TUNNELING THROUGH A BARRIER UNDER TRANSVERSE MAGNETIC FIELD AND *I-V* CHARACTERISTIC

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Abstract: The case whereby the transmission coefficient through a barrier, sandwiched by semiconductor reservoirs, under bias is provided by a general formula involving the logarithmic wave function derivative at the barrier entrance is now extended to include the influence of magnetic field perpendicular to the longitudinal barrier direction. Under the circumstances, the equation governing the logarithmic wave function derivative is appropriately modified via an effective potential energy which takes account of the magnetic field. Subsequently, the procedure for obtaining the transmission coefficient is applied to the case involving a smooth double, as well as quadruple, barrier for which the *I-V* characteristic is obtained. The results show reduction in current with increase in the magnetic field, up to a certain value of bias. Furthermore, increase in temperature exhibits increase in current as well as movement of the current peaks in the *I-V* curves towards lower bias.

Keywords: transmission coefficient, momentum-like quantity, *I-V* characteristic
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1. Introduction

In a previous paper, a general formula for the transmission coefficient through a barrier and application to *I-V* characteristic was obtained [1]. Presently, we extend the methodology, employed, to include magnetic field perpendicular to the longitudinal barrier direction confined within the barrier region. Such a state of affairs, on account of the extremely difficult if not impossible realization, as far as the experimental situation is concerned, it has to do with thought experiment [2]. However, the influence of transverse magnetic field within the barrier region plays an important role in the formation of the *I-V* characteristic in the case of an applied transverse magnetic field extended beyond the barrier region. Therefore, one expects useful information deriving from the study of the above thought experiment. At this point it is worth noting argument with respect to ignoring the influence of transverse magnetic field outside the barrier region based on ionized impurity scattering essentially destroys the coherence of the Landau motion [3]. A similar work to [3] as far as the methodology is concerned, namely use of transfer matrix procedure appears in [4]. A further work whereby the transverse magnetic field is dealt within the barrier region [5], employs the Wentzel-Kramers-Brillouin approximation for obtaining the transmission coefficient. In the above works there appears reduction in current in the *I-V* characteristic under the influence of magnetic field, a fact which L. Esaki and coworkers were aware of, as it appears through private communication [6], whereby experimental magneto-tunneling effects are presented. Further experimental results are presented in references [7–9].

In Section 2 we proceed to obtain the transmission coefficient in the case whereby the barrier region is acted upon by a constant transverse magnetic field to the longitudinal direction. Following the procedure developed [1] and taking into account the effect of the magnetic field we find that the transmission coefficient depends not only on the incoming kinetic energy, E_1, together with the applied voltage, V, in addition to the magnetic field, B, also on the wave vector component, k_2, perpendicular to the magnetic field as well as to the longitudinal direction.

Although, in the previous work it became possible to consider different effective masses for the carrier in the two reservoirs and the barrier region, presently we restrict the study to two effective masses, one in the longitudinal barrier direction and another for the reservoirs as well within the barrier region in direction perpendicular to both the magnetic field and the barrier direction.

2. Transmission coefficient

As previously, for the purpose of facilitating subsequent discussion we shall introduce a tri-orthogonal reference frame, $Oxyz$, relative to our nanostructure composed, as earlier stated, of a thin layer whereby the barrier resides together with two semiconducting reservoirs, each attached on either side of the obstruction layer. The x-axis is taken perpendicular to the thin layer and the origin occupies its middle. The y- and z-axes are taken so as to complete the tri-orthogonal system, with the z-axis taken parallel to the applied magnetic field and y-axis parallel to the thin layer. For reasons of subsequent communication we denote the left reservoir, the barrier region, agnd the right reservoir by (1), (o), (2) correspondingly. Assuming the thickness of the barrier region $2a$ and the barrier potential energy $U_o(x)$ the potential energy experienced by a carrier with zero magnetic field under bias V across the device can take the form

$$\begin{array}{llc} & & \text{Region} \\ U(x)=0, & x<-a & (1) \\ U(x)=U_o(x)-\dfrac{qV}{2a}(x+a), & -a\le x\le a & (\text{o}) \\ U(x)=-qV, & x>a & (2) \end{array}$$

(1), (2), (3)

where q stands for the carrier charge and V for the applied voltage.

Subsequently, we shall deal with the equations governing the carrier dynamics in the above three regions in the presence of transverse magnetic field, within the region (o), along the z-direction. As far as the effective masses are concerned we shall consider the case whereby the carriers in the two reservoirs have the same effective mass, $m=\mu m_c$, while the effective mass in the barrier region along the x-direction is taken $m_o=\mu_o m_c$ and along the y-direction equals m where m_c stands for the free carrier mass.

The Hamiltonian in the regions (1), (o), (2) takes the form

$$H_1=-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) \tag{4}$$

$$H_o=-\frac{\hbar^2}{2m_o}\frac{\partial^2}{\partial x^2}+\frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial y}-\frac{qB}{c}x\right)^2-\frac{\hbar^2}{2m_o}\frac{\partial^2}{\partial z^2}+U(x) \tag{5}$$

$$H_2=-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)-qV \tag{6}$$

The wave function form in the three regions for the purpose of transmission takes the form

$$\Psi_1(x,y,z)=\left(e^{ik_1x}+Re^{-ik_1x}\right)e^{ik_2y+ik_3z} \tag{7}$$

$$\Psi_o(x,y,z)=\Phi_o(x)e^{ik_2y+ik_3z} \tag{8}$$

$$\Psi_2(x,y,z)=T_1e^{iK_1x}e^{ik_2y+ik_3z} \tag{9}$$

where $\hbar k_1$ represents the incoming momentum towards the barrier, R the reflection amplitude and T_1 the transmission amplitude. At this point, we state for later use, the expression for the incoming kinetic energy, E_1, which is given via $E_1=\hbar^2k_1^2/2\mu m_c$. The function $\Phi_o(x)$ will be determined subsequently.

The Schrödinger equation in the regions (1), (o), (2) becomes

$$H_1\Psi_1=\frac{\hbar^2}{2\mu m_c}(k_1^2+k_2^2+k_3^2)\Psi_1 \tag{10}$$

$$H_o\Psi_o=\frac{\hbar^2}{2\mu m_c}(k_1^2+k_2^2+k_3^2)\Psi_o \tag{11}$$

$$H_2\Psi_2=\frac{\hbar^2}{2\mu m_c}(k_1^2+k_2^2+k_3^2)\Psi_2 \tag{12}$$

Clearly, we have the same eigenvalue in the three regions, for the Schrödinger equation. From (9) we can easily determine the wave vector component K_1 in (12) as

$$K_1=\frac{1}{\hbar}\sqrt{2\mu m_c(E_1+qV)} \tag{13}$$

On account of the fact the eigenfucntion in the regions (1), (o), (2) is expressed as product of the same function of y and z namely, $\exp(ik_2y+ik_3z)$ times a function of x it becomes possible to obtain the equations governing the transmission process in terms of the longitudinal variable, x, as follows:

$$-\frac{\hbar^2}{2\mu m_c}\frac{\partial^2}{\partial x^2}\left(e^{ik_1x}+Re^{-ik_1x}\right)=E_1\left(e^{ik_1x}+Re^{-ik_1x}\right) \tag{14}$$

$$-\frac{\hbar^2}{2\mu_o m_c}\frac{\partial^2}{\partial x^2}\Phi_o(x)+\left[U(x)+U_m(x)\right]\Phi_o(x)=E_1\Phi_o(x) \tag{15}$$

$$\left(-\frac{\hbar^2}{2\mu m_c}\frac{\partial^2}{\partial x^2}-qV\right)Te^{iK_1x}=E_1Te^{iK_1x} \tag{16}$$

where in (15) U_m constitutes effective potential energy, which takes account of the transverse magnetic field, B, and is given by

$$U_m(x)=\frac{1}{\mu}\left(\frac{m_c}{2}\omega^2x^2-\omega x\hbar k_2\right) \tag{17}$$

where ω in (17) stands for the cyclotron frequency associated with free carrier mass

$$\omega=\frac{qB}{m_c c} \tag{18}$$

and k_2 is the wave vector component associated with the y-direction.

Upon solution of the Equations (14)–(16), above, in the regions (1), (o), (2) taking account of the probability and current density continuity conditions at the barrier longitudinal boundaries we can be led to the required transmission coefficient,

$$T_c=\frac{K_1}{k_1}\left|T_1\right|^2=1-\left|R\right|^2 \tag{19}$$

It should be noted, here, that the possibility of acquiring Schrödinger equation depending on a single variable, x, is not generally feasible in the case of different effective masses. Presently our study is based on the choice of a case whereby the effective masses are equal in the y- and z-direction in the three regions (1), (o), (2). In what follows we shall proceed in accord with the circumstances provided by the restricted case, above.

Utilizing the Equations (14)–(16) we can proceed employing the sort of momentum related quantity formalism [1], developed earlier, for obtaining the relevant transmission coefficient, which now in the presence of magnetic field depends in addition to the incoming longitudinal energy, E_1, and the applied voltage, V, also on the wave vector component, k_2, in the y-direction, perpendicular to the magnetic field and the longitudinal one, as well as the magnetic field, B, *i.e.* we have $T_c=T_c(E_1,V,k_2,B)$. At this point one can draw an association between the pair of the voltage, V, together with the longitudinal component of the wave vector, k_1, and the pair of the magnetic field, B,together with the wave vector component k_2 in the direction perpendicular to the magnetic field and the longitudinal one.

Let us, now, introduce the longitudinal momentum-like quantity, in the region (o) as

$$p_o(x) = \frac{\hbar}{i} \frac{\Phi_o'(x)}{\Phi_o(x)} \tag{20}$$

where $\Phi_o'(x)$ in (20) stands for the derivative of $\Phi_o(x)$ with respect to x. With the aid of (15) we derive the equation governing $p_o(x)$, as

$$\frac{\hbar}{2m_o i} \frac{\partial p_o(x)}{\partial x} + \frac{p_o(x)^2}{2m_o} + \frac{U_o(x) - qV}{2a}(x+a) + \frac{1}{\mu}\left(\frac{m_c}{2}\omega^2 x^2 - \hbar\omega k_2 x\right) = E_1 \tag{21}$$

The magnetic field effect is incorporated in (21) through the terms involving the cyclotron frequency, ω.

Let us now proceed to obtain the transmission coefficient through the barrier, utilizing (21) under the probability and current density continuity conditions at the barrier boundaries, $-a$ and a. Introducing the notation

$$\Phi_1(x) = e^{ik_1 x} + Re^{-ik_1 x}, \quad \Phi_2(x) = T_1 e^{iK_1 x} \tag{22}$$

for the form of the eigenfunction in (14), (16) correspondingly in the regions (1) and (2) the continuity conditions are expressed, as

$$\Phi_1(-a) = \Phi_o(-a), \quad \frac{1}{\mu}\Phi_1'(-a) = \frac{1}{\mu_o}\Phi_o'(-a) \tag{23}$$

$$\Phi_o(a) = \Phi_2(a), \quad \frac{1}{\mu_o}\Phi_o'(a) = \frac{1}{\mu}\Phi_2'(a) \tag{24}$$

Combining (20) together with (24) and (13) we obtain the required expression, under the continuity conditions, for the pseudo-momentum at the boundary exit, $x = a$, as

$$p_o(a) = \frac{\mu_o}{\mu}\sqrt{2\mu m_c (E_1 + qV)} \tag{25}$$

Solving (21) under the condition (25) we can determine $p_o(-a)$, the required value for $p_o(x)$ at the at the barrier entrance, $x = -a$. Furthermore, through (23) and (22) in conjunction with (20) we are led to the equation providing the reflection amplitude, R, given by

$$p_o(-a) = \frac{e^{-ik_1 a} - Re^{ik_1 a}}{e^{-ik_1 a} + Re^{ik_1 a}} \frac{\mu_o}{\mu_1}\sqrt{2\mu_1 E_1} \tag{26}$$

Solving (26) with respect to R we obtain the transmission coefficient, T_c, via $T_c = 1 - |R|^2$, as

$$T_c = \frac{4\mu_o Re\left[p_o(-a)\right]\sqrt{2\mu_1 m_c E_1}}{\mu_1 \left|p_o(-a)\right|^2 + 2\mu_o Re\left[p_o(-a)\right]\sqrt{2\mu_1 m_c E_1} + 2\mu_o^2 m_c E_1} \tag{27}$$

In what follows we shall proceed obtaining *I-V* characteristics associated with the three region system, above, with the aid of the transmission coefficient.

3. *I-V* characteristic and numerical results

Once the transmission coefficient in the absence of magnetic field through a barrier nanostructure sandwiched between two reservoirs, is made available, we can obtain the system *I-V* characteristic following the procedure employed by Tsu-Esaki [10]. However, presently the transmission coefficient depends in addition to the incoming momentum, $\hbar k_1$, also to the momentum $\hbar k_2$ due to the applied magnetic field, $(0,0,B)$. Under the circumstances, taking account of the dependence of the transmission coefficient on k_2 the expression for the current density takes the form

$$J = \frac{q}{4\pi^3\hbar}\int\limits_{0}^{\infty} dE_1 \int\limits_{-\infty}^{\infty} dk_2 T_r(E_1,V,k_2,B)\int\limits_{-\infty}^{\infty} dk_3\,[f(E)-f(E')] \tag{28}$$

where

$$f(E) = \frac{1}{1+\exp\left[\left(E-E_f\right)/\kappa T\right]} \tag{29}$$

$$E = E_1 + \frac{\hbar^2 k_2^2}{2m_1} + \frac{\hbar^2 k_3^2}{2m_1}, \quad E' = E + qV \tag{30}$$

E_f in (29) stands for the chemical potential (energy) associated with temperature T.

Expression (28) for the current density takes account of the fact that the transmission coefficient depends on the parameter k_2, and this on the basis of the prevailing Fermi-Dirac statistics. As far as we know in the literature this is ignored, *e.g.* [3–5], considering a fixed value for k_2 or in general taking $k_2 = 0$ in the relevant expression for the transmission coefficient and subsequently proceeding via the Tsu-Esaki formalism [10], without relevant modification as in (28). The above procedures lead to differing results, as it will become evident in examples which follow in this section.

With the aid of (27) together with (28) and (29)–(30) we can proceed with numerical evaluation for obtaining the *I-V* characteristic for given magnetic field, B and temperature, T. The numerical procedure can be facilitated utilizing as basic energy unit $E_u = 0.1\,\text{eV} = 1.6021917\times10^{-13}$ erg, from which via $\hbar^2/m_c L_u^2 = E_u$ we obtain the unit of length as $L_u = \hbar/\sqrt{m_c E_u}$. The unit of momentum becomes $p_u = \hbar/L_u = \sqrt{m_c E_u}$. The unit of time is given by $T_u = \hbar/E_u$. The voltage unit is obtained as $V_u = E_u/q$, where q stands for the absolute value of the electron charge. Finally, we need the current density unit which takes the form $J_u = q/4\pi^3 T_u L^2{}_u$. It should be noted that the choice of the value of energy unit, E_u, is based on the fact that the usual barrier height is on the order of a few tenths of eV.

As far as the *I-V* characteristic is concerned it should be noted, here, that on account of the dependence of the transmission coefficient, T_c, on the wave vector component k_2 one cannot proceed integrating separately $f(E)$ and $f(E')$ with respect to k_2 and k_3, following the Tsue-Esaki procedure [10] valid for the case without magnetic field. In the case with transverse magnetic field one can only integrate $f(E)-f(E')$ separately over k_3, initially, as seen in (28). Definitely the choice of (28), whereby the transmission coefficient dependence on k_2 is necessary for more accurate results.

On the basis of the units exposed, earlier, one can proceed numerically via (28) whereby the current density, J, expressed in terms of the applied voltage, V, is employed for obtaining the *I-V* characteristic. In what follows we shall deal with a potential energy barrier given by

$$U_o = \frac{u_o}{4}\left\{\sin\left[\frac{2\pi}{\lambda}\left(x - \frac{\lambda}{4}\right)\right] + 1\right\}^2 \tag{31}$$

extending over the range $-a \le x \le a$, u_o being the barrier potential energy height. Formula (31) for $a = \lambda$ represents a smooth double barrier, while for $a = 2\lambda$ a quadruple barrier, both with same width. An example of smooth quadruple barrier as well as the effect of applied bias together with transverse magnetic field is depicted in Figure 1, below, in which the transmission coefficient associated with the data of the continuous curve in Figure 1(a).

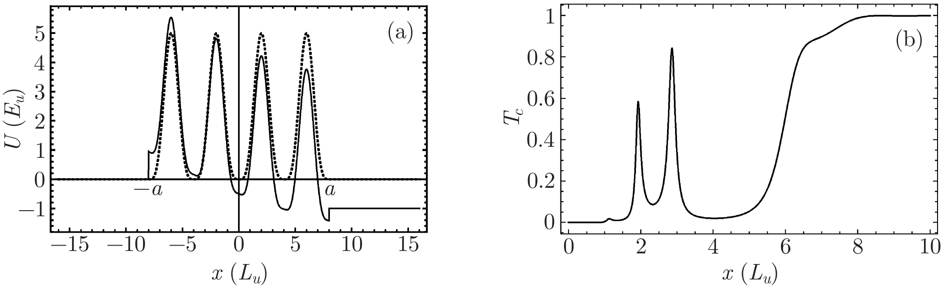

Figure 1. (a) Dashed curve shows barrier potential energy for smooth quadruple barrier, as obtained by (31), between $-a$ and a, $a = 2\lambda$, $\lambda = 4L_u$ and height $u_o = 5E_u$. Continuous curve shows potential energy barrier, above, in barrier region and part of the sandwiching reservoirs under transverse magnetic field in barrier region $B = 20$ Tesla, $k_2 = 0.24{L_u}^{-1}$ and bias $V = 0.1$ Volt. (b) Shows transmission coefficient corresponding to data in (a)

Subsequently, we provide figures, for either double or quadruple barrier, expressing the influence of magnetic field and temperature on the *I-V* characteristic. As far as the influence of transverse magnetic field, acting on the barrier region, on the *I-V* characteristic is exemplified in the cases of double and quadruple smooth barriers at low temperature and various values of the magnetic field, while the carrier density, in both cases, in the reservoirs remains the same.

Evidently, the state of affairs whereby we experience current reduction, on account of applied magnetic field, ceases at a certain value of applied voltage beyond which the situation gets reversed. It should be noted here that the magnetic field acts in the barrier region.

Finally, we shall proceed showing the effect of temperature on the current utilizing relevant *I-V* characteristics, as in Figure 3.

From Figures 3(a)(b) there appears that increase in temperature leads to higher current, appart from a minute bias region at low voltage in the double barrier case. Furthermore, from comparison of the *I-V* characteristics associated with double and quadruple barrier one sees that the quadruple barrier exhibits

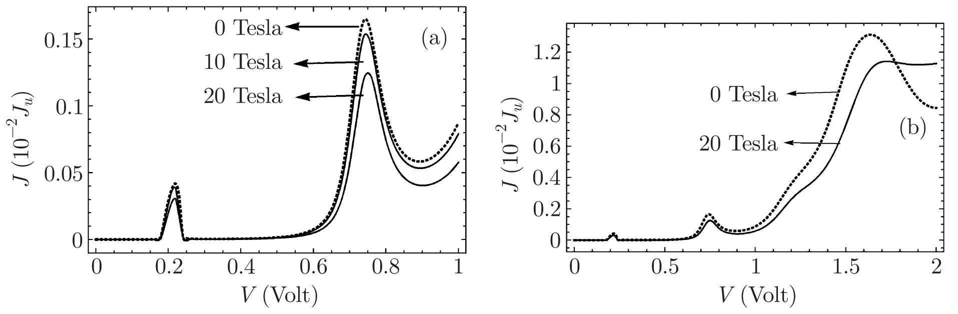

Figure 2. (a) Shows *I-V* characteristics for a double barrier, sandwiched by semiconducting reservoirs, under transverse magnetic fields, 0, 10, 20 Tesla. Rest of data: $u_o = 5E_u$, $a = \lambda$, $\lambda = 8L_u$, $\mu_o = 0.1$, $\mu = 0.065$, $T = 4.2°\text{K}$, $E_f = 0.289014 E_u$, $n \simeq 3.69846 \times 10^{17}/\text{cm}^3$. (b) Shows *I-V* characteristics for device as in (a) under 0 transverse magnetic field, and 20 Tesla with extended bias up to 2 Volt. Data as in (a)

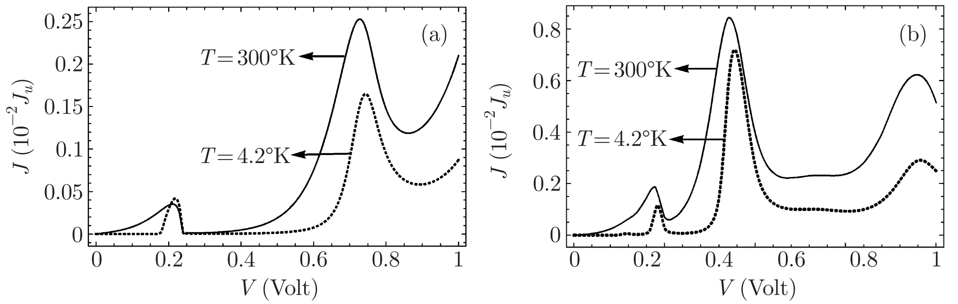

Figure 3. Shows influence of temperature on the *I-V* characteristics. (a) Case of double barrier, $\lambda = 8L_u$, under 0 magnetic field and (b) Case of quadruple barrier, $\lambda = 4L_u$, under 20 Tesla magnetic field. Data in common: Barrier height $u_o = 5E_u$, barrier width $2a = 16L_u$, carrier density in reservoirs $n \simeq 3.69846 \times 10^{17}/\text{cm}^3$, longitudinal effective mass coefficient $\mu_o = 0.1$, elsewhere $\mu = 0.065$. For above carrier density the chemical potential associated with temperature $T = 4.2°\text{K}$ is $E_f = 0.289014 E_u$, while for $T = 300°\text{K}$ becomes $E_f = 0.05 E_u$.

I-V characteristics with higher peaks and higher peak to valley ratios. The state of affairs, above, occurs in spite of the fact that the quadruple barrier is under strong transverse magnetic field, which causes reducion in current.

4. Conclusion

The procedure expounded in the present work for obtaining the transmission coefficient through a barrier under transverse magnetic field in terms of the applied bias, based on solving appropriate momentum-like equation can deal with every barrier and can lead to the corresponding *I-V* characteristic. The facility of handling all barriers is useful for choosing a suitable barrier for a given requirement, *e.g.* *I-V* characteristic exhibiting large negative differential resistance. An example for the above sort of choice we encountered in Figures 3(a)(b) whereby the quadruple barrier is more suitable in comparison with the double barrier.

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