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FORMAL MODELS OF GENERATING CHECKUP SETS FOR THE TECHNICAL CONDITION EVALUATION OF COMPOUND OBJECTS

MODELE FORMALNE GENEROWANIA ZBIORÓW SPRAWDZEŃ DLA OCENY STANU TECHNICZNEGO OBIEKTÓW ZŁOŻONYCH*

The paper refers to problems connected with building systems of computer-aided generation of evaluation sets of features necessary for the evaluation of compound objects' ability, and also the localization of imperfections of their component elements. In order to solve these problems, the usefulness of both the matrix method of determining sets of checkups and the cross-out method was analyzed. Binary and three-valued models of technical condition evaluation of the object's elements as well as the object's input and output features were formed. It allows then for the creation of the object matrix model which is used in both analyzed methods. For the matrix method, binary and three-valued evaluation models of distinguishing of the object technical condition were also defined. The binary models were used in a program which generates sets of features of the ability and localizing test. This program was written with the use of a Mathematica package. For a three-valued evaluation model for generating the feature checkup sets, the use of the cross-out method was proposed for both tests, with the technical state distinguishing conditions formed for this method. There was also presented an example of using this method to determine checkup sets of features which allow for the evaluation of the technical condition of part of the carriage brake pneumatic system.

Keywords: *compound objects, technical condition evaluation, formal models of objects, binary and multiple-valued evaluation, computer aiding.*

Praca dotyczy problemów związanych z budową systemów wspomaganego komputerowo generowania zbiorów sprawdzeń cech niezbędnych do oceny zdatności obiektów złożonych, a także lokalizacji niezdatności ich elementów składowych. Analizowano przydatność do tego celu macierzowej metody określania zbiorów sprawdzeń oraz metody skreśleń. Sformułowano binarne i trójwartościowe modele ocen stanu technicznego elementów obiektu oraz jego cech wejściowych i wyjściowych. Pozwala to wtedy na utworzenie macierzowego modelu obiektu, wykorzystywanego w obu analizowanych metodach. Dla metody macierzowej zdefiniowano także binarne i trójwartościowe modele oceny rozróżnialności stanów technicznych obiektu. Binarne modele wykorzystano w programie generującym zbiory cech testu zdatności i lokalizującego, napisanym przy użyciu pakietu Mathematica. Przy trójwartościowym modelu ocen do generowania zbiorów sprawdzeń cech dla obu testów zaproponowano użycie metody skreśleń i sformułowano dla niej warunki rozróżnialności stanów technicznych. Przedstawiono także przykład użycia tej metody do określenia zbiorów sprawdzeń cech pozwalających na ocenę stanu technicznego części układu pneumatycznego hamulca wagonu.

Słowa kluczowe: *obiekty złożone, ocena stanu technicznego, modele formalne obiektów, binarna i wielowartościowa ocena, wspomaganie komputerowe.*

1. Introduction

The use of advanced technologies in the construction of modern land vehicles does not give the vehicles' component elements protection against possibilities of wear occurrence and damage. The detection of these imperfections in numerous complex vehicle systems is more than once a difficult task to fulfill without using certain methods of technical condition identification. These methods described, for example, in [1, 3, 5, 6, 13, 15] enable us to choose sets of checkups necessary for the control of the object's proper functioning and for the localization of imperfections of the object's elements. The need for using such two stages of the technical condition examining is distinguished, among others, in [9, 15]. Some methods of the creation of checkup sets use the matrix model of an examined object. It is most frequently a model which is created on the basis of a binary evaluation of the object's features and on the technical condition of its component elements. The methods which are easy to use for a computer-aided generation of the checkup sets of the ability tests and the localization of damage are: a matrix method (Boolean matrices [3]) and a cross-out method (of a characteristic number [11]). The advantage of the latter method is the possibility of its effective application in the case of the multiple-valued evaluation of the object's features and the technical condition of its component elements. The issues connected with objects whose elements may take up a lot of technical conditions are discussed in a number of works [5, 7, 8, 10, 12, 16, 17], especially in reference to looking for optimal strategies of maintenance. For such objects, there is a need for applying a multiple-valued evaluation of the features which identify the objects' technical conditions [2].

This paper presents formal models of the matrix method and the cross-out method which may be used in the case of the binary or three-valued evaluations of input and output features and the technical condition of the object's compound elements. The possibilities of using both methods for a computer-aided generation of check-up sets of the ability test and the localizing test are also compared in the paper.

2. Binary models of evaluation

A technical object may be shown with the use of a functional model [3], which is built up from a certain number of distinguished boxes (rectangles) e_k representing certain sets, subsets, or elements of this

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** object. Specified external input features (w_i) or output features from other object's elements (y_{ej}) can act on a single functional box. Specified output features (y_j) are obtained on the output of the box. Every functional box can be shown as this one presented in Figure 1 [13].

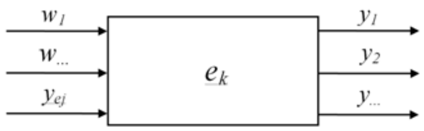

Fig. 1. A functional box as part of a functional model: e_k– element, w_1, $w_.$ – external input features, y_1, y_2, y – output features, y_{ej} – the output feature of the element e_j [13]

Both the element technical condition and the input and output features undergo evaluations in the process of examining of a given technical object. The acceptance of a given model of these evaluations also allows for going from an object's functional model to a matrix model.

For a binary evaluation of the technical condition of each object element, a two-valued characteristic function may be used. This function is used, for example, in [3] and [7] and it can be written in the following form:

$$Q_k = \varphi_2(e_k) = \begin{cases} 1, \ when\ e_k \Leftrightarrow S_0 \\ 0, \ when\ e_k \Leftrightarrow S_1 \end{cases} \tag{1}$$

where: Q_k – the evaluation variable of the technical condition of the element e_k,

$\varphi_2(...)$ – the binary characteristic function,

S_0, S_1 – technical conditions of the element e_k, respectively: ability and imperfection.

A two-valued function (1) assigns the logical value "1" to each variable Q_k which refers to the element e_k if this element is in the ability condition S_0, and "0" in the opposite case.

The basis for a binary evaluation of input and output features is checking whether an examined feature is within a determined range of values. Formally it is formulated by characteristic functions which, in order to unify the notation, can be shown as follows:

$$v_i = \varphi_2(w_i) = \begin{cases} 1, \ when \ (w_i)_{\min} \le w_i \le (w_i)_{\max} \\ 0, \ when \ w_i < (w_i)_{\min} \lor w_i > (w_i)_{\max} \end{cases} \tag{2}$$

$$z_j = \varphi_2(y_j) = \begin{cases} 1, \ when \ (y_j)_{\min} \le y_j \le (y_j)_{\max} \\ 0, \ when \ y_j < (y_j)_{\min} \lor y_j > (y_j)_{\max} \end{cases} \tag{3}$$

where: w_i – the external input feature for the element e_k,

y_j – the evaluated output feature,

v_i – the binary logical value of evaluation of the external input feature,

z_j – the binary logical value of evaluation of the output feature,

$(w_i)_{\min}$, $(w_i)_{\max}$ – boundary values of the input feature,

$(y_j)_{\min}$, $(y_j)_{\max}$ – boundary values of the output feature.

These functions assign the logical value „1" in reference to the situations in which input and output features are within the predicted ranges determined by the values $(w_i)_{\min}$, $(w_i)_{\max}$ and $(y_j)_{\min}$ and $(y_j)_{\max}$, and "0" in the opposite case.

If it is assumed that the values of each external input feature are within the standard range then, with the use of functions (1), (2) and (3), the matrix model of the object can be created [3]. The model shows the relations between the output features of the individual elements and the technical conditions of this object, i.e. the condition of ability and the conditions of imperfections induced by the appearance of imperfect elements in this object. In the case of a binary evaluation, the matrix of such a model is frequently referred to as the truth table.

The truth table is the basis for determining the sets of features which allow for the verification of the object's ability and the localization of the imperfections of its elements. It requires a twofold transformation of the truth table into the matrix of the ability test and the matrix of the localization test in the way described in a written form in [3]. In a formal perspective, these transformations require the use of two characteristic functions: one function evaluates the distinguishing of the ability condition of the examined object from any optional condition of imperfection; the other function allows for distinguishing the individual states of imperfection among themselves.

For the evaluation of the technical condition distinguishing in the first case, a two-valued function in the following form may be used:

$$z_j^{0,i} = \varphi_2(S_0, S_i) = \forall(i, j \in [1,k]) : \begin{cases} 1, \ when \ z_j(S_0) = 1 \land z_j(S_i) = 0 \\ 0, \ when\, z_j(S_0) = z_j(S_i) \end{cases} \tag{4}$$

where: $z_j^{0,i}$ – the logical variable of the evaluation of the ability and imperfection conditions' distinguishing,

$z_j(S_0)$ – the logical variable of evaluation of the feature y_j in the ability condition S_0,

$z_j(S_i)$ – the logical variable of evaluation of the feature y_j in the imperfection condition S_i.

A two-valued function of the evaluation of imperfection conditions' distinguishing is as follows:

$$z_j^{i,l} = \varphi_2(S_i, S_l) = \forall(i, l, k \in [1,k] \land (i \ne l)) : \begin{cases} 1, \ when \ z_j(S_i) \ne z_j(S_l) \\ 0, \ when \ z_j(S_i) = z_j(S_l) \end{cases} \tag{5}$$

where: $z_j^{i,l}$ – the logical variable of the evaluation of distinguishing imperfection conditions S_i and S_l on the basis of the evaluation of the feature y_j,

$z_j(S_i)$ – the logical variable of the evaluation of the feature y_j in the imperfection condition S_i,

$z_j(S_l)$ – the logical value of the evaluation of the feature y_j in the imperfection condition S_l.

Functions (4) i (5) allow for the transformation of the truth table successively into the matrix of the ability test and the matrix of localization test. Instead of these functions, an operation of adding modulo 2 may be used. This operation has to be applied for appropriate values from a two-valued truth table, i.e. the following formulae:

$$z_j^{0,i} = z_j(S_0) \oplus z_j(S_i) \tag{6}$$

$$z_j^{i,l} = z_j(S_i) \oplus z_j(S_l) \tag{7}$$

where: $\oplus$ – the modulo 2 sum.

The obtained matrices are the basis for determining respectively the sets of features for the object's ability test and for the localization test. Part 4 of this paper contains a description of the mode of generating such sets with the use of the matrix method.

3. Three-valued evaluation models

A binary evaluation model is insufficient in the case of features which have two or more ranges of boundary values, as well as for such object's elements which can take up various forms of imperfection [11, 13]. In the latter case, it refers to some elements of electric, hydraulic or pneumatic systems which constitute a given technical object. For such elements, even with one range of permissible values of the given input or output feature, it is important to have information which boundary value was exceeded: a maximum or minimum one.

In electric systems, for example, the appearance of such basic imperfections like a break in the circuit or the insulation punch-through can significantly influence the values of the object's features, i.e. generate distinct technical conditions [11]. In this case, for each *i* element of the object a set of technical conditions SU_i can be determined. This set contains, for example:

- the ability condition – S_i^0,
- the imperfection condition caused by the insulation punch-through – S_i^n,
- the imperfection condition of the element which results from a break in the circuit – S_i^d,

which is:

$$SU_i = \left\{S_i^0, S_i^n, S_i^d\right\} \tag{8}$$

Figure 2 shows a graph of walks between technical conditions of one such element of the set [11]. In operation it is possible to go from the ability condition to one of the two imperfection conditions and a return to the ability condition takes place after completing appropriate operational activities.

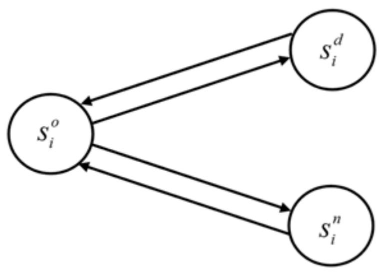

Fig. 2. Graph of walks for the set of the technical conditions of the exemplary element of the system that powers the rail-vehicle brake [11]

After the denotations of the individual technical conditions of a single element have been generalized, the following characteristic function may be proposed for this element's three-valued evaluation:

$$Q3_k = \varphi_3(e_k) = \begin{cases} 2, \ when \ e_k \Leftrightarrow S_2 \\ 1, \ when \ e_k \Leftrightarrow S_0 \\ 0, \ when \ e_k \Leftrightarrow S_1 \end{cases} \tag{9}$$

where: $Q3_k$ – a three-valued variable of the technical condition evaluation of the element e_k,

S_0, S_1, S_2 – technical conditions of the element: the ability, the 1st and 2nd form of the imperfection,

$\varphi_3(\ldots)$ – a three-valued characteristic function.

If the individual elements of the object can take up more than one imperfection form and when there is a possibility of the occurrence of an optional combination of imperfect elements then the size of the technical condition full set of such an object can be determined with the formula [13]:

$$l_m = (m+1)^k \tag{10}$$

where: l_m – the size of the set of the object's technical conditions,

m – the number of imperfection forms of each component element of the object,

k – the number of elements of the object.

For a three-valued evaluation of the external input features and the output features, the following characteristic functions can be applied [13]:

$$v3_i = \varphi_3(w_i) = \begin{cases} 2, \ when \ w_i < (w_i)_{\min} \\ 1, \ when \ (w_i)_{\min} \le w_i \le (w_i)_{\max} \\ 0, \ when \ w_i > (w_i)_{\max} \end{cases} \tag{11}$$

$$z3_j = \varphi_3(y_j) = \begin{cases} 2, \ when \ y_j < (y_j)_{\min} \\ 1, \ when \ (y_j)_{\min} \le y_j \le (y_j)_{\max} \\ 0, \ when \ y_j > (y_j)_{\max} \end{cases} \tag{12}$$

where: $v3_i$ – a three-valued variable of the evaluation of the measured external value of the input feature w_i,

$z3_j$ – a three-valued variable of the evaluation of the measured value of the output feature y_j.

The essence of the three-valued evaluation with the use of functions (11) and (12) is the assigning of various logical values when the upper or lower boundary values of the examined feature are exceeded, which is basically different from the proposition included in [2] in which the evaluation values were assigned in the following way: "2" – for the insignificant changes of the feature's value, "1" – for the significant changes of the feature's value, and "0" – for the unacceptable changes of the feature's value.

Similarly to a binary evaluation, functions (9), (11) i (12) allow for the creation of a three-valued matrix that constitutes the model of an object and which subsequently can be transformed into the matrix of the ability test and the matrix of the localization test.

An essential form of the distinguishing evaluation function of the pairs of the ability and imperfection conditions, i.e. pairs of the type $\langle S_0, S_i \rangle$ can be obtained by the modification of function (4):

$$z3_j^{0,i} = \varphi_2(S_0, S_i) = \forall (i, j \in [1,k]) : \begin{cases} 1, \ when \ z3_j(S_0) = 1 \wedge z3_j(S_i) \ne 1 \\ 0, \ when \ z3_j(S_0) = z3_j(S_i) \end{cases} \tag{13}$$

where: $z3_j^{0,i}$ – a three-valued logical variable of the distinguishing evaluation of the ability and imperfection conditions,

$z3_j(S_0)$ – a three-valued logical variable of the evaluation of the feature y_j in the ability condition S_0,

$z3_j(S_i)$ – a three-valued logical variable of the evaluation of the feature y_j in the imperfection condition S_i.

This function assigns the logical value "0" in such a situation when it is impossible to distinguish the ability and imperfection conditions on the basis of the value evaluation of the feature y_j, and "1" in the opposite situation. Since the variables $z3_j(S_0)$ and $z3_j(S_i)$ can take up one of the three logical values from the set $\{0,1,2\}$, the results obtained by using function (13) are impossible to be reached with the use of a two-argument modulo 2 sum operation working on the values of these variables. Moreover, with a three-valued model of the object's feature evaluation, the negation of the values of the variables $z3_j$, i.e. the one-argument operation $\overline{\overline{z3_j}}$ should be defined in the following way:

$$\overline{\overline{z3_j}} = \begin{cases} 0 \vee 2, when\, z3_j = 1 \\ 1, when\, z3_j = (0 \vee 2) \end{cases} \tag{14}$$

If, for the simplification of the notation, it is assumed that:

$$\begin{aligned} z3_j(S_0) &= a \\ z3_j(S_i) &= b \end{aligned} \tag{15}$$

then the form of the two-argument function $\Omega(a,b)$, which allows for the obtaining of the values consistent with those given by the function (13), is as follows:

$$z3_j^{0,i} = \Omega(a,b) = \left[\min(a,b) + a + b\right] \bmod 3 \tag{16}$$

Function (16) may be also used for the creation of the three-valued matrix of the localization test, but then its two arguments refer to the imperfection conditions, i.e. assuming that:

$$z3_j(S_l) = d \tag{17}$$

then $$z3_j^{i,l} = \Omega(b,d) = \left[\min(b,d) + b + d\right] \bmod 3 \tag{18}$$

where: $z3_j^{i,l}(S_l)$ – a three-valued logical variable of the evaluation of the feature y_j in the imperfection condition S_l,

$z3_j^{i,l}$ – a logical variable of the distinguishing evaluation of the pair of the imperfection conditions $\langle S_i, S_l \rangle$.

The values obtained with the use of function (18) correspond with the values of the distinguishing evaluation of the pairs of the imperfection conditions $\langle S_i, S_l \rangle$ obtained with the use of a three-valued characteristic function, whose form may be defined by using the simplifications of notation from the formulae (15) and (17) in the following way:

$$z3_j^{i,l} = \varphi_3(S_i, S_l) = \forall(i,l,k \in [1,k] \wedge (i \neq l)): \begin{cases} 0, when\, b = d \\ 1, when [b = 1 \wedge (d = (0 \vee 2))] \vee \\ \quad [d = 1 \wedge (b = (0 \vee 2))] \\ 2, when (b = 0 \wedge d = 2) \vee \\ \quad (d = 0 \wedge b = 2) \end{cases} \tag{19}$$

The matrices obtained with the use of functions: (13) or (16) and: (18) or (19) constitute the basis for further operations in order to distinguish the checkup sets of the ability test and the localization test.

4. An example of using a binary evaluation model for the generation of the checkup sets with the use of a matrix method

The process of selecting features for the checkup set of the object's ability with the use of a matrix method can be performed through indicating the determined columns of features on the basis of a certain criterion. It leads to the minimization of the matrix of the ability test. This criterion is, first of all, the uniqueness of the object's features in the range of the distinguishing of the technical condition pairs. It is expressed through the occurrence of one "1" in the row of this matrix. The column in which this value occurs indicates the feature belonging to the checkup set of the ability test because only this column feature allows for the distinguishing of the pairs of the ability and imperfection conditions assigned to this row. The use of this criterion leads to the choice of the features which are in the columns of the features representing the external outputs from a given object. In the next order, such factors as availability or measurement costs can act as the criteria for choosing the features for the ability test. After marking out the features which create the checkup set of the ability test it has to be controlled whether this set is sufficient to distinguish all pairs of the technical conditions. It is performed through crossing out the column of the selected features and all distinguishable rows from the matrix of the ability test. The crossing out of all the columns and rows allows for the closure of the checkup set of this test.

The method of looking for a set of the object's features necessary for the localization test is similar to that one employed in the ability test. The method is based on a subsequent choosing (for this set), first of all these object's features which, as the only ones, provide the distinguishing of certain pairs of the technical conditions $\langle S_i, S_l \rangle$. Then the ability of the distinguishing of all the remaining pairs of the imperfection conditions is checked. If the result of such a checkup is positive then the set of features can be closed. In the opposite case, one should continue the selection of features in a minimized matrix of the localization test; this matrix is formed after crossing out the columns of the selected features and the rows with the pairs of the technical conditions which can be distinguished on the basis of these features. The selection process is finished by a positive result of checking of the distinguishing abilities of all imperfection conditions' pairs. If, despite using the features which belong to the checkup set of the localization test, there are still undistinguishable pairs of the imperfection conditions then it may mean errors in the constructed model of the object or the need for using a multiple-valued evaluation of the input and output features.

An exemplary form of the truth table created for the railway carriage brake gear is shown in Table 1 [14]. Table 1 was created for a case in which the possibility of occurring at most one imperfect element of the brake gear was assumed. The table contains the evaluation values of the features $y_1 \div y_{21}$, i.e. the forces occurring in the brake gear

Table 1. The truth table for a railway carriage brake gear [14]

SU_k	z_j																				
	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	y_{18}	y_{19}	y_{20}	y_{21}
S_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S_2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S_3	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
S_4	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_5	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_6	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_7	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_8	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1
S_9	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
S_{10}	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1
S_{11}	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
S_{12}	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
S_{13}	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
S_{14}	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1
S_{15}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
S_{16}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
S_{17}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
S_{18}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
S_{19}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
S_{20}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
S_{21}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

in the particular feasible technical conditions of this system. The set of these technical conditions includes the ability condition of the gear S_0 and the imperfection conditions $S_1 \div S_{21}$ resulting from the imperfection of – respectively – the elements $e_1 \div e_{21}$.

The presented models of binary evaluations and the specified methodology of choosing the evaluation sets of the ability test and the localization test allows for their programmable creation. This type of a program which generates the checkup sets for the evaluation of the object's technical condition and which is written with the use of a Mathematica package is included in [14]. With the use of this program for a matrix model as in Table 1, the following input command should be employed:

```
Tp={{1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1},
    {0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},
    {1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},
    {1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},
    {1,1,1,0,0,0,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1},
    {1,1,1,1,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1},
    {1,1,1,1,1,0,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1},
    {1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1},
    {1,1,1,1,1,1,1,0,0,0,0,1,1,1,1,1,1,1,1,1,1},
    {1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,1,1,1},
    {1,1,1,1,1,1,1,1,1,0,0,1,1,1,1,1,1,1,1,1,1},
    {1,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,1},
    {1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0},
    {1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0},
    {1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,1,1,1,1},
    {1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1},
    {1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,1,1,1,1},
    {1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,1,1,1,1},
    {1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0},
    {1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,1,1},
    {1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0},
    {1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0}}
```

The checkup set of the ability test obtained due to the operation of the program is as follows:

```
Tds // question about the checkup set of the ability test

{y5,y7,y9,y11,y15,y17,y19,y21}  // response
```

and the checkup set of the localization test has the form:

```
Tdl // question about the checkup set of the localization test

{y1,y2,y6,y10,y12,y16,y20,y5,y7,y9,y11,y15,
y17,y19} // response
```

A such obtained solution allows for the localization of any optional imperfect element of the examined carriage brake gear.

5. The application of the cross-out method for generating sets of features for the evaluation of the technical condition of the compound object

A three-valued model of evaluation, described in Entry 3 of this paper, can be used, for example, for distinguishing a break in the electromagnetic circuit of the rail brake from the punch-through of the insulation of the brake's winding [11], i.e. the distinguishing of three classes of the technical condition specified by formula (8). The other example of the three-valued evaluation model application with the use of functions (9), (11) and (12) is a matrix model (Table 2) which can be created for part of the pneumatic system of the carriage brake on the basis of the computing model included in [4] and referring to a phase of filling up the system.

The matrix presented in Table 2 contains the values of evaluations of such features as: $y_0 \div y_4$ – i.e. the pressures in the system, and $y'_0 \div y'_4$ – massive intensity of the air flow in part of the carriage pneumatic system, in the technical conditions that belong to a set:

$$SU_r = \left\{ S_o, S_i^n, S_1^d, S_2^n, S_2^d, S_3^n, S_3^d, S_4^n, S_4^d, S_5^n, S_5^d \right\} \tag{20}$$

where: S_0 – the ability condition of the object,

$S_1^n \div S_5^n$ – technical conditions characteristic of leaks of the object's elements,

$S_1^n \div S_5^n$ – technical conditions characteristic of choking of air flow in the object's elements.

The transformation of a matrix which constitutes a three-valued model of an exemplary object (Table 2) into a matrix of the ability test with the use of functions (13) or (16) allows for the determination of the checkup set of this test in the same way as with a binary evaluation model. Thus the algorithmical generation of the set is possible. In the case of a matrix of the localization test obtained with the use of functions (18) or (19), the previously used methodology of the minimization of the checkup set is ineffective. It is easier to choose a minimal checkup set of the localization test with the use of a cross-out method.

Table 2. The matrix model of part of the pneumatic system of the carriage brake

Item number	Technical condition	Evaluations of the object's features - $z3_j$											
		y_0	y_0'	y_1	y_1'	y_2	y_2'	y_3	y_3'	y_4	y_4'	y_5	y_5'
1.	S_0	1	1	1	1	1	1	1	1	1	1	1	1
2.	S_1^n	1	0	1	1	1	1	1	1	1	1	1	1
3.	S_1^d	1	2	0	2	0	2	0	2	0	2	0	2
4.	S_2^n	1	0	1	1	2	2	1	1	1	1	1	1
5.	S_2^d	1	2	1	1	1	2	1	1	1	1	1	1
6.	S_3^n	1	0	1	1	1	1	2	0	2	0	2	0
7.	S_3^d	1	2	1	1	1	1	2	2	2	2	2	2
8.	S_4^n	1	0	1	0	1	1	1	0	2	0	1	1
9.	S_4^d	1	2	1	2	1	1	1	2	2	2	1	1
10.	S_5^n	1	0	1	0	1	1	1	0	1	1	2	0
11.	S_5^d	1	2	1	2	1	1	1	2	1	1	2	2

While analyzing the form of the object's matrix model it can be noticed that each row of this matrix constitutes a sequence of the evaluation values. This sequence can be interpreted as a characteristic number written in a binary, ternary or quaternary codes - it depends on the number of values used for the evaluation of the features of this object. If a conversion of this number into a decimal number is performed then it can be used for the evaluation of the uniqueness of the individual rows of this matrix. Such a decimal number d_r can be derived from the formula [13]:

$$d_r = \sum_{j=1}^{j_{\max}} z_j^r \cdot p^{j-1} \tag{21}$$

where: z_j^r – the evaluation value of the feature y_j in the row r of the object's matrix model,

j – the number of the column of the object's feature counting from the right side of this matrix,

p – the number of evaluation values which can be assigned to the individual features.

The condition for the distinguishing of each pair of the technical conditions $\langle S_r, S_s \rangle$ on the basis of the evaluation of all the features of the object (or of only a certain subset of the features) is the requirement that all characteristic numbers in the individual rows of the matrix (which constitutes the object's model) corresponding with the features are different, i.e. [13]:

$$\bigwedge_{r \neq s} S_r \neq S_s \Leftrightarrow d_r \neq d_s \tag{22}$$

where: s – the index of the matrix row different from r.

With a great number of the object's elements and the evaluated values, the conversion of the row of the matrix model is difficult without computer-aided calculations. Besides, the numbers obtained from formula (21) are so big that they only slightly facilitate the evaluation of their uniqueness. As a consequence, the evaluation of the distinguishing of the individual conditions of the analyzed object is also only slightly facilitated. In such a situation, when one decides to use a specialist computer program it is convenient to treat each row of this matrix as a conventional characteristic number represented by a sequence of the symbols c_r obtained in the following way [13]:

$$c_r = \sum_{j=j_{\max}}^{1} Str\left(z_j^r\right) \tag{23}$$

where: $\sum$ – the operator of the connection of symbols,

$Str()$ – the function converting the numerical value of the feature evaluation into a symbol.

By using such sequences, the distinguishing condition of the rows of this matrix can be formulated analogically to formula (22), i.e. [13]:

$$\bigwedge_{r \neq s} S_r \neq S_s \Leftrightarrow c_r \neq c_s \tag{24}$$

The substantial benefit of using symbol sequences is the ability of deploying a great number of the features of an object. It results from the acceptable lengths of the variables of the symbol type. The way of seeking the resulting checkup set of the localization test relies on the removing of the individual columns of the matrix of a given object, beginning from the left or the right side, and checking the distinguishing condition of the technical conditions which is included in formulae (22) or (24). In the case when this condition is fulfilled, one has to go to the next column and remove it. However, if the checkup is not successful, the previously removed column has to be restored and the feature from this restored column should be introduced into the set of the features of the localization test. Such an operation has to be continued until the list of features from the columns of this matrix is emptied. The values from the remaining columns form a minimal signature of each technical condition which is assigned to each row of the matrix.

In order to check the possibility of generation (with the use of the cross-out method) of the sets of features for the evaluation of the object's ability and for the localization of the imperfections of its elements, a computer program which performs the tasks of creating such sets was worked out. This computer application has a form with a window which is designed for the edition of the binary or multiple-valued matrix of a given object, and the buttons which start the generation of the checkup sets of the ability test and the localization test. Figure 3 contains a view of this form for the object whose three-valued matrix is shown in Table 2.

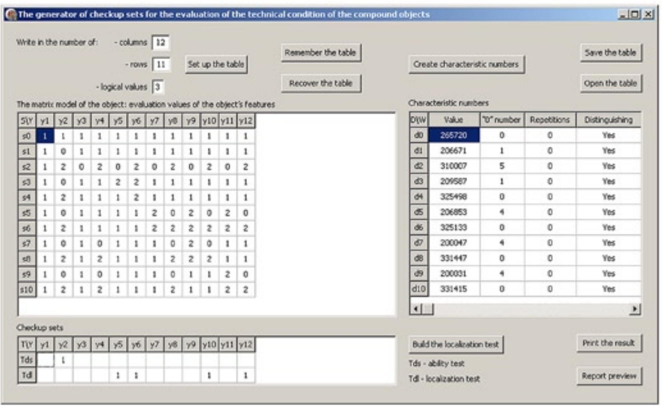

Fig. 3. A view of the form of the ability and localization tests' generator with the data as in Table 2

Before the input data is introduced, the size of a matrix has to be arranged and the number of logical values used for the evaluation of the object's features has to be defined. In evaluations like in Table 2, the application enables the checkup sets of the ability and localization tests to be generated. Thus the following report can be obtained:

A set of checkups of the ability test
Tds = { y2 }

A set of checkups of the localization test
Tdl = { y5,y6,y10,y12 }

The sum of checkup sets
Tdc = { y2,y5,y6,y10,y12 }

This report is a confirmation of the effectiveness of this method for creating feature sets which allow for identification of the technical condition of the compound object.

6. Summary

Binary and three-valued models of evaluations which are shown in this paper can be used for building systems of the computer-aided generation of feature sets whose examining allows for the checkup of the ability of a compound object and the localization of imperfections of its constituent elements. Such systems should choose only the necessary features out of the whole amount of the object's features. It can be done with the use of the matrix method or the cross-out method.

As it is shown in this paper, with the binary evaluation the task of choosing the features can be relatively easily accomplished with the use of a matrix method of creation of the checkup sets of the object's features. In order to achieve this, the functions of distinguishing evaluation of both the ability and imperfection conditions were formulated and an exemplary result of the functions' application in a program written with the use of a Matematica package was given. The matrix method may also be used for determining appropriate checkup sets for a case in which the object's features undergo a three-valued evaluation. Then, it is however necessary to admit other three-valued functions of the technical condition distinguishing evaluation of a compound object. It is also shown that in this case the criteria for the computer-aided selection of features into the checkup sets can't be uniquely distinguished, as it is in the binary evaluation of the object's features.

This fact was an impulse for creating another method of distinguishing checkup sets, i.e. the cross-out method. This method may be applied in both a binary and a multi-valued evaluation of the examined features. The method relies on subsequent attempts of eliminating of individual features and checking the truthfulness of the distinguishing condition of technical conditions of the analyzed object. The efficiency of this method was verified on the example which referred to part of the carriage brake pneumatic system.

The cross-out method can be applied with both the full and limited measurement availability of the features of a given object. By accepting the inability of checking of some features one can, however, obtain information about a possible undistinguishing of the technical conditions of a given object caused by the occurrence of certain imperfections of its constituent elements.

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