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Original research paper

Determination of correlation coefficient between geoid-to-quasigeoid separation calculated by the satellite data in Sjöberg's equation and GPS/Levelling method

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Abstract: In this paper, two techniques for calculating the geoid-to-quasigeoid separation are employed. One of them is GPS/Levelling customary method as a criterion where the geoid undulation and height anomaly are computed by subtracting the ellipsoid height attained via GPS from the orthometric height and normal height, respectively. Another approach is Sjöberg's equation. We have used of the ICGEM website for definition of the variables of the Sjöberg's equation, as the applied reference model is the EGM2008 global geopotential model and WGS84 reference ellipsoid. The investigations are performed over the stations of the GPS/Leveling network related to three selected areas in desert, mountain and flatland namely the Lout, Zagros and Khuzestan in Iran and afterward the correlation coefficient between the geoid-to-quasigeoid separation calculated using the satellite data in Sjöberg's equation and GPS/Levelling method is estimated. The results indicate a straight correlation between the estimated separations from the two methods as its value for the Lout is 0.754, for the Zagros is 0.497 and for the Khuzestan is 0.659. consequently, using the satellite data in Sjöberg's equation for the regions where there are not the GPS/Levelling and land gravity data, specially for the even areas, yield a satisfactory response of the geoid-to-quasigeoid separation.

Keywords: geoid, GPS/levelling, ICGEM, quasigeoid, Sjöberg's equation

1. Introduction

Traditionally, global and local gravimetric quasigeoid models are determined and, then, fitted to GPS/levelling data. The geoid is an equipotential surface of the Earth that corresponds to mean sea level, whereas the quasigeoid is a geometrical surface referred to a normal height system. The geoid undulation (N) is the separation between the ellipsoid and the geoid measured along a straight line between the geoid and ellipsoid which is perpendicular to the ellipsoid. The height anomaly (ζ) is the separation between the reference ellipsoid and quasigeoid along a perpendicular line to the ellipsoid. The Global Positioning System (GPS) has been extensively utilized in surveying and mapping applications worldwide. A combination of gravimetrically derived quasigeoid heights over a target area with ellipsoidal heights provided by GPS has become a standard procedure in quasigeoid and height reference surface modelling.

In recent decades, separation of geoid from quasigeoid has been of high interests in many geodetic and geophysical studies. For the realization of national and international height reference levels, and for accuracy height determination in geodetic engineering, it is essential to evaluate the geoid–quasigeoid separation with centimeter accuracy or better (Flury and Rummel, 2009). This matter is considered as a necessity to gain better understanding about height references recognized internationally.

In 1962, Molodensky formulated a geodetic boundary value problem for the physical surface of the earth and, consequently, defined quasigeoid and geoid in space (Figure 1). Tenzer et al. (2005) presented the expressions for an accurate computation of the mean gravity along the plumb line and derived corresponding relation between the evaluation of the mean actual and normal gravity values in definitions of the orthometric and normal heights. Sjöberg (2006) and Tenzer et al. (2006) derived the formula for the geoid-to-quasigeoid separation by means of applying Bruns' (1878) theorem. Kiamehr (2007) introduced a new height datum for Iran based on combination of the gravimetric and GPS/Levelling geoid models. Prutkin and Kless (2008) have solved Laplace equations to evaluate the separation between geoid and quasigeoid in high level areas accurately. Flury and Rummel (2009) develop a compact formulation for the rigorous treatment of topographic masses and apply it to determine the geoid–quasigeoid separation. Sjöberg (2010) proposed more accurate equation to separate geoid from quasigeoid. The expressions for calculating the geoid-to-quasi-geoid separation in spatial and spectral domains was summarized by Tenzer et al. (2015). Tenzer et al. (2016) applied the spectral expressions to estimate the geoid-to-quasi-geoid separation globally.

Regarding the fact that the northern neighbors of Iran employ quasigeoid as levelling reference, to unify and correlating levelling data in the area for local study purposes and also controlling them, we need to calculate a quasigeoid for Iran area. Accurate calculation of separation of quasigeoid from geoid seems to be essential. Therefore, in this paper, the separation are done by GPS/Levelling and the proposed method based on the Sjöberg's equation (2010) for three areas in Iran and finally they are compared. As the greater contribution to the geoid–quasigeoid separation is due to the effect of the distribution of topographic masses on the gravity field (Flury and Rummel, 2009), we have employed the Sjöberg's equation which is considered the topographic potentials at the geoid and surface points.

The main objective in this study is the determination of the Sjöberg's equation precision when the satellite data is applied for solving it. The numerical results obtained from GPS/Levelling method is consider as a criterion.

2. GPS/Levelling method

The geoid-to-quasigeoid separation exhibits the difference between the geoid and the quasi-geoid, or equivalently the difference between the normal and orthometric heights. The orthometric and normal heights can be estimated using the observed or normal gravity field data by the computational methods. The geometric GPS heights are related to physically meaningful heights through the geoid or the quaeigeoid. Thus we can compute geoid-to-quasigeoid separation $(N-\zeta)$ employing GPS/Levelling. The evaluated height by GPS is considered as ellipsoid height (distance of PQ_0 in Figure 1). The height anomaly ζ is obtained using GPS/Levelling datums via following formula:

$$\zeta_{\mathrm{GPS/Levelling}} = h_{\mathrm{GPS}} - H^{\mathrm{N}}, \tag{1}$$

where, H^{N} refers to normal height, i.e. QQ_0 drawn in Figure 1. The geoid height N, is computed using GPS/Levelling datums as:

$$N_{\mathrm{GPS/Levelling}} = h_{\mathrm{GPS}} - H^{\mathrm{O}}. \tag{2}$$

Here, H^{O} is orthometric height, i.e. PP_0 in Figure 1. Finally, $N-\zeta$ is calculated by subtracting the equations (1) from (2). Thus we can write:

$$N - \zeta = H^{\mathrm{N}} - H^{\mathrm{O}}. \tag{3}$$

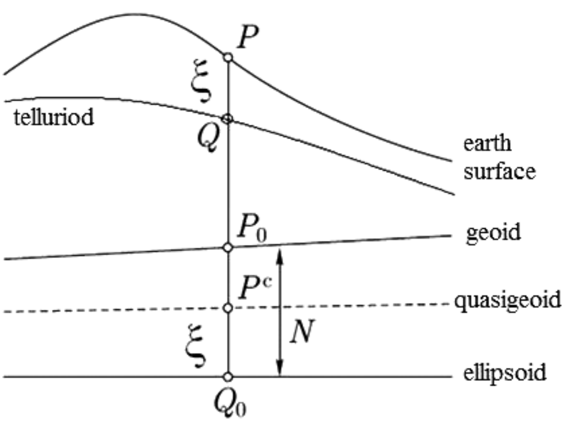

Fig. 1. Different geodetic levels. PP_0: orthometric height. PQ_0: ellipsoidal height. QQ_0: normal height. N: undulation. ζ: height anomaly

Although the $N-\zeta$ can directly obtain from $H^{\mathrm{N}}-H^{\mathrm{O}}$, but the estimated separation values by $H^{\mathrm{N}}-H^{\mathrm{O}}$ and ones are based on the subtraction between the $N_{\mathrm{GPS.Levelling}}$ and $\zeta_{\mathrm{GPS/Levelling}}$ have slightly difference numerically.

3. Determination of $N-\zeta$ using Sjöberg's equation

The second method to calculate $N-\zeta$ is accurate equation of Sjöberg (2010) as follows, Which consists of three parts:

$$N-\zeta \approx \frac{\Delta g_{\mathrm{P}}^{\mathrm{BO}}}{\bar{\gamma}} H^{\mathrm{O}} + \frac{vg^{T}-vp^{T}}{\bar{\gamma}} + \frac{\delta\bar{g}^{\mathrm{BO}}-\delta g_{\mathrm{P}}^{\mathrm{BO}}}{\bar{\gamma}}. \tag{4}$$

The first part is $\frac{\Delta g_{\mathrm{P}}^{\mathrm{BO}}}{\bar{\gamma}} H^{\mathrm{O}}$ known as approximate equation of Featherstone and Kirby (1998) in which $\Delta g_{\mathrm{P}}^{\mathrm{BO}}$ denotes (simple) Bouguer gravity anomaly at the computation point P on the earth surface obtained from the following equation (Heiskanen and Moritz, 1967)

$$\Delta g_{\mathrm{P}}^{\mathrm{BO}} = g + 0.3086H - 0.1119H - \gamma. \tag{5}$$

Here, H is orthometric height, g is observed gravity on the earth surface and γ refers to ellipsoidal normal gravity. It is worth mentioning, the equation (4) was proposed by Flury and Rummel (2009), but the last term of equation (4) was not clearly contained in their final formulation. In this paper, in order to compute Bouger gravity anomaly, the value of above mentioned variables, i.e. g, H and γ, have been gained from ICGEM website which are estimated based on a combination of satellite and land data. Also, the Bouger gravity anomaly at point P on the earth is accessible in ICGEM website directly.

$\bar{\gamma}$ is the average of normal gravity from geoid to reference ellipsoid or from earth surface to quasigeoid which can be computed by the following equation (Heiskanen and Moritz, 1967):

$$\bar{\gamma} = \gamma_{P0}\left[1-\left(1+f+m-2f+\sin^2\varphi\right)\frac{H}{a}+\frac{H^2}{a^2}\right], \tag{6}$$

where, $f = 0.00335281068118$ is geometrical flattening, $m = 0.00344978600308$ is Kloro constant, $a = 6356752.3141m$ is ellipsoid semi axis, φ is latitude of P and γ_{P0} normal gravity of ellipsoid and is determined using Pizzertti (1894) formula, $\lambda_e = \left(1+\beta_1\sin^2\varphi+\beta_2\sin^2 2\varphi\right)$, where λ_e refers to normal gravity at the equator and $\beta_1 = 0.0053024$ and $\beta_2 = 0.0000059$. In this paper, the value of ellipsoidal normal gravity has been gained from ICGEM website.

The second part is $\frac{vg^{T}-vp^{T}}{\bar{\gamma}}$ (Flury and Rummel, 2009), where vg^{T} is the topographic potential around the point located on the geoid and vp^{T} is the computed topographic potential around P on the earth surface which has been calculated using the prism formula proposed by Nagy et al. (2000) as will be elaborated further in the following section.

In order to obtain the second part of equation (3), $vg^T - vp^T$, topographic potential of masses positioned between earth surface and geoid needs to be estimated. Accordingly, the area around the point is divided into blocks based on the DTM data available in ICGEM website. As seen, a schematic view of topographic division above geoid into prisms is presented in Figure 2, whose centers referred to the computational points on the GPS/Levelling station.

b) a)

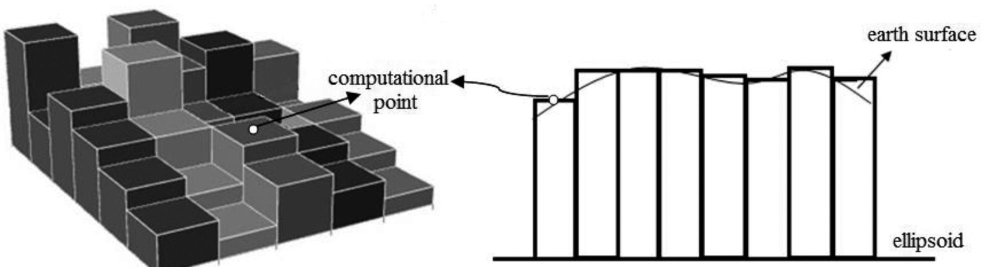

Fig. 2. Unreal sample of the area division around the measuring points a) two-dimensional model; b) the three dimensional model compose by contiguous prisms. The effect of gravimetric potential of blocks at measuring points, on the earth surface and corresponding point on the geoid is evaluated

The blocks surrounding the measuring point are divided into three parts. The first part is the potential effect of blocks with $30'' \times 30''$ spatial resolution which are located at a distance of 0.5° latitude and longitude from measurement point. In the second part the potential effect of blocks with $60'' \times 60''$ spatial resolution which are located at a distance of 0.5–1° latitude and longitude from measurement point and, for the third one, the potential effect of blocks with $1.5' \times 1.5'$ spatial resolution which are located at a distance of 1–2° latitude and longitude from measurement point is considered.

Due to trivial amount of the potential impact of further blocks, to reduce blocks number and, consequently, improve calculation speed and also running software in shorter time duration, the potential effect of further blocks has been ignored.

The gravity potential is regarded as the main superposition function. Hence, vp^T is summation of prisms potential with respect to reference point on the earth surface and vg^T is summation of all prisms potential with respect to reference point on geoid are subtracted. The mean density of 2.7 g/cm^3 has been prescribed.

Finally, the third part, $\dfrac{\delta\bar{g}^{\mathrm{BO}} - \delta g_{\mathrm{P}}^{\mathrm{BO}}}{\bar{\gamma}}$, in which is the difference between the average of gravity distribution between the earth surface and the geoid, and the gravity distribution at measuring point P as is normalized by the $\bar{\gamma}$. Bouger distribution on the earth surface is estimated as follows:

$$\delta g_{\mathrm{P}}^{\mathrm{BO}} = g_{\mathrm{P}} - \gamma_{\mathrm{P}}. \tag{7}$$

Here, g_{P} is the calculated gravity at P point on the earth and γ_{P} is measured normal gravity at P on the earth surface. To measure γ_{P} can be written in the following form (Heiskanen and Moritz, 1967):

$$\gamma_{\mathrm{P}} = \gamma_{\mathrm{P}_0}\left[1-2\left(1+f+m-f\sin^2\varphi\right)\frac{h}{a}+3\left(\frac{h}{a}\right)^2\right]. \tag{8}$$

Here, h is ellipsoidal height of the point. The value variables in recent formula have been gained from ICGEM website. It is worth mentioning that the WGS84 reference ellipsoid has been employed in all calculations. And also,

$$\delta\overline{g}^{\mathrm{BO}} = \frac{\delta g_{\mathrm{P}}^{\mathrm{BO}} + \delta g_{\mathrm{P}_0}^{\mathrm{BO}}}{2}, \tag{9}$$

$$\delta g_{\mathrm{P}}^{\mathrm{BO}} = g_{\mathrm{P}} + 0.3086h - 0.1119h - \gamma_{\mathrm{P}_0}, \tag{10}$$

where h is ellipsoidal height (gained from mentioned site) and γ_{P_0} has already been calculated.

4. The right angle prism formula

Considering Figure 3, gravity potential of right angle prism of ρ uniform density while dimension of $x_1 \le x \le x_2$, $y_1 \le y \le y_2$, $z_1 \le z \le z_2$ and equals to (Nagi et al., 2000):

$$u(p) = \int \frac{dxdydz}{r} = \int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2} \frac{dxdydz}{r}, \tag{11}$$

where,

$$r(x,y,z) = \sqrt{x^2+y^2+z^2}. \tag{12}$$

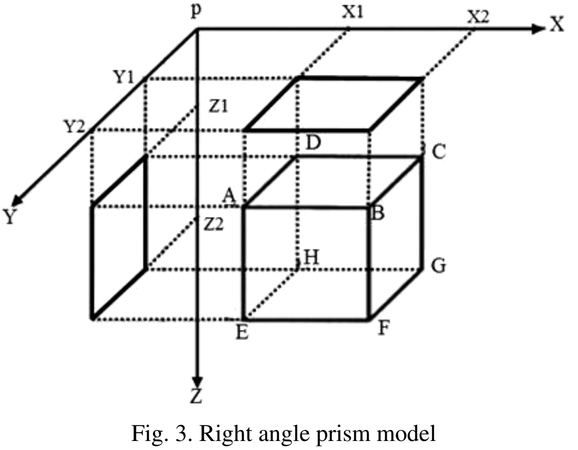

Fig. 3. Right angle prism model

The potential of prism at point P is equal to:

$$U(P) = u(P)G\rho. \tag{13}$$

Here, G denotes to universal gravitational constant. Integrating equation (10) concludes as follows:

$$u(p) = \Bigg\| xy\mathrm{Ln}(z+r) + yz\mathrm{Ln}(x+r) + zx\mathrm{Ln}(y+r) - \frac{x^2}{2}\tan^{-1}\frac{yz}{xr} - \frac{y^2}{2}\tan^{-1}\frac{zx}{yr} - \frac{z^2}{2}\tan^{-1}\frac{xy}{zr} \Bigg|\begin{matrix} x_2 \\ x_1 \end{matrix}\Bigg|\begin{matrix} y_2 \\ y_1 \end{matrix}\Bigg|\begin{matrix} z_2 \\ z_1 \end{matrix}. \tag{14}$$

5. Correlation coefficient

Correlation coefficient is considered as a measure used to determine association between two variables. The correlation coefficient illustrates the strength and type of relationship (straight or reverse). This coefficient ranges from 1 to -1 and in the case of nonlinear relationship between two variables, it is equal to 0. The coefficient for x and y series is computed using following equation:

$$correl(x,y) = \frac{\sum_{i=1}^{n}(x-\bar{x})(y-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x-\bar{x})^2 \sum_{i=1}^{n}(y-\bar{y})^2}}. \tag{15}$$

Here, $\bar{x}$ and $\bar{y}$ are the average of x and y series respectively. In this study, x and y are the separation between geoid and quasigeoid estimated by the two mentioned strategies on GPS stations. We consider the customary GPS/Levelling method as a criterion and apply the correlation coefficient as a statistical method to show the estimated separations from both methods how are similar and to interpret qualitatively the increasing and decreasing trend in the both series results.

The range of the correlation coefficient values is between -1 to 1. We have classified this range to four categories as is presented in table 1. To express the correlation value between two data set in percent, determination coefficient is used, as:

$$q = correl^2 \times 100. \tag{16}$$

Table 1. Qualitative description of correlation coefficient ranges. There is no defined relationship at point 0

Correlation range	1 to 0.5	0.5 to 0	0 to -0.5	-0.5 to -1
Qualitative description	Straight and good	Straight and mean	Reverse and weak	Reverse and bad

6. Areas under investigation

We have chosen three area for the investigation in Iran as are located in Lout dessert, Zagros ranges and Khuzestan plain where these districts are shown by the red rectangles in Figure 4. The information of the GPS/Levelling points including the observed gravity, ellipsoidal height, orthometric height and so on, have been provided from the National Cartographic Center of Iran. Figures 5, 6 and 7 indicate the GPS/Levelling benchmarks in the specified district in Lout dessert, Zagros ranges and Khuzestan plain, respectively.

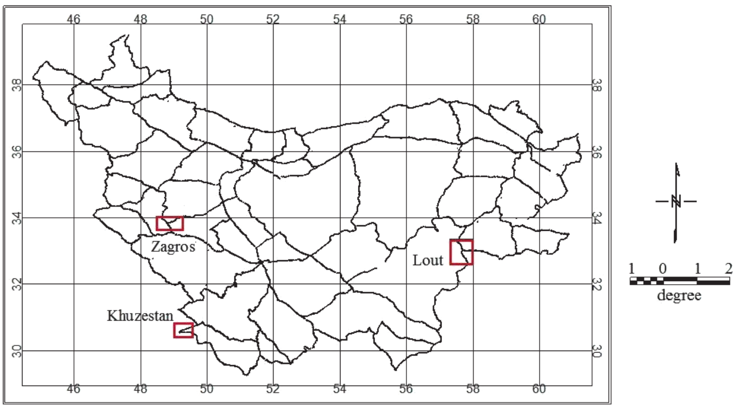

Fig. 4. The three areas under study (Lout desert, Zagros, Khuzestan) in Iran as specified with the red rectangular which including GPS/Levelling stations

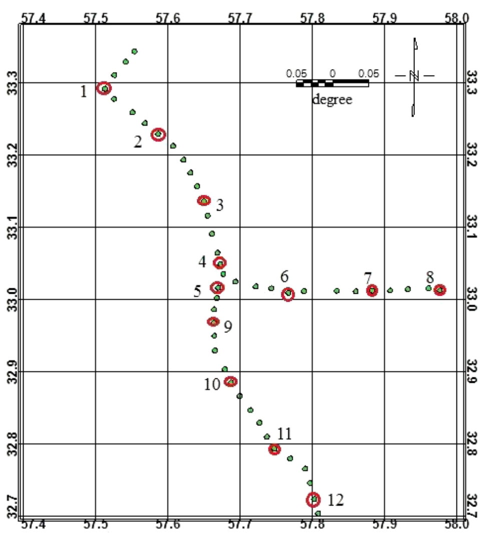

Fig. 5. Enlarged the area under study in Lout desert as shown in Fig. 4. Green points refer to GPS/Levelling benchmark while the points circled in red are the determined ones in order to measure correlation coefficient between Sjöberg method and GPS/Levelling data

In order to compare geoid-to-quasigeoid separation estimated by the GPS/Levelling and Sjöberg techniques, We considered 12 GPS station points in the Lout and Zagros and 10 GPS station points in the Khuzestan.

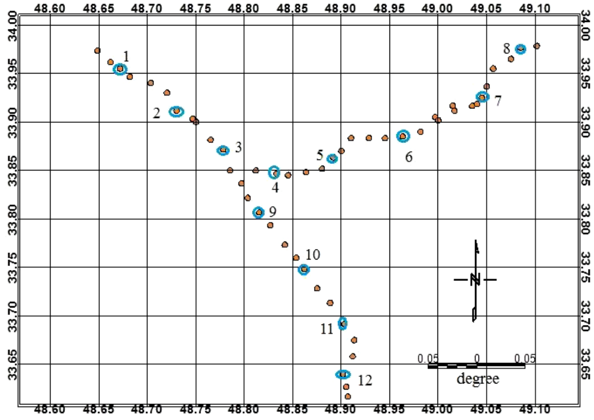

Fig. 6. Enlarged the under study area in Zagros as shown in Fig. 4. Orange points refer to GPS/Levelling benchmark while the points circled in green are the determined ones in order to measure correlation coefficient between Sjöberg method and GPS/Levelling data

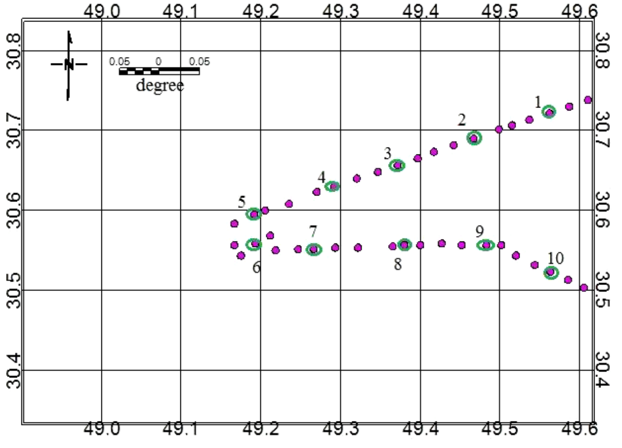

Fig. 7. Enlarged the under study area in Khuzestan as shown in Fig. 4. Purple points refer to GPS/Levelling benchmark while the points circled in green are the determined ones in order to measure correlation coefficient between Sjöberg method and GPS/Levelling data

7. Results

The inferred geoid-to-quasigeoid separation from the both approaches have been summarized in tables 2, 3 and 4. The correlation coefficient and determination coefficient between the obtained geoid-to-quasigeoid separations on the evaluation points were computed (Table 5). The results show a straight correlation between the computed separations using the both methods for the areas under study.

Table 2. Geoid – Quasigeoid separation computed using Sjöberg method and GPS/Levelling for different points plotted in Figure 5

Point number	$(N-\zeta)_{\text{Gps/Levelling}}$ (m)	$(N-\zeta)_{\text{Sjöberg}}$ (m)
1	−0.253485	−0.304652
2	−0.217359	−0.184835
3	−0.153753	−0.207485
4	−0.137416	−0.106248
5	−0.124352	−0.162744
6	−0.113693	−0.088462
7	−0.126352	−0.161462
8	−0.133641	−0.113624
9	−0.108525	−0.155273
10	−0.095735	−0.145374
11	−0.082749	−0.134726
12	−0.093627	−0.153725

Table 3. Geoid – Quasigeoid separation computed using Sjöberg and GPS/Levelling for different points plotted in Figure 6

Point number	$(N-\zeta)_{\text{Gps/Levelling}}$ (m)	$(N-\zeta)_{\text{Sjöberg}}$ (m)
1	−0.335275	−0.426273
2	−0.314527	−0.287344
3	−0.267351	−0.335237
4	−0.275631	−0.237463
5	−0.347583	−0.382631
6	−0.353784	−0.277846
7	−0.342693	−0.235735
8	−0.326482	−0.375367
9	−0.257465	−0.313517
10	−0.326835	−0.406427
11	−0.315354	−0.266378
12	−0.338134	−0.427316

Table 4. Geoid – Quasigeoid separation computed using Sjöberg and GPS/Levelling for different points shown in Figure 7

Point number	$(N-\zeta)_{\text{Gps/Levelling}}$ (m)	$(N-\zeta)_{\text{Sjöberg}}$ (m)
1	−0.001637	−0.003261
2	−0.001426	−0.000146
3	−0.000835	−0.000343
4	−0.000004	−0.000207
5	0.000024	0.000058
6	0.000375	0.000863
7	0.000024	0.000045
8	−0.000383	−0.000172
9	−0.000601	−0.000864
10	−0.000846	−0.001159

Table 5. Coefficient of correlation and determination computed for Lout desert, Zagros and Khuzestan

	Region		
Parameter	Lout desert	Zagros	Khuzestan
Correlation coefficient	0.754	0.497	0.659
Determination coefficient	60	25	43.4
Qualitative description	Straight and good	Straight and mean	Straight and good

Accordingly, all three areas under evaluation including Lout, Zagros and Khuzestan have been divided to networks of points of 0.07 degree interval and the amount of geoid-to-quasigeoid separation has been calculated using described method based on the Sjöberg's equation.

Figures 8a, 9a and 10a illustrate the maps of the geoid-to-quasigeoid separation resulting from Sjöberg method, as the position of the computational points are shown over them and Figures 8b, 9b and 10b show the DTM maps related to Lout, Zagros and Khuzestan, respectively. The recent Figures demonstrate that the higher area have the less geoid-to-quasigeoid separation, i.e. $N-\zeta$ and vice versa. This matter is specially visible in case of Lout and Zagros. For more accurate investigation, the topographic height and geoid-to-quasigeoid separation $N-\zeta$ data with the same coordinates were extracted with a interval 0.02 degree along the profiles AB (see Figures 8b, 9b and 10b), as are shown in Figure 11a, b and c and can be found that the relation between $N-\zeta$ and topographic height is not linear, i.e. correlation is negative.

a) b)

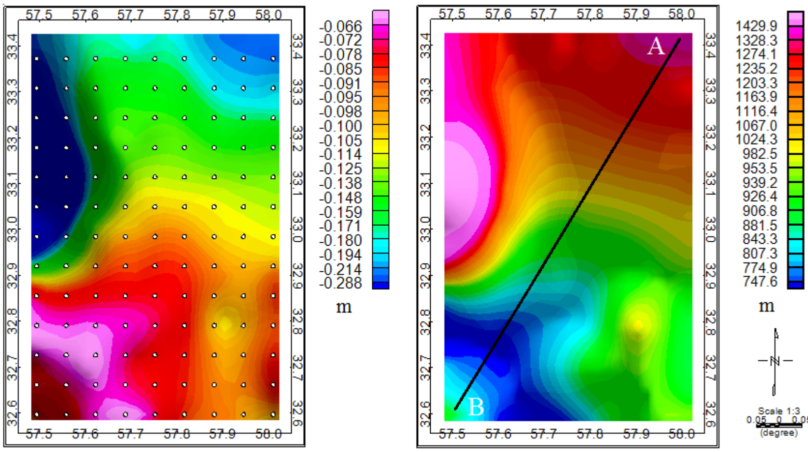

Fig. 8. a) Geoid-to-Quasigeoid separation map computed using Sjöberg method for Lout desert. The position of the computational points have been specified over the map. b) Topographic map of Lout desert. The location and trend of the profile AB has been shown

a) b)

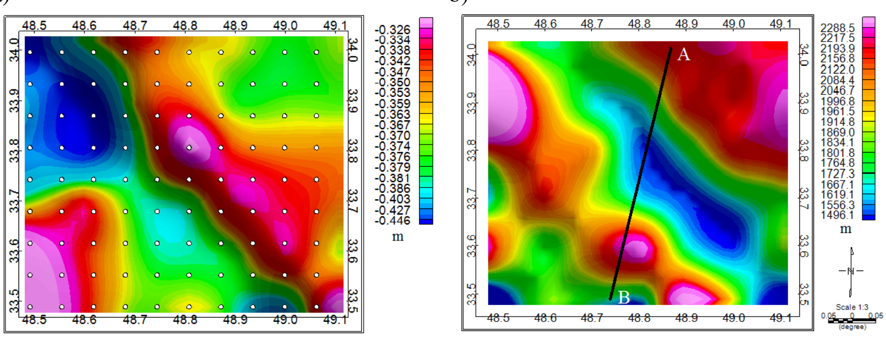

Fig. 9. a) Geoid-to-Quasigeoid separation map computed using Sjöberg method for Zagros region. The position of the computational points have been specified over the map. b) Topographic map of Zagros area. The location and trend of the profile AB has been shown

a) b)

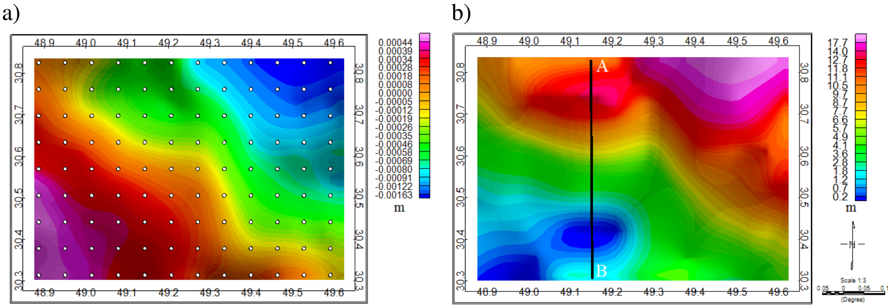

Fig. 10. a) Geoid-to-Quasigeoid separation map computed using Sjöberg method for Khuzestan province. The position of the computational points have been specified over the map. b) Topographic map of Khuzestan province. The location and trend of the profile AB has been shown

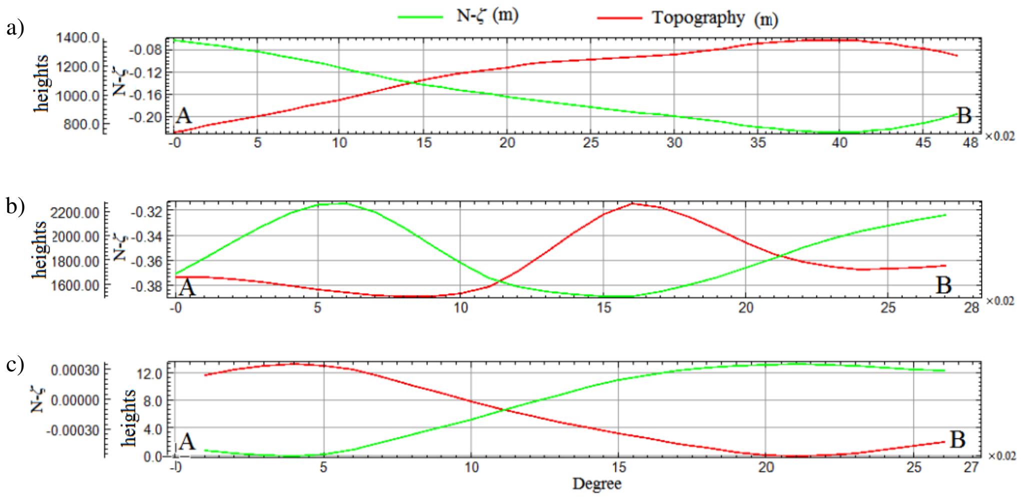

Fig. 11. The variations of the topographic height and geoid-to-quasigeoid separation $N-\zeta$ along the profiles AB for: a) Lout desert, b) Zagros region and c) Khuzestan province

8. Conclusion

Our main aims in this study are propounding a new approach for computing the topographic potentials at the geoid and surface points in the Sjöberg's equation and using ICGEM in determining the required components and comparing the separations evaluated by the both explained techniques.

In this paper, Sjöberg's equation was employed for estimating the geoid-to-quasigeoid separation as the requisite data is gotten from ICGEM website. Also, the separations obtained from GPS/Levelling prevalent approach was consider as a criterion for comparison. Regarding the evaluated correlation coefficient and determination coefficient values for the areas under study brought in table 5, the Sjöberg method resulted show an acceptable findings especially in the even parts, i.e. Lout desert and Khuzestan flatland where the computed correlation coefficient of 0.754 and 0.659 for the these districts indicate a straight and good correlation between the performance of the two methods. The amount of the correlation coefficient for the Zagros district is given as 0.497 which can be interpreted as a straight and mean correlation.

The maximum and minimum difference between the two strategies obtained for Lout desert is 0.060098 and 0.020017 m, for Zagros is 0.106958 and 0.027183 m and for Khuzestan is 0.001624 and 0.000021 respectively.

Although the satellite data obtained from ICGEM has been applied to solve Sjöberg's equation, the results have enough and acceptable precise. Therefore, to evaluate geoid-to-quasigeoid separation in areas without GPS/Levelling and gravity field data, the evaluations provided by Sjöberg's equation based on the existing information in the ICGEM website data can be useful and reliable.

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References

Bruns, H. (1878). *Die Figur der Erde*, Publ. Preuss. Geod. Inst., Berlin.

Featherstone, W.E. and Kirby, J.F. (1998). Estimates of the separation between the geoid and the quasigeoid over Australia. *Geomatics Research Australasia*, 68, 79–90.

Flury, J. and Rummel, R. (2009). On the geoid-to-quasigeoid separation inmountain areas. *J. Geod*, 83, 829–847. DOI: 10.1007/s00190-009-0302-9.

Heiskanen, W.A. and Moritz, H. (1967). *Physical geodesy*. W H Freeman and Co., San Francisco.

http://icgem.gfz-potsdam.de/ICGEM/

Kiamehr, R. (2007). A new height datum for Iran based on combination of the Gravimetric and GPS/Levelling geoid models. *J. Acta Geodaetica et Geophysica*, 42, 69–81. DOI: 10.1556/AGeod.42.2007.1.4.

Molodensky, M.S., Yeremeev, V.F. and Yurkina, M.I. (1960). Methods for study of the external gravitational field and figure of the Earth. *TRUDY Ts NIIGAiK*, 131, Geodezizdat, Moscow (in Russian).

Nagy, D., Papp, G. and Benedek, J. (2000). The gravitional and its derivation for the prism. *Journal of Geodesy*, 74, 552–560. DOI: 10.1007/s001900000116.

Pizzetti, P. (1894). Geodesia – Sulla espressione della gravita alla superficie del geoide, supposto ellissoidico. *Atti Reale Accademia dei Lincei*, 3, 166–172.

Prutkin, I. and Kless, R. (2008). On the non-uniqueness of local quasigeoids computed from terrestrial gravity anomalies. *Journal of Geodesy*, 82, 147–156. DOI: 10.1007/s00190-007-0161-1.

Sjöberg, L.E. (2006). A refined conversion from normal height to orthometric height. *Stud. Geophys. Geod.*, 50, 595–606. DOI: 10.1007/s11200-006-0037-5.

Sjöberg, L.E. (2010). A strict formula for geoid-to-quasigeoid separation. *J. Geod.*, 84, 699–702. DOI: 10.1007/s00190-010-0407-1.

Tenzer, R., Hirt, C.H., Claessens, S. and Novak, P. (2015). Spatial and spectral representations of the geoid-to-quasigeoid correction. *Surv. Geophys.*, 36, 627, DOI: 10.1007/s10712-015-9337-z.

Tenzer, R., Hirt, C.H., Novak, P., Pitonak, M. and Sprlak, M. (2015). Contribution of mass density heterogeneities to the geoid-to-quasigeoid separation. *J. Geod.*, 90, 65–80. DOI: doi.org/10.1007/s00190-015-0858-5.

Tenzer, R., Moore, P., Novak, P., Kuhn, M. and Vanicek, P. (2006). Explicit formula for the geoid-to-quasigeoid separation, *Stud. Geoph. Geod.*, 50, 607–618. DOI: 10.1007/s11200-006-0038-4.

Tenzer, R., Vanicek, P., Santos, M., Featherstone, W.E. and Kuhn, M. (2005). The rigorous determination of orthometric heights. *J. Geod.*, 79, 82–92. DOI: 10.1007/s00190-004-0374-5.