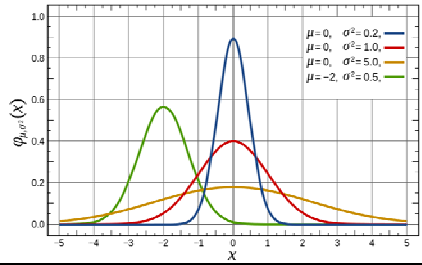

GAUSSIAN LAW AND ITS APPLICATION IN RESOLVING URBAN PLANNING PROBLEMS

Anna Hoblyk
Ph.D., Associate Professor

Kyiv National University of Construction and Architecture
City Development Department

ABSTRACT

The article is dedicated to the problem of computer modeling of distribution of the field of intensity of gravitational pull on population to the retail outlets using results of experimental studies. The main result of the work is the established correspondence of graphic regularities obtained experimentally, compliance with the Gaussian law or normal distribution. The practical value of this work lies in the possibility of substantiation of the required capacity of the retail outlets taking into consideration quantitative regularities of formation of service areas for these objects, as well as the conditions of placement of the projected retail outlets.

Key words: Gaussian law, computer model, intensity of gravitational pull on population, retail outlet, city.

INTRODUCTION

One of the greatest mathematicians of all times Johann Carl Friedrich Gauss, while still being a college student in Braunschweig (Germany), began his research in the field of the so-called "normal distribution" that was later called after his name. In the course of his work he derived the Gaussian curve or simply Gaussian – a symmetric parabolic curve, whose shape resembles a bell, that in most cases appears when showing a series of results on the frequency graph (Normal distribution; Saul Stahl, 2006). According to the conclusions of the great mathematician, any quantity X which is subject to numerous random multidirectional perturbations must comply with the Gaussian law. That is, distribution of values x_i of this quantity must have the expression $\exp\left[-(x-\mu)^2/\sigma^2\right]$, while "$\mu$" quantity determines position of the top of the conventional "bell", and quantity "σ" determines its width. "μ" and "σ" themselves depend on the concrete measured quantity, but the functional expression of the law is unchanged (Fig. 1).

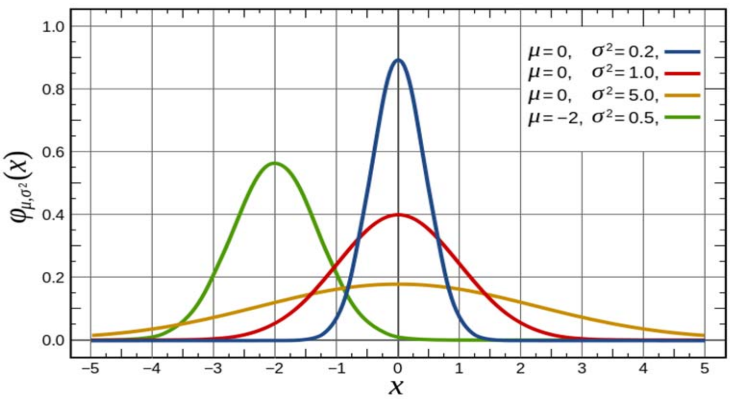

Fig. 1. Gaussian law. Source: http://en.wikipedia.org/wiki/normal_distribution

Gaussian law, known also as the normal law of distribution, appears to be valid for many phenomena in engineering, biology, sociology, psychology and other fields of human knowledge. For example, projectiles fall near the aiming point by Gaussian law, probability of detecting a particle in any point of space is distributed by Gaussian law, arterial blood pressure in a population is distributed by Gaussian law, the size of ball-bearing balls produced at a ball-bearing plant is distributed according to Gaussian law, etc (Nikonov, 2005).

It should be noted that some social-spatial phenomena characteristic of urbanized territories, like distribution of intensity of gravitational pull on population to a shopping center depending on the distance obey the Gaussian law. Identification of this quantitative regularity became possible owing to the colossal by their scale experimental studies in the 90's of the twentieth century conducted under supervision of Professor N. Dyomin (Dyomin, 1991). Further studies of the nature of graphs of the obtained dependencies constructed on basis of statistical data, led to the conclusion that the functional pattern of the obtained curves corresponds to the Gaussian function graph (Dyomin & Hoblyk, 2013).

It should be noted that the studied regularities are not only of a great scientific-application importance, but represent also a pressing issue for solving the problems connected with spatial organization of the domain of public services. However, extensive application of the obtained results in urban planning is possible only with the use of computer modeling methods, for these are the methods that permit to "project" regularities of the formation of zones with an increased activity of population (or zones for servicing sales outlets) onto the practical plane when elaborating projects of territorial-planning organization of public services.

However, the use of computer modeling methods during the above said studies (Dyomin, 1991) was limited because of the rather insufficient level of development of computer hard- and software. Only now, owing to the rapid development of GIS-technologies and computer mathematics packages, new possibilities have been opened up to implement in full the idea of such studies aiming at development of the tools for modeling and analysis of social-spatial phenomena, whose application in urban planning will raise the level of feasibility of the design solutions.

The aim of this work is to develop a computer model of distribution of the field of intensity of gravitational pull ($\vec{I}_m$) on population to a shopping center using results of the experimental studies, obtained in the work (Dyomin, 1991), calibration of this model, substantiation of numerical values of the Gaussian curve parameters.

EXPERIMENTAL STUDY: QUANTITATIVE REGULARITIES OF THE PROCESSES OF FORMATION OF SALES OUTLETS SERVICE ZONES

One of the most important problems of the theory and practice of urban planning is the problem of rational territorial-planning organization of the sphere of public services. A great number of scientific works (Merlin, 1973; Zablotskiy, 1977; Gutnov, 1984; Radomskiy, 1984; Dyomin, 1991; Sosnovskiy, 2006) deal with the problem of developing methodological principles of spatial organization of the sphere of public services. Of paramount importance in solving the problem of rational placement of public service facilities is information on the processes and regularities of the formation of such facilities' service zones. In the result of analysis of data on the intensity of cultural-community links correlated with a certain service zone it is possible to define a territory characterized by a higher level of activity. Selection of the zone with a higher level of activity of cultural-community links is of not only scientific, but also of considerable practical importance, since it is here that the contingent of population is formed that produces essential influence on the designed capacity of the service establishments and on the conditions of their placement in the planning structure of populated areas (Dyomin, 1991). The study of such social-spatial phenomena is connected with laborious gathering and processing of vast statistical material, which in its turn presents a separate task, difficult of accomplishment.

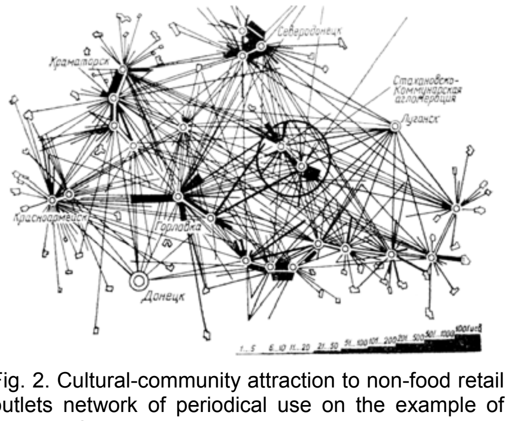

Fig. 2. Cultural-community attraction to non-food retail outlets network of periodical use on the example of Donbas. Source: Dyomin, 1991.

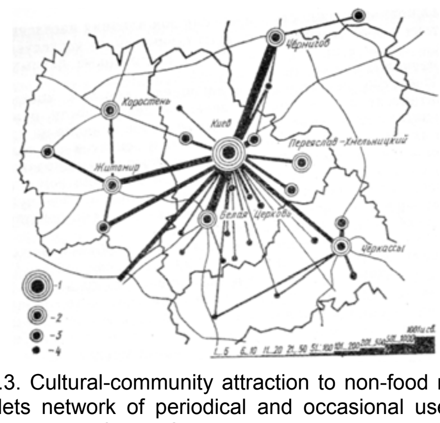

Fig.3. Cultural-community attraction to non-food retail outlets network of periodical and occasional use on the example of Kiev. Source: Dyomin, 1991.

A group of experts headed by Professor Dyomin N. carried out a unique by its magnitude and organization study in the 1990's: in a number of large and larger cities of Ukraine they studied regularities of the formation of the service zones of shopping centers. To study the service zones of shopping centers they used materials of survey by questionnaire of customers in the central city department stores that are the centers of mass attraction of customers from other cities (Dyomin, 1991). Results of analysis of the materials of this survey of retailers are shown in Fig. 2-3.

Proceeding from the experimental data there have been established functional dependencies linking, on the one hand, intensity of the flow of customers to the shopping centers of various levels and, on the other hand, the distance of the ride from the place of permanent residence (Fig. 4).

DESCRIPTION OF INTENSITY OF GRAVITATIONAL PULL ON POPULATION TO THE SALES OUTLETS BY THE GAUSSIAN FUNCTION

The preliminary stage of the development of a computer model of distribution of the field of intensity of gravitational pull on population to the retail outlets in MATLAB medium was identification of the function in analytical form, whose graph (Fig. 5) with a high degree of accuracy coincides with the experimental graphs of the work (Dyomin, 1991) (see Fig. 4).

We shall specify, that $\vec{I}_m$ is a vector quantity whose modulus characterizes the amount of gravitational pull on population at a particular point of urban space to a services center (a public services object).

Vector $\vec{I}_m$ modulus is measured by the ratio of the number of rides N per 1000 residents of a certain locality (Dyomin, 1991).

Comparing further at any point M (x, y, z) in the system of spatial coordinates (x, y, z) vector quantity $\vec{I}_m$, we shall obtain the vector field by definition (Kalnitskiy, 1976).

For modeling such a field in the MATLAB system there was found the function in analytical form whose graph (Fig. 5) coincides with a high degree of accuracy with the experimental graphs of the work (Dyomin, 1991) (Fig. 4), describing dependence of vector $\vec{I}_m$ modulus quantity on the distance to the center of services. Such function appeared to be the Gaussian function described by expression (1):

$$\varphi(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} + \delta, \qquad (1)$$

where parameters μ and σ are real numbers and δ – calibration constant.

The high degree of approximation of the experimental data on intensity of gravitational pull by the Gaussian curves (Fig. 4-5) is ensured by the appropriate choice of μ and σ parameters of the formula (1). It turned out, that calibration quantities of these parameters must have the following values: μ = 0, σ = 2.8. In such case, the root-mean-square deviation of the Gaussian curve from the experimental curves does not exceed 0.001%.

COMPUTER MODEL OF DISTRIBUTION OF THE FIELD OF INTENSITY OF THE FIELD OF GRAVITATIONAL PULL ON POPULATION TO THE RETAIL OUTLETS

A part of the program, namely the script file, developed on the basis of function (1) found in analytical form is presented below (Fig. 6). Next is a graph constructed in Matlab environment and describing distribution of the field of intensity of gravitational pull on the population to the retail outlets (Fig. 7)

The resulting model allows us to determine value of the intensity of gravitational pull on population to the shopping center in case of the same radial (along the straight line) spatial-temporal accessibility. However in practice actual configuration of the transport network, types of transport, quality of the roads render a considerable influence on the time of accessibility of population to the shopping center and, correspondingly, produce also an effect on the intensity of customers' flow. This is why a pressing issue for the urban planning tasks is the spatial visualization of field distribution I_t (x, y, z), taking into consideration actual configuration of the road networks and transport accessibility for the population.

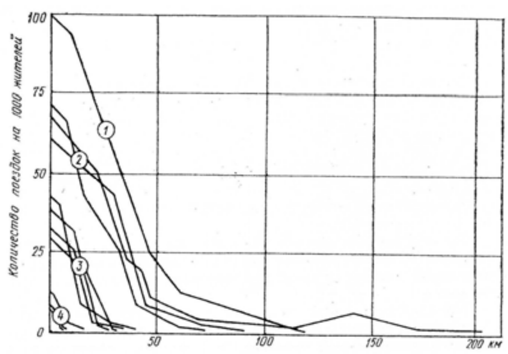

Fig.4. Gravitation intensity (number of rides per 1000 residents) 1-4 –region (inter-regional), regional (inter-district), district (local) primary centers respectively. Source: Dyomin, 1991.

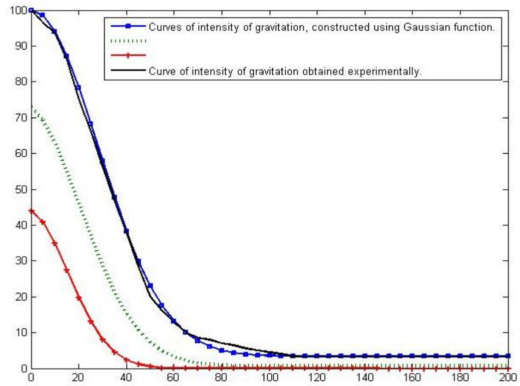

Fig.5. Graphs of Gaussian function describing intensity of gravitation (number of rides per 1000 residents). Source: A. Hoblyk.

CONCLUSION

One of the important conclusions of the work is, that the experimentally obtained graphic regularity describing distribution of the values of intensity of gravitational pull on population to the shopping center corresponds to the Gaussian law with parameters calibrated on the experimental results.

Owing to the function found in the analytical form whose graph coincided with a high degree of accuracy with the experimental graphs contained in professor N. M. Dyomin's work, we have managed to construct a computer model in MATLAB environment that reflects spatially continuous phenomenon - surface distribution of intensity of gravitation of the population to the shopping center.

The novelty and value of the proposed method are in the fact that for any territory in which it is planned to build public service establishments, it is possible to create a model of distribution of the field of intensity of gravitation taking into account actual studied quantitative regularities of the processes of formation of service zones for such objects. Such modeling will permit to determine more accurately the required capacity of public service establishments, justify conditions of placement of the proposed facility and, if necessary, substantiate development of transport network in order to improve the level of transport accessibility.

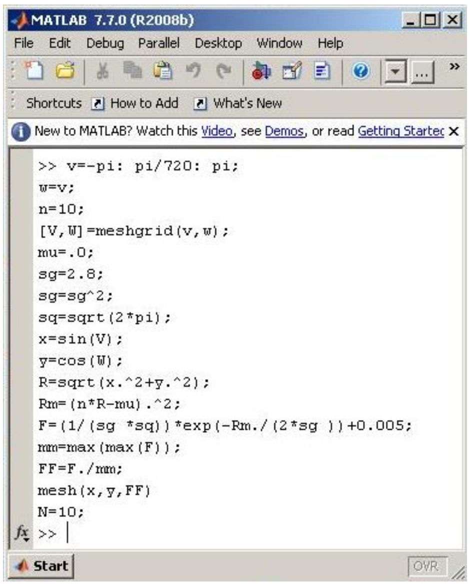

Fig. 6. Script file. Source: A. Hoblyk.

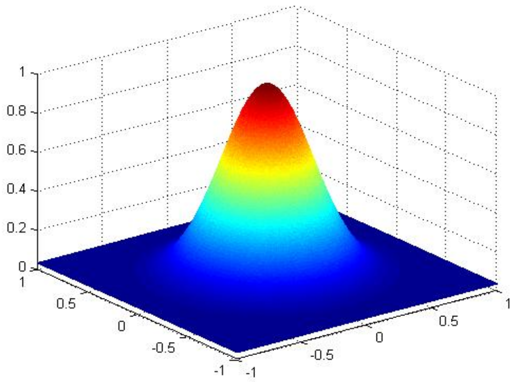

Fig.7. Spatial distribution of the field of gravitation intensity potential (3D graph). Source: A. Hoblyk.

REFERENCES

[1] Gutnov A. *Evolyutsiya gradostroitelstva*, Moscow, Stroyizdat, 1984.

[2] Dyomin N., *Upravlenie razvitiem gradostroitelnykh sistem*, Kyiv, Budivelnyk, 1991.

[3] Dyomin N., Hoblyk A. O primenenii metodov teorii potentsiala v gradostroitelnykh zadachakh, w: *Mistobuduvannya ta terytorialne planuvannya,* nr 50, Kyiv, Kyiv National University of Construction and Architecture, 2013, 140 – 154 s.

[4] Zablotskiy G. Printsypy opredeleniya granits i chleneniya territorialnykh obektov, w: *Matematicheskie metody resheniya kompleksnykh zadach gradostroitelnogo proektirovaniya* Moscow, 1977, 14 – 17 s.

[5] Kalnitskiy L. *Spetsialny kurs vysshey matematiki: prikladnye voprosy analiza*, Moscow, Vysshya shkola, 1976.

[6] Merlin P., *Methodes quantitatives et espace urbain*, Paris, Masson, Collection de Géographie applicable, 1973.

[7] Nikonov A. *Apgreyd obezyany. Bolshaya istoriya malenkoy singulyarnosti*, Piter, Enas, 2005.

[8] Normal distribution, http://en.wikipedia.org/wiki/Normal_distribution

[9] Radomskiy V. Sovershenstvovanie territorialnoy organizatsii bytovogo obsluzhyvaniya naseleniya, Manuscript, Thesis for the degree of PhD in specialty 08.00.04 – Ekonomica rayonov USSR, razmeshcheniya proizvoditelnykh sil USSR, of Kyiv, Academy of Science of Ukrainian SSR, 1984.

[10] Saul Stahl. The Evolution of the Normal Distribution, in: *Mathematics magazine*, Vol. 79, 2006, 96 – 113 p.

[11] Sosnovskiy V. *Prikladnye metody gradostroitelnykh issledovaniy*, Moscow, Arkhitektura – S, 2006.

AUTHOR'S NOTE

The author is an architect, urban planner, PhD. Associate professor of Lviv Polytechnic National University, Institute of Architecture, Department of Urban Planning. She is carrying out postdoctoral research at Kyiv National University of Construction and Architecture since 2012 year. Principal sphere of her scientific interests is: urban development, mathematical and computer modeling of urban planning systems; spatial planning. E-mail: a.hoblyk@gmail.com