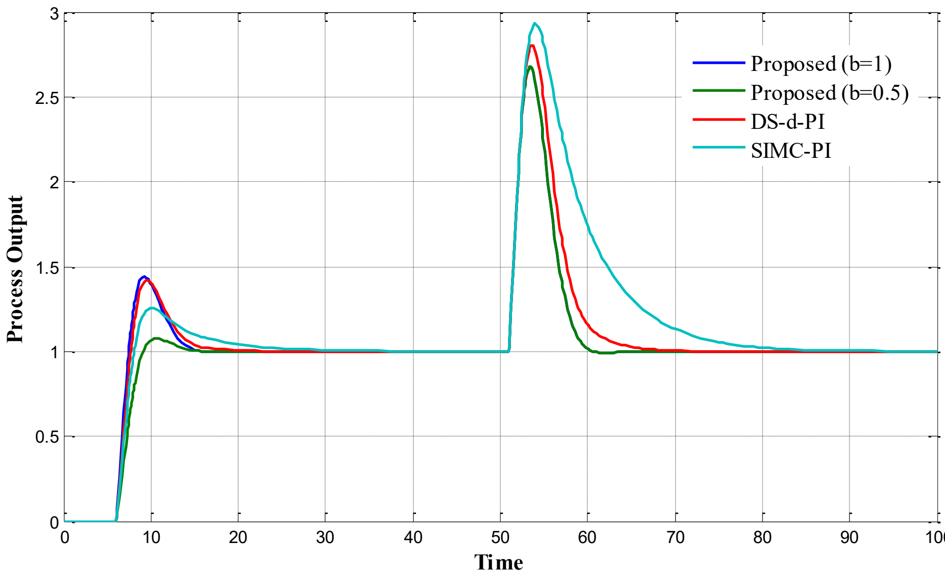


Figure 10: Process output responses for FOPDT process with $L/\tau = 0.01$

are tabulated in Table 2. From the results, it can be observed that the time delayed PI controller shows lower IAE value for the regulatory response when compared to DS-d-PI and SIMC-PI controllers. Also, the setpoint weighted time delayed PI controller provides lower IAE value for reference tracking response when compared to other controllers. The time delayed PI controller shows equal robustness of DS-d based PI controller.

Example 3

Consider the following first order process with a long dead-time $G_m(s) = \dfrac{1.02}{1.7s+1}e^{-8.2s}$, the time delayed PI controller filter time constant is set to 2.73 ($L_m/3$) for better robustness against uncertainty in dead-time. The PI controller MSP and FSP primary is tuned to get proportional gain of 0.9804 and constant integral time equivalent to time constant process. MSP filter employed is given as [16],

$$F_r(s) = \frac{1.7s+1}{s+1}\,.$$

FSP filter utilized is expressed as [15],

$$F_r(s) = \frac{(1.698s+1)(1.7s+1)}{(s+1)^2}\,.$$

To evaluate the control techniques' robustness the Nyquist curve is drawn and exhibited in Fig. 11.

From Nyquist curves, it is concluded that the time delayed PI controller has one unit circle intersection point whereas MSP and FSP techniques have a number of unit circle intersection points. The system is highly sensitive due to multiple unit circle interaction points for dead-time variation. Thus, robustness filter designed for FSP in [16] and MSP [3] has not considered the multiple unit circle interaction which is triggered by the DTC technique. The servo and disturbance responses of the time delayed PI controller, MSP and FSP control techniques are presented in Fig. 12. It is noticed that MSP and FSP control techniques' operation are comparable for long dead time processes. The time delayed PI controller techniques exhibit a sluggish regulatory performance than MSP and FSP. The objective of time delayed PI controller filter model is to neglect the varied unit circle interactions, thereby uncertainty in dead-time robustness is increased and degraded function when differentiated to other control techniques.

To evaluate the operation of control techniques in the presence of dead-time uncertainty, an uncertainty of +25% in dead-time process than actual loop dead-time have been taken, and simulated. The MSP and FSP techniques servo responses of time delayed PI controller, are presented in Fig. 13. The time delayed

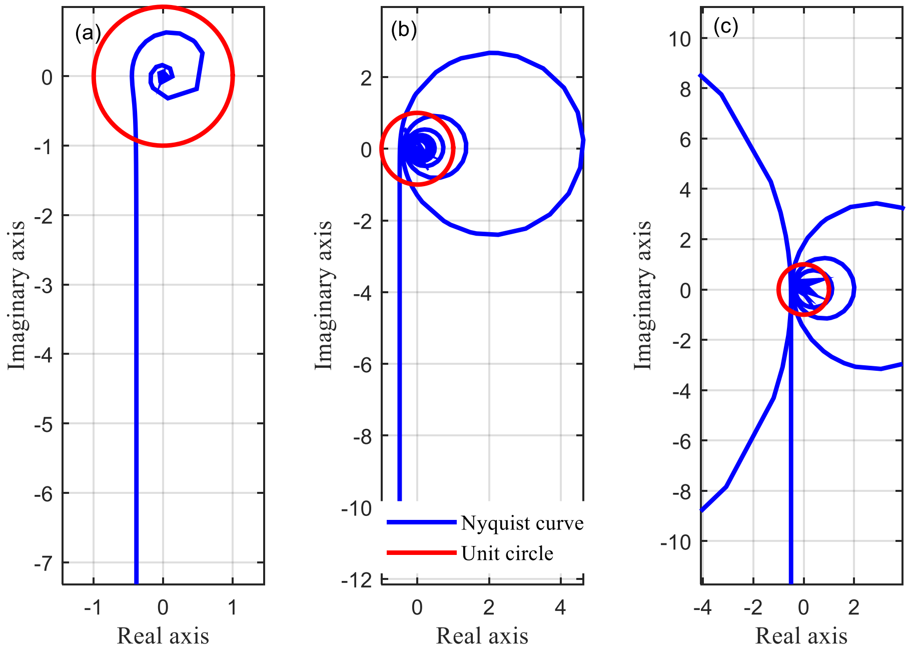


Figure 11: Nyquist curves for long dead-time first order process: (a) time delayed PI, (b) MSP, and (c) FSP

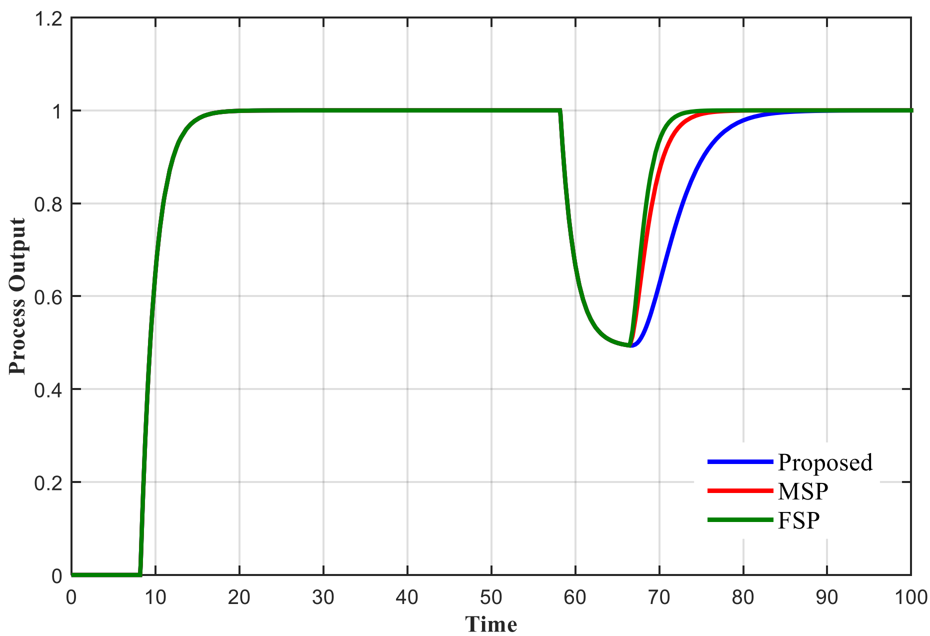


Figure 12: Servo and regulatory response of DTC schemes for long dead-time first order process

PI controller exhibits a stable performance in the presence of uncertainty in the process dead-time, whereas the MSP and FSP control techniques presents an unstable performance. The presence of multiple unit circle interactions in the Nyquist curve of MSP and FSP schemes loop transfer function makes the techniques sensitive to dead-time. Hence, the methodologies exhibit unstable response for process with dead-time uncertainty. It should be noted that the design of robustness filter of MSP and FSP methodologies should consider the removal of multiple unit circle interactions in the loop Nyquist curve when a large delay margin is needed.

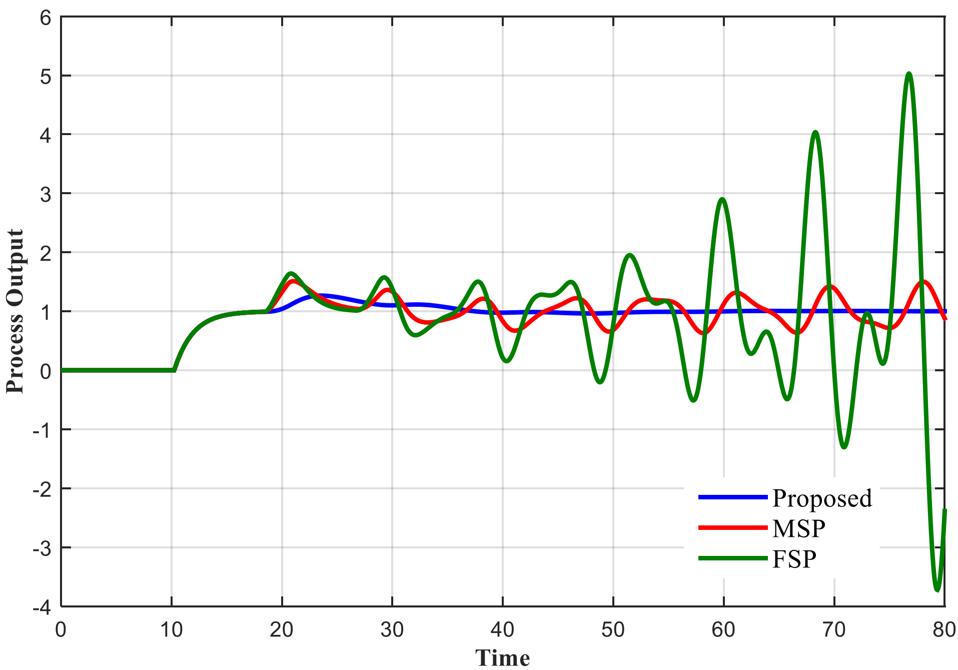


Figure 13: Servo response of DTC schemes for long dead-time first-order process with dead-time uncertainty

4.1. Extension to other processes

In this section, the applicability of the developed model to high-order, Integral plus dead-time and non-minimum phase processes are illustrated with simulation examples. Also, experimental evaluation is presented. High-order processes may be approximated as FOPDT model using suitable model reduction techniques, and then the controller has been designed based on the approximated model. In this paper, half rule model reduction technique proposed by Skogestad is used. Integral plus time delay process can be considered as lag dominated process and controller can be modelled to gain margin specification.

Example 4

To evaluate the applicability of the time delayed PI controller on a high-order process, the following transfer function has been considered $G_p(s) = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)}$. The high-order model was approximated to a FOPDT model by utilizing Skogestad Half rule and is shown as $\frac{1}{1.1s+1}e^{-0.148s}$. The time delayed PI controller, DS-d-PI controller, and SIMC-PI controllers are designed using the approximated FOPDT model. The obtained parameters are presented in Table 3.

Table 3: Performance and robustness index of time delayed PI controller, Ds-d-PI and SIMC-PI controllers for high-order process

Method	k_c	τ_i	GM	PM	Ms	Setpoint		Load disturbance	
						IAE	TV	IAE	TV
Proposed ($b = 1$)	4.67	0.56	7.0	49.85	1.61	0.55	10.54	0.49	5.54
Proposed ($b = 0.5$)	4.67	0.56	7.0	49.85	1.61	0.35	5.47	0.49	5.54
DS-d-PI	2.94	0.71	7.45	46.75	1.61	0.55	6.61	0.72	4.56
SIMC-PI	3.72	1.1	6.69	51.15	1.57	0.45	8.18	0.89	4.23

The process of control techniques for the reference and input disturbance changes are presented in Fig. 14. The evaluated function and robustness indexes are exhibited in Table 3. The time delayed PI controller gives good regulatory response when compared to DS-d and SIMC PI controllers. The time delayed PI controller with setpoint weighting gives a good setpoint response when compared to DS-d and SIMC PI controller.

Example 5

Consider an integrating plus dead-time process model $G_p(s) = \frac{0.2}{s}e^{-7.4s}$, and the time delayed PI controller is designed for this integrating process. Integrating process can be considered as a lag dominated process and τ_i value can be taken as the ultimate period. Then, the controller gain k_c is determined for a gain margin specification of 2.5. For comparison, DS-d-PI and Tyreus-Luyben PI controller is considered and presented in Table 4 [4].

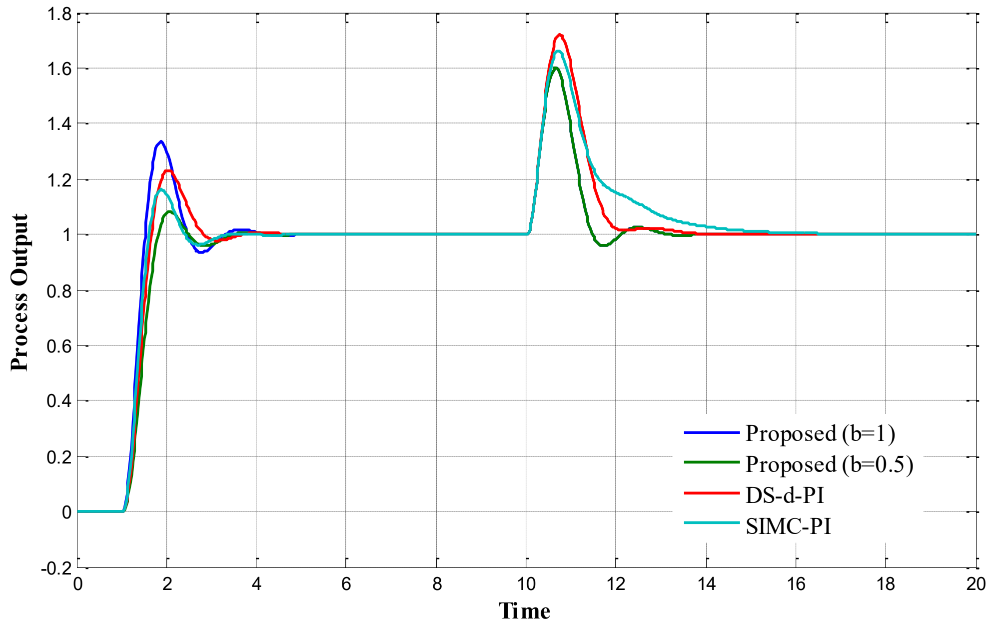


Figure 14: Process output responses for high-order process

Table 4: Performance and robustness index of time delayed PI controller, DS-d-PI and TL-PI controllers for integrating plus dead-time process

Method	k_c	τ_i	GM	PM	Ms	Setpoint		Load disturbance	
						IAE	TV	IAE	TV
Proposed ($b = 1$)	0.48	29.6	2.5	35.77	1.96	25.66	1.14	39.46	0.97
Proposed ($b = 0.5$)	0.48	29.6	2.5	35.77	1.96	17.15	0.62	39.46	0.97
DS-d-PI	0.37	37.4	2.57	37.85	1.94	27.09	1.05	50.13	0.93
TL-PI	0.33	64.7	3.05	48.43	1.67	29.49	0.79	97.95	0.75

The process of the control schemes for reference and input disturbance changes are given in Fig. 15. The computed performance and robustness indexes are presented in Table 4. From the results, it is concluded that the time delayed PI controller shows lower IAE value for load disturbance changes when compared to DS-d-PI and TL-PI controllers. The setpoint weighted time delayed PI controller gives lower IAE value for reference changes when compared to other controllers.

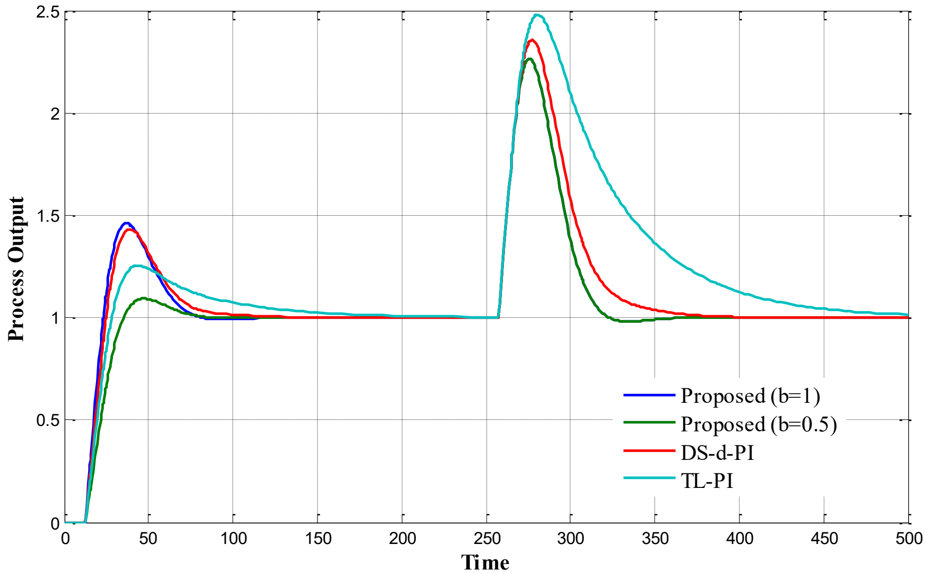


Figure 15: Process output responses for integrating plus dead-time process

Example 6

To evaluate the function of time delayed PI controller on a high-order process with RHP zero and dead-time, the following transfer function has been considered $G_p(s) = \dfrac{(-s+1)}{(s+1)^5}e^{-2s}$. The high-order model was approximated to FOPDT model using Skogestad Half rule and is expressed as $\dfrac{1}{1.5s+1}e^{-6.5s}$.

The time delayed PI controller, DS-d-PI controller, and SIMC-PI controllers are designed using the approximated FOPDT model. The obtained parameters are presented in Table 5. The process of the control schemes for reference and

Table 5: Performance and robustness index of time delayed PI controller, Ds-d-PI and SIMC-PI controllers for high-order process with RHP zero and dead-time

Method	k_c	τ_i	τ_f	GM	PM	Ms	Setpoint		Load disturbance	
							IAE	TV	IAE	TV
Time delayed PI – Proposed	1	1.5	2.17	2.6	62.68	1.67	8.232	1.3	10.4	1.05
DS-d-PI ($\tau_c = 2.5$)	0.139	1.38		2.57	52.95	1.83	13.98	1.4	13.77	1.38
SIMC-PI ($\tau_c = 6.5$)	0.115	1.5		3.35	61.61	1.56	14.18	1.09	14.21	1.1

input disturbance changes are given in Fig. 16. The computed performance and robustness indexes are presented in Table 5. The time delayed PI controller gives good servo-regulatory response when compared to DS-d and SIMC PI controllers.

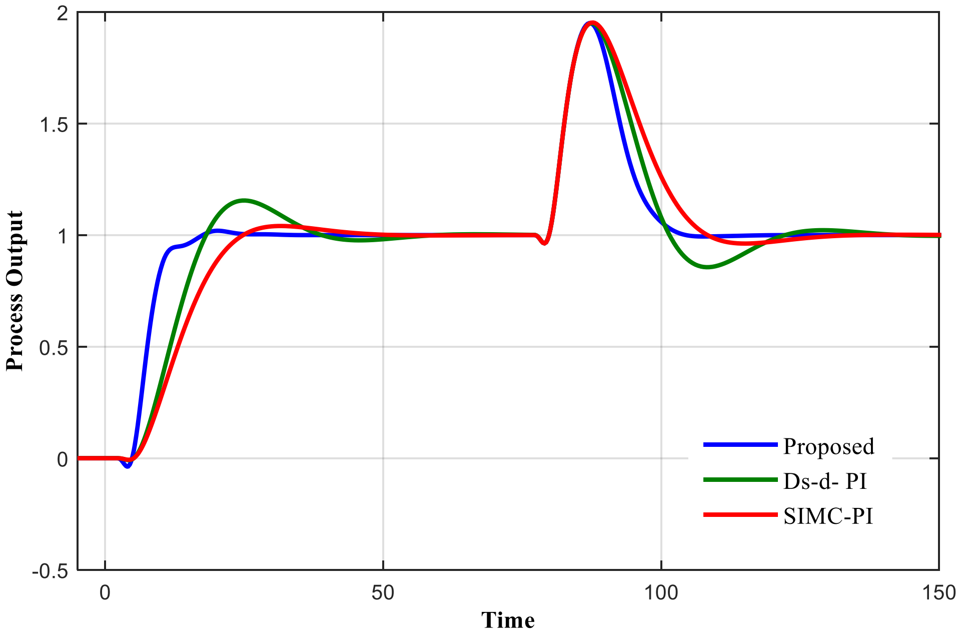


Figure 16: Process output responses for high-order process with RHP zero and dead-time

Example 7

The proposed scheme is evaluated on a two-tank non-interacting system. The actual set-up is a five-tank system, and each tank has a circular area of 67 cm^2 and a height of 25 cm. The experimental setup is shown in Fig. 17. For this study, the piping arrangement of the set-up has been altered in such a way that the five-tank system will behave like a two-tank non-interacting system.

The piping and instrument diagram for the two-tank non-interacting tank is given in Fig. 18. In this control system, the level of Tank 2 is the Process Variable (PV) which is controlled by adjusting the inflow of Tank 1. Note that the inflow of Tank 1 is manipulated only by the voltage of the first pump (P1) which is the manipulated variable (MV).

The pump and the level transmitter are interfaced with a personal computer for online implementation using a NIUSB Data Acquisition Card (NI USB DAQ 6001). The algorithms for the control schemes were developed and implemented in real time using the LabVIEW software package. Using the process reaction curve method, the process transfer function around an operating point, relating

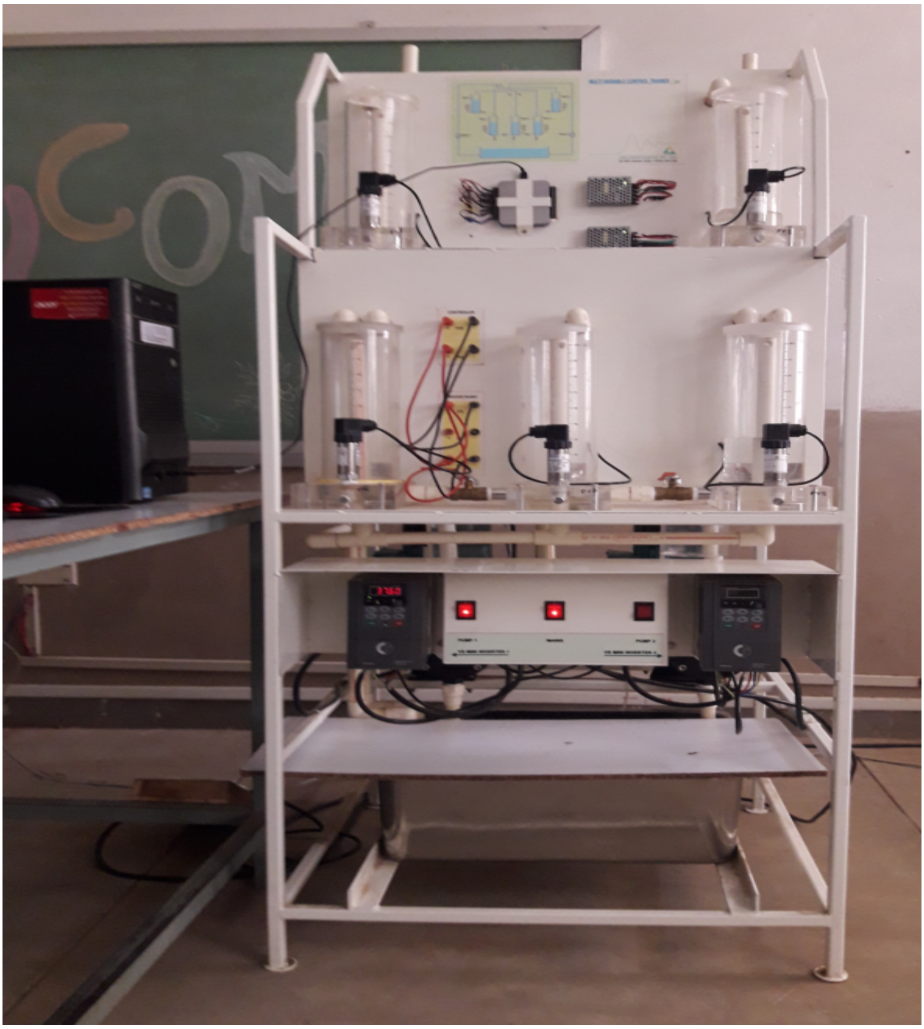


Figure 17: Experimental setup

the MV to PV is obtained by,

$$G_m(s) = \frac{1.5}{70s + 1} e^{-10s}.$$

The time delayed PI controller is designed for gain margin specification of 4. The control schemes are implemented in real-time, and the obtained step servo response and regulatory response are shown in Fig. 19. The time delayed PI controller performance is compared to the popular SIMC PI controller. The computed performance and robustness measures are reported in Table 6.

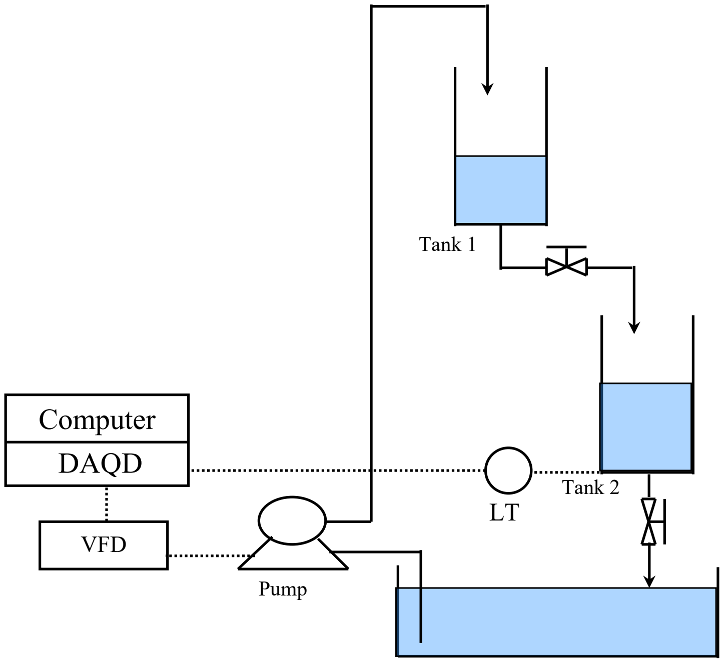


Figure 18: Schematic diagram of the two-tank non-interacting system

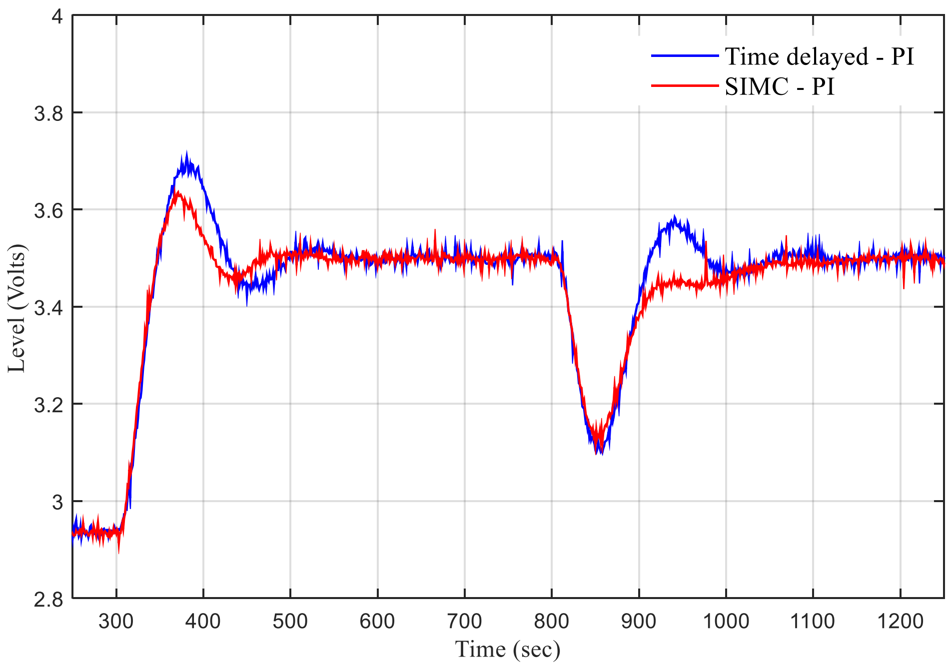


Figure 19: Servo-regulatory response for two-tank non-interacting process

Table 6: Performance and robustness index of time delayed PI controller and SIMC-PI controllers for two tanks non-interacting process

Method	k_c	τ_i	GM	PM	Ms	Setpoint		Load disturbance	
						IAE	TV	IAE	TV
Time delayed PI – Proposed	2.22	37.9	4.0	54.93	1.44	30.23	10.89	29.19	9.44
SIMC-PI (τ_c = 10)	2.33	70	3.14	61.36	1.59	24.09	11.63	30.39	9.36

The time delayed PI controller offers high gain margin when compared to SIMC-PI controller. The time delayed PI controller shows slightly higher overshoot servo response and improved regulatory performance when compared to SIMC-PI controller.

5. Concluding remarks

FOPDT process models are classified as dead-time and lag dominant based on the dead-time to time constant ratios. A unified design procedure for time delayed PI control scheme has been proposed in this paper for different dead-time/time-constant ratios of the FOPDT process model. A first order filter is used to ensure the robustness of time delayed PI controller for the dead-time dominant process. The integral time constant is taken as the ultimate period of the FOPDT process model, and this choice gives improved disturbance response for the lag dominated process. A reference filter is inserted to reduce the servo response overshoot for the lag dominated processes. Through extensive simulation analysis, it is found that the time delayed PI controller provides enhanced servo and regulatory performances. The proposed design of time delayed controller is applicable for processes having different characteristics such as integrating, high-order and right half plane zero.

Simulation examples clearly demonstrate the servo-regulatory performance improvement of the proposed design method of time delayed PI controller when compared to PI controller tuning methods for FOPDT processes with different dead-time to time constant ratios. Further, the proposed design method of time delayed PI controller gives equal robustness as that of PI controller and improved performance. The presence of time delay element in the controller structure enhances the performance, and proper design of filter ensures the robustness against dead-time uncertainty for dead-time dominant processes. The modified integral time constant for dominant lag process improves the disturbance rejection performance when compared to PI controller. Also, the proposed design method shows better performance for process having medium dead-time to the time constant ratio processes. Simulation examples vividly show that the proposed design method can be easily extended to high-order, integrating and non-minimum phase time delay processes.

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