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Monte Carlo simulation for reliability assessment and optimization of an object subjected to varying operation conditions

Keywords

operation process, reliability, prediction, optimization, Monte Carlo simulation

Abstract

This paper presents the computer simulation technique related to the reliability of an object under variable operation conditions. The considered object operation process is modelled using semi-Markov processes and its reliability is analysed by application of the conditional reliability functions in its different operation states. The backgrounds and procedures of the Monte Carlo simulation method application to an object at variable condition reliability analysis are proposed and applied to reliability evaluation of an exemplary object. Consequently, under arbitrarily assumed the parameters of this exemplary object operation process and its conditional reliability functions, using the proposed Monte Carlo simulation procedures, the transient probabilities of the exemplary object operation process at the particular operation states and its unconditional reliability function are determined. Further, the linear programming is introduced and proposed to reliability optimization of an object at variable operation conditions and the optimal transient probabilities of the exemplary object operation process at the particular operation states and its optimal unconditional reliability function are determined. Finally, some practical suggestions on the modification of the exemplary object operation process improving its reliability are proposed.

1. Introduction

The reliability function of an object subjected to varying in time its operation process analytical determination very often leads to complicated formulae and therefore it is sometimes difficult to implement modeling, prediction and optimization using this way [1]- [7]. The Monte Carlo simulation method is a tool that sometimes allows to simplify solving this problem [8]. The analytical approach to systems reliability analysis is shortly presented and next the background of the computer simulation modelling method for such objects reliability assessment are is given. The Monte Carlo method is practically applied to examine the reliability of an exemplary object at variable operation conditions. This way, the main reliability and operation process characteristics of this exemplary object are found. Further, the optimal values of those characteristics are determined.

2. An object operation process

We assume that an object during its operation at the fixed moment t, $t \in \langle 0,+\infty)$, may be at one of ν, $\nu \in N$, different operations states z_b, $b = 1,2,...,\nu$. Consequently, we mark by $Z(t)$, $t \in \langle 0,+\infty)$, the object operation process, that is a function of a continuous variable t, taking discrete values at the set $\{z_1, z_2,..., z_\nu\}$ of the object operation states. We assume a semi-Markov model [2], [4] of the object operation process $Z(t)$ and we mark by θ_{bl} its random conditional sojourn times at the operation states z_b, when its next operation state is z_l, $b, l = 1,2,..., \nu$, $b \neq l$.

Consequently, the operation process may be described by the following parameters:

- the vector of the initial probabilities of the object operation process $Z(t)$ staying at the particular operations states at the moment $t = 0$

$$[p_b(0)]_{1\times\nu} = [p_1(0), p_2(0),..., p_\nu(0)], \qquad (1)$$

where

$$p_b(0) = P(Z(0) = z_b),\ b = 1,2,...,\nu\ ; \tag{2}$$

- the matrix of the probabilities of the object operation process $Z(t)$ transitions between the operation states z_b and z_l, $b,l = 1,2,...,\nu,\ b \neq l$

$$[p_{bl}]_{\nu\times\nu} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1\nu} \\ p_{21} & p_{22} & \cdots & p_{2\nu} \\ \vdots & \vdots & \ddots & \vdots \\ p_{\nu1} & p_{\nu2} & \cdots & p_{\nu\nu} \end{bmatrix} \tag{3}$$

where $p_{bb} = 0$ for $b = 1,2,...,\nu;$

- the matrix of the conditional distribution functions of the object operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states

$$[H_{bl}(t)]_{\nu\times\nu} = \begin{bmatrix} H_{11}(t) & H_{12}(t) & \cdots & H_{1\nu}(t) \\ H_{21}(t) & H_{22}(t) & \cdots & H_{2\nu}(t) \\ \vdots & \vdots & \ddots & \vdots \\ H_{\nu1}(t) & H_{\nu2}(t) & \cdots & H_{\nu\nu}(t) \end{bmatrix}, \tag{4}$$

where

$$H_{bl}(t) = P(\theta_{bl} < t),\ H_{bb}(t) = 0, \tag{5}$$

for $b,l = 1,2,...,\nu,\ b \neq l$.

Having identified the probabilities p_{bl} defined by (3) of transitions between the operation states and the distributions of conditional sojourn times θ_{bl}, the mean values M_b of the object operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,...,\nu$, at the particular operation states can be determined by

$$M_b = E[\theta_b] = \sum_{l=1}^{\nu} p_{bl} M_{bl}\ ,\ b = 1,2,...,\nu, \tag{6}$$

where M_{bl} are the mean values of the conditional sojourn times θ_{bl} given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t h_{bl}(t)dt, \tag{7}$$

$b,l = 1,2,...,\nu,\ b \neq l,$ and

$$h_{bl}(t) = \frac{dH_{bl}(t)}{dt},\ b,l = 1,2,...,\nu,\ b \neq l, \tag{8}$$

are the conditional density functions of the object operation process $Z(t)$ conditional sojourn times θ_{bl}, $b,l = 1,2,...,\nu,\ b \neq l,$ at the particular operation states corresponding to the distribution functions $H_{bl}(t)$.

Further, the limit values of the object operation process $Z(t)$ transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b)\ ,\ b = 1,2,...,\nu,$$

can be determined from the following relationship

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{\nu} \pi_l M_l},\ b = 1,2,...,\nu, \tag{9}$$

where M_b are given by (6), while the steady probabilities π_b of the vector $[\pi_b]_{1x\nu}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{\nu} \pi_l = 1, \end{cases} \tag{10}$$

where $[\pi_b] = [\pi_1, \pi_2, ..., \pi_\nu]$ and the matrix $[p_{bl}]$ is defined by (3).

Other practically interesting characteristics of the object operation process $Z(t)$ when the operation time θ is sufficiently large, are its total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, $b = 1,2,...,\nu,$ during the fixed object opetation time that have approximately normal distribution with the expected value given by

$$\hat{M}_b = E[\hat{\theta}_b] = p_b \theta,\ b = 1,2,...,\nu, \tag{11}$$

where p_b are given by (9).

3. Reliability of an object subjected to varying operation conditions

We assume that every operation state of the object operation process $Z(t)$, $t \in \langle 0,+\infty)$, described in section 2, have an influence on the object reliability [4]. Therefore, the object reliability at the particular operation state z_b, $b = 1,2,...,\nu$, can be described using the conditional reliability function

$$\boldsymbol{R}^{(b)}(t) = P(T^{(b)} > t \mid Z(t) = z_b), \tag{12}$$

for $t \in \langle 0, +\infty)$, $b = 1,2,...,\nu$, that is the conditional probability that the object conditional lifetime $T^{(b)}$ is greater than t, while the object operation process $Z(t)$ is at the operation state z_b, $b = 1,2,...,\nu$ [4].

The relationship between the distribution function $\boldsymbol{F}^{(b)}(t)$ of the object conditional lifetime $T^{(b)}$ and the object conditional reliability function $\boldsymbol{R}^{(b)}$ is given by

$$\boldsymbol{F}^{(b)}(t) = P(T^{(b)} < t \mid Z(t) = z_b) = 1 - \boldsymbol{R}^{(b)}(t), \tag{13}$$

for $t \in \langle 0, +\infty)$, $b = 1,2,...,\nu$,
Further, we denote the object unconditional lifetime by T and the unconditional reliability function of the object by

$$\boldsymbol{R}(t) = P(T > t), \quad t \in \langle 0, +\infty). \tag{14}$$

The relationship between the distribution function $\boldsymbol{F}(t)$ of the object unconditional lifetime T and the object unconditional reliability function $\boldsymbol{R}(t)$ is given by

$$\boldsymbol{F}(t) = P(T \le t) = 1 - P(T > t) = 1 - \boldsymbol{R}(t), \tag{15}$$

for $t \in \langle 0, +\infty)$.
In the case when the object operation time θ is large enough, the unconditional reliability function of the object is approximated by [4]

$$\boldsymbol{R}(\mathrm{t}) \cong \sum_{\mathrm{b}=1}^{\mathrm{v}} \mathrm{p_b} \boldsymbol{R}^{(\mathrm{b})}(\mathrm{t}), \quad t \in \langle 0, +\infty), \tag{16}$$

where p_b, $b = 1,2,...,\nu$, are the object operation process limit transient probabilities given by (9). Hence the mean value of the object unconditional lifetime T is given by

$$\mu \cong \sum_{b=1}^{\nu} p_b \mu_b, \tag{17}$$

where μ_b are the mean values of the object conditional lifetimes $T^{(b)}$ at the operation state z_b, $b = 1,2,...,\nu$, given by

$$\mu_b = \int_0^{+\infty} \boldsymbol{R}^{(\mathrm{b})}(t)\mathrm{dt}, \quad b = 1,2,...,\nu, \tag{18}$$

$\boldsymbol{R}^{(b)}(t)$, $b = 1,2,...,\nu$, are defined by (12) and p_b are given by (9).

Whereas, the standard deviation of the object unconditional lifetime T is given by

$$\sigma = \sqrt{D[T]} = \sqrt{2\int_0^{\infty} t\, \boldsymbol{R}(t)dt - \mu^2}, \tag{19}$$

where $\boldsymbol{R}(t)$ is given by (14) and μ is given by (17).

4. Monte Carlo simulation approach to an object operation process modelling

We denote by $z_b(q)$, $b = 1,2,...,\nu$, the realization of the object operation process initial operation state at the moment $t = 0$ generated from the distribution defined by (1)- (2). This realization is generated according to the formula

$$z_b(q) = \begin{cases} z_1, & 0 \le q < p_1(0), \\ z_2, & p_1(0) \le q < p_1(0) + p_2(0), \\ \vdots & \qquad \vdots \\ z_\nu, & \sum_{i=1}^{\nu-1} p_i(0) \le q \le 1, \end{cases} \tag{20}$$

where q is a randomly generated number from the uniform distribution on the interval $\langle 0, 1 \rangle$.
We denote by $z_{bl}(g)$, $l = 1,2,...,\nu$, $b \ne l$, the sequence of the realizations of the object operation process consecutive operation states generated from the distribution defined by (3). Those realizations are generated according to the formula

$$z_{1l}(g) = \begin{cases} z_2, & 0 \le g < p_{12}, \\ z_3, & p_{12} \le g < p_{12} + p_{13}, \\ \vdots & \qquad \vdots \\ z_\nu, & \sum_{i=1}^{\nu-1} p_{bi} \le g \le 1, \end{cases} \tag{21}$$

$$z_{bl}(g) = \begin{cases} z_1, & 0 \le g < p_{b1}, \\ \vdots & \vdots \\ z_{b-1}, & \sum_{i=1}^{b-2} p_{bi} \le g < \sum_{i=1}^{b-1} p_{bi}, \\ z_{b+1}, & \sum_{i=1}^{b-1} p_{bi} \le g < \sum_{i=1}^{b+1} p_{bi}, \\ \vdots & \vdots \\ z_{\nu}, & \sum_{i=1}^{\nu-1} p_{bi} \le g \le 1, \end{cases} \tag{22}$$

for $b = 2, 3, ..., \nu$,

$$z_{\nu l}(g) = \begin{cases} z_1, & 0 \le g < p_{\nu 1}, \\ z_2, & p_{\nu 1} \le g < p_{\nu 1} + p_{\nu 2} \\ \vdots & \vdots \\ z_{\nu-1}, & \sum_{i=1}^{\nu-2} p_{\nu i} \le g \le 1, \end{cases} \tag{23}$$

where g is a randomly generated number from the uniform distribution on the interval $\langle 0, 1 \rangle$.

We denote by $\theta_{bl}^{(i)}$, $b, l = 1,2,...,\nu, b \ne l$, $i = 1,2,...,n_{bl}$, the realizations of the conditional sojourn time θ_{bl} of the object operation process generated from the distribution H_{bl}, defined by (4), where n_{bl} is the number of those sojourn time realizations during the experiment time $\tilde{\theta}$. Those realizations are generated according to the formulae

$$\theta_{bl} = H_{bl}^{-1}(h),\ b, l = 1,2,...,\nu, b \ne l, \tag{24}$$

where $H_{bl}^{-1}(h)$ is the inverse function of the distribution function $H_{bl}(t)$ and h is a randomly generated number from the uniform distribution on the interval $\langle 0, 1 \rangle$, which in the case of exponential distribution

$$H_{bl}(t) = 1 - \exp[-\alpha_{bl}\, t],\ t \ge 0, \tag{25}$$

takes the following form

$$\theta_{bl} = -\frac{1}{\alpha_{bl}} \ln(1-h),\ \ b, l = 1,2,...,\nu, b \ne l. \tag{26}$$

The exemplary realizations of the considered object operation process including the realisations

$$\theta_{bl}^{(1)},\ \theta_{bl}^{(2)},\ \dots,\ \theta_{bl}^{(n_{bl})},\ b, l = 1,2,...,\nu, b \ne l \tag{27}$$

is presented in *Figure 1*.

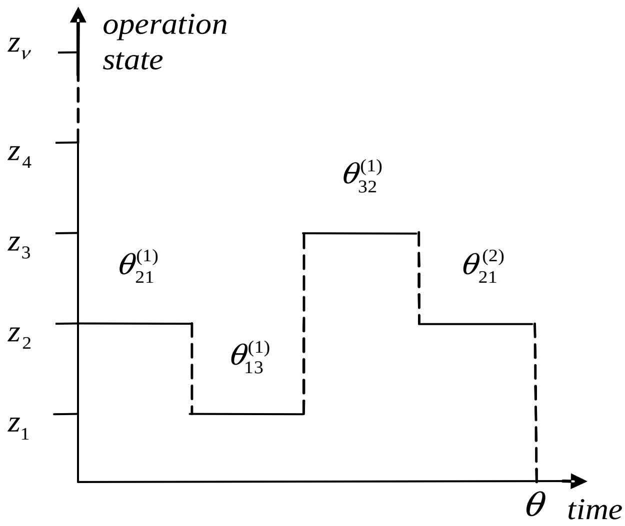

Figure 1. The exemplary sojourn times of the object operation process

Having those realisations, it is possible to determine approximately the total sojourn time at the operation state z_b during the time of the experiment $\tilde{\theta}$ applying the formula

$$\tilde{\theta}_b = \sum_{\substack{l=1 \\ l \ne b}}^{\nu} \sum_{j=1}^{n_{bl}} \theta_{bl}^{(i)}. \tag{28}$$

and the object operation process unconditional mean sojourn times are given by

$$M_b = \frac{1}{n_b} \tilde{\theta}_b,\ n_b = \sum_{l=1}^{\nu} n_{bl}. \tag{29}$$

Further, the limit transient probabilities defined by (9) can be approximately obtained using the formula

$$p_b = \frac{\tilde{\theta}_b}{\tilde{\theta}},\ b = 1,2,...,\nu,\ \tilde{\theta} = \sum_{b=1}^{\nu} \tilde{\theta}_b. \tag{30}$$

5. Monte Carlo simulation approach to an object reliability modelling

The realizations of the object conditional lifetimes $t^{(b)}$ are generated according to the distribution (13), i.e. they are generated by the sampling formula

$$t^{(b)} = \left(\boldsymbol{F}^{(b)}(f)\right)^{-1} = \left(1 - \boldsymbol{R}^{(b)}(f)\right)^{-1}, \tag{31}$$

where $\left(\boldsymbol{F}^{(b)}(f)\right)^{-1}$ is the inverse function of the distribution function $\boldsymbol{F}^{(b)}(t)$ of the object conditional lifetime $T^{(b)}$ defined by (13) which in the case of exponential distribution takes the following form

$$\boldsymbol{F}^{(b)}(t) = 1 - \exp[-\lambda^{(b)}\, t],\ t \ge 0,\ b = 1,2,...,\nu, \quad (32)$$

In the case of the above exponential distribution the realisations of the object conditional lifetimes take the following form

$$t^{(b)} = -\frac{1}{\lambda^{(b)}}\ln(1-f),\ b = 1,2,...,\nu. \quad (33)$$

where $\lambda^{(b)}$, are the failure rates according to (32) and f is a randomly generated number from the uniform distribution on the interval $\langle 0,1\rangle$.

6. Procedures of Monte Carlo simulation application to operation and reliability of an object characteristics determination

The procedure is illustrated in *Figure 2*. At the beginning, we fix the following parameters:

- the number $N \in \boldsymbol{N} \setminus \{0\}$ of iterations (runs of the simulation) equal to the number of the lifetime realizations;
- the vector of the initial probabilities $[p_b(0)]$, $b = 1,2,...,\nu$, of the object operation process $Z(t)$ at the moment $t = 0$ defined by (2);
- the matrix of the probabilities $[p_{bl}]$, $b, l = 1,2,...,\nu, b \ne l$, of the object operation process $Z(t)$ transitions between the various object operation states defined by (3).
- the matrix $[\alpha_{bl}]$, $\alpha_{bl} \in \langle 0,\infty)$, $b, l = 1,2,...,\nu$, $b \ne l$, of the intensities of the object operation process transitions between the operation states existing in (25);
- the vector $[\lambda^{(b)}]$, $b = 1,2,...,\nu$, of the failure rates $\lambda^{(b)} \in \langle 0,\infty)$ existing in (32);

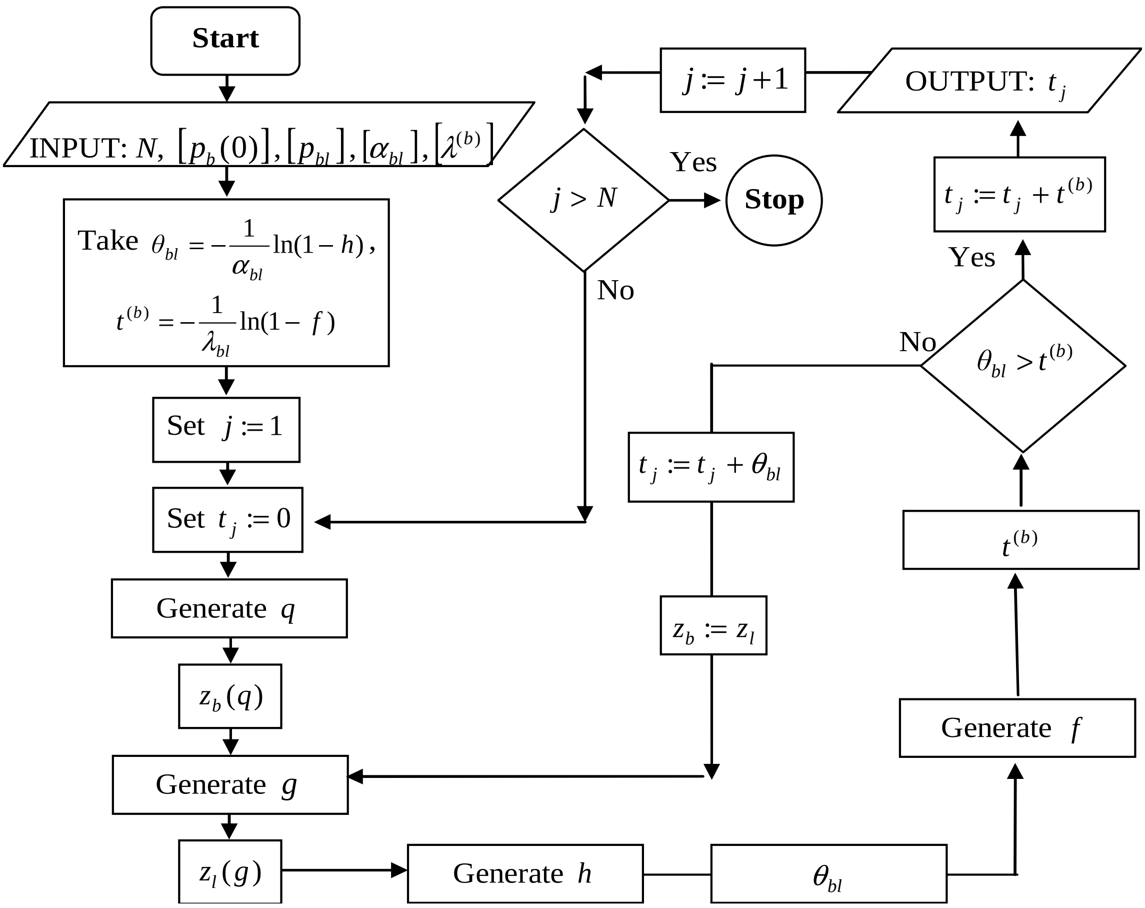

Figure 2. Monte Carlo algorithm for an object reliability evaluation

Next, we generate the realizations of the conditional sojourn times θ_{bl}, $b,l = 1,2,...,\nu, b \neq l$, of the object operation process defined by (24) according to the formula (26). Further, we generate the realisations $T^{(b)}$, $b = 1,2,...,\nu$, of the object conditional lifetimes according to the formula (33).
In the next step we introduce:

- $j \in \boldsymbol{N} \setminus \{0\}$ as the subsequent iteration in the main loop and set $j = 1$;
- $t_j \in \langle 0, \infty)$, $j = 1,2,...,N$ as the object unconditional lifetime realization and set $t_j = 0$.

As the algorithm progresses, we draw a random number q from the uniform distribution on the interval $\langle 0,1 \rangle$. Based on this random value, the realization

$$z_b(q), \ b = 1,2,...,\nu,$$

of the object operation process initial operation state at the moment $t = 0$ is generated according to the formula (20).
Next, we draw a random number g uniformly distributed on the unit interval. Concerning this random value, the realization

$$z_l(g), \ l = 1,2,...,\nu, l \neq b,$$

of the object operation process consecutive operation state is generated according to the formula (21).
Further, we generate a random number h from the uniform distribution on the interval $\langle 0, 1 \rangle$, which we put into the formula (26) obtaining the realisation θ_{bl}, $b, l = 1,2,...,\nu, b \neq l$. Subsequently, we generate a random number f uniformly distributed on the unit interval, which we put into the formula (33) obtaining the realisation $t^{(b)}$, $b = 1,2,...,\nu$. If the realization of the empirical conditional sojourn time is not greater than the realization of the object conditional lifetime, we add to the object unconditional lifetime realization t_j the value θ_{bl}. The realization t_j is recorded and z_l is set as the initial operation state.
We generate another random numbers g, h, f from the uniform distribution on the interval $\langle 0, 1 \rangle$ obtaining the realizations $z_l(g)$, θ_{bl} and $t^{(b)}$, $b, l = 1,2,...,\nu, b \neq l$. Each time we compare the realization of the conditional sojourn time θ_{bl} with the realization of the object conditional lifetime $t^{(b)}$. If θ_{bl} is greater than $t^{(b)}$, we add to the sum of the realizations of the conditional sojourn times θ_{bl} the realisation $t^{(b)}$, $b = 1,2,...,\nu$, and we obtain and record an object unconditional lifetime realization t_j. Thus, we can proceed replacing j with $j+1$ and shift into the next iteration in the loop if $j < N$. In the other case, we stop the procedure.

7. Optimization of an object operation process and reliability at variable operation conditions

The object operation process has significant influence on its reliability [4]. According to (17), the mean value μ, of the object unconditional lifetimes is determined by the limit values of transient probabilities $p_b, b = 1,2,...,\nu$, of the object operation process at the operation states given by (30) and the mean values $\mu_b, b = 1,2,...,\nu$, of the object conditional lifetimes given by (18). The corresponding optimal values $\dot{p}_b, b = 1,2,...,\nu$, of the transient probabilities may be found to maximize the mean value μ of the unconditional object lifetimes.
Therefore, the optimization problem can be formulated as a linear programming model with the objective function of the following form

$$\mu = \sum_{b=1}^{\nu} p_b \mu_b, \tag{34}$$

where $\sum_{b=1}^{\nu} p_b = 1$, $\mu_b \geq 0$, $b = 1,2,...,\nu$, with the following bound constraints for the unknown transient probabilities p_b, $b = 1,2,...,\nu$,

$$\breve{p}_b \leq p_b \leq \hat{p}_b, \ b = 1,2,...,\nu, \tag{35}$$

where

$$0 \leq \breve{p}_b \leq 1, \ 0 \leq \hat{p}_b \leq 1, \ \breve{p}_b \leq \hat{p}_b, b = 1,2,...,\nu. \tag{36}$$

The optimal value of the transient probabilities p_b, $b = 1,2,...,\nu$, can be found after completing a few steps.

First, we arrange the object conditional lifetime mean values μ_b, $b = 1,2,...,\nu$, in non-increasing order

$$\mu_{b_1} \geq \mu_{b_2} \geq \ldots \geq \mu_{b_\nu}, \tag{37}$$

where

$$b_i \in \{1,2,\ldots,\nu\}, \text{ for } i = 1,2,\ldots,\nu. \tag{38}$$

Next, we substitute

$$x_i = p_{b_i}, \ \breve{x}_i = \breve{p}_{b_i}, \ \hat{x}_i = \hat{p}_{b_i}, \tag{39}$$

for $i = 1,2,\ldots,\nu$, and we maximize with respect to x_i, $i = 1,2,\ldots,\nu$, the linear form (34) that after this transformation takes the form

$$\mu = \sum_{i=1}^{\nu} x_i \mu_{b_i}, \tag{40}$$

with the following bound constraints

$$\breve{x}_i \leq x_i \leq \hat{x}_i, \ i = 1,2,\ldots,\nu, \tag{41}$$

$$\sum_{i=1}^{\nu} x_i = 1, \tag{42}$$

where μ_{b_i}, $\mu_{b_i} \geq 0$, $i = 1,2,\ldots,\nu$, are fixed mean values of the object conditional lifetimes arranged in non-increasing order and

$$\begin{aligned} &\breve{x}_i, \ 0 \leq \breve{x}_i \leq 1, \\ &\hat{x}_i, \ 0 \leq \hat{x}_i \leq 1, \\ &\breve{x}_i \leq \hat{x}_i, \ i = 1,2,\ldots,\nu, \end{aligned} \tag{43}$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,\ldots,\nu$, respectively.
To find the optimal values of x_i, $i = 1,2,\ldots,\nu$, we define

$$\breve{x} = \sum_{i=1}^{\nu} \breve{x}_i, \ \ \hat{y} = 1 - \breve{x} \tag{44}$$

and

$$\breve{x}^0 = 0, \ \hat{x}^0 = 0, \ \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i, \ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \tag{45}$$

for $I = 1,2,\ldots,\nu$.

Next, we find the largest value $I \in \{0,1,\ldots,\nu\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y}, \tag{46}$$

and we fix the optimal solution that maximize (39) in the following way:
- if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = 2,3,\ldots,\nu; \tag{47}$$

- if $0 < I < \nu$, the optimal solution is

$$\begin{aligned} &\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,\ldots,I, \ \dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1}, \\ &\dot{x}_i = \breve{x}_i \text{ for } i = I+2, I+3,\ldots,\nu; \end{aligned} \tag{48}$$

- if $I = \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,\ldots,\nu. \tag{49}$$

Finally, after making the inverse to (39) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \text{ for } i = 1,2,\ldots,\nu, \tag{50}$$

that maximize the object mean lifetime defined by the linear form (34), giving its maximum value in the following form

$$\dot{\mu} = \sum_{b=1}^{\nu} \dot{p}_b \mu_b. \tag{51}$$

8. An exemplary object operation and reliability evaluation and optimization

8.1. An object operation process

8.1.1. Analytical approach to an exemplary object operation process analysis

We consider an exemplary object operating at $\nu = 4$ operation states z_1, z_2, z_3 and z_4. The probabilities of the initial operation states of this object operation process are fixed arbitrarily in the following way

$$[p_b(0)] = [0.21, 0.10, 0.29, 0.40]. \tag{52}$$

The probabilities of the exemplary object operation process $Z(t)$ transitions between the operation states z_b and z_l, $b,l = 1,2,3,4$, $b \neq l$ are also fixed arbitrarily [4] and given in the matrix below

$$[p_{bl}] = \begin{bmatrix} 0 & 0.22 & 0.32 & 0.46 \\ 0.20 & 0 & 0.30 & 0.50 \\ 0.12 & 0.16 & 0 & 0.72 \\ 0.48 & 0.22 & 0.30 & 0 \end{bmatrix}. \tag{53}$$

Moreover, we assume that the distribution functions of the exemplary object operation process conditional sojourn times measured in days are as follows:

$$H_{12}(t) = 1 - \exp[-0.0052t],$$
$$H_{31}(t) = 1 - \exp[-0.0011t],$$
$$H_{13}(t) = 1 - \exp[-0.0021t],$$
$$H_{32}(t) = 1 - \exp[-0.0021t],$$
$$H_{14}(t) = 1 - \exp[-0.005t],$$
$$H_{34}(t) = 1 - \exp[-0.0033t],$$
$$H_{21}(t) = 1 - \exp[-0.0104t],$$
$$H_{41}(t) = 1 - \exp[-0.0031t],$$
$$H_{23}(t) = 1 - \exp[-0.0123t],$$
$$H_{42}(t) = 1 - \exp[-0.0020t],$$
$$H_{24}(t) = 1 - \exp[-0.0182t],$$
$$H_{43}(t) = 1 - \exp[-0.0023t], \quad (54)$$

for $t \in \langle 0, +\infty)$.

Applying (54) and (8) to the conditional distributions given by (7), the conditional mean values $M_{bl} = E[\theta_{bl}]$, $b, l = 1, 2, 3, 4,$ of the exemplary object sojourn times at the particular operation states measured in days are fixed as follows:

$$M_{12} = 192, \quad M_{13} = 480, \quad M_{14} = 200,$$
$$M_{21} = 96, \quad M_{23} = 81, \quad M_{24} = 55,$$
$$M_{31} = 870, \quad M_{32} = 480, \quad M_{34} = 300,$$
$$M_{41} = 325, \quad M_{42} = 510, \quad M_{43} = 438. \quad (55)$$

Based on the formula (6) and applying (53)- (55), the object operation process unconditional mean sojourn times θ_b, $b = 1,2,...,v$, measured in days at the particular operation states are given by

$$M_1 = E[\theta_1] = 287.84, \quad M_2 = E[\theta_2] = 71.00,$$
$$M_3 = E[\theta_3] = 397.20, \quad M_4 = E[\theta_4] = 399.60. \quad (56)$$

Further, according to (10), the system of equations

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4] = [\pi_1, \pi_2, \pi_3, \pi_4][p_{bl}]_{4x4} \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1, \end{cases}$$

after considering (53), takes the form

$$\begin{cases} \pi_1 = 0.20\pi_2 + 0.12\pi_3 + 0.48\pi_4 \\ \pi_2 = 0.22\pi_1 + 0.16\pi_3 + 0.22\pi_4 \\ \pi_3 = 0.32\pi_1 + 0.30\pi_2 + 0.30\pi_4 \\ \pi_4 = 0.46\pi_1 + 0.50\pi_2 + 0.72\pi_3 \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1. \end{cases}$$

The approximate solutions of the above system of equations are:

$$\pi_1 \cong 0.236, \quad \pi_2 \cong 0.169,$$
$$\pi_3 \cong 0.234, \quad \pi_4 \cong 0.361. \quad (57)$$

Further, applying (9) and (57), the limit values of the object operation process transient probabilities $p_b(t)$, $b = 1,2,...,v$, at the operations states z_b can be found after completing a few steps described in [4] and get

$$p_1 \cong 0.214, \quad p_2 \cong 0.038,$$
$$p_3 \cong 0.293, \quad p_4 \cong 0.455. \quad (58)$$

Hence, applying (11), the object operation process expected values $E[\hat{\theta}_b]$, of the total sojourn times $\hat{\theta}_b$, $b = 1, 2, 3, 4,$ measured in days, at the particular operation states z_b, $b = 1,2,...,v$, and during the fixed operation time $\theta = 1$ year $= 365$ days are given by

$$E[\hat{\theta}_1] = 0.214 \times 365 = 78.1,$$
$$E[\hat{\theta}_2] = 0.038 \times 365 = 13.9,$$
$$E[\hat{\theta}_3] = 0.293 \times 365 = 106.9,$$
$$E[\hat{\theta}_4] = 0.455 \times 365 = 166.1. \quad (59)$$

8.1.2. Monte Carlo approach to an exemplary object operation process analysis

The simulation is performed according to data given in section 8.1.1. The first step is to select the initial operation state $z_b(g)$, $b = 1, 2, 3, 4,$ at the moment $t = 0,$ using formula (20), which is given by

$$z_b(g) = \begin{cases} z_1, & 0 \le g < 0.21 \\ z_2, & 0.21 \le g < 0.31 \\ z_3, & 0.31 \le g < 0.60 \\ z_4, & 0.60 \le g \le 1, \end{cases}$$

where g is a randomly generated number from the uniform distribution on the interval $\langle 0, 1\rangle$. The next operation state z_l, $l = 1, 2, 3, 4$, is generated according to (21)- (23), from $z_{bl}(g)$, $b = 1,2,3,4$, defined as

$$z_{1l}(g) = \begin{cases} z_2, & 0 \le g < 0.22 \\ z_3, & 0.22 \le g < 0.54 \\ z_4, & 0.54 \le g \le 1, \end{cases}$$

$$z_{2l}(g) = \begin{cases} z_1, & 0 \le g < 0.2 \\ z_3, & 0.2 \le g < 0.5 \\ z_4, & 0.5 \le g \le 1, \end{cases}$$

$$z_{3l}(g) = \begin{cases} z_1, & 0 \le g < 0.12 \\ z_2, & 0.12 \le g < 0.28 \\ z_4, & 0.28 \le g \le 1, \end{cases}$$

$$z_{4l}(g) = \begin{cases} z_1, & 0 \le g < 0.48 \\ z_2, & 0.48 \le g < 0.7 \\ z_3, & 0.7 \le g \le 1. \end{cases}$$

For instance, if $z_b(g) = z_1$, then the next operation state would be z_2, z_3 or z_4 generated from $z_{1l}(g)$. Applying (24), the realizations of the empirical conditional sojourn times are generated according to the formulae

$$\theta_{12}(H) = -192\ln[1-H],\ \theta_{13}(H) = -480\ln[1-H],$$
$$\theta_{14}(H) = -200\ln[1-H],\ \theta_{21}(H) = -96\ln[1-H],$$
$$\theta_{23}(H) = -81\ln[1-H],\ \theta_{24}(H) = -55\ln[1-H],$$
$$\theta_{31}(H) = -870\ln[1-H], \theta_{32}(H) = -480\ln[1-H],$$
$$\theta_{34}(H) = -300\ln[1-H],\ \theta_{41}(H) = -325\ln[1-H],$$
$$\theta_{42}(H) = -510\ln[1-H], \theta_{43}(H) = -438\ln[1-H],$$

where H is a randomly generated number from the uniform distribution on the interval $\langle 0, 1\rangle$.

The object operation process characteristics are calculated using the Monte Carlo method with time of the experiment fixed as $\tilde{\theta} = 18\,250$ days.

Applying (30) the limit values of the object operation process transient probabilities at the operation states z_b are as follows:

$$p_1 = 0.233,\quad p_2 = 0.037,$$
$$p_3 = 0.278,\quad p_4 = 0.452\,. \tag{60}$$

Based on the formula (29) and applying (60), the object operation process unconditional mean sojourn times θ_b, $b = 1,2,...,v$, measured in days at the particular operation states are given by

$$M_1 = 289.45,\quad M_2 = 68.71,$$
$$M_3 = 349.62,\quad M_4 = 391.48. \tag{61}$$

Hence, applying (11) and according to (60), the exemplary object operation process expected values $E[\hat{\theta}_b]$ of the total sojourn times $\hat{\theta}_b$, $b = 1,2,3,4$, at the particular operation states z_b, $b = 1,2,3,4$, and during the fixed operation time $\theta = 1$ year $= 365$ days are given by

$$E[\hat{\theta}_1] = 0.233 \cdot 365 = 85.0,$$
$$E[\hat{\theta}_2] = 0.037 \cdot 365 = 13.5,$$
$$E[\hat{\theta}_3] = 0.278 \cdot 365 = 101.5,$$
$$E[\hat{\theta}_4] = 0.452 \cdot 365 = 165.0. \tag{62}$$

8.2. An exemplary object reliability

8.2.1. Analytical approach to an exemplary object reliability analysis

For the considered exemplary object, we assume that the conditional reliability functions defined by (12) are different at various operation states and have the exponential forms

$$\boldsymbol{R}^{(1)}(t) = \exp[-0.00206667\,t] \text{ at } z_1,$$
$$\boldsymbol{R}^{(2)}(t) = \exp[-0.00144001\,t] \text{ at } z_2,$$
$$\boldsymbol{R}^{(3)}(t) = \exp[-0.00261069\,t] \text{ at } z_3,$$
$$\boldsymbol{R}^{(4)}(t) = \exp[-0.00393887\,t] \text{ at } z_4, \tag{63}$$

for $t \in \langle 0, +\infty)$.

According to (14), considering (60) and (63), the object unconditional reliability function is given by

$$\boldsymbol{R_a}(t) \cong 0.233\exp[-0.00206667t] + 0.037\exp[-0.001440001\,t] + 0.278\exp[-0.00261069\,t] + 0.452\exp[-0.00393887\,t], \tag{64}$$

for $t \in \langle 0, +\infty)$.

The mean values of the object conditional lifetimes at the particular operation states measured in days are calculated according to (63) and given by

$$\mu_1 = 483.87 \text{ days},\qquad \mu_2 = 694.44 \text{ days}$$
$$\mu_3 = 383.04 \text{ days}\qquad \mu_4 = 253.88 \text{ days}$$

The object operation time is large enough to apply the formula (17) and get

$$\mu \cong 0.233 \cdot 483.87 + 0.037 \cdot 694.44 + 0.278 \cdot 383.04 + 0.452 \cdot 253.88 = 359.67487 \text{ days}. \quad (65)$$

8.2.2. Monte Carlo approach to an object reliability analysis

The realizations of the object conditional lifetimes $t^{(b)}$, $b = 1,2,3,4,$ are generated from the exponential distribution according to (31)- (33), (63) given by

$$t^{(1)} = -483.87 \ln(1 - f),$$
$$t^{(2)} = -694.44 \ln(1 - f),$$
$$t^{(3)} = -383.04 \ln(1 - f),$$
$$t^{(4)} = -253.88 \ln(1 - f),$$

where f is a randomly generated number from the uniform distribution on the interval $\langle 0, 1\rangle$.

The histogram of the exemplary object lifetimes is illustrated in *Figure 3*.

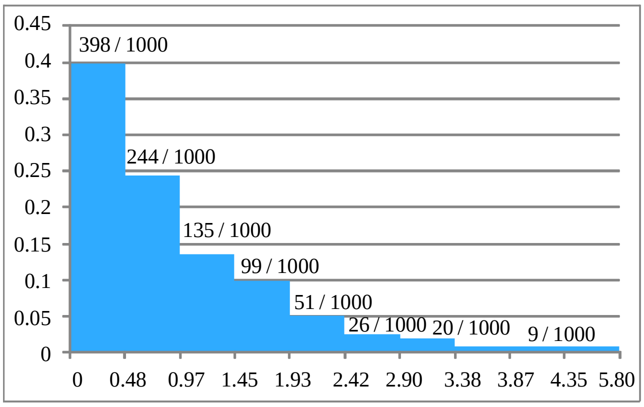

Figure 3. The graph of the histogram of the exemplary object lifetime

After analyzing and comparing the histogram with the graph of exponential distribution density function, we formulate the null hypothesis:
H_0: The lifetime of the exemplary object has the exponential distribution with the density function

$$f(t) = \begin{cases} 0, & t < 0 \\ \lambda \exp[-\lambda t], & t \geq 0, \end{cases} \quad (66)$$

where $\lambda \in \langle 0, +\infty)$.

Further, we estimate the unknown parameter λ of the density function (66) of the hypothetical exponential distribution and we obtain

$$\lambda = \frac{1}{\overline{T}} \cong \frac{1}{341.83} \cong 0.00292545,$$

where

$$\overline{T} = \frac{1}{N}\sum_{j=1}^{N} t_j,$$

is the empirical mean value of the object unconditional lifetime *T*, whereas N is the number of the lifetime realizations and t_j, $j = 1,2,\ldots,N$ are its realizations.
Hence, we get the following form of the object unconditional reliability function

$$\boldsymbol{R}_s(t) = \begin{cases} 0, & t < 0 \\ \exp[-0.00292545\, t], & t \geq 0. \end{cases} \quad (67)$$

To verify the hypothesis, we find the realization of the χ^2 (chi-square)-Pearson's, calculated according to the formula given in [4], which amounts $u_n \cong 5.10$. Assuming the significance level $\alpha = 0.05$ for $\bar{r} - l - 1 = 10 - 1 - 1 = 8$ degrees of freedom, from the tables of the χ^2-Pearson's distribution we find the value $u_\alpha = 15.51$. The obtained value u_n belongs to the acceptance domain, i.e.

$$u_n = 5.10 \leq u_\alpha = 15.51.$$

Therefore, at the significance level $\alpha = 0.05$, we do not reject the hypothesis H_0 stating that the exemplary object unconditional reliability function is exponential of the form.
The mean value of the object unconditional lifetime T obtained by using Monte Carlo method according to (67) is given by

$$\mu_s = \int_0^{+\infty} \boldsymbol{R}_s(t)\,dt = 341.83 \text{ days}.$$

The standard deviation is given according to (19) as follows

$$\sigma_s = 341.83 \text{ days}.$$

8.3. An exemplary object reliability and operation optimization

8.3.1 An exemplary object reliability optimization

The object characteristics can be improved by changing the parameters of its operations process. According to (65), the objective function defined by (34) takes the form

$$\mu = p_1 \cdot 483.87 + p_2 \cdot 694.44 + p_3 \cdot 383.04 + p_4 \cdot 253.88 \quad (68)$$

where $p_b,\ b = 1, 2, 3, 4,$ are the transient probabilities we want to optimize. Arbitrarily assumed, the bound constraints of the transient probabilities p_b respectively are:

$$0.201 \le p_1 \le 0.351, \quad 0.03 \le p_2 \le 0.105,$$
$$0.245 \le p_3 \le 0.395, \quad 0.309 \le p_4 \le 0.459,$$
$$\sum_{b=1}^{4} p_b = 1.$$

The object conditional lifetime mean values μ_b, $b = 1,2,3,4,$ arranged in non-increasing order are as follows

$$\mu_2 \ge \mu_1 \ge \mu_3 \ge \mu_4. \quad (69)$$

Further, according to (69), we substitute

$$x_1 = p_2,\ x_2 = p_1,\ x_3 = p_3,\ x_4 = p_4, \quad (70)$$

and we maximize with respect to x_i, $i = 1,2,3,4,$ the linear form (68) that according to (40)- (42) takes the form

$$\mu = x_1 \cdot 694.44 + x_2 \cdot 483.87 + x_3 \cdot 383.04 + x_4 \cdot 253.88,$$

with the following bound constraints

$$\breve{x}_1 = 0.03 \le x_1 \le 0.105 = \hat{x}_1,$$
$$\breve{x}_2 = 0.201 \le x_2 \le 0.351 = \hat{x}_2,$$
$$\breve{x}_3 = 0.245 \le x_3 \le 0.395 = \hat{x}_3,$$
$$\breve{x}_4 = 0.309 \le x_4 \le 0.459 = \hat{x}_4,$$
$$\sum_{i=1}^{4} x_i = 1.$$

Therefore, according to (44), we calculate

$$\breve{x} = \sum_{i=1}^{4} \breve{x}_i = 0.785,$$
$$\hat{y} = 1 - \breve{x} = 1 - 0.785 = 0.215, \quad (71)$$

and according to (45) we determine

$$\breve{x}^0 = 0, \quad \hat{x}^0 = 0, \quad \hat{x}^0 - \breve{x}^0 = 0,$$
$$\breve{x}^1 = 0.03, \quad \hat{x}^1 = 0.105, \quad \hat{x}^1 - \breve{x}^1 = 0.075,$$
$$\breve{x}^2 = 0.231, \quad \hat{x}^2 = 0.456, \quad \hat{x}^2 - \breve{x}^2 = 0.225,$$
$$\breve{x}^3 = 0.476, \quad \hat{x}^3 = 0.851, \quad \hat{x}^3 - \breve{x}^3 = 0.375,$$
$$\breve{x}^4 = 0.785, \quad \hat{x}^4 = 1.31, \quad \hat{x}^4 - \breve{x}^4 = 0.525.$$

From the above, as according to (71), the inequality (46) takes the form

$$\hat{x}^I - \breve{x}^I < 0.215,$$

it follows that the largest value $I \in \{0,1,2,3,4\}$ such that this inequality holds is $I = 1$.
Therefore, we fix the optimal solution that maximizes linear function (68) according to the rule (48) and we get

$$\dot{x}_1 = \hat{x}_1 = 0.105,$$
$$\dot{x}_2 = \hat{y} - \hat{x}^1 + \breve{x}^1 + \breve{x}_2$$
$$= 0.215 - 0.105 + 0.03 + 0.201 = 0.341,$$
$$\dot{x}_3 = \breve{x}_3 = 0.245,$$
$$\dot{x}_4 = \breve{x}_4 = 0.309. \quad (72)$$

Finally, after making the inverse to (70) substitution, we get the optimal transient probabilities

$$\dot{p}_2 = \dot{x}_1 = 0.105, \quad \dot{p}_1 = \dot{x}_2 = 0.341,$$
$$\dot{p}_3 = \dot{x}_3 = 0.245, \quad \dot{p}_4 = \dot{x}_4 = 0.309, \quad (73)$$

that maximize the exemplary system mean lifetime μ expressed by the linear form (68) giving, according to (51) and (73), its optimal value

$$\dot{\mu} = \dot{p}_1 \cdot 483.87 + \dot{p}_2 \cdot 694.44 + \dot{p}_3 \cdot 383.04 + \dot{p}_4 \cdot 253.88$$
$$= 0.341 \cdot 483.87 + 0.105 \cdot 694.44 + 0.245 \cdot 383.04 + 0.309 \cdot 253.88$$
$$\cong 410.21 \text{ days}, \quad (74)$$

which is greater than that before optimization given by (65).

Substituting the optimal solution (73) into the formula (16) the corresponding optimal unconditional reliability function of the object is of the form

$$\dot{\boldsymbol{R}}(t) = 0.341 \cdot \boldsymbol{R}^{(1)}(t) + 0.105 \cdot \boldsymbol{R}^{(2)}(t) + 0.245 \cdot \boldsymbol{R}^{(3)}(t) + 0.309 \cdot \boldsymbol{R}^{(4)}(t),$$

for $t \geq 0$, where $\boldsymbol{R}^{(b)}(t)$, $b = 1, 2, 3, 4$, are given by (63). The graph of the exemplary object optimal reliability function $\dot{\boldsymbol{R}}(t)$ is presented in *Figure 4*.

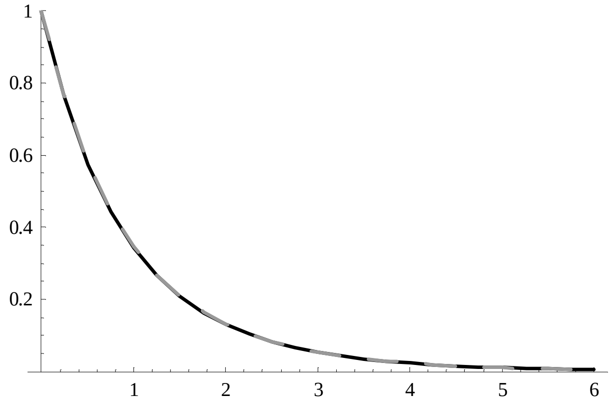

Figure 4. The graphs of the object reliability functions $\dot{\boldsymbol{R}}(t)$ and $\boldsymbol{R}_s(t)$.

Further, according to (19), the corresponding optimal standard deviation of the object unconditional lifetimes is given by

$$\dot{\sigma} \cong 0.9392.$$

8.3.2. An exemplary object operation process optimal characteristics

Having the values of the optimal transient probabilities determined by (73), it is possible to find the optimal unconditional mean values of the sojourn times of the object operation process at the operation states. Substituting the optimal transient probabilities at operation states

$$\dot{p}_1 = 0.341, \quad \dot{p}_2 = 0.105, \quad \dot{p}_3 = 0.245, \quad \dot{p}_4 = 0.309,$$

determined by (73) and considering the steady probabilities determined by (57), we get the following system of equations

$$\begin{aligned}
-0.1555\dot{M}_1 + 0.0576\dot{M}_2 + 0.0798\dot{M}_3 + 0.1231\dot{M}_4 = 0 \\
0.0248\dot{M}_1 - 0.1513\dot{M}_2 + 0.0246\dot{M}_3 + 0.0379\dot{M}_4 = 0 \\
0.0577\dot{M}_1 + 0.0413\dot{M}_2 - 0.1768\dot{M}_3 + 0.0883\dot{M}_4 = 0 \\
0.0729\dot{M}_1 + 0.0522\dot{M}_2 + 0.0723\dot{M}_3 - 0.2495\dot{M}_4 = 0
\end{aligned}$$

with the unknown optimal mean values $\dot{M}_b$.

Since the determinant of the main matrix of the above system of equations is equal to 0, then its rank is less than 4 and there are non-zero solutions of this system of equations that are ambiguous and dependent on one or more parameters. Thus, we may fix some of them and determine the remaining ones. In our case, according to (61), we conclude that it is sensible to assume $\dot{M}_4 = 400$. Consequently, from [4], the obtained optimal mean values of the object unconditional sojourn times at the operation states, are as follows

$$\dot{M}_1 \cong 675, \quad \dot{M}_2 \cong 290, \quad \dot{M}_3 \cong 490, \quad \dot{M}_4 = 400. \tag{75}$$

It can be seen that these solutions differ much from the values M_1, M_2, M_3, and M_4 given by (61).

Having these solutions, it is also possible to look for the optimal values $\dot{M}_{bl}$ of the mean values M_{bl} of the conditional sojourn times at operation states. Namely, substituting the probabilities of the object operation process transitions between the operation states, determined by (53) and the optimal mean values $\dot{M}_b$ given by (75), we get the following system of equations

$$\begin{aligned}
0.22\dot{M}_{12} + 0.32\dot{M}_{13} + 0.46\dot{M}_{14} = 675 \\
0.20\dot{M}_{21} + 0.30\dot{M}_{23} + 0.50\dot{M}_{24} = 290 \\
0.12\dot{M}_{31} + 0.16\dot{M}_{32} + 0.72\dot{M}_{34} = 490 \\
0.48\dot{M}_{41} + 0.22\dot{M}_{42} + 0.30\dot{M}_{14} = 400
\end{aligned}$$

with the unknown optimal values $\dot{M}_{bl}$ we want to find.

As the solutions of the above system of equations are ambiguous, then we arbitrarily fix some of them because, for instance because of practically important reasons, and we find the remaining ones. In this case we proceed as follows:

- we fix $\dot{M}_{12} = 200$, $\dot{M}_{13} = 500$ and we find $\dot{M}_{14} \cong 1024$;
- we fix $\dot{M}_{21} = 100$, $\dot{M}_{23} = 100$ and we find $\dot{M}_{24} \cong 480$;
- we fix $\dot{M}_{31} = 900$, $\dot{M}_{32} = 500$ and we find $\dot{M}_{34} \cong 419$;

- we fix $\dot{M}_{41} = 300,\ \dot{M}_{42} = 500$
 and we find $\dot{M}_{43} \cong 487.$ (76)

It can be seen that these solutions differ much from the mean values of the object conditional sojourn times at the particular operation states before its operation process optimization given by (55).
Another very useful and much easier to be applied in practice tool that can help in planning the operation process of an object are the system operation process optimal mean values of the total sojourn times at the particular operation states during the system operation time that by the assumpion is equal to $\theta = 1$ year = 365 days. Under this assumption, after aplying (11), we get their optimal values

$$\dot{E}[\hat{\theta}_1] = \dot{p}_1\theta = 0.341 \cdot 365 \cong 124.5,$$
$$\dot{E}[\hat{\theta}_2] = \dot{p}_2\theta = 0.105 \cdot 365 \cong 38.3,$$
$$\dot{E}[\hat{\theta}_3] = \dot{p}_3\theta = 0.245 \cdot 365 \cong 89.4,$$
$$\dot{E}[\hat{\theta}_4] = \dot{p}_4\theta = 0.309 \cdot 365 \cong 112.8, \quad (77)$$

that differ much from the values of $E[\hat{\theta}_1],\ E[\hat{\theta}_2],$ $E[\hat{\theta}_3],\ E[\hat{\theta}_4]$ determined by (59).
In practice, the knowledge of the optimal values of $\dot{M}_b$ $\dot{M}_{bl}$ and $\dot{E}[\hat{\theta}_b]$ given respectively by (75)-(77), can be very important and helpful for an object operation process planning and improving in order to make the object operation more reliable and safer.

8.3.3. Comments on the object operation and reliability new strategy

The comparison of the selected object characteristics before the object operation process optimization given by (55), (61) and (59) with their values after the object operation process optimization respectively given by (75)- (77) justifies the sensibility of the performed object operation process optimization.
From the above it can be suggested to organize the object operation process in the way that causes the replacing approximately the conditional mean sojourn times M_{bl} of the object at the particular operation states before the optimization given by (55) by their optimal values $\dot{M}_{bl}$ after the optimization given by (76). However, the fulfilling this suggestion of the operation process parameters changing is not easy in practice.
It seems to be practically a bit easier way to change the object operation process characteristics by the reorganizing the operation process that results in replacing approximately the unconditional mean sojourn times M_b of the object at the particular operation states before the optimization given by (61) by their optimal values $\dot{M}_b$ after the optimization given by (75).
The easiest way of the object operation process reorganizing is that leading to the replacing approximately the total sojourn times $E[\hat{\theta}_b]$ before the optimization given by (59) by their optimal values $\dot{E}[\hat{\theta}_b]$ after the optimization given by (77).

9. Conclusions

The achieved results may be considered as an illustration of the possibilities of the proposed Monte Carlo simulation method application to the system operation and reliability prediction and optimization. The obtained evaluation may be useful in complex systems reliability analysis and improving, especially during the design and when planning their operation processes effectiveness.

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