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STRESS IN THE ECCENTRIC ELLIPTICAL GEARING TOOTH

Summary. In calculating the stress in the gear based on a number of assumptions and calculates the so. carrier current stress that is very useful if used in calculating the results and findings from research and practice in the determination of any effects that affect the actual teeth stress. The complicated shape of the teeth is the theoretical determination of stress in the teeth difficult. The starting assumption is highly idealized notion of a linear displacement of the tooth for and tooth load is considered as a beam loaded by bending. The article is devoted to problems of stress examining in a dangerous section of the foot tooth in gearing with variable gear ratio solution by finite element method.
Keywords. Gear transmission, stress, FEM.

NAPRĘŻENIE W ZAZĘBIENIU PRZEKŁADNI ELIPTYCZNEJ UŁOŻONEJ MIMOŚRODOWO

Streszczenie. Podczas obliczania naprężenia w zazębieniu wychodzi się z wielu założeń oraz uwzględnia tzw. naprężenie wyrównawcze, które jest wykorzystywane w przypadku, kiedy w obliczeniach są stosowane wyniki i wiadomości oparte na praktyce i badaniach przy określaniu wszystkich wpływów, które powodują realne obciążenie zazębienia. Ze względu na skomplikowany kształt zębów, teoretyczne określenie naprężenia zazębienia jest trudne. Założeniem wyjściowym najczęściej bywa mocno wyidealizowany obraz zależności liniowej ugięcia zęba w wyniku obciążenia, a ząb jest uważany za nośnik poddawany ugięciu. Artykuł jest poświęcony problematyce badania naprężenia w niebezpiecznym przekroju podstawy zęba przekładni zębatej ze zmiennym przełożeniem za pomocą metody elementów skończonych.
Słowa kluczowe. Przekładnia zębata, naprężenie, MES.

1. INTRODUCTION

In calculating the stress of gearing can be stress determined by calculating and the actual values vary significantly. The stress calculated in a standard way according to available formulas is approximate because some impacts cannot be determined with sufficient

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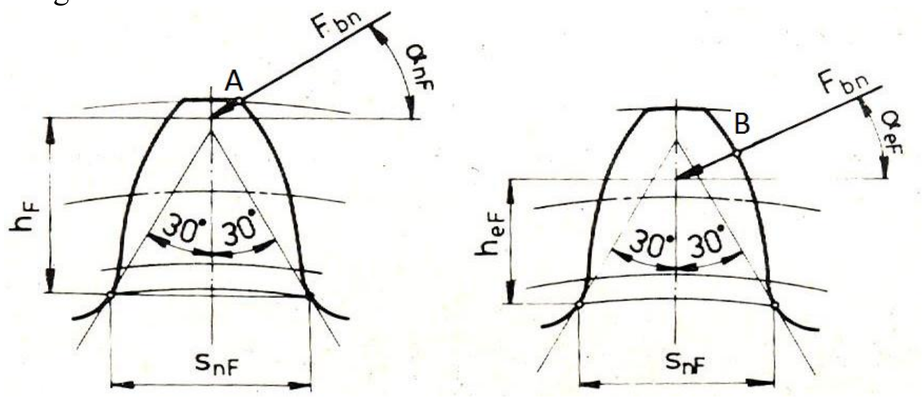
accuracy. The tooth has a complex shape and mesh conditions that affect the size and locations of transmitting power, arm bending, position and size of the dangerous section, while the mesh is variable and dependent on the precision gearing and assembly. The maximum stress in the teeth is influenced by factors that determine the strength of teeth, for example effect of nick type distortion and the like.

Recently, at ever faster evolving computer technology, the available literature, we can meet with modern numerical methods, such as finite element method (FEM), which can serve as one of the methods for the determination of deflection gearing. The article is devoted to problems determining of the stress in a dangerous section of tooth foot using FEM. The problem is solved for elliptical, eccentric gear with a continuously variable gear ratio to a range from 0,5 through 1 to 2.

2. THE BENDING STRESS IN A DANGEROUS SECTION OF GEAR TOOTH

According to STN 01 4686 was calculated bending stress in the foot spur gear teeth for these assumptions:

- the requirements for accuracy of calculating the resultant force acting on a tooth side effects on the lateral edge a tooth and is introduced into the calculation of impact factor mesh (Fig. 1-a), or the resultant force acts on a lonely spot mesh (Fig. 1-b),
- considering only the bending load component,
- dangerous cross-sections for tangential points of the tangents to the transition curves, at an angle of 30 ° to the axis of the tooth.



a) b)

Fig. 1. The tooth load
Rys. 1. Obciążenie zęba
Source: Boháček F.: Části a mechanizmy strojů III, VUT Brno 1987.

Calculation of the local bending stress in a dangerous section of the gear tooth where the normal force is applied to the head a tooth (the mesh point A) is computed:

$$\sigma_{Fn} = \frac{F_t}{b_w \cdot m_n} \cdot Y_{Fa} \cdot Y_{\varepsilon} \cdot Y_{\beta} \tag{1}$$

In cases where the force acts on a lonely mesh point (point B) bending stress in the dangerous section of the tooth foot calculate the relation (2):

$$\sigma_{Fn} = \frac{F_t}{b_w \cdot m_n} \cdot Y_F \cdot Y_{\beta} \tag{2}$$

where:

F_t – circumferential forse [N];

b_w – gear width to calculate the bending [mm];
m_n – module in the normal plane [mm];
Y_{Fa} – coefficient of tooth shape [-];
Y_F – coefficient of tooth shape [-];
Y_ε – coefficient of profile mesh impact [-];
Y_β – coefficient inclination of the tooth [-].

In place of dangerous section is a stress concentration. Shape foot transition curve, tension in the surface layer, surface finish and nick, resulting in grinding prominences without affecting the stress and the stress peak production. It is therefore recommended in the calculation of the maximum local stress include the coefficient of stress concentration.

This standard is applicable to the spur gear teeth without corrections and modifications. To determine the stress in the foot the tooth for other gear, such as an unbalanced profile of this approach is insufficient. One way to determine the tension in a dangerous section of the tooth foot stress determination by FEM.

3. GEAR WITH CONTINUOUSLY VARIABLE GEAR RATIO

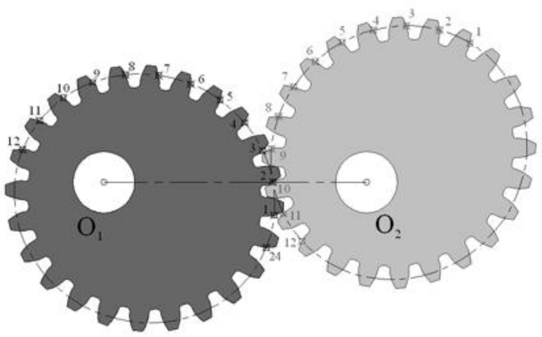


Fig. 2. Designed gear in AutoCAD
Rys. 2. Model kół stworzony w AutoCAD

To sponsor the work of the private sector has been created using CAD model of gear with variable transmission in the range $u = 0,5$ to $2,0$,with the number of teeth $z_1 = z_2 = 24$ and gearing module $m_n = 3,75$ *mm*, the axial distance $a = 90$ *mm* and for a one sense of rotation. Grantor work said gears and made to say "roughly" to illustrate the problem.

To create this gear is analyzed in detail in the literature [4] and [8]. The gears for a given variable transmission have been proposed as elliptical - eccentrically placed (*Fig. 2*), so that conditions were right shot.

In pursuit of kinematic ratios on the proposed gearings we assume from the right mesh conditions. Kinematic conditions were processed for a gear 1 (the center of rotation at point O_1) and the gear 2 (with the center of rotation at point O_2). The two gears are shown in a kinematic dependence one graph (on the horizontal axis of the wheel teeth first).

In Fig. 3 shows the radius of teeth on the gear points 1 and 2 for a pair of teeth, depending on the instantaneous position of teeth relative to the center of rotation. In the Fig. 4 is process continuously changing the gear ratio in the examined of gear.

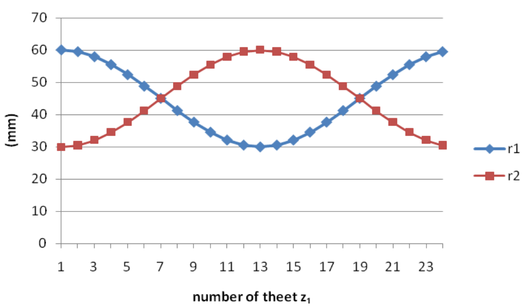


Fig. 3. Radius of mesh points of gear
Rys. 3. Średnice punktów zazębiających się kół

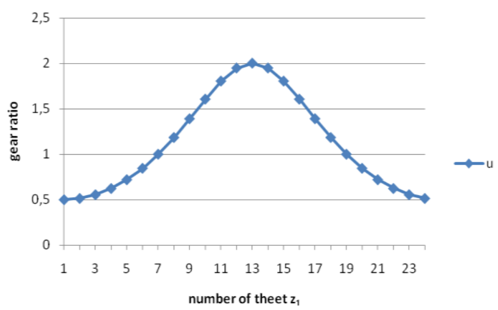


Fig. 4. Changing the gear ratio
Rys.4. Zmieniający się stosunek przełożenia

Real of load gear teeth with variable gear ratio is not constant. By way of illustration is given unit input torque (driven) spur gear $M_{k1} = 100Nm$. In Fig. 5 shows the course of torque M_{k1} on the input gear and torque M_{k2} on the output (driven) gear ($M_{k2i} = M_{k1}.u_i$). In Fig.6 are value of changing tangential tooth load the driver and driven gear $F_{01} = F_{02}$ $(F_{01}=M_{k1}/r_{1i})$, radial force $F_{r1} = F_{r2}$ $(Fr_1 = F_{01}.tg\alpha)$ and resultant force acting on the side of the tooth $F_{N1} = F_{N2}$ $(F_{N1} = F_{01} / \cos\alpha)$, where α is an angle of action to $20°$.

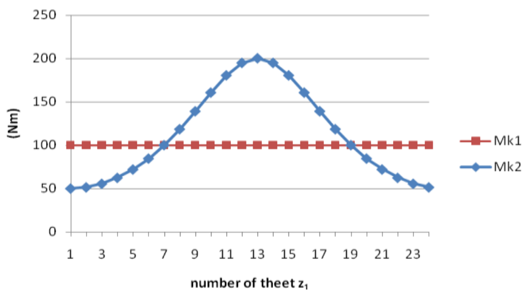


Fig. 5. The course of torque
Rys. 5. Przebieg momentów obrotowych

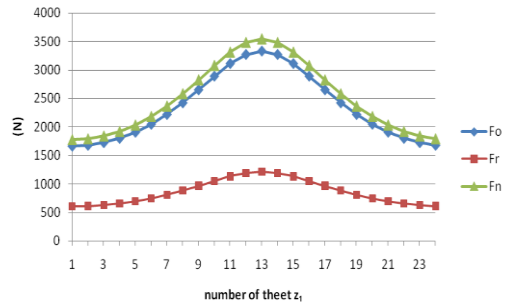


Fig. 6. The course of force in gearing
Rys. 6. Przebieg sił w badanym obiekcie

4. STRESS IN A THE GEARING SOLVED BY FEM

Create a geometric model of the gear is considered the first step to deal with tooth deformation FEM. Universal user to create geometry computer model does not exist an effective procedure is the transfer of geometry from any CAD system (such as. AutoCAD, Bentley, ProEngineer, I-DEAS, Solid Works, etc.). The first part for the contractor role was to develop a functional model gear designated for the production of gears gearing for NC machine to electrospark cutting. It is this suitably modified dxf format describing the shape of gears was used to create a geometric model. To determine the computer model for studying deformation of the teeth using FEM was necessary to determine the material constants, define the type of finite element, and selecting appropriate boundary conditions (geometry and power).

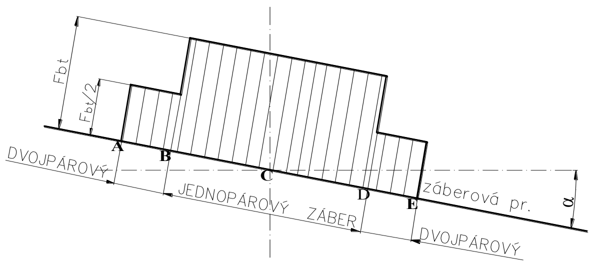


Fig. 7. Separation of loading on the line of contact
Rys. 7. Rozłożenie obciążenia wdłuż odcinka przyporu

To determine the stress at the foot of the tooth for loads according to Fig.1 it is necessary to know the distribution of load on individual pairs of teeth in the mesh. To start with let us consider the simplest, the ideal distribution of the load when the load-pair mesh are divided in half for each pair of frame (Fig. 7).

The problem is solved with the gear continuously variable transmission numbers. Stress in a dangerous section of the tooth is solved using the finite element method for driving gear, the gear teeth to reach the number 0.5, 1 and 2. Tooth load is shown in Fig.8, the resulting stress in dangerous cross-section is considered at points X and Y after the width of the teeth. Because it is expedient to solve problems around the gear, as shown in Fig.9 gear segments studied.

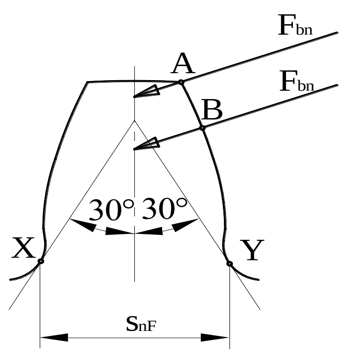


Fig. 8. The tooth load
Rys. 8. Obciążenie zęba

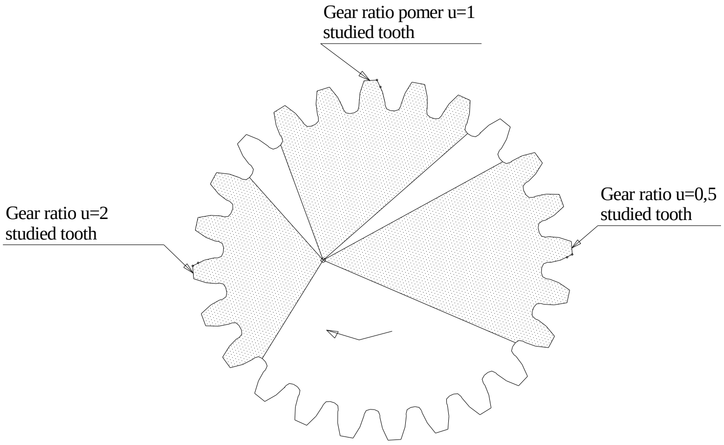


Fig. 9. Segments of studied gear
Rys. 9. Segmenty badanego koła zębatego

In Tab. 1 are results of stress in the dangerous section of tooth solution by FEM for segments (Fig. 8) driver elliptical gear set with continuously variable gear ratio. Width of teeth is *10 mm*, the driving torque is $M_{k1}=100$ *Nm.*

Table 1

Table name

Gear ratio	Mesh point	Force F_{bn} [N]	Medium Stress in point Y [MPa]	Max. Stress in point Y [MPa]	Min. Stress in point Y [MPa]	Medium Stress in point X [MPa]	Max. Stress in point X [MPa]	Min. Stress in point X [MPa]
0,5	A	886,81	80,81	113,98	41,04	90,77	126,23	49,20
	B	1773,63	126,87	168,02	69,02	146,58	188,86	85,44
1	A	1182,42	100,16	140,24	50,12	114,04	144,78	62,43
	B	2364,84	155,77	196,51	87,37	179,86	244,11	108,41
2	A	1773,63	161,64	190,60	128,73	182,89	213,65	151,27
	B	3547,26	243,03	307,28	141,07	281,52	384,70	159,99

In Fig.10 are results solutions to stress in gear by FEM for the gear segment with gear ratio $u = 1$ (Fig. 9), the distribution of load after Fig. 7 under load according to Fig. 8.

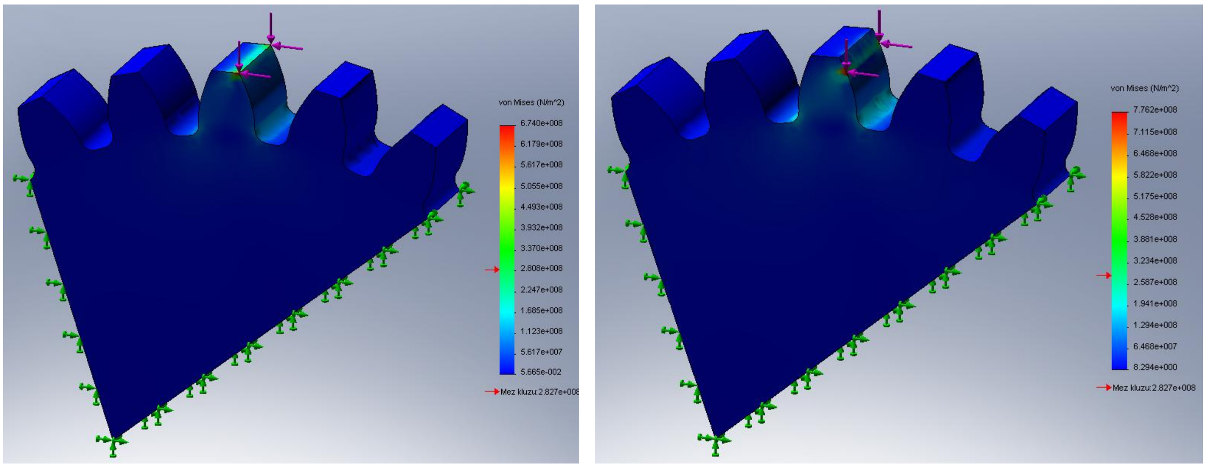


Fig. 10. Sample solutions to stress in gear by FEM
Rys. 10. Przykład rozwiązania naprężenia w zazębieniu za pomocą MES

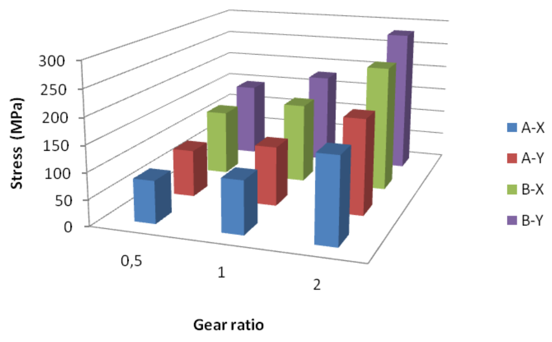


Fig. 11. Medium stress in the tooth foot
Rys. 11 Średnie naprężenie w podstawie zęba

Graphic representation of the medium stress in a dangerous section of tooth gear segments for the gear ratio u = 0.5, u = 1 and = 2 (Fig. 9) is shown in Figure 11. The stress at the load of the tooth point A on the side load force (Fig. 8) has be specification A-X. The results show that the stress in a dangerous section of teeth on the load side and on the opposite side (at point Y and point X Fig. 9) is different. The stress in the foot the tooth drive gear increases with gear ratio.

5. CONCLUSION

Calculation of stress in a dangerous section of tooth spur gear by STN 014686 is provided according to specific conditions. This calculation is not suitable for elliptical spur gear with variable gear ratio. The complex shape of the teeth is the theoretical determination of stress in the teeth difficult. One way to determine the stress in a dangerous section of the tooth is a solution to this problem using the finite element method.

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