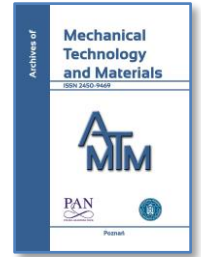


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Multi-scale curvature tensor analysis of machined surfaces

Tomasz Bartkowiak^{a*}, Christopher Brown^b^a Poznan University of Technology, Institute of Mechanical Technology, Pl. M. Skłodowskiej-Curie 5, 60-965 Poznan, Poland^b Worcester Polytechnic Institute, Surface Metrology Laboratory, Mechanical Engineering Department, USA

*Corresponding author, Tomasz.bartkowiak@put.poznan.pl

ARTICLE INFOReceived 00 Month 0000
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This paper demonstrates the use of multi-scale curvature analysis, an areal new surface characterization technique for better understanding topographies, for analyzing surfaces created by conventional machining and grinding. Curvature, like slope and area, changes with scale of observation, or calculation, on irregular surfaces, therefore it can be used for multi-scale geometric analysis. Curvatures on a surface should be indicative of topographically dependent behavior of a surface and curvatures are, in turn, influenced by the processing and use of the surface. Curvatures have not been well characterized previously. Curvature has been used for calculations in contact mechanics and for the evaluation of cutting edges. In the current work two parts were machined and then one of them was ground. The surface topographies were measured with a scanning laser confocal microscope. Plots of curvatures as a function of position and scale are presented, and the means and standard deviations of principal curvatures are plotted as a function of scale. Statistical analyses show the relations between curvature and these two manufacturing processes at multiple scales.

1. INTRODUCTION

The objective of this paper is to demonstrate the use of the new multi-scale curvature analysis and characterization methods for areal surface topography. Curvature can be characterized as a second order tensor as a function of scale and position from areal topographic measurements, i.e., surfaces heights ($z=z(x,y)$). This kind of areal, multi-scale curvature analysis has been recently developed to better understand surface topographies, how they are manufactured and how they behave.

Surface textures are commonly characterized using a simple height parameter, such as average roughness, the mean of the absolute values of the heights measured from a mean line, Ra (for profiles) or Sa (for surfaces). Typically characterization analysis is applied to topographic measurements after removing form and waviness by applying a Gaussian filter [13]. The average roughness can be shown to be most sensitive to the longest wavelengths remaining after filtering, as wavelength amplitudes tend to increase with their lengths for the majority of commonly measured surfaces. A characterization method is most valuable for assisting process

and product design when it can help to discover correlations between manufacturing processes and the resulting topographies or between topographies and performance, such as adhesion or wetting behavior. When these kinds of conventional height parameters are tested for correlations with behavioral measures or process parameters with conventional filtering with large band passes for waviness and roughness relations with specific texture scales are missed, especially at the finer scales. However, conventional parameters which showed weak correlations with traditional filtering have been shown to correlate strongly when a narrow band-pass filter is applied at an appropriate scale, i.e., center wavelength or spatial frequency [3]. The conventional height parameters also do not provide lateral information, e.g., characterization of the spacing or sequence of the heights. The motif analysis, that was developed in the 1980s in France, proved to be successful in the characterization of functional properties of surfaces, especially in friction and contact problems [4][8]. Today, motif parameters are less used but the conclusions regarding the relationship between function and specification still remain crucial.

There are other surface parameters that can better characterize complex surfaces to assist in providing functional correlations. It has been shown for friction [3][12], fracture surfaces [6], and adhesion [5] that the strength of functional correlations can depend on using the appropriate scale from a multi-scale characterization of the surface area. These works used multi-scale geometric characterizations and multi-scale regression analyses; specifically, they evaluated the relative areas over a range of scales and then performed regression analyses over these same scales to determine the strength of the correlations between friction, fracture, and adhesion with relative areas, all with respect to scale.

Multi-scale curvature analysis of profiles has recently been developed and proved useful in fatigue [10] and multi-scale curvature characterization. Multi-scale curvature tensor analysis method is an addition to the aforementioned characterization. It was successfully used to describe complex new types of textures that are created by additive manufacturing [2].

Moreover, the curvature tensor can be a powerful tool in surface metrology as curvature, unlike conventional height parameters, is independent of orientation. Therefore, levelling the measured surface data before computation is no longer an important issue. In addition, curvature can indicate convex and concave regions. The former are important in contact mechanics and the latter can be significant in fatigue and in fracture mechanics. Curvature analysis can identify microgrooves and holes, which act as stress concentrators. It can be also helpful in identifying lay, or surface directionality, i.e., anisotropy, by analyzing the orientation of maximal and minimal curvature vectors. Finally, because curvature is a tensor, it is possible to calculate gradient, divergence and curl, which can help in better characterization of particular surface features.

2. METHODOLOGY OF RESEARCH

To illustrate the surface curvature analysis two surfaces were measured. First, steel (S275) cuboidal parts were milled on a conventional machine tool, with a 500 $\mu\text{m}/\text{rev}$ feed at 1,000 rpm with a 80 mm diameter face mill. Then, one part was ground with an aluminum oxide (A97) wheel.

Four measurements were made of each surface with an Olympus scanning laser confocal microscope (OLS4100) with a 20x lens. Initially, each surface measurement consisted of over a million (1024x1024) height samples, over a region of 620x620 μm . The region is reduced to 500x500 height samples to facilitate the calculation of the 3D curvature tensors, which is computationally intensive.

A 3D normal based method is used to calculate the curvatures. First, normals are computed using a covariance matrix method on the closest 3x3 neighborhood. The height samples on the edge are excluded from further analysis because a complete 3x3 neighborhood is not available. The curvature tensor \mathbf{T} is a symmetric 3x3 matrix, denoted as

$$\mathbf{T} = \mathbf{P} \cdot \mathbf{D} \cdot \mathbf{P}^{-1}, \quad (1)$$

where $\mathbf{P} = (\mathbf{k}_1, \mathbf{k}_2, \mathbf{n})$ and

$$\mathbf{D} = \begin{pmatrix} \kappa_1 & 0 & 0 \\ 0 & \kappa_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (2)$$

The eigenvalues κ_1 and κ_2 are the principal curvatures. The eigenvectors \mathbf{k}_1 , \mathbf{k}_2 are the corresponding principal directions for the principal curvatures and \mathbf{n} is the surface normal.

Theisel et al. showed a new technique for estimating curvature tensor \mathbf{T} in a triangular mesh that incorporates calculation of Weingarten curvature matrix [11] based on triangular patches [9]. That method shows better error behavior than a cubic fitting (Goldfeather et al. [7]) and is independent of rotations of the mesh and does not involve any parameterization or fitting. What is important, the accuracy of this method is dependent mostly on the accuracy of the estimation of the normals. The selection of points, i.e., height samples (with their positions), for the estimation of normals and the creation of triangular patches at a particular scale is an important issue.

The calculation method used here is derived directly from the work of Theisel et al. [9]. For the estimation of curvature at each scale in the analysis used here, height samples are taken from the original measurement. Down sampling is used for approximating the appropriate, corresponding scale. The point cloud, a regular array in x and y of measured height samples (z), is tiled to create triangular patches. Each triangle is right-angled and isosceles, in its (x,y) projection, and lengths of both catheti are equal to the scale of the calculation [1].

3. RESULTS AND DISCUSSION

Renderings of the four measurements of the milled and the ground surfaces are shown in Figures 1 a) and b) respectively. Note that the vertical scale for the milled surface is about that of the ground surface (there is 2.5-fold difference). The difference grinding makes is striking.

Conventional height parameters for both surfaces are shown in Table 1. The machined and milled surfaces are clearly different. The arithmetic mean and root mean square of the heights, S_a and S_q , for milled surfaces are more than five times larger than for the ground. The peak to valley distance, peak heights and valley depths, S_z , S_p and S_v , for the milled surface are more than twice the ground. The skew of the heights, S_{sk} , changes from positive to negative with grinding, as expected because the grinding tends to remove the peaks. These conventional analyses are consistent with the curvature results at the largest scales, shown and discussed below. However they do not provide fine scale lateral information, like multi-scale methods do, nor do they supply lateral information describing the more complex geometric nature of the surface, like the curvature does.

Maximum principal curvatures κ_1 calculated for two different scales from as-milled and as-ground region are shown in Fig. 2. The vertical scale is the same in all the figures to make them easier to compare.

Different scales can show different kinds of features and anisotropies. Positive curvatures indicate concavity and negative convexity. At any particular scale, if at some location both principle curvatures are positive, i.e., if κ_2 is positive, then this location is a pit, at that scale. If both are negative at some location, i.e., if κ_1 is negative, then this is a peak at that scale. And, if one principle curvature is positive and the other is negative it represents a saddle, also at that scale.

Table 1. Conventional roughness parameters calculated on the measured samples

	MILLED				GROUND				
	<i>M.1</i>	<i>M.2</i>	<i>M.3</i>	<i>M.4</i>	<i>G.1</i>	<i>G.2</i>	<i>G.3</i>	<i>G.4</i>	
Sq	3.24	4.49	3.61	4.51	0.826	0.767	0.904	0.668	μm
Ssk	0.893	0.353	0.668	0.307	-0.471	-0.767	-0.113	-0.356	
Sku	2.55	2.48	2.61	1.93	3.37	3.97	3.05	3.30	
Sp	12.4	14.1	15.8	21.5	6.27	7.41	10.2	5.01	μm
Sv	7.23	17.7	21.1	10.7	5.67	4.43	6.38	7.21	μm
Sz	19.7	31.8	36.8	32.1	11.9	11.8	16.6	12.2	μm
Sa	2.76	3.71	3.09	3.97	0.651	0.591	0.737	0.534	μm

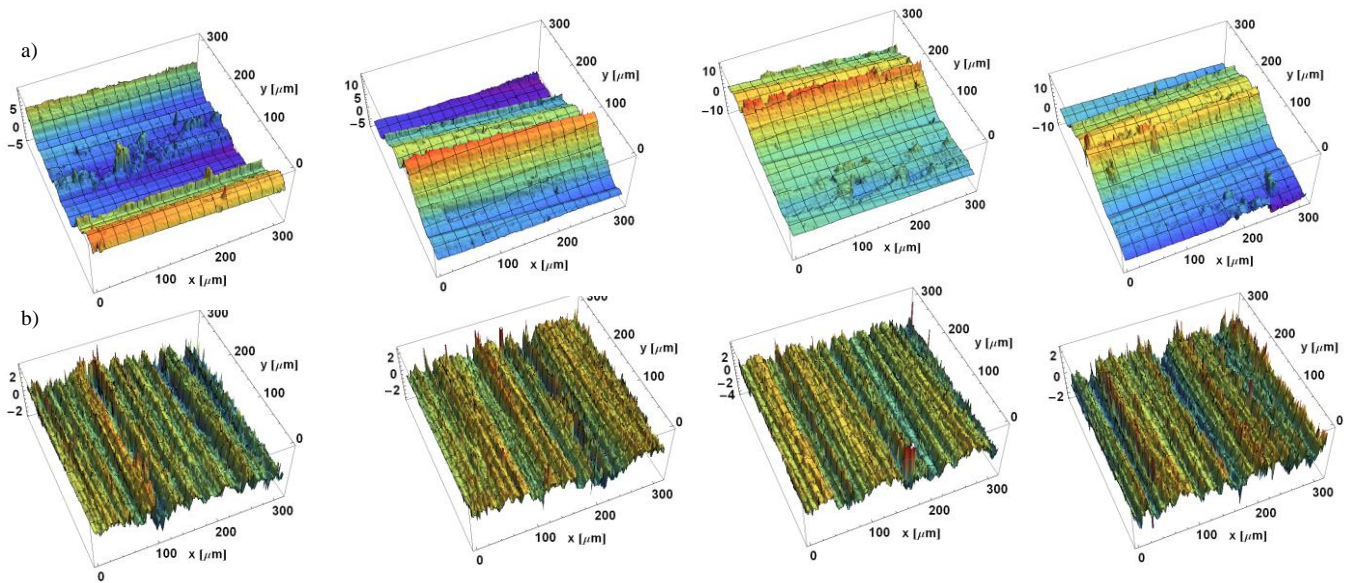
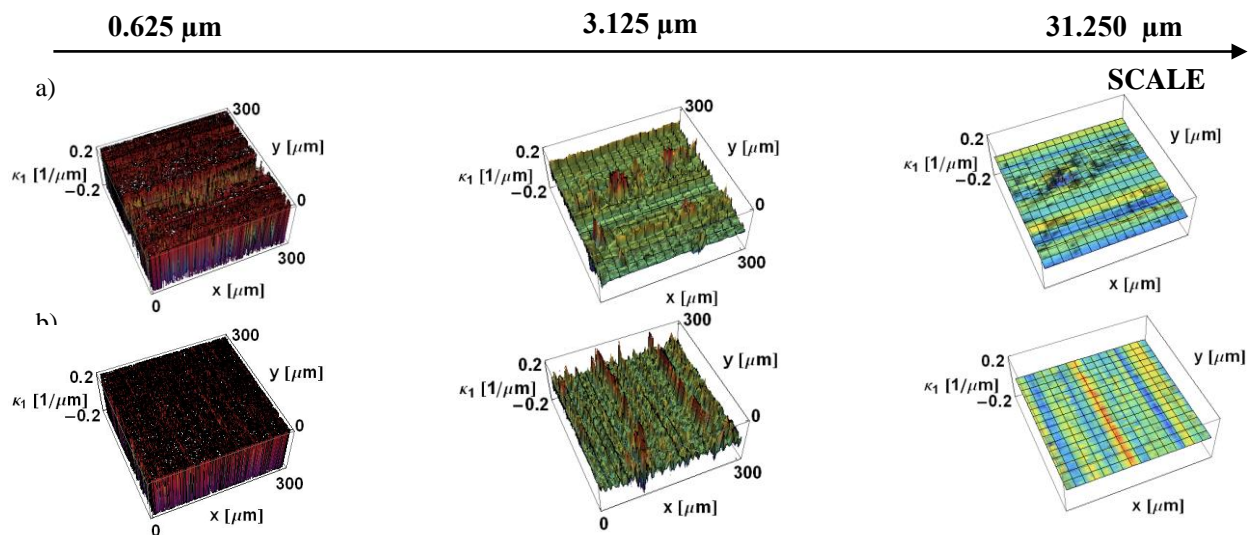


Fig. 1. Renders of surfaces created by different methods: a) milling and b) grinding

Fig. 2. Maximum principle curvature \mathcal{K}_1 calculated at each position for a) as-milled and b) as-ground surfaces for three different scales: 0.625, 3.125 and 31.250 μm (from left to right). Please note that positive curvature regions are colored with red and negative with blue.

Grinding removes the much of the large scale concavity and, compared to the machined surface, adds more fine scale complexity, i.e., changes in curvature per region, or details. Grooves visible are visible on the fine scale κ_1 curvature plot.

The minimum principal curvatures κ_2 presented in Fig. 3 show the same tendencies as the maximums. Note that the absolute values are significantly lower as compared to κ_1 . This indicates that these surfaces have valleys that are sharper than the peaks. The influence of the grinding can also be seen at the finer scales, where the curvature values are higher. Grinding has created some fine scale grooves and ridges.

Distributions of κ_1 principal curvatures are calculated for both the milled and the ground surfaces at scales of $6.250\mu\text{m}$ and $12.500\mu\text{m}$ and presented in Fig. 4. It can be seen that the distributions change with grinding. The distributions for the ground surfaces are narrower and smoother, with smaller curvatures at both scales. At the larger scale both surfaces have the most positive curvatures, or concave regions.

When the direction of the principle curvatures is aligned from location to location on a surface, then this is an indication of anisotropy, which would also be specific to that scale. Alignment of positive curvatures on a surface is an indication of a valley, or groove, and alignment of negative, a ridge. The vectors \mathbf{k}_1 and \mathbf{k}_2 indicate the direction about

which surface curves the most and the least. These vectors can indicate anisotropy. Fig. 5 shows examples of \mathbf{k}_1 for both surfaces at $3.125\mu\text{m}$ and $12.500\mu\text{m}$. Regions indicating ridges and synclines that are clearly anisotropic can be easily visible on those plots. Please note that the size of the arrows is only relative to show where surface curves the most.

Fig. 6 a) and b) shows the means and standard deviations of the maximum principal curvature, κ_1 , as a function of scale. The machined surface has a greater mean principal curvature κ_1 at the larger scales. Whereas, for the ground surface at the finer scales the mean maximum curvatures are higher on average. Standard deviations follow the same pattern. It can be understood that grinding removes large scale features created by the geometry of the milling tool nose and adds features derived from the geometry of the abrasive grains. At the larger scales, the ground surface is flatter which is evident because the mean and standard deviation of maximum curvatures trends to zero. The standard deviations increase markedly with decreasing scale and are larger for the ground surfaces at the finer scales, perhaps because shapes of the abrasive grains vary at these scales.

Fig. 7 a) and b) shows the means and standard deviations of the minimum principal curvature, κ_2 , as a function of scale.

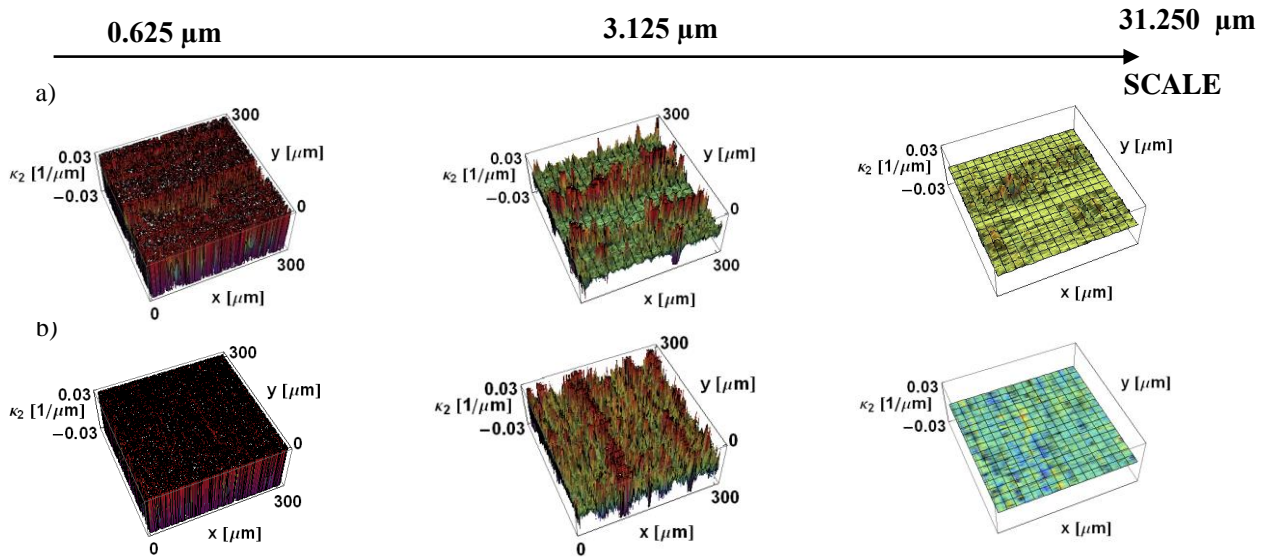


Fig. 3. Minimum principle curvature κ_2 calculated at each position for a) as-milled and b) as-ground surfaces for three different scales: 0.625 , 3.125 and $31.250\mu\text{m}$ (from left to right). Please note that positive curvature regions are colored with red and negative with blue.

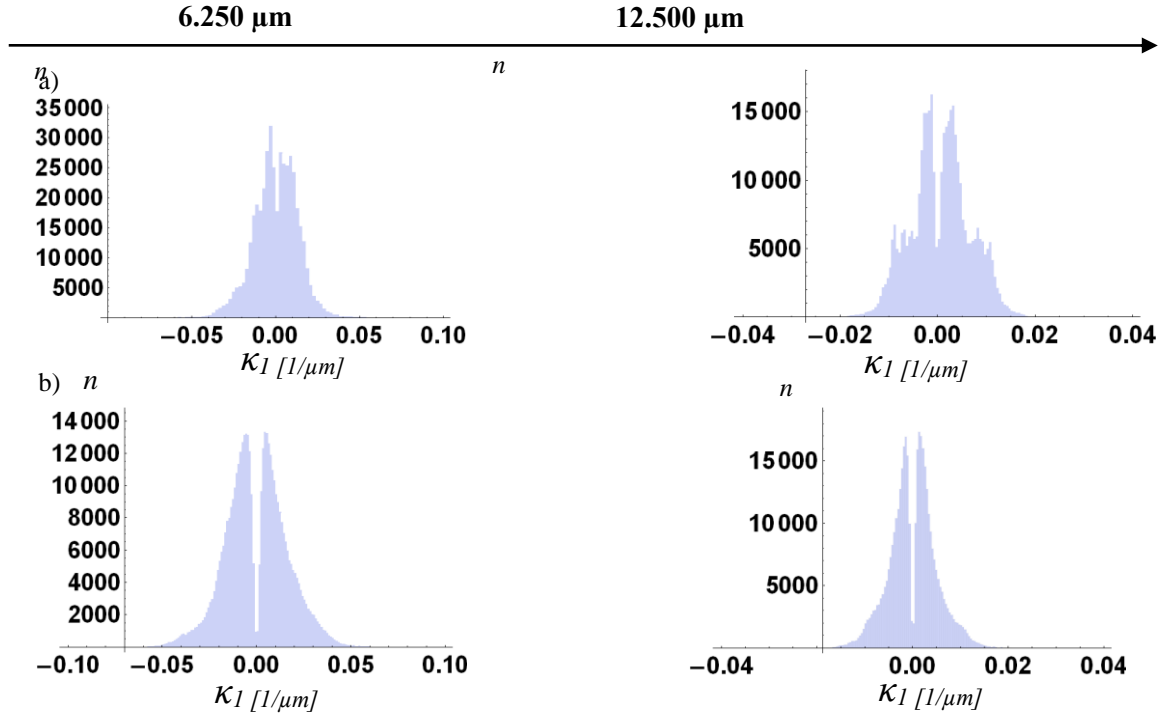


Fig. 4. Distributions of maximum principal curvatures \mathcal{K}_1 for a) milled and b) ground surfaces computed at two different scales 6.250 μm (left column) and 12.500 μm (right column) note the difference in the scales

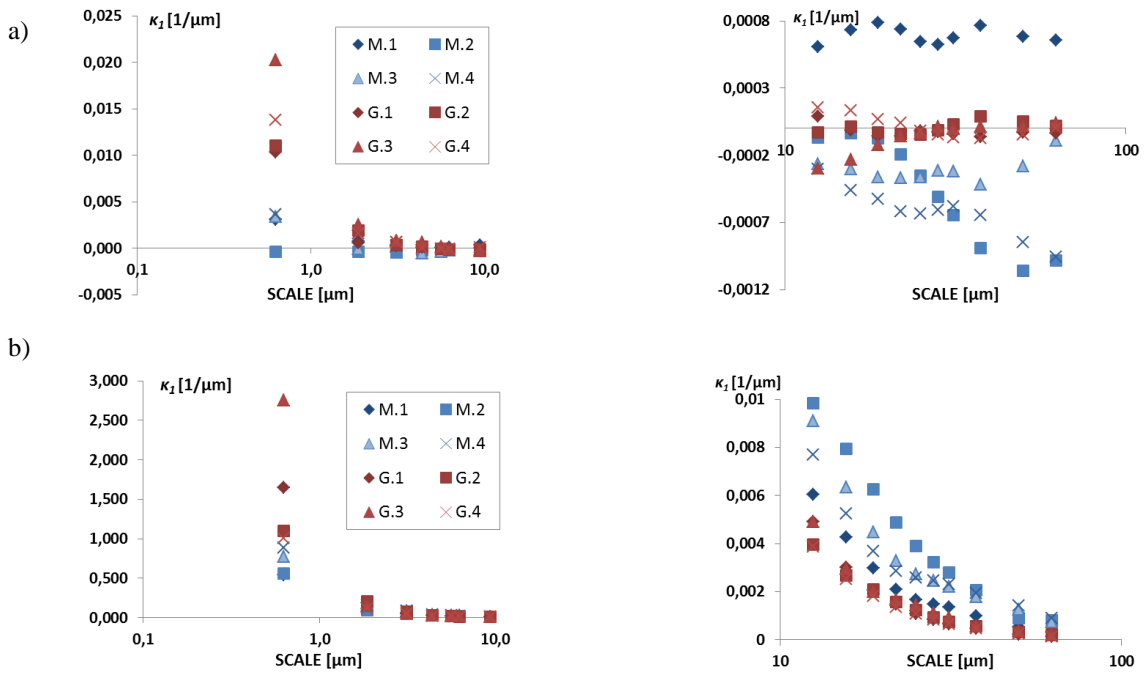


Fig. 6. Distribution parameters: a) mean and b) std of maximum principle curvature \mathcal{K}_1 deviation of minimum principle curvature \mathcal{K}_2 displayed in two ranges of scales

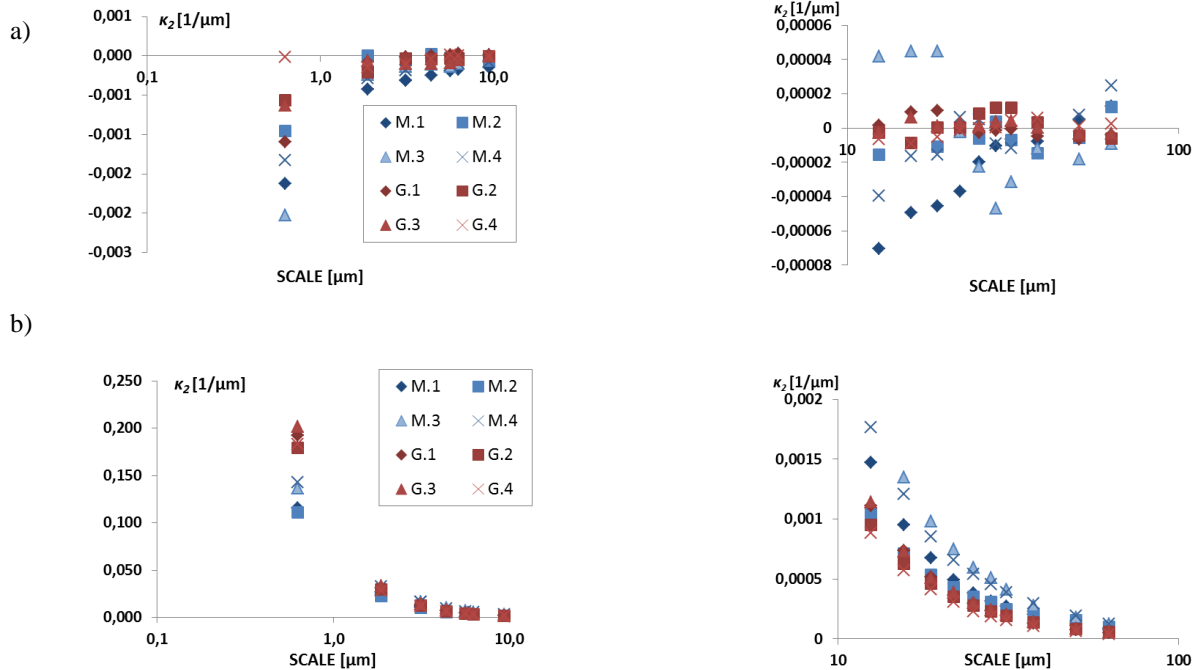


Fig. 6. Distribution parameters: a) mean and b) standard deviation of minimum principle curvature κ_2 displayed in two ranges of scales

4. CONCLUSIONS

The plots of curvature versus position at different scales show things about the character of the differences between surfaces peaks, pits, grooves and ridges as functions of scale which cannot be found with conventional height parameters. The conventional height parameters tend to be sensitive to the largest amplitude wavelengths and contain no spatial information and therefore miss the transitions in and natures of the topographies that are evident in multi-scale evaluation of the curvatures.

Curvature describes surface topographies in a way that can identify convex and concave regions of a measured surface. Concave and convex features can interact differently with the environment which can be evident in things like wetting behavior, fatigue, cracking, and charge distribution. Different scales of calculation can show different surface features, e.g., fine scale analysis can indicate micro groves and holes, whereas large scale analysis can show the general shape of the surface. What is more, the proposed method can indicate anisotropy because it can determine the orientation of the curvature. Multi-scale curvature analysis can work to identify particular types of features at specific scales.

Distribution of principal curvatures can change during grinding of a machined surface. Both maximum and minimum principal curvature values decreased after abrasive processing. At the finer scales, ground surfaces tended to be more curved than the milled surfaces. This could be because geometry of the abrasive for grinding is more varied than the nose of the cutting. The ground surfaces are flatter, whereas milled surfaces reflect the geometry created by feed and the nose tool.

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