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Novel Theory of Mathematical Pendulum Part 1

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„Master content, no matter the words”
Katon the Elder

ABSTRACT

In the paper, a new adequate theory of a simple mathematical pendulum is presented. This paper consists of two parts. In Part 1, the behaviour of pendulum in particular points, that is in central and terminal/extremum ones have been analyzed very carefully in detail. System of forces in these points was considered with a special attention turned towards the terminal points where the equilibrium of forces occurs and in the next moment the lack of that equilibrium takes place with the proof of the open polygon of forces as the condition of beginning of accelerated free variable motion. Part 2 of the paper is to be devoted to the kinetics of the pendulum weight presented by separating in it the descriptions of differentiated motion of this body in the consecutive neighbouring space-times corresponding with particular quarter-periods. In the conclusion, further elaborations in the subject are forecasted, regarding both dynamics and energy of the flat mathematical pendulum.

***Keywords*:** Mathematical pendulum; Inertia; Force characteristics; Potential field; Stable state; Unstable state

1. INTRODUCTION

The paper is devoted to the simple mathematical pendulum which is oscillating around the position of a stable equilibrium in the vertical plane. Existent theory of the mathematical pendulum is commonly known [1,2]; it is contained in textbooks covering different fields of science, for instance; physics [3,4], mechanics [5-8], theory of vibrations [9,10], strength of materials [11]. Analysis of some exemplary references, covering also latest references [7,8], indicates clearly that the fundamentals of description of mathematical pendulum have not changed. Fundamentals of the pendulum theory created by Galileo Galilei (1564-1642), Italian physicist, astronomer and philosopher [12], remained with its essence until today.

The first part of the paper considers motivation of the subject to follow with an adequate force characteristics of the mathematical pendulum in its particular points of the track. Part 2 of the work will result in the solution with the path length and kinetic derivatives of the pendulum weight leaving the energy and dynamics problems for consideration in the future.

2. MOTIVATION FOR THE SUBJECT

The subject of the work has been undertaken due to inadequacy of the mathematical pendulum theory with its nature. Description of the mathematical pendulum motion does not correspond with reality. This is the general motivation of the work. Many detailed essential drawbacks of the theory have been presented [1,2]. The theory of mathematical pendulum which considers introducing components of the gravity force proves of its bifurcation [13,14].

One should clearly state that the gravity force cannot be decomposed in any measure. Characteristics of that force is univocal and results from its nature; the reason is terrestrial gravitation which is the natural Earth ability to attract any material bodies. The gravity force has just radial direction into the Earth core. In a small scale it is treated as the vertical direction [13-17]. Motion of the simple mathematical pendulum does not result from a tangent action of gravitation as it used to be presented in existent references. This is the direction of inertia of mathematical pendulum. It makes the pendulum oscillates around the position of its stable equilibrium. The real force of inertia should be in the place where the tangent of the gravity force was until now [18-21]. Existent theory of a simple mathematical pendulum does not take into consideration all real forces constituting the measure of components of elements structure actions of pendulum which moves with flat harmonic free motion [13,14]. For instance, centrifugal force of inertia has not been taken into account, substituting it by a normal component of the gravity force. The theory assumes the condition of force equilibrium and, based on this the equation of pendulum motion, being differential record of the so called dynamic equilibrium motion, is derived. It appears, however, in the free mathematical pendulum motion there is no, and *cannot be* any equilibrium. The mentioned here main shortcomings of the existent theory provide a sufficient motivation to undertake the subject studies. Then there will be a new theory of mathematical pendulum presented, the theory without the drawbacks of up-to-date approach to describe the real pendulum behaviours.

3. ADEQUATE FORCE CHARACTERISTICS OF THE PENDULUM IN PARTICULAR POINTS OF ITS TRACK

Now the force characteristics of pendulum in particular peculiar path points will be presented. There are three points of this type: two terminal and one central. Terminal points refer to the turning positions of pendulum, whereas the central one is related with the lowest position of the device. In the central position a stable static equilibrium of the pendulum takes place; it has been reflected by the force system (**Fig. 1**), in which the gravity force *Q* is balanced by the tension force *S* of thread/string of length *l*. A solid/body of mass *m*, hung on weightless and inextensible thread/string, is the considered mathematical pendulum, in accordance with its common definition, provided in the literature [9,10]. In that position of pendulum, when no external stimulus act on it, there is still no inertia of the body hung on the thread detected/revealed. It may appear in the moment of action of a determined external stimulus. An external stimulus begins to act on the body of mass *m*; it acts on the body with a determined force *F* (**Fig. 2**). The nature of the body is such that it reacts with a self-resistance, being the inertia. The measure of this inertia is tangent force of inertia B_τ; that real external force is directed in the opposite side, directed in the same tangent to the track of weight, that is $\tau-\tau$. Normal direction to this track is marked by symbols $n-n$, whereas the vertical direction, on which the gravity force *Q* is situated – by letters $w-w$. The body is still in the stable static equilibrium, on the stable static potential field *SSPF* [22-24]. All the forces (Q, S, F, B_τ) have equal values in this position.

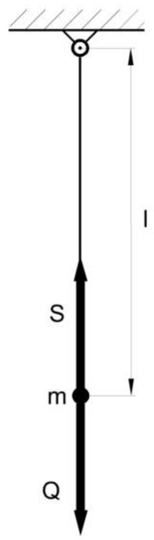

Fig. 1. System of force vectors corresponding with the stable static equilibrium of a simple mathematical pendulum.

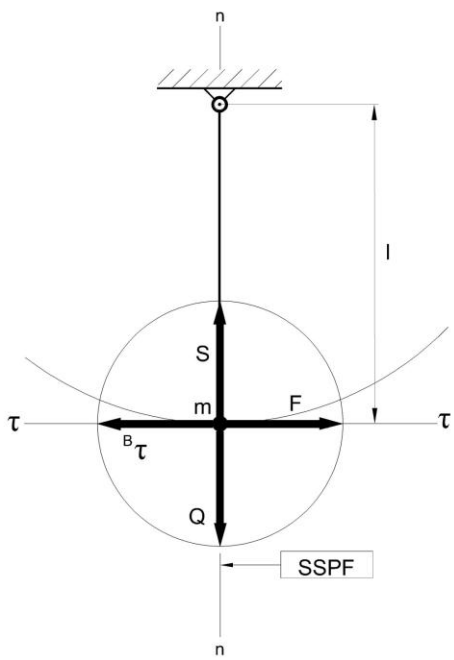

Fig. 2. System of force vectors corresponding with the initial moment of external stimulus/incentive on the pendulum weight.

By deflection of the body of angle φ^* (**Fig. 3a**) an unstable static equilibrium was assumed and by this it was situated on an unstable static potential field *ASPF*. As it can be seen, the system of component forces underwent an essential change. Tangent force of inertia B_τ^*, force of thread tension S^* and external force F^* now have quite different characteristics. Only the inertia force Q (apart from the fixation point/application of force) remained the same. The closed polygon of forces (**Fig. 3b**), corresponding with that terminal position of pendulum, reflects exactly the unstable static equilibrium of weight.

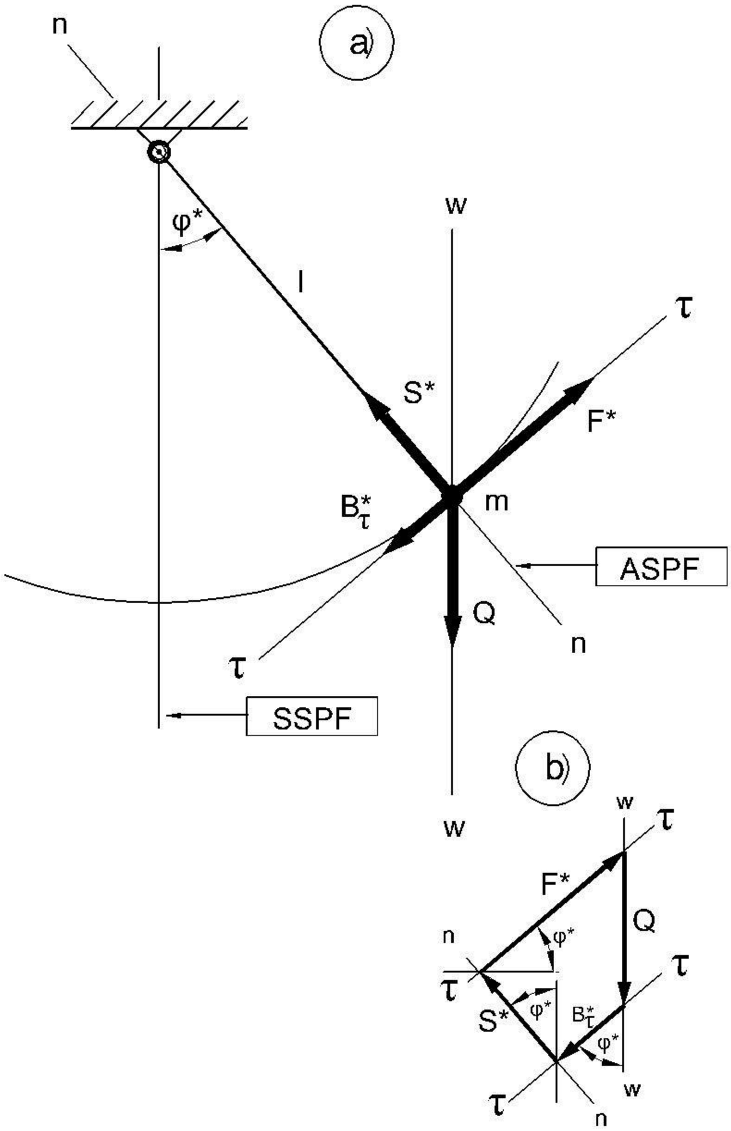

Fig. 3. Star-rosette (a) and quadrilateral of forces (b) in extreme position of pendulum at the unstable static potential field.

After release of the action of external stimulus, that means lack of force F^*, the pendulum weight begins its accelerated free motion. In the first moment of that motion (**Fig. 4a**) the centrifugal inertia appears with the measure being the centrifugal force of inertia B_n^0. Tangent force of inertia B_τ^0 has still the same characteristics. It concerns also the force of inertia *Q*. All that is happening on the instantaneous unstable static potential field $(ASPF)^*$. Polygon of forces (**Fig. 4b**), corresponding with that situation, is an open polygon, as there is not any fifth component, i.e. external force F^*. The body will be tending now to the central position, corresponding with instantaneous yet stable static potential field $(SSPF)^*$.

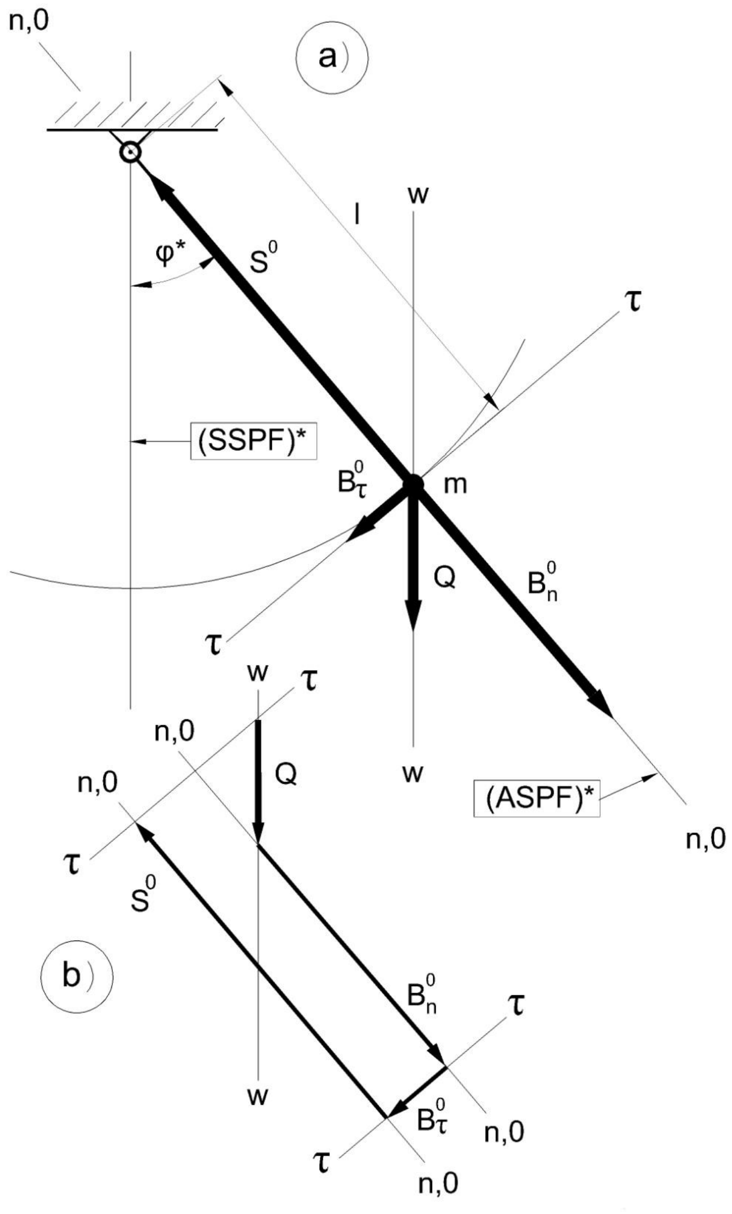

Fig. 4. Star-rosette (a) and open quadrilateral of forces (b) in the moment of start pendulum motion.

On the mentioned instantaneous stable static potential field $(SSPF)^*$ the body remains just one moment, changing in this place its hitherto existing accelerated motion into a free retarded motion. That potential field, referred to the direction $n-n$, has been denoted by symbols $1-1$. As can be seen, (**Fig. 5a**), characteristics of the gravity force is yet the same, and the remaining forces $\left(B_\tau^1, B_n^1, S^1\right)$ possess higher values, respectively. This situation corresponds with the open polygon of particular forces (**Fig. 5b**).

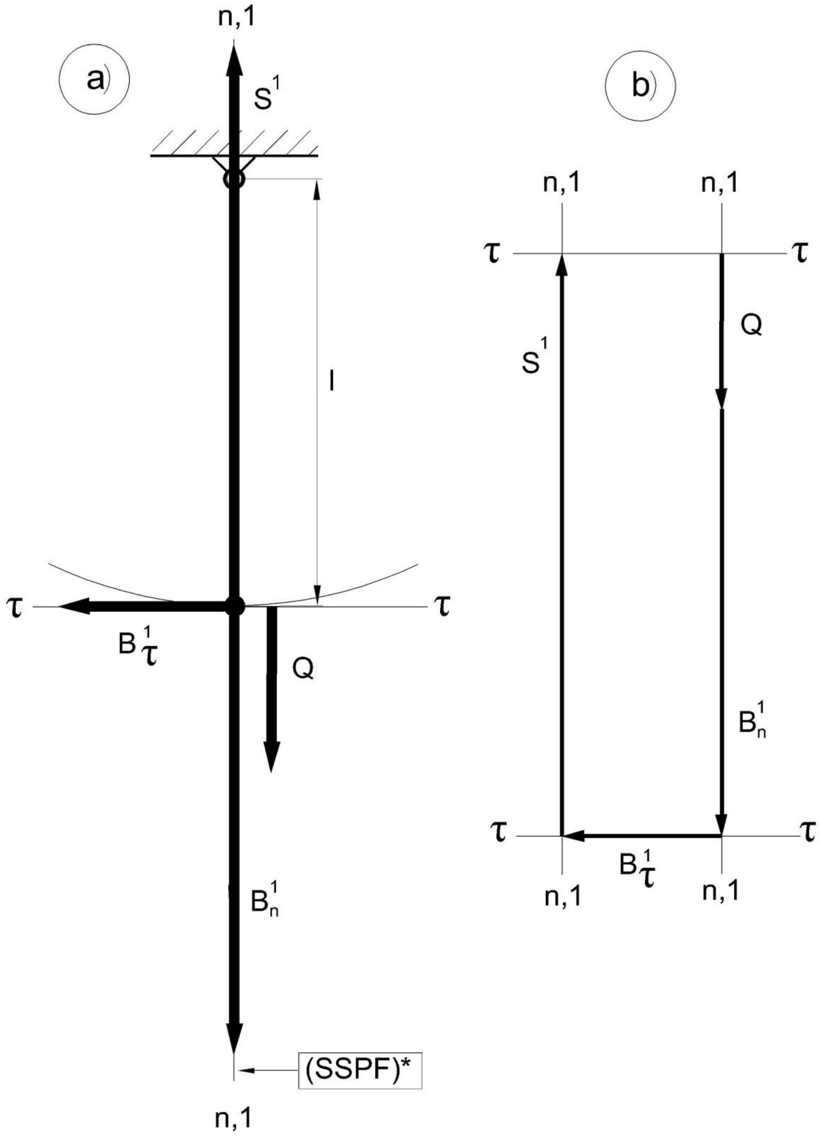

Fig. 5. Star-rosette (a) and open quadrilateral of forces (b) for central instantaneous position of pendulum.

Further on, after transition of the body through the central position, it transfers with a retarded free motion, tending to an instantaneous unstable state of equilibrium. That instantaneous equilibrium (**Fig. 6a**) occurs on an instantaneous unstable static potential field $(ASPF)^*$, denoted by symbols $2-2$, and overlapping with a normal direction $n-n$. As can be seen, the tangent force of inertia B_τ^2 did not change its sense, but decreased to the value corresponding with the initial position of pendulum (see Fig. 5a). The remaining forces $\left(B_n^2, S^2, Q\right)$ also have the same unchanged characteristics. Corresponding with the situation, the quadrilateral of forces is now closed (**Fig. 6b**).

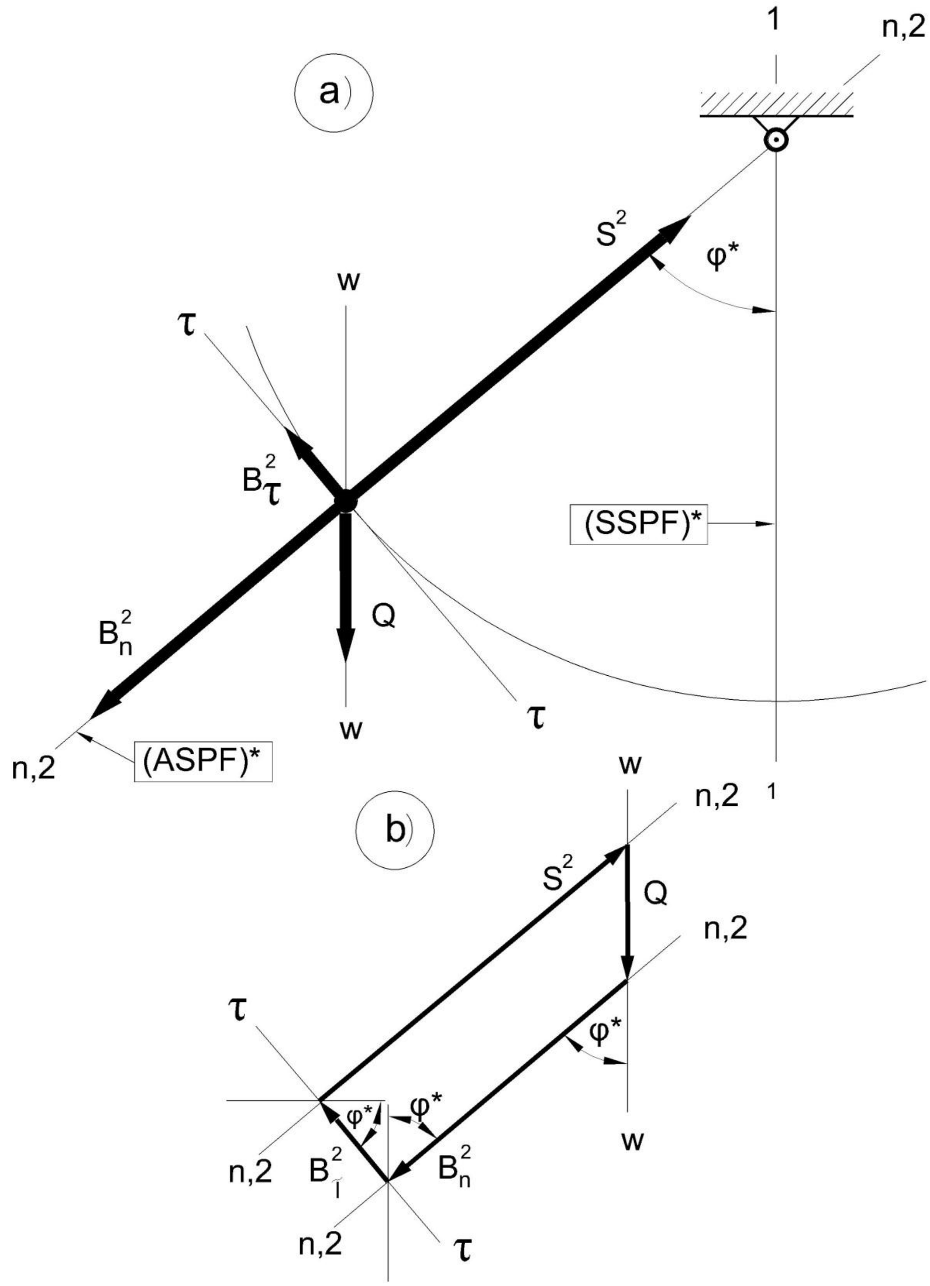

Fig. 6. Star-rosette (a) and closed quadrilateral of forces (b) in the neighbouring turning point of pendulum free motion.

Furthermore, the tangent force of inertia B_{τ}^{2} changes its sense (**Fig. 7a**), which corresponds with an open polygon of the component forces (**Fig. 7b**). Therefore the pendulum repeats its motion cycle (accelerated-retarded), returning to the initial/primary position where again the change of sense of tangent inertia force will take place.

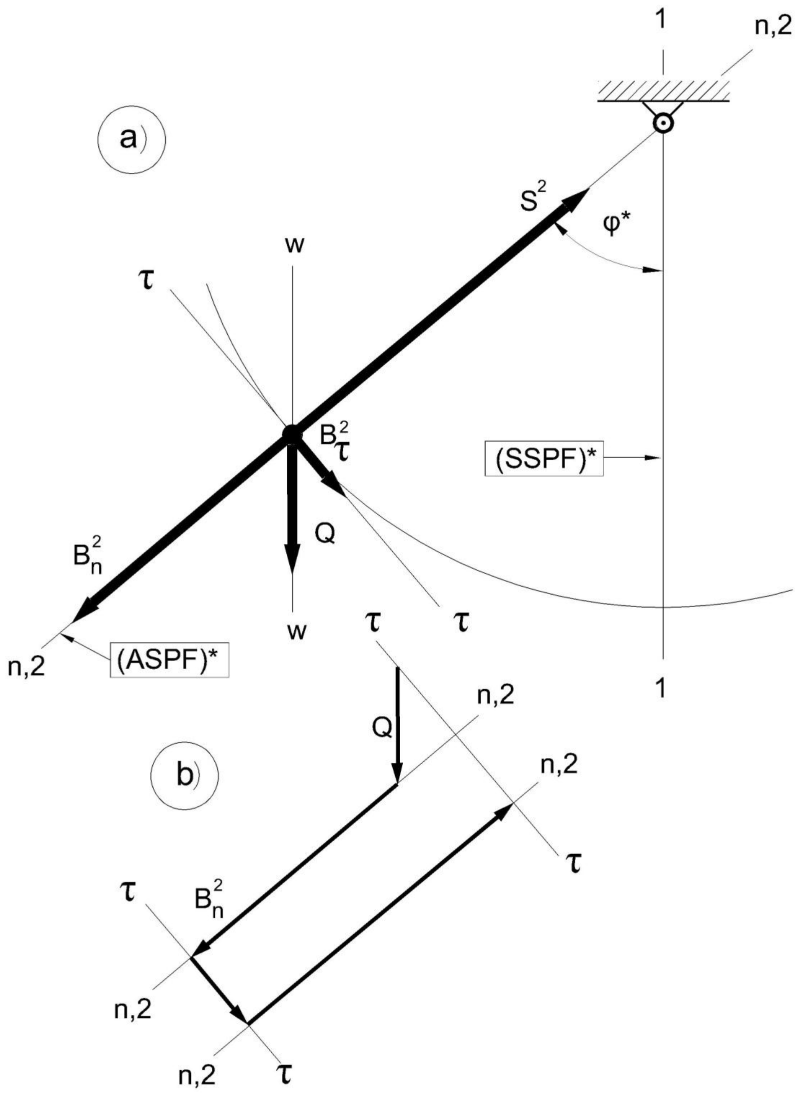

Fig. 7. Star-rosette (a) and open quadrilateral of forces (b) in the moment of start returning pendulum to the point of departure/origin.

4. CONCLUSION

This Part 1 of the paper was directed to the behaviour of a simple mathematical pendulum in particular points, that is in central and terminal/extremum ones and they have been analyzed in detail. System of forces in these points was considered with a special attention turned towards the terminal points where the equilibrium of forces occurs and in the next moment the lack of that equilibrium took place with the proof of the open polygon of forces as the condition of beginning of accelerated free variable motion. The kinetics and the dynamics will be considered in the next part of the paper. In the future, the energy and dynamics problems will be discussed in the following elaborations.

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