VOLUME 16, N° 3 2022
Journal of Automation, Mobile Robotics and Intelligent Systems

Robust H_∞ fuzzy approach design via Takagi-Sugeno descriptor model. Application for 2-DOF serial manipulator tracking control

Submitted: 12th May 2022; accepted: 20th June 2022

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DOI: 10.14313/JAMRIS/3-2022/21

Abstract:
This paper focuses on trajectory tracking control for robot manipulators. While much research has been done on this issue, many other aspects of this field have not been fully addressed. Here, we present a new solution using feedforward controller to eliminate parametric uncertainties and unknown disturbances. The Takagi-Sugeno fuzzy descriptor system (TSFDS) is chosen to describe the dynamic characteristics of the robot. The combination of this fuzzy system and the robust $\mathcal{H}_\infty$ performance makes the system almost isolated from external factors. The linear matrix inequalities based on the theory of Lyapunov stability is considered for control design. The proposed method has proven its effectiveness through simulation results.

Keywords: *Tracking control, serial manipulator robot, fuzzy control, Takagi-Sugeno, Lyapunov stability*

1. Introduction

The output regulation issue, often known as one of the core problems in control theory, involves following specified tracking signals and rejecting undesired disturbances in a dynamical system's output while preserving closed-loop stability. Up to now, numerous research investigations have been devoted to the tracking control using a fuzzy approach in literature [1, 2]. Let us mention for examples, the works [3] applied on spacecraft system, [4] on robotic manipulator systems, [5] on stochastic synthetic biology systems, [6] on air-breathing hypersonic vehicle, [7] on servo motors, [8] on linear motor systems, [9] on tower cranes, [10] on nuclear reactor, [11] on memristive recurrent neural network, [12] on electrically driven free-floating space manipulators, [13] on networked control systems. Our goal is to propose a new design framework in robot tracking control in order to ensure trajectory tracking and disturbance rejection.

In the experimental environment, the influence from the external disturbance on the system is inevitable. Researchers have also studied this issue very carefully, commonly using active disturbance rejection control (ADRC) controllers. Many applications utilized the ADRC controller and its variations have shown positive results. For instance, a linear ADRC has been applied in an Electro-Mechanical Actuator (EMA) [14], an ADRC-based backstepping control for fractional-order systems [15], and in sliding mode ADRC in the trajectory tracking of a quadrotor UAV [16]. Another approach, which is the same one used in this paper, is to use $\mathcal{H}_\infty$ performance.

The paper presents the problem for the manipulator tracking control using Takagi-Sugeno (T-S) fuzzy approach [17–19]. In this paper, the T-S model in descriptor form has been introduced [18, 20] and is used in many robotics applications [21]. Although both the standard form and the descriptor form can describe the object model well, the latter shows the advantage in reducing the complexity of the description equations. The trajectory tracking issue has been one of the foci in the controls field for many decades, and intensive research on this topic has yielded productive results [1, 2, 13]. These researches solve the tracking problem with $\mathcal{H}_\infty$ tracking performance [22–24]. Nevertheless, it is not possible to ensure tracking and rejection specifications using the $\mathcal{H}_\infty$ performance in the case of nonlinear closed loop systems [25]. As the objective is to ensure tracking and rejection specifications for a nonlinear system, we propose the new control structure, which includes a feedback part and a feedforward one. The feedforward part holds a vital role to reject reference input and the feedback part maintains the closed-loop tracking error stability. If a suitable Lyapunov function is chosen, the closed system will be proved to be stable and hence use the linear matrix inequalities (LMI) to rewrite the conditions. The LMI matrix is built from pre-existing conditional equations and its calculation can be done through some simple programming steps. Once we have the feedback gains from the LMI, a distributed compensation controller (PDC) [26], which is commonly used in T-S fuzzy system, is generated as the feedback part of the control structure. There are many researches using feedforward which do not bring high efficiency in the tracking control problem. In the case without disturbance of a dynamical system, the robot tracking control is asymptotically stable. And in the case of any bounded disturbance, we can reduce a minimal value tracking error.

This paper provides some major contributions:

1) Used descriptor equations for modelling the control object. It is also noticed that when using the descriptor model, the number of fuzzy rules is less than when utilizing the standard one.

2) This paper proposed new formulations to generate a new control law that includes a feedforward part and a feedback part. Hence, the control signal now not only stabilizes the system but also can dismiss the disturbance of the reference model.

3) The problem of disturbance rejection for the fuzzy system is also handled using $\mathcal{H}_\infty$ performance.

The paper is organized as follows: Section 2 mentions the T-S fuzzy descriptor system with external disturbance. Problem formulation and a new control approach are represented in section 3. Section 4 is associated with linear matrix inequality and PDC controller. Simulation results when applying the proposed control theory in the 2-DoF manipulator are shown in section 5. Some conclusions of this paper are in section 6.

2. Fuzzy system description

The T-S fuzzy descriptor system can be derived as follows:

$$\mathbb{E}_{\mathfrak{v}}\dot{\mathbf{x}}(t) = \mathbb{A}_{\mathfrak{h}}\mathbf{x}(t) + \mathbb{B}_{\mathfrak{h}}\mathbf{u}(t) + \mathbb{D}_{\mathfrak{h}}\mathbf{w}(t) \quad (1)$$

$$\mathbf{y}(t) = \mathbb{C}_{\mathfrak{h}}\mathbf{x}(t) \quad (2)$$

with

$$\mathbb{E}_{\mathfrak{v}} = \sum_{k=1}^{l_{le}} \mathfrak{v}_k(z(t))\mathbb{E}_k \quad (3)$$

$$\mathbb{A}_{\mathfrak{h}} = \sum_{i=1}^{l_{ri}} \mathfrak{h}_i(z(t))\mathbb{A}_i \quad (4)$$

$$\mathbb{B}_{\mathfrak{h}} = \sum_{i=1}^{l_{ri}} \mathfrak{h}_i(z(t))\mathbb{B}_i \quad (5)$$

$$\mathbb{D}_{\mathfrak{h}} = \sum_{i=1}^{l_{ri}} \mathfrak{h}_i(z(t))\mathbb{D}_i \quad (6)$$

$$\mathbb{C}_{\mathfrak{h}} = \sum_{i=1}^{l_{ri}} \mathfrak{h}_i(z(t))\mathbb{C}_i \quad (7)$$

where in (1) and (2), $\mathbf{x}(t)$ is a state vector, $\mathbf{u}(t)$ represents control input, $\mathbf{y}(t)$ is the system output, and $\mathbf{w}(t)$ represents the additional disturbance. $k \in \mho_{l_{le}}$={$1, 2, .., l_{le}$}, $i \in \mho_{l_{ri}}$={$1, 2, .., l_{ri}$}. The variable $z(t)$ is the premise vector which consists of premise variables. If vector $z(t)$ has n elements, which means there are n premise variables, the system will have 2^n fuzzy rules in total. $\mathfrak{h}_i(z(t))$ and $\mathfrak{v}_k(z(t))$ are membership functions in the right sides and the left one of these above equations, respectively. Also note that the number of fuzzy rules in the right-hand parts of (1) is l_{ri} and in the left-hand parts is l_{le}. The membership functions can be calculated as follows:

$$\mathfrak{h}_i(z(t)) = \prod_{j=1}^{log_2(l_{ri})} w_j^{k_j}(z_j(t)). \quad (8)$$

$$\mathfrak{v}_k(z(t)) = \prod_{j=1}^{log_2(l_{le})} w_j^{k_j}(z_j(t)). \quad (9)$$

Assumption 1. *There exists a positive number ϑ as the constraint for the disturbance function $\mathbf{w}(t)$:*

$$\|\mathbf{w}(t)\| \leqslant \vartheta. \quad (10)$$

Remark 1. *To reject the disturbance of the model $\mathbf{w}(t)$, the inequality of $\mathcal{H}_\infty$ performance [27] is considered and has the following form:*

$$\int_0^\infty p(t)^T p(t)dt \leqslant \zeta^2 \int_0^\infty \mathbf{w}(t)^T \mathbf{w}(t)dt. \quad (11)$$

where $p(t)$ is the desired control signal and ζ is in the set of real numbers.

The augmented form of the system in (1) can be inferred in the following equations:

$$\mathbb{E}^\star\dot{\mathbf{x}}^\star(t) = \mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}}\mathbf{x}^\star(t) + \mathbb{B}^\star_{\mathfrak{h}}\mathbf{u}(t) + \mathbb{D}^\star_{\mathfrak{h}}\mathbf{w}(t) \quad (12)$$

$$\mathbf{y}(t) = \mathbb{C}^\star_{\mathfrak{h}}\mathbf{x}^\star(t) \quad (13)$$

with

$$\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} = \sum_{i=1}^{l_{ri}}\sum_{k=1}^{l_{le}} \mathfrak{h}_i(z(t))\mathfrak{v}_k(z(t))\mathbb{A}^\star_{ik} \quad (14)$$

$$\mathbb{B}^\star_{\mathfrak{h}} = \sum_{i=1}^{l_{ri}} \mathfrak{h}_i(z(t))\mathbb{B}^\star_i \quad (15)$$

$$\mathbb{C}^\star_{\mathfrak{h}} = \sum_{i=1}^{l_{ri}} \mathfrak{h}_i(z(t))\mathbb{C}^\star_i \quad (16)$$

$$\mathbb{D}^\star_{\mathfrak{h}} = \sum_{i=1}^{l_{ri}} \mathfrak{h}_i(z(t))\mathbb{D}^\star_i \quad (17)$$

where $\mathbf{x}^\star(t) = \begin{bmatrix}\mathbf{x}(t)\\ \dot{\mathbf{x}}(t)\end{bmatrix}$, $\mathbb{E}^\star = \begin{bmatrix} I & 0\\ 0 & 0\end{bmatrix}$, $\mathbb{A}^\star_{ik} = \begin{bmatrix} 0 & I\\ \mathbb{A}_i & -\mathbb{E}_k\end{bmatrix}$, $\mathbb{B}^\star_i = \begin{bmatrix}0\\ \mathbb{B}_i\end{bmatrix}$, $\mathbb{D}^\star_i = \begin{bmatrix}0\\ \mathbb{D}_i\end{bmatrix}$ and $\mathbb{C}^\star_i = \begin{bmatrix}\mathbb{C}_i & 0\end{bmatrix}$.

Remark 2. *Although (1) and (12) are two similar TSFDSs, the conversion from (1) to (12) will make the process of calculating and proving the formulas below more convenient. Many studies have also applied this transformation [28, 29].*

Assumption 2. *The descriptor reference model can be presented in the following form:*

$$\mathbb{E}_r\dot{\mathbf{x}}_r = \mathbb{A}_r\mathbf{x}_r + \mathbb{B}_r\mathbf{r}. \quad (18)$$

This model can be used as a sample trajectory for the tracking control problem.

In the same way to achieve (12), the T-S fuzzy system (18) are rewritten as:

$$\mathbb{E}^\star\dot{\mathbf{x}}^\star_r(t) = \mathbb{A}^\star_r\mathbf{x}^\star_r(t) + \mathbb{B}^\star_r\mathbf{r}(t) \quad (19)$$

where

$$\mathbf{x}^\star_r(t) = \begin{bmatrix}\mathbf{x}_r(t)\\ \dot{\mathbf{x}}_r(t)\end{bmatrix}, \mathbb{E}^\star = \begin{bmatrix} I & 0\\ 0 & 0\end{bmatrix}, \mathbb{A}^\star_r = \begin{bmatrix} 0 & I\\ \mathbb{A}_r & -\mathbb{E}_r\end{bmatrix},$$

$$\mathbb{B}^\star_r = \begin{bmatrix}0\\ \mathbb{B}_r\end{bmatrix}.$$

3. Problem formulation and control

In this section, some transformations have been done to generate new formulations and ensure reference tracking control for the system (1). From (12) and (19), we have:

$$\mathbb{E}^\star \dot{\mathbf{e}}^\star(t) = \mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} \mathbf{e}^\star(t) + \mathbb{B}^\star_{\mathfrak{h}} \mathbf{u}(t) + \mathbb{D}^\star_{\mathfrak{h}} \mathbf{w}(t) + (\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{A}^\star_r)\mathbf{x}^\star_r(t) - \mathbb{B}^\star_r \mathbf{r}(t) \quad (20)$$

where $\mathbf{e}^\star(t) = \begin{bmatrix} \mathbf{e}(t) \\ \dot{\mathbf{e}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{x}(t) - \mathbf{x}_r(t) \\ \dot{\mathbf{x}}(t) - \dot{\mathbf{x}}_r(t) \end{bmatrix}$, in which $\mathbf{e}(t)$ is the tracking error that needs converging to zero.

Remark 3. *The control input $\mathbf{u}(t)$ can be synthesized from two components: the feedforward signal $\mathbf{u}_{ff}$ and the feedback signal $\mathbf{u}_{fb}$ in Fig. 1.*

$$\mathbf{u}(t) = \mathbf{u}_{ff} + \mathbf{u}_{fb}. \quad (21)$$

The main objective of the feedforward part is to reject the reference input and the feedback one is used to maintain the closed-loop tracking error stability.

From (20) and (21):

$$\mathbb{E}^\star \dot{\mathbf{e}}^\star = \begin{bmatrix} 0 & I \\ \mathbb{A}_{\mathfrak{h}} & -\mathbb{E}_{\mathfrak{v}} \end{bmatrix} \mathbf{e}^\star + \begin{bmatrix} 0 \\ \mathbb{B}_{\mathfrak{h}} \end{bmatrix} \mathbf{u}_{ff} + \begin{bmatrix} 0 \\ \mathbb{B}_{\mathfrak{h}} \end{bmatrix} \mathbf{u}_{fb} + \begin{bmatrix} 0 \\ \mathbb{D}_{\mathfrak{h}} \end{bmatrix} \mathbf{w} + \begin{bmatrix} 0 & 0 \\ \mathbb{A}_{\mathfrak{h}} - \mathbb{A}_r & \mathbb{E}_r - \mathbb{E}_{\mathfrak{v}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_r \\ \dot{\mathbf{x}}_r \end{bmatrix} - \begin{bmatrix} 0 \\ \mathbb{B}_r \end{bmatrix} \mathbf{r}. \quad (22)$$

Assumption 3. *In order to have reference input rejected, we assume that:*

$$\begin{bmatrix} 0 \\ \mathbb{B}_{\mathfrak{h}} \end{bmatrix} \mathbf{u}_{ff} + \begin{bmatrix} 0 & 0 \\ \mathbb{A}_{\mathfrak{h}} - \mathbb{A}_r & \mathbb{E}_r - \mathbb{E}_{\mathfrak{v}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_r \\ \dot{\mathbf{x}}_r \end{bmatrix} - \begin{bmatrix} 0 \\ \mathbb{B}_r \end{bmatrix} \mathbf{r} = 0. \quad (23)$$

The feedforward part now can be inferred as follows:

$$\mathbf{u}_{ff} = \mathbb{B}^{-1}_{\mathfrak{h}} (\mathbb{E}_{\mathfrak{v}} \dot{\mathbf{x}}_r - \mathbb{A}_{\mathfrak{h}} \mathbf{x}_r). \quad (24)$$

Substituting (24) to (20):

$$\mathbb{E}^\star \dot{\mathbf{e}}^\star(t) = \mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} \mathbf{e}^\star(t) + \mathbb{B}^\star_{\mathfrak{h}} \mathbf{u}_{fb}(t) + \mathbb{D}^\star_{\mathfrak{h}} \mathbf{w}(t). \quad (25)$$

It is obvious that the system (25) has a similar form to the TSFDS in (1), then the following parallel distributed compensation (PDC) control law can be considered:

$$\mathbf{u}_{fb}(t) = -\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}} \mathbf{e}^\star(t) \quad (26)$$

with:

$$\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}} = \sum_{i=1}^{l_{ri}} \sum_{k=1}^{l_{le}} \mathfrak{h}_i(z(t)) \mathfrak{v}_k(z(t)) \mathbb{F}^\star_{ik} \quad (27)$$

where the local control gains $\mathbb{F}^\star_{ik} = \begin{bmatrix} \mathbb{F}_{ik} & 0 \end{bmatrix}$ are to be designed.

The closed-loop descriptor T-S fuzzy system in extended form can be rewritten:

$$\mathbb{E}^\star \dot{\mathbf{e}}^\star(t) = (\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{B}^\star_{\mathfrak{h}} \mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}}) \mathbf{e}^\star(t) + \mathbb{D}^\star_{\mathfrak{h}} \mathbf{w}(t). \quad (28)$$

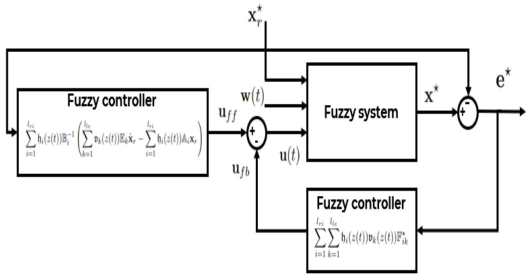

Fig. 1. The block diagram of fuzzy controller for the TSFDS

4. Main result

The stability of the system is an extremely important factor and must be satisfied in the design of the controller. For a fuzzy system, LMI is an effective solution to find the stability conditions. From there, the LMI-based control gains for the 2-DoF robot can be calculated.

Theorem 1. *The closed-loop descriptor system in (28) with the feedback control signal $\mathbf{u}_{fb}(t) = -\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}} \mathbf{e}^\star(t)$ is asymptotically stable if there exists matrices $\mathbb{P}_3$, $\mathbb{P}_4$, $\mathbb{M}_{ik}$ in appropriate dimensions and a positive matrix $\mathbb{P}_1$ such that:*

$$\Psi_{iik} = \begin{bmatrix} \Xi_{ii} & \star & \star & \star & \star & \star \\ \Phi_{iik} & \Pi_{iik} & \star & \star & \star & \star \\ \mathbb{D}_i^T & 0 & -\zeta^2 I & \star & \star & \star \\ 0 & 0 & 0 & -\zeta^2 I & \star & \star \\ P_1 & 0 & 0 & 0 & -I & \star \\ 0 & P_4 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (29)$$

where $\Xi_{ii} = -\mathbb{P}_3 - \mathbb{P}_3^\top + 2\alpha\mathbb{P}_1$, $\Phi_{iik} = \mathbb{A}_i\mathbb{P}_1 - \mathbb{B}_i\mathbb{M}_{ik} + \mathbb{E}_k\mathbb{P}_3 + \mathbb{P}_4^\top$, $\Pi_{iik} = -\mathbb{E}_k\mathbb{P}_4 - (\mathbb{E}_k\mathbb{P}_4)^\top$. Furthermore, the control gains of the PDC controller (26) can be computed as follows:

$$\mathbb{F}_{ik} = \mathbb{M}_{ik}\mathbb{P}_1^{-1}. \quad (30)$$

Proof. Let $\mathbb{P} = \begin{bmatrix} \mathbb{P}_1 & 0 \\ -\mathbb{P}_3 & \mathbb{P}_4 \end{bmatrix}$.

Consider the following Lyapunov function candidate:

$$V(\mathbf{e}^\star) = \mathbf{e}^{\star\top} \mathbb{E}^\star \mathbb{P}^{-1} \mathbf{e}^\star \quad (31)$$

Based on the defined matrices $\mathbb{E}^\star$ and $\mathbb{P}$, (31) has the time-derivative as follows:

$$\dot{V}(\mathbf{e}^\star) = \dot{\mathbf{e}}^{\star\top} \mathbb{E}^{\star\top} \mathbb{P}^{-1} \mathbf{e}^\star + \mathbf{e}^{\star\top} (\mathbb{P}^{-1})^\top \mathbb{E}^\star \dot{\mathbf{e}}^\star \quad (32)$$

Then

$$\begin{aligned} \dot{V}(\mathbf{e}^\star) &= \left[(\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{B}^\star_{\mathfrak{h}}\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}})\mathbf{e}^\star + \mathbb{D}\mathbf{w}\right]^\top \mathbb{P}^{-1}\mathbf{e}^\star \\ &+ \mathbf{e}^{\star\top}(\mathbb{P}^{-1})^\top \left[(\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{B}^\star_{\mathfrak{h}}\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}})\mathbf{e}^\star + \mathbb{D}\mathbf{w}\right] \\ &= \mathbf{e}^{\star\top}[(\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{B}^\star_{\mathfrak{h}}\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}})^\top \mathbb{P}^{-1} \\ &+ (\mathbb{P}^{-1})^\top (\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{B}^\star_{\mathfrak{h}}\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}})]\mathbf{e}^\star \\ &+ \mathbf{w}^\top\mathbb{D}^\top\mathbb{P}^{-1}\mathbf{e}^\star + \mathbf{e}^{\star\top}(\mathbb{P}^{-1})^\top\mathbb{D}\mathbf{w} \end{aligned} \quad (33)$$

Since the control objective is for the tracking errors to converge at zero, the $p(t)$ term in (11) now equals to $\mathbf{e}^\star$. Then the stability condition of the closed-loop system (28) can be inferred as follows:

$$\dot{V}(\mathbf{e}^\star) + \mathbf{e}^{\star\top}\mathbf{e}^\star - \zeta^2\mathbf{w}^\top\mathbf{w} < -2\alpha V(\mathbf{e}^\star) \quad (34)$$

Some simple computation leads to

$$\Psi_{iik} = \begin{bmatrix} \hat{\xi} & \star & \star \\ \mathbb{D}_i^{\star\top} & -\zeta^2 I & 0 \\ \mathbb{C}_z^\star\mathbb{P} & 0 & -I \end{bmatrix} \quad (35)$$

where $\mathbb{C}_z^\star = [I\ 0\ 0\ 0]$ and note that:

$$\hat{\xi} = (\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{B}^\star_{\mathfrak{h}}\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}})^\top\mathbb{P}^{-1} + \alpha\mathbb{E}^\star\mathbb{P}^{-1} + (\mathbb{P}^{-1})^\top(\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{B}^\star_{\mathfrak{h}}\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}}) + \alpha(\mathbb{P}^{-1})^\top\mathbb{E}^\star < 0 \quad (36)$$

Consider the expression in the right side of (36), multiplying it on the left and right by $\mathbb{P}^\top$ and $\mathbb{P}$, respectively, we get:

$$\mathbb{P}^\top(\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{B}^\star_{\mathfrak{h}}\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}})^\top + \alpha\mathbb{P}^\top\mathbb{E}^\star + (\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}} - \mathbb{B}^\star_{\mathfrak{h}}\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}})\mathbb{P} + \alpha\mathbb{E}^\star\mathbb{P} < 0 \quad (37)$$

then

$$\mathbb{A}^\star_{\mathfrak{h}\mathfrak{v}}\mathbb{P} - \mathbb{B}^\star_{\mathfrak{h}}\mathbb{F}^\star_{\mathfrak{h}\mathfrak{v}}\mathbb{P} + \alpha\mathbb{E}^\star\mathbb{P} + \mathbb{A}^{\star\top}_{\mathfrak{h}\mathfrak{v}}\mathbb{P}^\top - \mathbb{P}^\top\mathbb{F}^{\star\top}_{\mathfrak{h}\mathfrak{v}}\mathbb{B}^{\star\top}_{\mathfrak{h}} + \alpha\mathbb{P}^\top\mathbb{E}^\star < 0 \quad (38)$$

□

5. Illustrative Results and Discussions

In this paper, one applied the $\mathcal{H}_\infty$ performance and the new fuzzy control in the form of T-S descriptor system for the 2-DoF robot. The model of the robot and its parameters was referred from a study [30]. Dynamic equations of this manipulator were converted to the form descriptor model as (1) with these following matrices:

$$\mathbb{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha + 2m_2r_1L_2z_5(\mathbf{x}) & \beta + m_2L_1r_2z_5(\mathbf{x}) \\ 0 & 0 & \beta + m_2L_1r_2z_5(\mathbf{x}) & \beta \end{bmatrix}$$

$$\mathbb{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ z_3(\mathbf{x}) & z_4(\mathbf{x}) & 2z_1(\mathbf{x}) - f_{v_1} & z_1(\mathbf{x}) \\ z_4(\mathbf{x}) & z_4(\mathbf{x}) & z_2(\mathbf{x}) & -f_{v_2} \end{bmatrix}$$

$$\mathbb{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbb{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \mathbb{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

with $\mathbf{x} = \begin{bmatrix}\theta_1 & \theta_2 & \dot{\theta}_1 & \dot{\theta}_2\end{bmatrix}^\top$, $\alpha = m_1r_1^2 + I_1 + m_2L_1^2 + m_2r_2^2 + I_2$, $\beta = m_2r_2^2 + I_2$, $z_1(\mathbf{x}) = m_2L_1r_2\dot{\theta}_2 sin\theta_2$, $z_2(\mathbf{x}) = -m_2L_1r_2\dot{\theta}_1 sin\theta_2$, $z_3(\mathbf{x}) = -(m_1gr_1 + m_2gL_1)\frac{sin\theta_1}{\theta_1} - m_2gr_2\frac{sin\theta_{12}}{\theta_{12}}$, $z_4(\mathbf{x}) = -m_2gr_2\frac{sin\theta_{12}}{\theta_{12}}$, $z_5(\mathbf{x}) = cos(\theta_2)$.

Matrices $\mathbb{E}$ and $\mathbb{A}$ accordingly have 1 and 4 variable z, then they have 2 and 16 rules, respectively. For instance, one can perform $\mathbb{E}$ as follows:

$$\mathbb{E}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha + 2m_2r_1L_2z_{5max} & \beta + m_2L_1r_2z_{5max} \\ 0 & 0 & \beta + m_2L_1r_2z_{5max} & \beta \end{bmatrix}$$
$$\mathbb{E}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha + 2m_2r_1L_2z_{5min} & \beta + m_2L_1r_2z_{5min} \\ 0 & 0 & \beta + m_2L_1r_2z_{5min} & \beta \end{bmatrix} \quad (39)$$

The remaining $\mathbb{A}$ matrices will be represented in the same way.

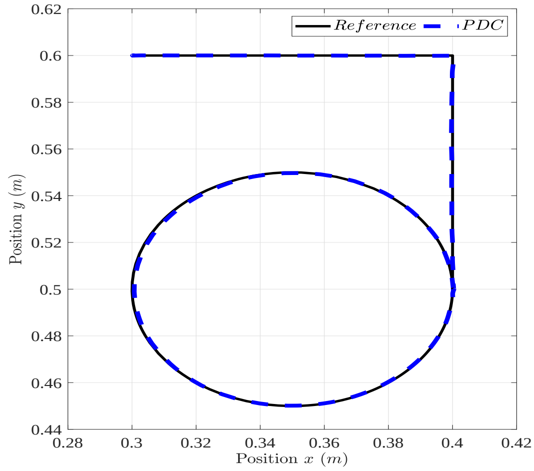

Fig. 2. Trajectory tracking

The sample trajectories are designed so that the end effector moves sideways, then perpendicular, and finally into a circle. From Fig. 2, it is clear that the simulated trajectory achieved coincides with the sample trajectory. Using inverse kinematics, it is possible to construct sample angular and velocity trajectories for two joint angles. Simulation results in Figs. 3 and 4 have shown that in order for the end effector to move along the set path, the two joints of the robot also change almost identically with the results calculated from the reverse kinematics. Figs. 5 and 6 clearly show

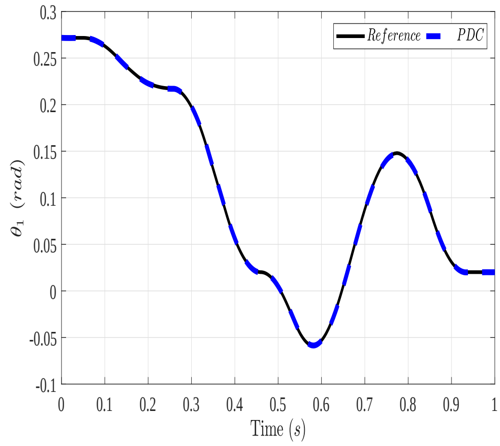

Fig. 3. Angular position of Joint 1

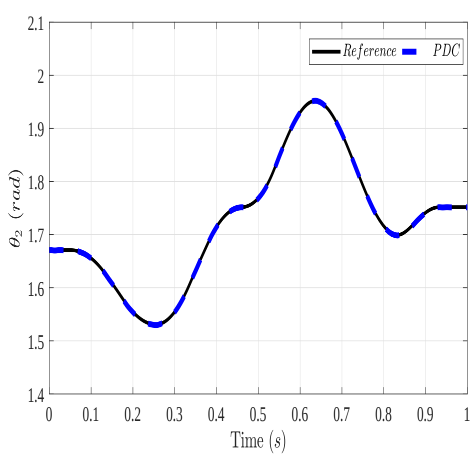

Fig. 4. Angular position of Joint 2

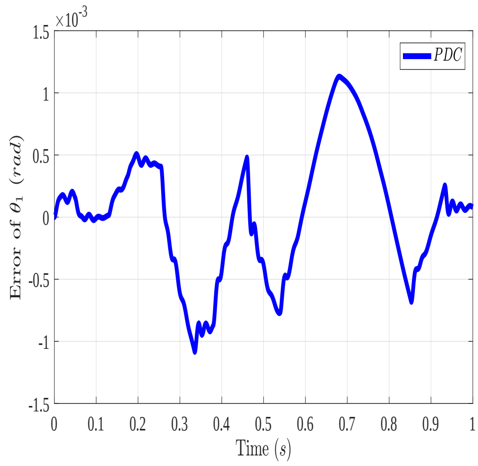

Fig. 5. Tracking errors of angular position at Joint 1

the orbital tracking ability of the system. The magnitude of the errors is also specified through Root Mean Square Error (RMSE). The errors of the variables are calculated in detail and presented in Tab. 1 . Even with the effect of the disturbance, the errors in angles of two joints are tiny with only 4.6657×10^{-4} (rad) and 4.8859×10^{-4} (rad), respectively. The PDC controller does an excellent job in stabilizing the whole system, and the feedforward component also shows strength in eliminating reference disturbance.

Error	RMSE
θ_1 (rad)	4.6657×10^{-4}
θ_2 (rad)	4.8859×10^{-4}
$\dot{\theta}_1$ (rad/s)	0.0176
$\dot{\theta}_2$ (rad/s)	0.0361

Tab. 1. Root Mean Square Errors

As seen in Figs. 7 and 8, the travel velocities of the joints also follow the calculated velocities. The deviation between the actual and the reference is minimal and is close to zero, see Figs. 9 and 10. Tab. 1 also gives the RMSE of the two-joint velocity with error in the first joint is 0.0176 (rad/s) and in the second one is 0.0361 (rad/s).

The oscillation torque characteristic of both joints in Fig. 11 is not too large, indicating that the system will not jerk during motion. At the same time, it also shows that this control method is suitable for the actual model and can be applied well in controlling robotic mechanisms.

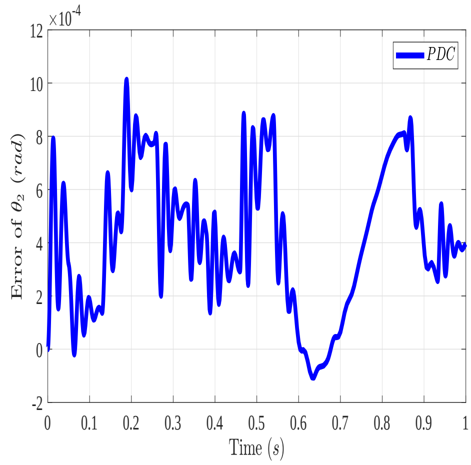

Fig. 6. Tracking errors of angular position at Joint 2

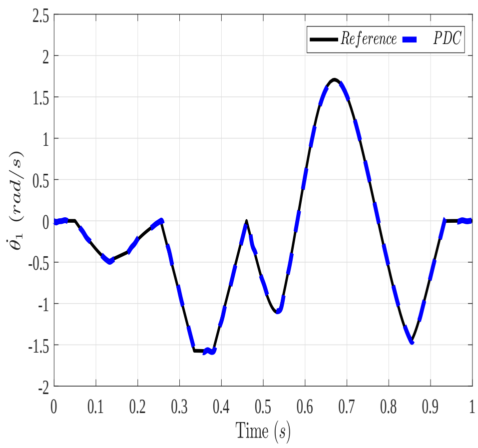

Fig. 7. Velocity profile of Joint 1

6. Conclusions

This study was conducted with the primary objective of trajectory tracking control for the robot manipulators. With many advantages over the conventional T-S fuzzy system, the T-S fuzzy descriptor model was chosen to describe the dynamic behaviour of the control object. The article also considers external factors such as the disturbance of the model and uses $\mathcal{H}_\infty$ performance to handle that problem. The novelty of this paper is the replacement of a common controller into two separate controllers with different functions, respectively. The feedforward controller is intended to remove influences of the reference model, and the feedback controller is utilized to stabilize the system. By a few simple transformations, the feedforward component can be easily deduced. Meanwhile, the feedback part, which is the PDC controller, is de-

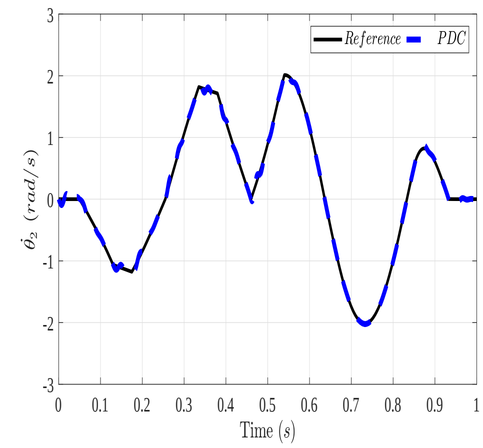

Fig. 8. Velocity profile of Joint 2

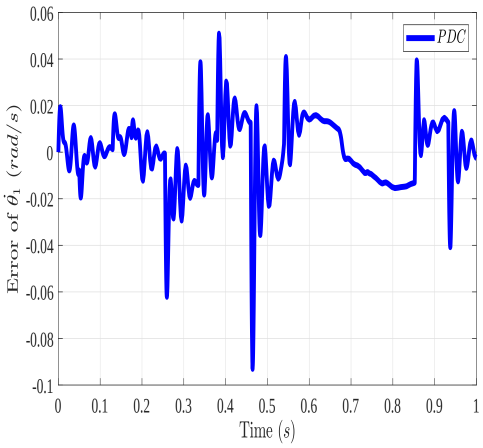

Fig. 9. Tracking errors of velocity profile at Joint 1

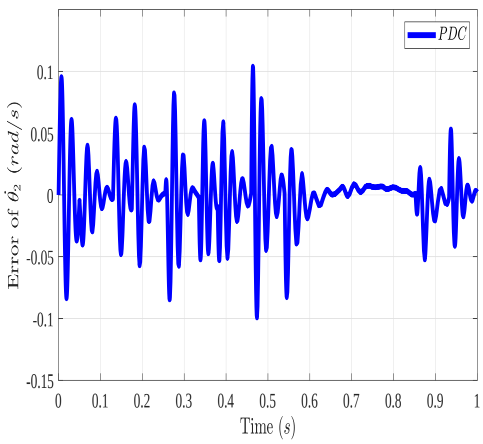

Fig. 10. Tracking errors of velocity profile at Joint 2

signed based on the Lyapunov and LMI stability conditions. We have obtained many successful results by applying those control theories to the 2-DoF robot model and performing simulations. Not only is the trajectory of the end effector almost wholly coincident with the sample trajectory, but also such components as the positions or velocities of the joints are strictly followed.

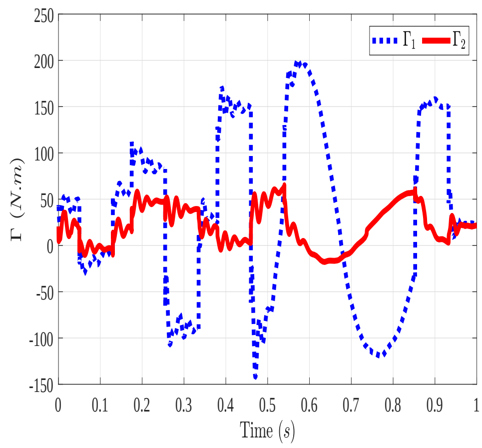

Fig. 11. Torques at Joints 1 and 2

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ACKNOWLEDGEMENTS

This research is funded by Hanoi University of Science and Technology (HUST) under project number T2021-TT-002.

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