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EXTENDED CLARKE TRANSFORMATION FOR *n*-PHASE SYSTEMS

ABSTRACT *The article describes how to convert space vectors written in a stationary multiphase system, consisting of a number of phases where n > 3, to the stationary $\alpha\beta$ orthogonal coordinate system. The transformation of vectors from a stationary n-phase system to the stationary $\alpha\beta$ orthogonal coordinate system is defined. The inverse transformation of a vector written in the $\alpha\beta$ orthogonal coordinate system to a stationary n-phase system is also defined. The application of the extended Clarke transformation allows control calculations to be performed in both stationary $\alpha\beta$ or rotating dq orthogonal coordinate systems. This gives the possibility of performing different control strategies. It has a practical application for drive systems with five-phase, six-phase or dual three-phase motors.*

Keywords: *Clarke transformation, multiphase system, electric drive*
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1. INTRODUCTION

Three-phase AC induction or synchronous motors are currently the most widely used electric drives. But in recent years, in the literature describing electrical drive systems, there have been more and more publications related to multiphase systems, i.e. systems where the number of motor phases is greater than 3. This applies to induction and synchronous motors [1, 2, 3, 4]. Because they have more than three phases such motors cannot be powered directly from a three-phase grid, so they must be supplied by a transistor inverter, with the number of branches corresponding to the number of motor phases.

The construction of motors with a number of phases greater than 3 is justified by the fact that for the given rated power of a multiphase motor the rated values of the phase currents are smaller than in a three-phase motor. This has an impact on the design of the motor and transistor inverter, for which transistors with a lower current limit

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value can be used. Multiphase induction motors have further advantageous properties such as less torque variation in the motor, smaller harmonic currents in the motor phases and thus lower energy loss [7, 8]. With a number of phases greater than 3 it is still possible for the motor and the inverter transistor to work with one phase of the motor or the inverter branches damaged. This ensures the drive's greater reliability [5, 6, 7, 8, 9].

In electrical engineering, in multiphase systems, vector notation is used for the analysis of waveforms of different physical quantities such as voltage, current and flux [10, 11]. Space vectors are defined by physical quantities (voltages, currents, flux linkages) in the individual phases of motor [12]. Space vectors $\boldsymbol{X}_n$ are written in a stationary n-phase coordinate system for a multiphase drive. Torque and motor speed are controlled by the control system, implementing appropriate feedback. Most often drives are built with an overarching speed control loop, with torque or current control loops used in subordination to this. It is convenient to write the control procedures in orthogonal coordinate systems: stationary $\alpha\beta$ or rotating dq synchronized with the rotor or stator flux. Control strategies synchronized with the rotor (ROC – Rotor oriented control) or flux vector (FOC – Field oriented control) are implemented in this way.

The commonly used vector transformation from a three-phase stationary system $\boldsymbol{X}_{\mathrm{ABC}}$ to a stationary orthogonal $\boldsymbol{X}_{\alpha\beta}$ system is done by the Clarke Transformation [13], in which the transformation matrix is:

$$\boldsymbol{T}_{\alpha\beta/\mathrm{ABC}} = \frac{2}{3}\cdot\begin{bmatrix} 1 & \cos(\frac{2\pi}{3}) & \cos(\frac{4\pi}{3}) \\ 0 & -\sin(\frac{2\pi}{3}) & -\sin(\frac{4\pi}{3}) \end{bmatrix} = \frac{2}{3}\cdot\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \tag{1}$$

and the reverse Clarke transformation, where the transformation matrix is:

$$\boldsymbol{T}_{\mathrm{ABC}/\alpha\beta} = \begin{bmatrix} 1 & 0 \\ \cos(\frac{2\pi}{3}) & -\sin(\frac{2\pi}{3}) \\ \cos(\frac{4\pi}{3}) & -\sin(\frac{4\pi}{3}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \tag{2}$$

The aim of this study is to present a transformation from multiphase systems to the stationary $\alpha\beta$ orthogonal coordinate system and its application to AC drive control systems. In this work the transformation of the space vector from the stationary n-phase system to the stationary $\alpha\beta$ orthogonal system, which is an extension of the Clarke Transformation, for systems with a number of phases where $n > 3$, is defined. The inverse transformation of the space vector from the stationary $\alpha\beta$ orthogonal system to the stationary n-phase system is also defined.

2. ASSUMPTIONS FOR THE *n*-PHASE SYSTEM

We will consider an n-phase system, making the following assumptions (Fig. 1):
1. The system consists of n phases.
2. The actual spatially distributed motor windings are treated as concentrated winding.
3. We assume the electric and magnetic symmetry of the n-phase system.
4. The multiphase stator winding is star connected with an isolated neutral point.

5. The electric angle between the axes of the phase windings is: $\theta_n = \frac{2\pi}{n}$
6. We will use the notation of the vector in the stationary phase coordinate system:

$$\boldsymbol{X}_n = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \tag{3}$$

where: $x_1(t), x_2(t),..., x_n(t)$ are the instantaneous values of the phase quantities.

7. The condition allowing the vector to be written is when the sum of the instantaneous values of the phase quantities is equal to zero:

$$\sum_{k=1}^{k=n} x_k(t) = 0 \tag{4}$$

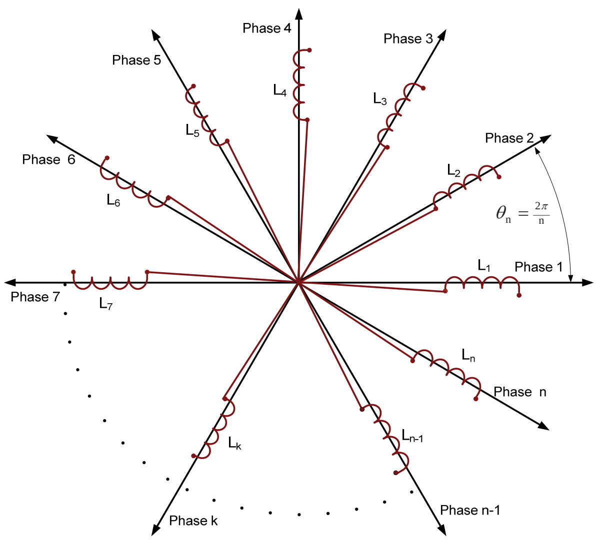


Fig. 1. Simplified diagram of the stator windings of an n-phase motor

In the literature [5, 15] drive systems are described with hysteresis controllers of the phase currents of the stator (Fig. 2). Typically drives work with a speed control loop. The actual speed of the motor ω is compared with the reference speed ω_{ref}. The difference is entered to the input of the PI speed controller, which generates a reference value for the current component i_{qref}. The value of the component i_{dref} is usually taken to be zero. The current reference components i_{dref}, i_{qref} are converted into $i_{\alpha ref}$, $i_{\beta ref}$ in the stationary $\alpha\beta$ orthogonal coordinate system, using Park's transformation. The reference values of the phase currents are compared with the measured values of the currents in the *n*-phase hysteresis controller. This structure of control requires a comparison of the *n*-phase values.

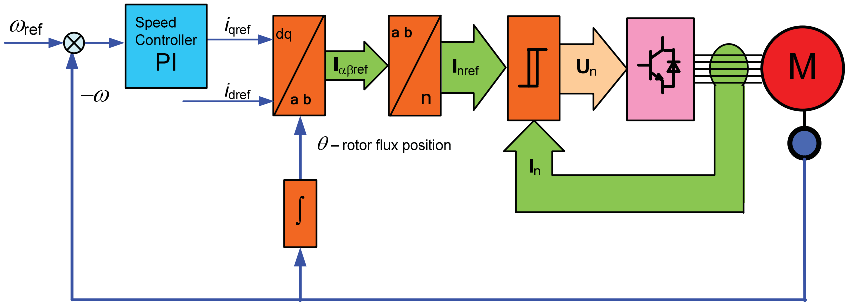


Fig. 2. Block diagram of a drive system with an n-phase hysteresis current controller

The proposed *n*-phase drive system consists of an n-phase motor, with an inverter composed of *n* transistor branches, and a processor control system performing the control procedures (Fig. 3).

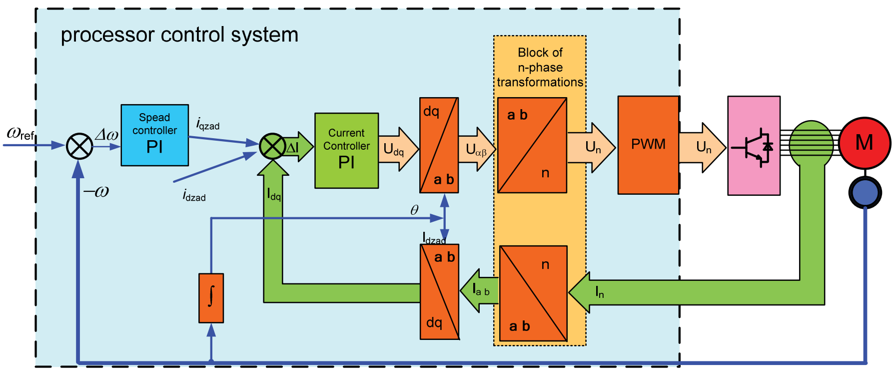


Fig. 3. Block diagram of an n-phase drive with a PI current regulator

This configuration allows the use of a PI current regulator and a PWM circuit. This allows constant frequency of the switching transistors, and quieter and smoother operation of the drive waveforms of the phase currents, to be achieved.

An essential element of the drive is an *n*-phase transformation block:

- The transformation of vector $\boldsymbol{X}_n$ from a stationary *n*-phase coordinate system to the stationary $\alpha\beta$ orthogonal coordinate system is accomplished by means of the matrix $T_{\alpha\beta/n}$. This transformation applies the current $\boldsymbol{I}_n$ or voltage $\boldsymbol{U}_n$ vectors.
- The inverse transformation, i.e. the conversion from vector $\boldsymbol{X}_{\alpha\beta}$ in a stationary $\alpha\beta$ orthogonal system to vector $\boldsymbol{X}_n$ in a stationary n-phase system uses matrix $\boldsymbol{T}_{n/\alpha\beta}$.

The voltage vector is converted in a Pulse Width Modulated block and served on the gate of multiphase inverter transistors. A system consisting of an n-phase transformation block and a PWM block can be called an *n*-phase voltage modulator. The most common voltage modulator converting the voltage vector components u_d, u_q to *n*-phase voltages is a current (torque) and speed control loop.

A processor control system executes control procedures. Calculations are performed in the stationary $\alpha\beta$ or rotating *dq* orthogonal coordinate systems. Different control strategies are possible: ROC – rotor oriented control or FOC – field oriented control.

3. TRANSFORMATION OF SPACE VECTORS FROM A STATIONARY n-PHASE SYSTEM TO A STATIONARY $\alpha\beta$ ORTHOGONAL SYSTEM

The transformation of vector $\boldsymbol{X}_n$ from the stationary *n*-phase coordinate system to a stationary $\alpha\beta$ orthogonal coordinate system is performed by the matrix:

$$\boldsymbol{T}_{\alpha\beta/n} = \frac{2}{n}\cdot\begin{bmatrix} 1 & \cos(\frac{2\pi}{n}) & \cos(\frac{4\pi}{n}) & \ldots & \cos(\frac{2(n-1)\pi}{n}) \\ 0 & -\sin(\frac{2\pi}{n}) & -\sin(\frac{4\pi}{n}) & \ldots & -\sin(\frac{2(n-1)\pi}{n}) \end{bmatrix} \tag{5}$$

This involves the left-sided multiplication of vector $\boldsymbol{X}_n$ by a matrix $\boldsymbol{T}_{\alpha\beta/n}$ whereby we obtain vector $\boldsymbol{X}_{\alpha\beta}$ in the $\alpha\beta$ orthogonal coordinate system:

$$\boldsymbol{X}_{\alpha\beta} = \boldsymbol{T}_{\alpha\beta/n}\cdot\boldsymbol{X}_n \tag{6}$$

Vector $\boldsymbol{X}_n$ can be written in a stationary *n*-phase coordinate system:

$$\boldsymbol{X}_n = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_k(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \left|\boldsymbol{X}(t)\right|\cdot\begin{bmatrix} \cos(\omega t+\alpha) \\ \cos(\omega t+\alpha+\frac{2\pi}{n}) \\ \cos(\omega t+\alpha+\frac{4\pi}{n}) \\ \ldots \\ \cos(\omega t+\alpha+\frac{2k\pi}{n}) \\ \ldots \\ \cos(\omega t+\alpha+\frac{2(n-1)\pi}{n}) \end{bmatrix} \tag{7}$$

where:

ω – the instantaneous value of the pulsation,
α – initial angle position of the vector,
$\dfrac{2k\pi}{n}$ – the angle position of the phase *k*.

Using the above notation we can do the following:

$$\boldsymbol{X}_{\alpha\beta} = \boldsymbol{T}_{\alpha\beta/\mathrm{n}} \cdot \boldsymbol{X}_n = \frac{2}{n} \cdot \begin{bmatrix} 1 & \cos(\frac{2\pi}{n}) & \cos(\frac{4\pi}{n}) & \dots & \cos(\frac{2(n-1)\pi}{n}) \\ 0 & -\sin(\frac{2\pi}{n}) & -\sin(\frac{4\pi}{n}) & \dots & -\sin((\frac{2(n-1)\pi}{n}) \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \tag{8}$$

Substituting equation (7) and performing the following operations we get:

$$\boldsymbol{X}_{\alpha\beta} = \boldsymbol{T}_{\alpha\beta/n} \cdot \boldsymbol{X}_n = \left|\boldsymbol{X}(t)\right| \cdot \frac{2}{n} \cdot \begin{bmatrix} \sum\limits_{\mathrm{k}=0}^{k=n-1} \cos(\frac{2k\pi}{n}) \cdot \cos(\omega t + \alpha + \frac{2k\pi}{n}) \\ -\sum\limits_{k=0}^{k=n-1} \sin(\frac{2k\pi}{n}) \cdot \cos(\omega t + \alpha + \frac{2k\pi}{n}) \end{bmatrix} \tag{9}$$

Vector coefficients (9) converted to the form:

$$\begin{aligned} &\sum_{k=0}^{k=n-1} \cos(\tfrac{2k\pi}{n}) \cdot \cos(\omega t + \alpha + \tfrac{2k\pi}{n}) = \\ &= \sum_{k=0}^{k=n-1} \left[\cos^2(\tfrac{2k\pi}{n}) \cdot \cos(\omega t + \alpha) - \cos(\tfrac{2k\pi}{n}) \cdot \sin(\tfrac{2k\pi}{n}) \cdot \sin(\omega t + \alpha) \right] = \\ &= \cos(\omega t + \alpha) \cdot \sum_{k=0}^{k=n-1} \cos^2(\tfrac{2k\pi}{n}) - \sin(\omega t + \alpha) \cdot \sum_{k=0}^{k=n-1} \cos(\tfrac{2k\pi}{n}) \cdot \sin(\tfrac{2k\pi}{n}) \end{aligned} \tag{10}$$

$$\begin{aligned} &-\sum_{k=0}^{k=n-1} \sin(\tfrac{2k\pi}{n}) \cdot \cos(\omega t + \alpha + \tfrac{2k\pi}{n}) = \\ &= \sum_{k=0}^{k=n-1} \left[\sin^2(\tfrac{2k\pi}{n}) \cdot \sin(\omega t + \alpha) - \cos(\tfrac{2k\pi}{n}) \cdot \sin(\tfrac{2k\pi}{n}) \cdot \cos(\omega t + \alpha) \right] = \\ &= \sin(\omega t + \alpha) \cdot \sum_{k=0}^{k=n-1} \sin^2(\tfrac{2k\pi}{n}) - \cos(\omega t + \alpha) \cdot \sum_{k=0}^{k=n-1} \cos(\tfrac{2k\pi}{n}) \cdot \sin(\tfrac{2k\pi}{n}) \end{aligned} \tag{11}$$

because:

$$\sum_{k=0}^{k=n-1} \cos(\tfrac{2k\pi}{n}) \cdot \sin(\tfrac{2k\pi}{n}) = 0 \tag{12}$$

$$\sum_{k=0}^{k=n-1} \cos^2(\tfrac{2k\pi}{n}) = \frac{n}{2} \tag{13}$$

$$\sum_{k=0}^{k=n-1} \sin^2(\tfrac{2k\pi}{n}) = \frac{n}{2} \tag{14}$$

We receive the space vector in the stationary $\alpha\beta$ orthogonal coordinate system:

$$\boldsymbol{X}_{\alpha\beta} = |\boldsymbol{X}(t)| \cdot \begin{bmatrix} \cos(\omega t + \alpha) \\ \sin(\omega t + \alpha) \end{bmatrix} = \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} \tag{15}$$

In the literature describing multiphase drive systems, it is assumed that the number of phases of the motor can be odd, even, or a multiple of 3 or 5 [14, 15]. As a rule, motor phase windings are in a star connection with an insulated neutral point, which makes the sum of the instantaneous values of the phase currents zero.

This is a necessary condition in order to apply the vector methods of current control. When the number of phases *n* is not a prime number it is also possible to divide them into sections. In these cases, the number of isolated neutral points of the multiphase windings is equal to the number of sections into which the coil has been divided. In this way dual three-phase windings are combined (from here on denoted 2x3).

The main problem in applying this transformation is that the matrix coefficients are stored in the form of trigonometric functions. It is convenient to write the matrix coefficients in the form of finite formulas using basic arithmetic and square roots of integers. This writing of the trigonometric functions for the arguments as $k\pi/n$ is possible only if, in the simplified fraction π/n, the number *n* is the product of the power of number 2 and various Fermat prime numbers. The condition for number *n* is the same as the condition for constructing the regular *n*-angle polygon by compass and straightedge according to the theorem of Gauss-Wantzel.

The Gauss-Wantzel theorem says that the regular *n*-angle polygon can be constructed by compass and straightedge only if:

$$n = 2^k \tag{16}$$

or

$$n = 2^k \cdot F_0 \cdot F_1 \cdot \ldots \cdot F_s \tag{17}$$

where:

$F_0,\ F_1, \ldots, F_s$ are various Fermat prime numbers.

Fermat numbers are numbers of the form:

$$F_s = 2^{2^s} + 1 \tag{18}$$

Where $s = 0,\ 1,\ 2, \ldots$ are other natural numbers.

For the following values of s, the Fermat numbers assume the values:

$$s = 0 \quad \Rightarrow \quad F_0 = 2^{2^0} + 1 = 2^1 + 1 = 3 \tag{19}$$

$$s = 1 \quad \Rightarrow \quad F_1 = 2^{2^1} + 1 = 2^2 + 1 = 5 \tag{20}$$

$$s = 2 \quad \Rightarrow \quad F_2 = 2^{2^2} + 1 = 2^4 + 1 = 17 \tag{21}$$

$$s = 3 \quad \Rightarrow \quad F_3 = 2^{2^3} + 1 = 2^8 + 1 = 257 \tag{22}$$

$$s = 4 \quad \Rightarrow \quad F_4 = 2^{2^4} + 1 = 2^{16} + 1 = 65537 \tag{23}$$

$$s = 5 \quad \Rightarrow \quad F_5 = 2^{2^5} + 1 = 2^{32} + 1 = 4294967297 \; = \; 641 \cdot 6700417 \tag{24}$$

Known Fermat prime numbers are: 3, 5, 17, 257, and 65537.

In practical applications of the design of electrical machines, we can consider the Fermat prime numbers F_0 = 3 and F_1 = 5. Moreover, for practical reasons, the number n of the motor phases should be less than 10.

For k = 0, when $2^k = 1$, we obtain from equation (17) systems with an odd number of phases, i.e. three-phase and five-phase systems.

For k = 1, when $2^k = 2$, we obtain from equation (17) systems with an even number of phases, i.e. six-phase, and ten-phase systems.

In addition, for $k = 2$, when $2^2 = 4$ we obtain from equation (16) a four-phase system, and for $k = 3$ when 2^3 = 8, we obtain an eight-phase system.

TABLE 1
The number of phases of the motor stator winding which are possible with the dependencies: $n = 2^k$ $\quad n = 2^k F_0$ $\quad n = 2^k F_1$

	$k = 0$	$k = 1$	$k = 2$	$k = 3$
$n = 2^k$	1	2	4	8
$n = 2^k F_0$	3	6	12	24
$n = 2^k F_1$	5	10	20	40

In the above table we have discounted one-phase and two-phase systems as well as systems with a number of stages where n > 10. In order to to apply the systems with a number of phases between 3 and 10 and at the same time have the matrix coefficients written in the form of finite formulas using basic arithmetic and square roots of integers, we should choose a system with the number of phases: 3, 4, 5, 6, 8. For the 7- and 9-phase systems the matrix coefficients are written in real numbers. Circuit diagrams of high-current converters and systems for windings of phase numbers 3, 4, 5, 6 and 2x3, are shown below (Fig. 4 – 8).

Table 3 shows the matrices $\boldsymbol{T}_{\alpha\beta/n}$ for the transformation of the vector $\boldsymbol{X}_n$ from the stationary coordinate systems 3, 4, 5, 6 and 2x3-phase to the stationary $\alpha\beta$ orthogonal coordinate system.

TABLE 2
Electrical angles between the axes of the phase windings for multiphase stator winding systems

Number of phases	electrical angles
3	$\theta_3 = \frac{2\pi}{3} = 120^{\circ}$
4	$\theta_4 = \frac{2\pi}{4} = \frac{\pi}{2} = 90^{\circ}$
5	$\theta_5 = \frac{2\pi}{5} = 72^{\circ}$
6	$\theta_6 = \frac{2\pi}{6} = \frac{\pi}{3} = 60^{\circ}$
2x3	$\theta_{2x3} = \frac{2\pi}{12} = \frac{\pi}{6} = 30^{\circ}$

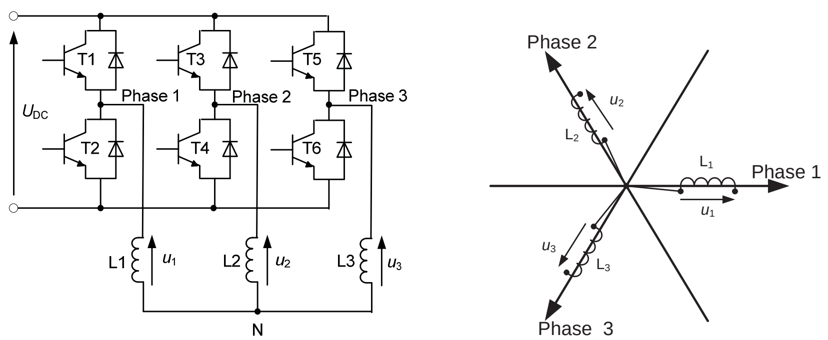


Fig. 4. Schematic diagram of the inverter system and three-phase windings

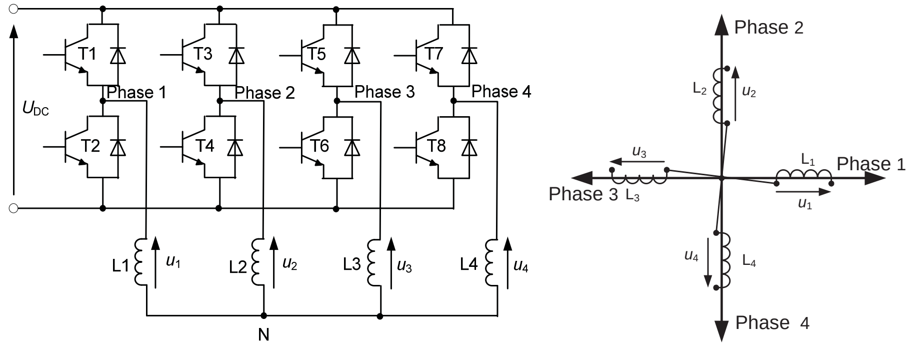


Fig. 5. Schematic diagram of the inverter system and four-phase windings

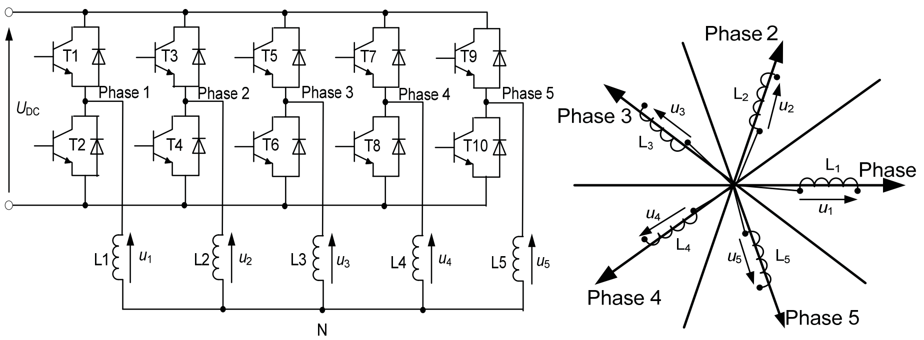


Fig. 6. Schematic diagram of the inverter system and five-phase windings

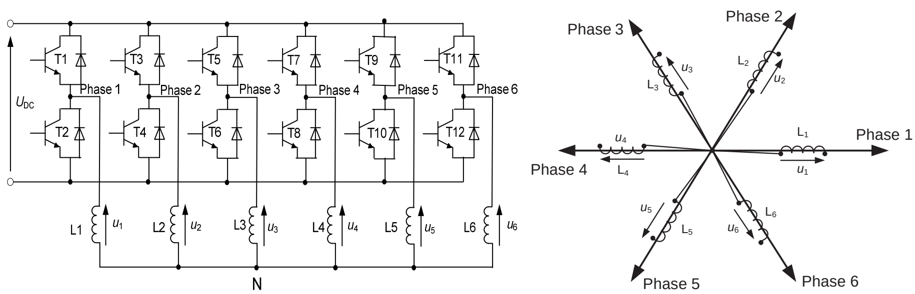


Fig. 7. Schematic diagram of the inverter system and six-phase windings

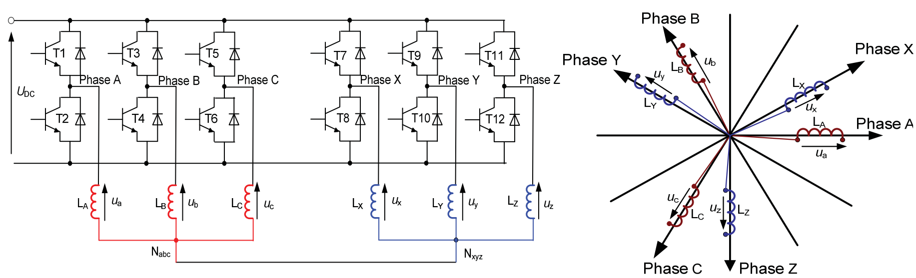


Fig. 8. Schematic diagram of the inverter system and dual three-phase windings

TABLE 3
Matrices $\boldsymbol{T}_{\alpha\beta/n}$ transforming vector $\boldsymbol{X}_n$ from the n-phase stationary coordinate system to the stationary $\alpha\beta$ orthogonal coordinate system

Number of phases	$\boldsymbol{T}_{\alpha\beta/n}$
3	$\frac{2}{3}\cdot\begin{bmatrix} 1 & \cos(\frac{2\pi}{3}) & \cos(\frac{4\pi}{3}) \\ 0 & -\sin(\frac{2\pi}{3}) & -\sin(\frac{4\pi}{3}) \end{bmatrix} = \frac{2}{3}\cdot\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
4	$\frac{1}{2}\cdot\begin{bmatrix} 1 & \cos(\frac{\pi}{2}) & \cos(\pi) & \cos(\frac{3\pi}{2}) \\ 0 & -\sin(\frac{\pi}{2}) & -\sin(\pi) & -\sin(\frac{3\pi}{2}) \end{bmatrix} = \frac{1}{2}\cdot\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$
5	$\frac{2}{5}\cdot\begin{bmatrix} 1 & \cos(\frac{2\pi}{5}) & \cos(\frac{4\pi}{5}) & \cos(\frac{6\pi}{5}) & \cos(\frac{8\pi}{5}) \\ 0 & -\sin(\frac{2\pi}{5}) & -\sin(\frac{4\pi}{5}) & -\sin(\frac{6\pi}{5}) & -\sin(\frac{8\pi}{5}) \end{bmatrix} =$ $= \frac{2}{5}\cdot\begin{bmatrix} 1 & \frac{\sqrt{5}-1}{4} & -\frac{\sqrt{5}+1}{4} & -\frac{\sqrt{5}+1}{4} & \frac{\sqrt{5}-1}{4} \\ 0 & -\frac{\sqrt{10+2\sqrt{5}}}{4} & -\frac{\sqrt{10-2\sqrt{5}}}{4} & \frac{\sqrt{10-2\sqrt{5}}}{4} & \frac{\sqrt{10+2\sqrt{5}}}{4} \end{bmatrix}$
6	$\frac{1}{3}\cdot\begin{bmatrix} 1 & \cos(\frac{\pi}{3}) & \cos(\frac{2\pi}{3}) & \cos(\pi) & \cos(\frac{4\pi}{3}) & \cos(\frac{5\pi}{3}) \\ 0 & -\sin(\frac{\pi}{3}) & -\sin(\frac{2\pi}{3}) & -\sin(\pi) & -\sin(\frac{4\pi}{3}) & -\sin(\frac{5\pi}{3}) \end{bmatrix} =$ $= \frac{1}{3}\cdot\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} & -1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
2x3	$\frac{1}{3}\cdot\begin{bmatrix} 1 & \cos(\frac{2\pi}{3}) & \cos(\frac{4\pi}{3}) & \cos(\frac{\pi}{6}) & \cos(\frac{5\pi}{6}) & \cos(\frac{9\pi}{6}) \\ 0 & -\sin(\frac{2\pi}{3}) & -\sin(\frac{4\pi}{3}) & -\sin(\frac{\pi}{6}) & -\sin(\frac{5\pi}{6}) & -\sin(\frac{9\pi}{6}) \end{bmatrix} =$ $= \frac{1}{3}\cdot\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}$

4. TRANSFORMATION OF SPACE VECTORS FROM THE STATIONARY $\alpha\beta$ ORTHOGONAL SYSTEM TO THE STATIONARY n-PHASE SYSTEM

The inverse transformation, i.e. the conversion from vector $\boldsymbol{X}_{\alpha\beta}$ written in the stationary $\alpha\beta$ orthogonal coordinate system to vector $\boldsymbol{X}_n$ written in the n-phase stationary coordinate system is performed using the matrix $\boldsymbol{T}_{n/\alpha\beta}$.

TABLE 4

Matrices $\boldsymbol{T}_{n/\alpha\beta}$ transforming vector $\boldsymbol{X}_{\alpha\beta}$ from the stationary $\alpha\beta$ orthogonal coordinate system to the stationary n-phase coordinate system

Number of phases	$\boldsymbol{T}_{n/\alpha\beta}$
3	$\begin{bmatrix} 1 & 0 \\ \cos(\frac{2\pi}{3}) & -\sin(\frac{2\pi}{3}) \\ \cos(\frac{4\pi}{3}) & -\sin(\frac{4\pi}{3}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
4	$\begin{bmatrix} 1 & 0 \\ \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ \cos(\pi) & -\sin(\pi) \\ \cos(\frac{3\pi}{2}) & -\sin(\frac{3\pi}{2}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}$
5	$\begin{bmatrix} 1 & 0 \\ \cos(\frac{2\pi}{5}) & -\sin(\frac{2\pi}{5}) \\ \cos(\frac{4\pi}{5}) & -\sin(\frac{4\pi}{5}) \\ \cos(\frac{6\pi}{5}) & -\sin(\frac{6\pi}{5}) \\ \cos(\frac{8\pi}{5}) & -\sin(\frac{8\pi}{5}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{\sqrt{5}-1}{4} & -\frac{\sqrt{10+2\sqrt{5}}}{4} \\ -\frac{\sqrt{5}+1}{4} & -\frac{\sqrt{10-2\sqrt{5}}}{4} \\ -\frac{\sqrt{5}+1}{4} & \frac{\sqrt{10-2\sqrt{5}}}{4} \\ \frac{\sqrt{5}-1}{4} & \frac{\sqrt{10+2\sqrt{5}}}{4} \end{bmatrix}$
6	$\begin{bmatrix} 1 & 0 \\ \cos(\frac{\pi}{3}) & -\sin(\frac{\pi}{3}) \\ \cos(\frac{2\pi}{3}) & -\sin(\frac{2\pi}{3}) \\ \cos(\pi) & -\sin(\pi) \\ \cos(\frac{4\pi}{3}) & -\sin(\frac{4\pi}{3}) \\ \cos(\frac{5\pi}{3}) & -\sin(\frac{5\pi}{3}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
2x3	$\begin{bmatrix} 1 & 0 \\ \cos(\frac{2\pi}{3}) & -\sin(\frac{\pi}{3}) \\ \cos(\frac{4\pi}{3}) & -\sin(\frac{2\pi}{3}) \\ \cos(\frac{\pi}{6}) & -\sin(\frac{\pi}{6}) \\ \cos(\frac{5\pi}{6}) & -\sin(\frac{5\pi}{6}) \\ \cos(\frac{9\pi}{6}) & -\sin(\frac{9\pi}{6}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & 1 \end{bmatrix}$

$$\boldsymbol{T}_{\mathrm{n}/\alpha\beta} = \begin{bmatrix} 1 & 0 \\ \cos(\frac{2\pi}{n}) & -\sin(\frac{2\pi}{n}) \\ \cos(\frac{4\pi}{n}) & -\sin(\frac{4\pi}{n}) \\ \vdots & \vdots \\ \cos(\frac{2(n-1)\pi}{n}) & -\sin(\frac{2(n-1)\pi}{n}) \end{bmatrix} \tag{25}$$

The inverse transformation is based on left-multiplying the vector $\boldsymbol{X}_{\alpha\beta}$ by matrix $\boldsymbol{T}_{n/\alpha\beta}$.

$$\boldsymbol{X}_n = \boldsymbol{T}_{n/\alpha\beta} \cdot \boldsymbol{X}_{\alpha\beta} \tag{26}$$

So, in more precise notation, after multiplication and execution of trigonometric transformation, we get vector $\boldsymbol{X}_n$ in an n-phase system:

$$\boldsymbol{X}_n = \left|\boldsymbol{X}(t)\right| \cdot \begin{bmatrix} 1 & 0 \\ \cos(\frac{2\pi}{n}) & -\sin(\frac{2\pi}{n}) \\ \cos(\frac{4\pi}{n}) & -\sin(\frac{4\pi}{n}) \\ \vdots & \vdots \\ \cos(\frac{2(n-1)\pi}{n}) & -\sin(\frac{2(n-1)\pi}{n}) \end{bmatrix} \cdot \begin{bmatrix} \cos(\omega t + \alpha) \\ \sin(\omega t + \alpha) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_{\mathrm{n}}(t) \end{bmatrix} \tag{27}$$

5. EXAMPLE SIMULATIONS OF THE WORK OF AN *n*-PHASE VOLTAGE MODULATOR

The transformations of vectors from a stationary n-phase coordinate system to a stationary $\alpha\beta$ orthogonal coordinate system, and vice versa, allows the n-phase voltage modulator, which is a fundamental part of the drive system, to be built. Most often the modulator is covered by current, torque and speed control loops. The following shows simulations of a modulator working in open and closed current loop systems. As the load modulator, a multiphase *RL* circuit in a star connection was adopted, with the parameters in Table 5.

5.1. The work of the modulator in an open current control loop

In an open system (Fig. 9) the reference signals are the voltage vector components u_{dzad}, u_{qzad} written in a rotating dq orthogonal coordinate system:

$$\boldsymbol{U}_{\mathrm{dqref}} = \begin{bmatrix} u_{dref} \\ u_{qref} \end{bmatrix} = \left|\boldsymbol{U}\right| \cdot \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} \tag{28}$$

and pulsation reference ω.
α – initial angle position of the current vector reference.

The components of the voltage vectors u_{dref}, u_{qref} are converted to components u_α, u_β written in a stationary $\alpha\beta$ orthogonal coordinate system using the Park Transformation.

$$\boldsymbol{U}_{\alpha\beta} = \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = |\boldsymbol{U}| \cdot \begin{bmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{bmatrix} \cdot \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} \tag{29}$$

After multiplication and performing trigonometric transformations we get the desired voltage vector components.

$$\boldsymbol{U}_{\alpha\beta} = \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = |\boldsymbol{U}| \cdot \begin{bmatrix} \cos(\omega t + \alpha) \\ \sin(\omega t + \alpha) \end{bmatrix} \tag{30}$$

Next, the voltage vector $\boldsymbol{U}_{\alpha\beta}$ is transformed to vector $\boldsymbol{U}_n$ written in an n-phase stationary coordinate system by means of the extended Clarke Transformation (25, 26). Phase voltages $\boldsymbol{U}_n$ force the phase currents $\boldsymbol{I}_n$ in the load circuits.

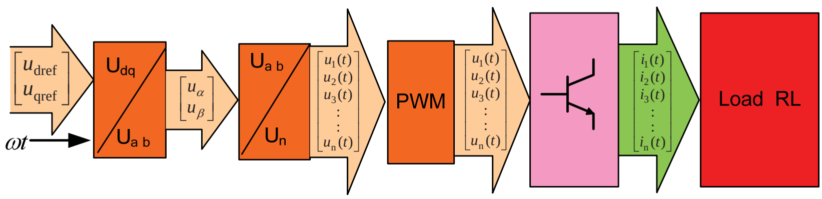


Fig. 9. Block diagram of an *n*-phase voltage modulator

TABLE 5
Circuit simulation parameters

Parameter	Value
Resistance of phase load	$R_F = 1\ \Omega$
Inductance of phase load	$L_F = 0.01$ H
Intermediate circuit voltage	$U_{DC} = 600$ V
PWM carrier frequency	$f_{\mathrm{PWM}} = 10$ kHz
Pulsation of voltage	$\omega = 2\pi f = 126\ \frac{\mathrm{rad}}{\mathrm{s}}$

Calculations were performed for the number of phases 3, 4, 5, 6 and 2x3.
The following were assumed as reference signals:

- Pulsation of the voltage modulator:

$$\omega = 126\ \frac{\mathrm{rad}}{\mathrm{s}} \tag{31}$$

- Components of the voltage vector:

$$u_{dref} = 100\ \mathrm{V} \tag{32}$$

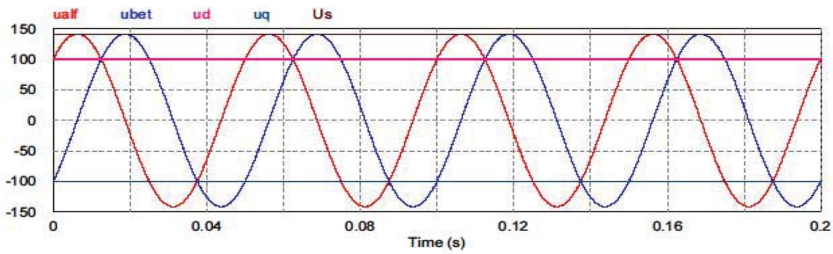
$$u_{qref} = -100\ \mathrm{V} \tag{33}$$

The module for the voltage vector reference is:

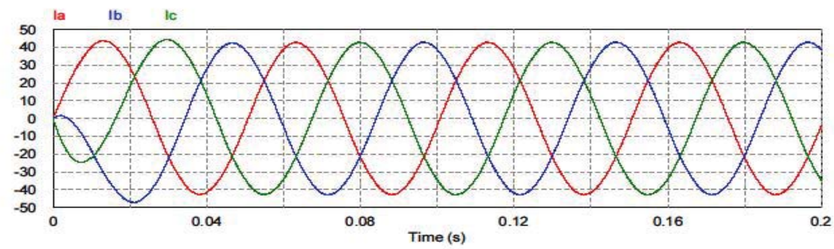
$$\left|\boldsymbol{U}_S\right| = \sqrt{u_{dref}^2 + u_{qref}^2} = 100 \cdot \sqrt{2} = 141\,\mathrm{V} \tag{34}$$

The output signals were the calculated components of output voltage vectors u_α, u_β, u_d, u_q, the module of the output voltage vector $|\boldsymbol{U}_s|$ and the output phase currents in the *n*-phase systems: three-phase, four-phase, five-phase, six-phase and dual three-phase (Fig. 10).

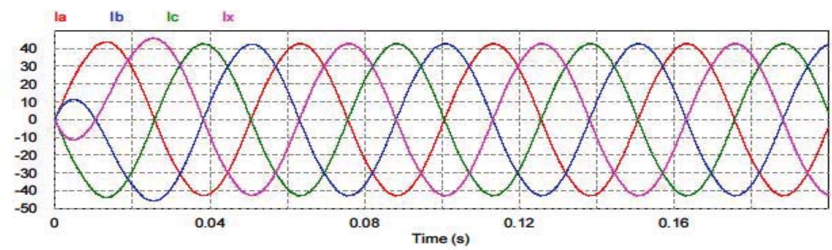
a)



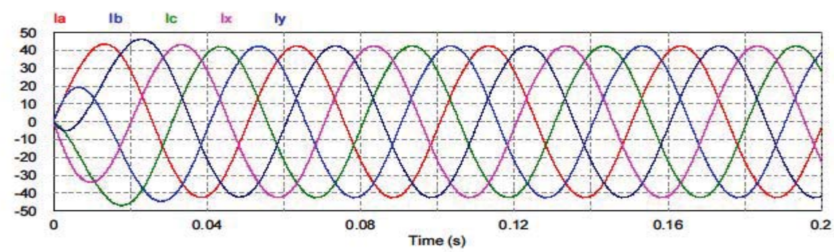
b)



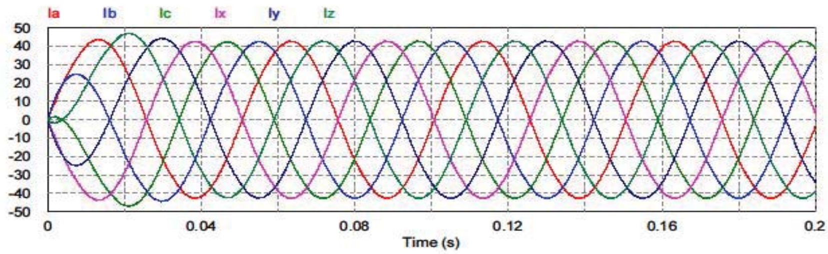
c)



d)



e)



f)

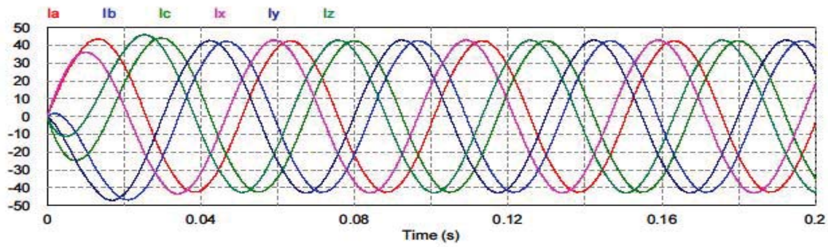


Fig. 10. Example results of the simulation of the work of the modulator in the form of waveforms of voltages and currents: a) the components of the reference voltage vector u_{dref}, u_{qref}; the components of the voltage vector u_{α}, u_{β}; module reference voltage vector $|\boldsymbol{U}_s|$.
The output phase currents in an n-phase system: b) three-phase; c) four-phase; d) five-phase; e) six-phase; f) dual three-phase.

5.2. The work of the modulator in a closed current control loop

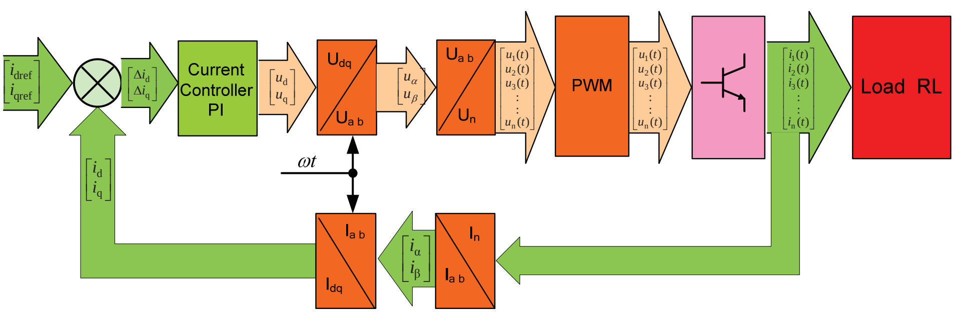


Fig. 11. Block diagram of an *n*-phase modulator in a current control system

The work of the modulator in a closed current control loop comprises:

- Measurement of phase currents $i_1, i_2, \ldots, i_n$ defining the current vector $\boldsymbol{I}_n$.
- Transformation of the current vector $\boldsymbol{I}_n$ to the form $\boldsymbol{I}_{\alpha\beta}$ in the $\alpha\beta$ coordinate system using the matrix $\boldsymbol{T}_{\alpha\beta/n}$ (5). The result is:

$$\boldsymbol{I}_{\alpha\beta} = \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = |\boldsymbol{I}| \cdot \begin{bmatrix} \cos(\omega t + \beta) \\ \sin(\omega t + \beta) \end{bmatrix} \tag{35}$$

- Transform the current vector $\boldsymbol{I}_{\alpha\beta}$ to form $\boldsymbol{I}_{dq}$ using the Park Transformation:

$$\boldsymbol{I}_{dq} = \begin{bmatrix} i_d \\ i_q \end{bmatrix} = |\boldsymbol{I}| \cdot \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \cdot \begin{bmatrix} \cos(\omega t + \beta) \\ \sin(\omega t + \beta) \end{bmatrix} \tag{36}$$

After multiplication and performing trigonometric transformations we get the desired current vector components:

$$\boldsymbol{I}_{dq} = \begin{bmatrix} i_d \\ i_q \end{bmatrix} = |\boldsymbol{I}| \cdot \begin{bmatrix} \cos(\beta) \\ \sin(\beta) \end{bmatrix} \tag{37}$$

In the current control system (Fig. 11) the reference signals are the current vector components i_{dref}, i_{qref} in a rotating dq coordinate system:

$$\boldsymbol{I}_{\mathrm{dqref}} = \begin{bmatrix} i_{dref} \\ i_{qref} \end{bmatrix} = |\boldsymbol{I}| \cdot \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} \tag{38}$$

and pulsation reference ω.
α – initial angle position of the reference current vector.

Current vectors $\boldsymbol{I}_{dq}$ and $\boldsymbol{I}_{\mathrm{dqref}}$ are compared with each other and their difference is the control error.

$$\Delta\boldsymbol{I} = \boldsymbol{I}_{dqref} - \boldsymbol{I}_{\mathrm{dq}} = \begin{bmatrix} i_{dref} - i_d \\ i_{qref} - i_q \end{bmatrix} = \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix} \tag{39}$$

The control error $\boldsymbol{\Delta I}$ is entered to the input of the PI current regulator, which calculates the coordinates of the voltage vector $\boldsymbol{U}_{dq}$. Next the modulator operation proceeds as described in section. 5.1.
Calculations were performed for the phase numbers 3, 4, 5, 6 and 2x3.
As reference signals it was assumed:

- Pulsation of the voltage modulator:

$$\omega = 126 \, \frac{\mathrm{rad}}{\mathrm{s}} \tag{40}$$

- Components of the current vector:

$$i_{dref} = 50 \text{ A} \tag{41}$$

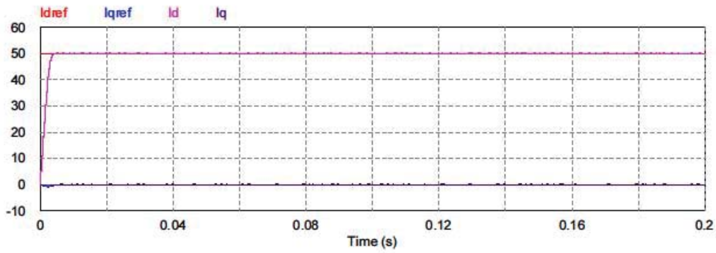
$$i_{qref} = 0 \text{ A} \tag{42}$$

The module of the current vector reference is:

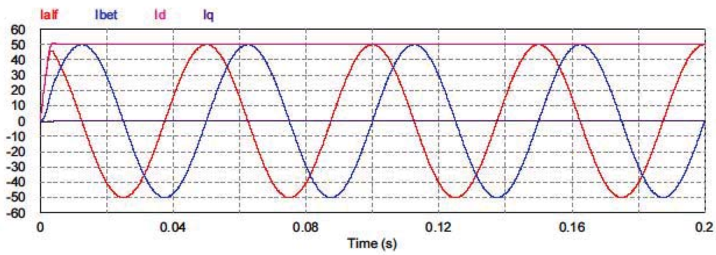
$$\left|\boldsymbol{I}_S\right| = \sqrt{i_{dref}^2 + i_{qref}^2} = 50 \text{ A} \tag{43}$$

The output signals were the calculated components of output current vectors i_α, i_β, i_d, i_q, the module of the output current vector $\boldsymbol{I}_s$ and the output phase currents in the *n*-phase systems: three-phase, four-phase, five-phase, six-phase and dual – three-phase (Fig. 12).

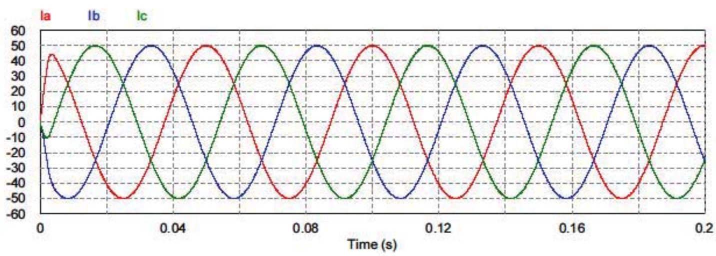
a)



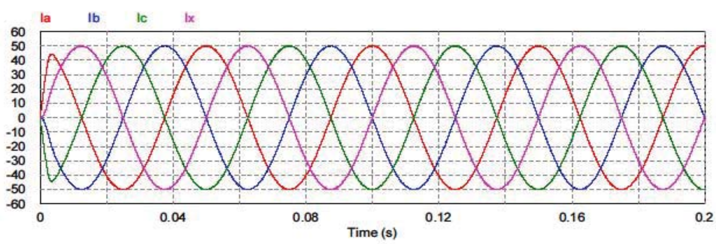
b)



c)



d)



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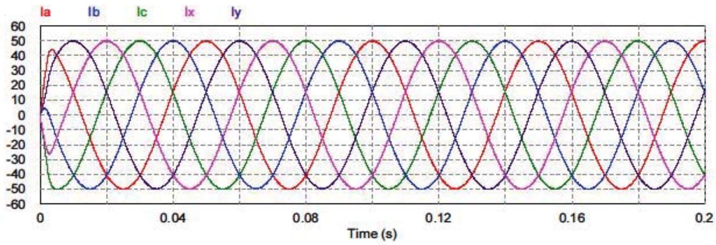

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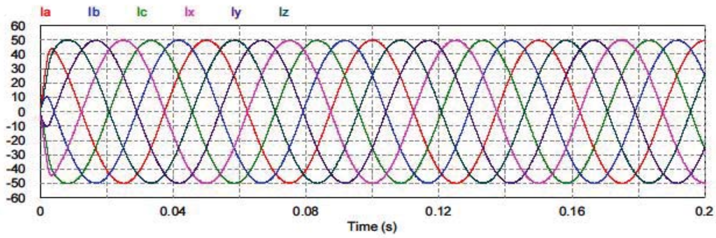

g)

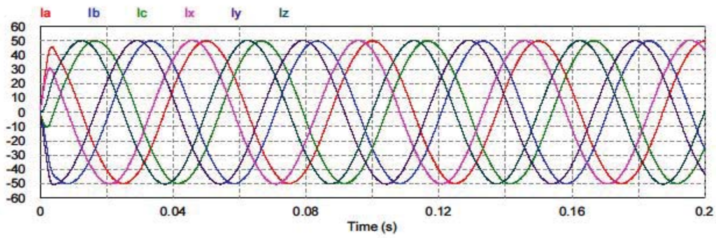

Fig. 12. Example results of the simulation of the work of the modulator in the form of waveforms of voltages and currents: a) the components of the reference current vector i_{dref}, i_{qref}; the components of the output current vector i_d, i_q; b) the components of the output current vector i_α, i_β; i_d, i_q.
The output phase currents in the *n*-phase systems: c) three-phase; d) four-phase; e) five-phase; f) six-phase; g) dual three-phase.

6. CONCLUSIONS

The article presents an extended Clarke Transformation for motors and drive systems with a number of phases where $n > 3$, called multiphase systems.

- It applies to induction, synchronous and synchronous permanent magnet and brushless motors.
- Because motors with more than 3 phases cannot be supplied directly from a three-phase grid, they are therefore supplied from transistor converters with the number of branches corresponding to the number of motor phases.

- The Extended Clarke Transformation can transform space vectors written in the n-phase system:
  - The transformation of vector $\boldsymbol{X}_n$ from a stationary n-phase coordinate system into a stationary $\alpha\beta$ orthogonal coordinate system is accomplished by means of the matrix $\boldsymbol{T}_{\alpha\beta/n}$ (5). This transformation concerns the vectors of current $\boldsymbol{I}_n$ or voltage $\boldsymbol{U}_n$.
  - The inverse transformation from the vector written $\boldsymbol{X}_{\alpha\beta}$ in the stationary $\alpha\beta$ orthogonal coordinate system to vector $\boldsymbol{X}_n$ in the stationary n-phase coordinate system is accomplished by means of the matrix $\boldsymbol{T}_{n/\alpha\beta}$ (25). This transformation applies the voltage vector $\boldsymbol{U}_n$ and allows you to apply n control signals to the n-phase inverter.

- Application of the extended Clarke Transformation allows calculations to be performed in orthogonal coordinate systems, whether stationary $\alpha\beta$ or rotating dq. This allows the use of different control strategies in rotor-oriented or field-oriented control.
- As shown (Tab. 1) the practical application have drive systems with five-phase, six-phase or dual-three phase motors.

LITERATURE

1. Rogalski A., Bienkowski K.: Multiphase windings of induction machines. 39 International Symposium on Electrical Machines SME 2003, 9 – 11 June, 2003 r., Gdańsk – Jurata.
2. Rogalski A., Bienkowski K.: Do induction motors have to be three-phased? 41 International Symposium on Electrical Machines SME 2005, 14 – 17 June, Jarnołtówek.
3. Pienkowski K.: Analysis and control of multi-phase squirrel–cage induction motor. Scientific papers of the Institute of Electrical Machines, Drives and Measurements, Wroclaw University of Technology, No. 65, pp. 305-319, 2011.
4. Jakubiec B.: The drive of electric vehicle with multi-phase permanent magnet synchronous motor. Przegląd Elektrotechniczny, ISSN 0033-2097, No. 12, pp. 125–128, 2015.
5. Rolak M., Che H.S., Malinowski M.: Modeling and Fault-tolerant control of 5-phase induction machine. Bulletin of Polish Academy of Sciences, Technical Sciences, Vol. 63, No. 4, pp. 997–1006, 2015.
6. Min Kang, Jin Huang, Hai-bo Jiang, Jia-qiang Yang: Principle and Simulation of a 5-phase Bearingless Permanent Magnet-Type Synchronous Motor. International Conference on Electrical Machines and Systems, ICEMS 2008, 17–18 October, 2008 r.
7. Chang Liuchen, Muszynski Jerzy (Jerry): Design of a 5-Phase Permanent Magnet Brushless DC Motor for Automobiles. Vehicular Technology Conference, 2003. VTC 2003-Fall. 2003 IEEE 58th, 6-9 October, 2003.
8. Ren Yuan, Zhu Z. Q.: Reduction of Both Harmonic Current and Torque Ripple for Dual Three-Phase Permanent-Magnet Synchronous Machine Using Modified Switching-Table-Based Direct Torque Control. IEEE Transaction on Industrial Electronics, Vol. 62, No. 11, pp. 6671–6683, November, 2015.

9. Kontarcek A., Bajec P., Nemec M.: Cost-Effective Three-Phase PMSM Drive Tolerant to Open-Phase Fault. IEEE Transaction on Industrial Electronics, vol. 62, No 11, pp. 6708–6718, November, 2015.
10. Janaszek M.: New method of direct reactive energy and torque control for permanent magnet synchronous motor. Bulletin of the Polish Academy of Sciences. Technical Sciences, Vol. 54, No. 3, pp. 299-305, 2006.
11. Janaszek M.: Experimental drive with direct torque control of permanent magnet synchronous motor. Archives of Electrical Engineering, Vol. 50, No. 1. pp. 73–92; 2001.
12. Kazmierkowski M.P., Krishnan R., Blaabjerg F.: Control in Power Electronics: Selected Problems. San Diego: Academic Press. 2002.
13. Duesterhoeft W. C., Schulz M. W., Clarke E.: Determination of Instantaneous Currents and Voltages by Means of Alpha, Beta, and Zero Components. AIEE Transaction 70, July, pp. 1248–1255, 1951.
14. Figna W., Pieńkowski K.: Vector control of induction motor with dual stator Winding. Electrical Machines - Transaction Journal, No. 86, pp. 67–70, 2010.
15. Listwan J., Pieńkowski K.: Analysis of vector control of multi-phase induction motor. Electrical Machines - Transaction Journal, No. 3, pp. 235–240, 2014.
16. Navarro J.: Secret life of numbers. Mathematical curiosities. BUKA Books 2012.
17. Navarro J.: Fleeting ideas, the eternal assertion. Big problems of mathematics. BUKA Books, 2012.

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ROZSZERZONA TRANSFORMACJA CLARKE DLA UKŁADÓW *n*-FAZOWYCH

Michał JANASZEK

STRESZCZENIE *W artykule przedstawiono sposób przeliczania wektorów przestrzennych zapisanych w układzie składającym się z liczby faz* $n > 3$ *do zapisu w układzie prostokątnym* $\alpha\beta$ *będący rozszerzeniem transformacji Clarke dla układów wielofazowych. Zdefiniowano również przekształcenie odwrotne wektora z zapisu w układzie prostokątnym* $\alpha\beta$ *do zapisu w układzie n-fazowym. Zastosowanie rozszerzonej transformacji Clarke pozwala na wykonywanie obliczeń regulacyjnych w układach współrzędnych prostokątnych, stacjonarnym* $\alpha\beta$ *lub wirującym dq. Możliwe są realizacje różnych strategii sterowania. Praktyczne zastosowanie mają napędy z silnikami 5-fazowymi, 6-fazowymi lub dualnymi 3-fazowymi.*

Słowa kluczowe: *napęd elektryczny, układy wielofazowe, transformacja Clarke*

Michał JANASZEK, Ph.D. – graduated from the Faculty of Electrical Engineering at the Warsaw University of Technology with a specialization in automation in 1974. In October 1974 he began working at the Electrotechnical Institute in the Department of Electric Machine-Tool Drives. He specialized in DC drives with transistor power amplifiers for numerically controlled machine tools and industrial robots. He dealt with the identification of the dynamic parameters of direct current machines, and the numerical modeling of the dynamics in the synthesis of drive systems. He participated in the development and preparation for production of drives for industrial robots of the IRb type. Since the eighties he has been involved in drives with synchronous motors, in particular focusing on the problems of optimization control. He defended his doctoral thesis "The problems of direct torque and flux control of synchronous motors with a permanent magnet," in 2001. He is the author or co-author of several publications on electric drives. For several years he has dealt with drives designed for passenger vehicles, trucks and buses. He is a member of the Polish Society of Theoretical and Applied Electrical Engineering.