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Reliability, risk and availability based optimization of complex technical systems operation processes
Part 2
Application in port transportation

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reliability, availability, complex system, operation process, optimization, piping transport

Abstract

The joint general model of reliability and availability of complex technical systems in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to system reliability and availability analysis and linear programming considered in the paper Part 1 are applied in maritime industry to reliability, risk and availability optimization of a port piping oil transportation system.

5. Reliability, risk and availability evaluation of a port oil piping transportation system

The oil terminal in Dębogórze is designated for the reception from ships, the storage and sending by carriages or cars the oil products. It is also designated for receiving from carriages or cars, the storage and loading the tankers with oil products such like petrol and oil.
The considered system is composed of three terminal parts *A*, *B* and *C*, linked by the piping transportation systems. The scheme of this system is presented in *Figure 1* [7].
The unloading of tankers is performed at the piers placed in the Port of Gdynia. The piers is connected with terminal part *A* through the transportation subsystem S_1 built of two piping lines composed of steel pipe segments with diameter of 600 mm. In the part *A* there is a supporting station fortifying tankers pumps and making possible further transport of oil by the subsystem S_2 to the terminal part *B*. The subsystem S_2 is built of two piping lines composed of steel pipe segments of the diameter 600 mm. The terminal part *B* is connected with the terminal part *C* by the subsystem S_3. The subsystem S_3 is built of one piping line composed of steel pipe segments of the diameter 500 mm and two piping lines composed of steel pipe segments of diameter 350 mm. The terminal part *C* is designated for the loading the rail cisterns with oil products and for the wagon sending to the railway station of the Port of Gdynia and further to the interior of the country.
The oil pipeline system consists three subsystems:
- the subsystem S_1 composed of two identical pipelines, each composed of 178 pipe segments of length 12m and two valves,
- the subsystem S_2 composed of two identical pipelines, each composed of 717 pipe segments of length 12m and to valves,
- the subsystem S_3 composed of three different pipelines, each composed of 360 pipe segments of either 10 m or 7,5 m length and two valves.

The subsystems S_1, S_2, S_3 are forming a general port oil pipeline system reliability series structure. However, the pipeline system reliability structure and the subsystems reliability depend on its changing in time operation states [7].
Taking into account the varying in time operation process of the considered system we distinguish the following as its eight operation states:

- an operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines in subsystem S_3,
- an operation state z_2 – transport of one kind of medium from the terminal part C (from carriages) to part B using one out of three pipelines in subsystem S_3,
- an operation state z_3 – transport of one kind of medium from the terminal part B through part A to piers using one out of two pipelines in subsystem S_2 and one out of two pipelines in subsystem S_1,
- an operation state z_4 – transport of two kinds of medium from the piers through parts A and B to part C using one out of two pipelines in subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines in subsystem S_3,
- an operation state z_5 – transport of one kind of medium from the piers through part A to B using one out of two pipelines in subsystem S_1 and one out of two pipelines in subsystem S_2,
- an operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines in subsystem S_3, and simultaneously transport one kind of medium from the piers through part A to B using one out of two pipelines in parts S_1 and one out of two pipelines in subsystem S_2,
- an operation state z_7 – lack of medium transport (system is not working)
- an operation state z_8 – transport of one kind of medium from the terminal part B to C using one out of three pipelines in part S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines in part S_3.

At the moment because of the luck of sufficient statistical data about the oil terminal operation process it is not possible to estimate its all operational characteristics. However, on the basis the still limited data, given in [7], the transient probabilities p_{bl} from the operation state z_b into the operations state z_l for $b, l = 1,2,...,8,$ $b \neq l,$ were preliminary evaluated. Their approximate values are included in the matrix below

$$[p_{bl}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.06 & 0.06 & 0.86 & 0.02 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.125 & 0 & 0 & 0 & 0 & 0.125 & 0.687 & 0.063 \\ 0.4 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0 \\ 0.82 & 0 & 0 & 0 & 0.16 & 0 & 0 & 0.02 \\ 0.67 & 0 & 0 & 0 & 0 & 0 & 0.33 & 0 \end{bmatrix} \quad (42)$$

Unfortunately, it was not possible yet to determine the matrix of the conditional distribution functions $[H_{bl}(t)]_{8x8}$ of the sojourn times θ_{bl} for $b, l = 1,2,...,8,$ $b \neq l,$ [1], [6] and further consequently, according to (2.1), it was also not possible to determine the vector $[H_b(t)]_{1x8}$ of the unconditional distribution functions of the sojourn times θ_b of this operation process at the operation states z_b, $b = 1,2,...,8.$ However, on the basis of the preliminary statistical data coming from experiment it was possible to evaluate approximately the conditional mean values $M_{bl} = E[\theta_{bl}],$ $b, l = 1,2,...,8,$ $b \neq l,$ of sojourn times in the particular operation states defined by (3). On the basis of the statistical data given in Tables 1-10 in [7] (Appendix 1A) their approximate evolutions are as follows:

$$M_{15} = 720,\ M_{16} = 420,\ M_{17} = 698.95,\ M_{18} = 480,$$

$$M_{51} = 750,\ M_{56} = 564,\ M_{57} = 748.7,\ M_{58} = 540,$$

$$M_{61} = 360,\ M_{65} = 360,\ M_{71} = 975.3,\ M_{75} = 872.4,$$

$$M_{78} = 600,\ M_{81} = 900,\ M_{87} = 420. \quad (43)$$

Hence, by (2), the unconditional mean lifetimes in the operation states are

$$M_1 = E[\theta_1] = p_{15}M_{15} + p_{16}M_{16} + p_{17}M_{17} + p_{18}M_{18}$$

$$= 0.06 \cdot 720 + 0.06 \cdot 420 + 0.86 \cdot 698.95$$

$$+ 0.02 \cdot 480 \cong 679.1,$$

$$M_5 = E[\theta_5] = p_{51}M_{51} + p_{56}M_{56} + p_{57}M_{57} + p_{58}M_{58}$$

$$= 0.125 \cdot 750 + 0.125 \cdot 564 + 0.687 \cdot 748.7$$

$$+ 0.063 \cdot 540 \cong 712.63,$$

$$M_6 = E[\theta_6] = p_{61}M_{61} + p_{65}M_{65}$$

$$= 0.4 \cdot 360 + 0.6 \cdot 360 = 360,$$

$$M_7 = E[\theta_7] = p_{71}M_{71} + p_{75}M_{75} + p_{78}M_{78}$$

$$= 0.82 \cdot 975.3 + 0.16 \cdot 872.4 + 0.02 \cdot 600$$

$$\cong 951.33,$$

$$M_8 = E[\theta_8] = p_{81}M_{81} + p_{87}M_{87}$$

$$= 0.67 \cdot 900 + 0.33 \cdot 420 \cong 741.6. \qquad (44)$$

Since from the system of equations (5) given here in the form

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8] \\ = [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8][p_{bl}] \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 + \pi_8 = 1, \end{cases}$$

we get

$$\pi_1 = 0.396, \ \pi_2 = 0, \ \pi_3 = 0, \ \pi_4 = 0, \ \pi_5 = 0.116,$$

$$\pi_6 = 0.038, \ \pi_7 = 0.435, \ \pi_8 = 0.015, \qquad (45)$$

then the limit values of the transient probabilities $p_b(t)$ at the operational states z_b, according to (4), are given by

$$p_1 = 0.34, \ p_2 = 0, \ p_3 = 0, \ p_4 = 0, \ p_5 = 0.1,$$

$$p_6 = 0.02, \ p_7 = 0.53, \ p_8 = 0.01. \qquad (46)$$

From the above result, according to (34)-(35), the unconditional multistate (three-state) reliability function of the system is of the form

$$\overline{\boldsymbol{R}}_3(t,\cdot) = [1, \ \overline{\boldsymbol{R}}_3(t,1), \ \overline{\boldsymbol{R}}_3(t,2)], \qquad (47)$$

with the coordinates given by

$$\overline{\boldsymbol{R}}_3(t,1) = 0.34 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(1)} + 0 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(2)}$$

$$+ 0 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(3)} + 0 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(4)}$$

$$+ 0.01 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(5)} + 0.02 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(6)}$$

$$+ 0.53 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(7)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(8)}] \qquad (48)$$

for $t \geq 0$,

$$\overline{\boldsymbol{R}}_3(t,2) = 0.34 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(1)} + 0 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(2)}$$

$$+ 0 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(3)} + 0 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(4)}$$

$$+ 0.1 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(5)} + 0.02 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(6)}$$

$$+ 0.53 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(7)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(8)}] \qquad (49)$$

for $t \geq 0$,

where $[\overline{\boldsymbol{R}}(t,1)]^{(b)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(b)}$, $b = 1,2,...,8,$ are fixed in [7].
In [7] (Appendix 1B), it is also fixed that the mean values of the system unconditional lifetimes in the particular reliability state subsets $\{1,2\}$ and $\{2\}$ are:

$$\mu_1(1) = 0.364, \ \mu_1(2) = 0.304,$$

$$\mu_2(1) = 0.807, \ \mu_2(2) = 0.666,$$

$$\mu_3(1) = 0.307, \ \mu_3(2) = 0.218,$$

$$\mu_4(1) = 0.079, \ \mu_4(2) = 0.058,$$

$$\mu_5(1) = 0.307, \ \mu_5(2) = 0.218,$$

$$\mu_6(1) = 0.079, \ \mu_6(2) = 0.058,$$

$$\mu_7(1) = 0.110, \ \mu_7(2) = 0.085,$$

$$\mu_8(1) = 0.364, \ \mu_8(2) = 0.079. \qquad (50)$$

After considering (46)-(50) and applying (11), the mean values of the system unconditional lifetimes in the reliability state subsets $\{1,2\}$ and $\{2\}$, before the optimization, respectively are:

$$\mu(1) = p_1\mu_1(1) + p_2\mu_2(1) + p_3\mu_3(1) + p_4\mu_4(1)$$

$$+ p_5\mu_5(1) + p_6\mu_6(1) + p_7\mu_7(1) + p_8\mu_8(1)$$

$$= 0.34 \cdot 0.364 + 0.00 \cdot 0.807 + 0.00 \cdot 0.307$$

$$+ 0.00 \cdot 0.079 + 0.10 \cdot 0.307 + 0.02 \cdot 0.079$$

$$+ 0.53 \cdot 0.110 + 0.01 \cdot 0.364 \cong 0.218, \qquad (51)$$

$$\mu(2) = p_1\mu_1(2) + p_2\mu_2(2) + p_3\mu_3(2) + p_4\mu_4(2)$$

$$+ p_5\mu_5(2) + p_6\mu_6(2) + p_7\mu_7(2) + p_8\mu_8(2)$$

$$= 0.34 \cdot 0.304 + 0.00 \cdot 0.666 + 0.00 \cdot 0.218$$

$$+ 0.00 \cdot 0.058 + 0.10 \cdot 0.218 + 0.02 \cdot 0.058$$

$$+ 0.53 \cdot 0.083 + 0.01 \cdot 0.304 \cong 0.173, \quad (52)$$

and according to (14), the mean values of the system lifetimes in the particular reliability states $u = 1$ and $u = 2$, before the optimization, respectively are

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.045,$$

$$\bar{\mu}(2) = \mu(2) = 0.173. \quad (53)$$

Further, according to (13), the variances and standard deviations of the system unconditional lifetimes in the system reliability state subsets are

$$\sigma^2(1) = 2\int_0^\infty t \; \overline{\boldsymbol{R}}_3(t,1)dt - [\mu(1)]^2 \cong 0.0520,$$

$$\sigma(1) \cong 0.228, \quad (54)$$

$$\sigma^2(2) = 2\int_0^\infty t \; \overline{\boldsymbol{R}}_3(t,2)dt - [\mu(2)]^2 \cong 0.0342,$$

$$\sigma(2) \cong 0.185, \quad (55)$$

where $\overline{\boldsymbol{R}}_3(t,1)$, $\overline{\boldsymbol{R}}_3(t,2)$ are given by (48)-(49) and $\mu(1)$, $\mu(2)$ are given by (51)-(52).

If the critical safety state is r =1, then the system risk function, according to (7), is given by

$$\boldsymbol{r}(t) = 1 - \overline{\boldsymbol{R}}_3(t,1) \text{ for } t \geq 0, \quad (56)$$

where $\boldsymbol{R}_3(t,1)$ is given by (48).

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (8), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.011 \text{ years.} \quad (57)$$

Further, assuming that the oil pipeline system is repaired after its failure and that the time of the system renovation is ignored, applying *Theorem 3.1*, we obtain the following results:

i) the distribution of the time $S_N(1)$ until the *Nth* exceeding of reliability critical state 1 of this system, for sufficiently large *N*, has approximately normal distribution $N(0.218N, 0.228\sqrt{N})$, i.e.,

$$F^{(N)}(t,1) = P(S_N(1) < t) \cong F_{N(0,1)}\left(\frac{t - 0.218N}{0.228\sqrt{N}}\right),$$

$$t \in (-\infty, \infty),$$

ii) the expected value and the variance of the time $S_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$$E[S_N(1)] = 0.218N, \; D[S_N(1)] = 0.0519N,$$

iii) the distribution of the number $N(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0$, for sufficiently large *t*, is approximately of the form

$$P(N(t,1) = N) \cong F_{N(0,1)}\left(\frac{0.218N - t}{0.4883\sqrt{t}}\right)$$

$$- F_{N(0,1)}\left(\frac{0.218(N+1) - t}{0.4884\sqrt{t}}\right), \; N = 0,1,2,...,$$

iv) the expected value and the variance of the number $N(t,1)$ of exceeding the reliability critical state 1 of this system at the moment $t, t \geq 0$, for sufficiently large *t*, approximately take respectively forms

$$H(t,1) = 4.587t, \; D(t,1) = 5.0095t.$$

Further, assuming that the oil pipeline system is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value $\mu_0(1) = 0.005$ and the standard deviation $\sigma_0(1) = 0.005$, applying *Theorem 3.2*, we obtain the following results:

i) the distribution function of the time $\overline{\overline{S}}_N(1)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(0.223N, 0.2279\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)}(t,1) = P(\overline{\overline{S}}_N(1) < t) \cong F_{N(0,1)}\left(\frac{t - 0.223N)}{0.2279\sqrt{N}}\right),$$

$$t \in (-\infty, \infty), \; N = 1,2,...,$$

ii) the expected value and the variance of the time $\overline{\overline{S}}_N(1)$ until the *Nth* system's renovation take respectively forms

$$E[\bar{\bar{S}}_N(1)] \cong 0.223N, \quad D[\bar{\bar{S}}_N(1)] \cong 0.0519N,$$

iii) the distribution function of the time $\bar{S}_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system takes form

$$\bar{F}^{(N)}(t,1) = P(\bar{S}_N(1) < t)$$

$$= F_{N(0,1)}(\frac{t - 0.223N + 0.005}{\sqrt{0.0519N - 0.000025}}),$$

$t \in (-\infty, \infty), \ N = 1,2,...,$

iv) the expected value and the variance of the time $\bar{S}_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$$E[\bar{S}_N(1)] \cong 0.218N + 0.005(N-1),$$

$$D[\bar{S}_N(1)] \cong 0.0519N + 0.000025(N-1),$$

v) the distribution of the number $\bar{\bar{N}}(t,1)$ of system's renovations up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{\bar{N}}(t,1) = N) \cong F_{N(0,1)}(\frac{0.223N - t}{0.482\sqrt{t}})$$

$$- F_{N(0,1)}(\frac{0.223(N+1) - t}{0.482\sqrt{t}}) \ N = 1,2,...,$$

vi) the expected value and the variance of the number $\bar{\bar{N}}(t,1)$ of system's renovations up to the moment $t, t \geq 0,$ take respectively forms

$$\bar{\bar{H}}(t,1) \cong 4.484t, \quad \bar{\bar{D}}(t,1) \cong 4.68t,$$

vii) the distribution of the number $\bar{N}(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{N}(t,1) = N) \cong F_{N(0,1)}(\frac{0.223N - t - 0.005}{0.482\sqrt{t + 0.005}})$$
$$- F_{N(0,1)}(\frac{0.223(N+1) - t - 0005}{0.482\sqrt{t - 0.005}}), \ N = 1,2,...,$$

viii) the expected value and the variance of the number $\bar{N}(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0,$ are respectively given by

$$\bar{H}(t,1) \cong \frac{t + 0.005}{0.223}, \quad \bar{D}(t,1) \cong 4.68(t + 0.005),$$

ix) the availability coefficient of the system at the moment *t* is given by the formula

$$A(t,1) \cong 0.9776, \quad t \geq 0,$$

x) the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ is given by the formula

$$A(t,\tau,1) \cong 4.484 \int_{\tau}^{\infty} \bar{\boldsymbol{R}}_3(t,1)dt, \quad t \geq 0, \quad \tau > 0.$$

6. Reliability, risk and availability optimization of a port oil piping transportation system

The objective function (15), in this case as the critical state is $r = 1$, takes the form

$$\mu(1) = p_1 \cdot 0.364 + p_2 \cdot 0.807 + p_3 \cdot 0.307$$
$$+ p_4 \cdot 0.079 + p_5 \cdot 0.307 + p_6 \cdot 0.079$$
$$+ p_7 \cdot 0.110 + p_8 \cdot 0.364. \quad (58)$$

The lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown limit transient probabilities p_b, $b = 1,2,...,8,$ coming from experts are respectively:

$\breve{p}_1 = 0.25$, $\breve{p}_2 = 0.01$, $\breve{p}_3 = 0.01$. $\breve{p}_4 = 0.01$,
$\breve{p}_5 = 0.08$, $\breve{p}_6 = 0.01$, $\breve{p}_7 = 0.40$, $\breve{p}_1 = 0.01$;

$\hat{p}_1 = 0.50$, $\hat{p}_2 = 0.05$, $\hat{p}_3 = 0.05$, $\hat{p}_4 = 0.05$,
$\hat{p}_5 = 0.20$, $\hat{p}_6 = 0.05$, $\hat{p}_1 = 0.75$, $\hat{p}_1 = 0.05$.

Therefore, according to (16)-(18), we assume the following bound constraints

$$\sum_{b=1}^{8} p_b = 1, \quad (59)$$

$$0.25 \leq p_1 \leq 0.50, \ 0.01 \leq p_2 \leq 0.05,$$
$$0.01 \leq p_3 \leq 0.05, \ 0.01 \leq p_4 \leq 0.05,$$
$$0.08 \leq p_5 \leq 0.20, \ 0.01 \leq p_6 \leq 0.05,$$
$$0.40 \leq p_7 \leq 0.75, \ 0.01 \leq p_8 \leq 0.05. \quad (60)$$

Now, before we find optimal values $\dot{p}_b$ of the limit transient probabilities p_b, $b = 1,2,...,\nu$, that maximize the objective function (58), we arrange the system conditional lifetime mean values $\mu_b(1)$, $b = 1,2,...,8$, in non-increasing order

$$\mu_2(1) \ge \mu_1(1) \ge \mu_8(1) \ge \mu_3(1) \ge \mu_5(1) \ge \mu_7(1) \ge$$

$$\mu_4(1) \ge \mu_6(1).$$

Next, according to (19), we substitute

$$x_1 = p_2 = 0.00,\ \ x_2 = p_1 = 0.34,\ \ x_3 = p_8 = 0.01,$$
$$x_4 = p_3 = 0.00,\ \ x_5 = p_5 = 0.10,\ \ x_6 = p_7 = 0.53,$$
$$x_7 = p_4 = 0.00,\ \ x_8 = p_6 = 0.02 \quad (61)$$

and

$$\breve{x}_i = 0.01,\ \hat{x}_i = 0.95 \text{ for } i = 1,2,...,\nu \quad (62)$$

and we maximize with respect to x_i, $i = 1,2,...,8$, the linear form (52) that according to (20) takes the form

$$\mu(1) = x_1 \cdot 0.807 + x_2 \cdot 0.364 + x_3 \cdot 0.364$$
$$+ x_4 \cdot 0.307 + x_5 \cdot 0.307 + x_6 \cdot 0.110$$
$$+ x_7 \cdot 0.079 + x_8 \cdot 0.079 \quad (63)$$

with the following bound constraints

$$\sum_{i=1}^{8} x_i = 1, \quad (64)$$

$$0.01 \le x_1 \le 0.05,\ \ 0.25 \le x_2 \le 0.50,$$
$$0.01 \le x_3 \le 0.05,\ \ 0.01 \le x_4 \le 0.05,$$
$$0.08 \le x_5 \le 0.20,\ \ 0.40 \le x_6 \le 0.75,$$
$$0.01 \le x_7 \le 0.05,\ \ 0.01 \le x_8 \le 0.05. \quad (65)$$

where

$$\breve{x}_1 = 0.01,\ \breve{x}_2 = 0.25,\ \breve{x}_3 = 0.01,\ \breve{x}_4 = 0.01,$$
$$\breve{x}_5 = 0.08,\ \breve{x}_6 = 0.40,\ \breve{x}_7 = 0.01,\ \breve{x}_8 = 0.01;$$

$$\hat{x}_1 = 0.05,\ \hat{x}_2 = 0.50,\ \hat{x}_3 = 0.05,\ \hat{x}_4 = 0.05,$$
$$\hat{x}_5 = 0.20,\ \hat{x}_6 = 0.75,\ \hat{x}_7 = 0.05,\ \hat{x}_8 = 0.05. \quad (66)$$

are lower and upper bounds of the unknown limit transient probabilities x_i, $i = 1,2,...,8$, respectively. According to (24), we find

$$\breve{x} = \sum_{i=1}^{8} \breve{x}_i = 0.78,\ \ \hat{y} = 1 - \breve{x} = 1 - 0.78 = 0.22 \quad (67)$$

and according to (25), we find

$$\breve{x}^0 = 0,\ \ \hat{x}^0 = 0,\ \ \hat{x}^0 - \breve{x}^0 = 0,$$
$$\breve{x}^1 = 0.01\ \ \hat{x}^1 = 0.05,\ \ \hat{x}^1 - \breve{x}^1 = 0.04$$
$$\breve{x}^2 = 0.26\ \ \hat{x}^2 = 0.55,\ \ \hat{x}^2 - \breve{x}^2 = 0.29,$$
$$...$$
$$\breve{x}^8 = 0.78\ \ \hat{x}^8 = 1.70,\ \ \hat{x}^8 - \breve{x}^8 = 0.92. \quad (68)$$

From the above, as according to (67), the inequality (26) takes the form

$$\hat{x}^I - \breve{x}^I < 0.22, \quad (69)$$

then it follows that the largest value $I \in \{0,1,...,8\}$ such that this inequality holds is $I = 1$.
Therefore, we fix the optimal solution that maximize (63) according to the rule (28). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.05,$$

$$\dot{x}_2 = \hat{y} - \hat{x}^1 + \breve{x}^1 + \breve{x}_2$$
$$= 0.22 - 0.05 + 0.01 + 0.25 = 0.43, \quad (70)$$

$$\dot{x}_3 = \breve{x}_3 = 0.01,\ \dot{x}_4 = \breve{x}_4 = 0.01,\ \dot{x}_5 = \breve{x}_5 = 0.08,$$
$$\dot{x}_6 = \breve{x}_6 = 0.40,\ \dot{x}_7 = \breve{x}_7 = 0.01,\ \dot{x}_8 = \breve{x}_8 = 0.01. \quad (71)$$

Finally, after making the inverse to (61) substitution, we get the optimal limit transient probabilities

$$\dot{p}_2 = \dot{x}_1 = 0.05,\ \ \dot{p}_1 = \dot{x}_2 = 0.43,\ \ \dot{p}_8 = \dot{x}_3 = 0.01,$$
$$\dot{p}_3 = \dot{x}_4 = 0.01,\ \ \dot{p}_5 = \dot{x}_5 = 0.08,\ \ \dot{p}_7 = \dot{x}_6 = 0.40,$$
$$\dot{p}_4 = \dot{x}_7 = 0.01,\ \ \dot{p}_6 = \dot{x}_8 = 0.01 \quad (72)$$

that maximize the system mean lifetime in the reliability state subset $\{1,2\}$ expressed by the linear form (58) giving, according to (31) and (72), its optimal value

$$\dot{\mu}(1) = \dot{p}_1 \cdot 0.364 + \dot{p}_2 \cdot 0.807 + \dot{p}_3 \cdot 0.307$$
$$+ \dot{p}_4 \cdot 0.079 + \dot{p}_5 \cdot 0.307 + \dot{p}_6 \cdot 0.079$$
$$+ \dot{p}_7 \cdot 0.110 + \dot{p}_8 \cdot 0.364$$
$$= 0.43 \cdot 0.364 + 0.05 \cdot 0.807 + 0.01 \cdot 0.307$$
$$+ 0.01 \cdot 0.079 + 0.08 \cdot 0.307 + 0.01 \cdot 0.079$$
$$+ 0.40 \cdot 0.110 + 0.01 \cdot 0.364 = 0.274. \quad (73)$$

Further, according to (32), substituting the optimal solution (72) in (52), we obtain the optimal solutions for the mean values of the system unconditional lifetimes in the reliability state subset {2}

$$\dot{\mu}(2) = \dot{p}_1 \cdot 0.304 + \dot{p}_2 \cdot 0.666 + \dot{p}_3 \cdot 0.218 + \dot{p}_4 \cdot 0.058 + \dot{p}_5 \cdot 0.218 + \dot{p}_6 \cdot 0.058 + \dot{p}_7 \cdot 0.085 + \dot{p}_8 \cdot 0.304 = 0.43 \cdot 0.304 + 0.05 \cdot 0.666 + 0.01 \cdot 0.218 + 0.01 \cdot 0.058 + 0.08 \cdot 0.218 + 0.01 \cdot 0.058 + 0.40 \cdot 0.085 + 0.01 \cdot 0.079 = 0.220. \quad (74)$$

and according to (36), the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states

$$\dot{\bar{\mu}}(1) = \dot{\mu}(1) - \dot{\mu}(2) = 0.054,$$
$$\dot{\bar{\mu}}(2) = \dot{\mu}(2) = 0.220. \quad (75)$$

Moreover, according to (34)-(35) and (47)-(49), the corresponding optimal unconditional multistate reliability function of the system is of the form

$$\dot{\boldsymbol{R}}_3(t,\cdot) = [1, \dot{\boldsymbol{R}}_3(t,1), \dot{\boldsymbol{R}}_3(t,2)], \quad (76)$$

with the coordinates given by

$$\dot{\boldsymbol{R}}_3(t,1) = 0.43 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(1)} + 0.05 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(2)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(3)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(4)} + 0.08 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(5)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(6)} + 0.40 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(7)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(8)}], \quad (77)$$

$$\dot{\boldsymbol{R}}_3(t,2) = 0.43 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(1)} + 0.05 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(2)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(3)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(4)} + 0.08 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(5)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(6)} + 0.40 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(7)} + 0.01 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(8)}] \quad (78)$$

for $t \geq 0$, where $[\overline{\boldsymbol{R}}(t,1)]^{(b)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(b)}$, $b = 1,2,...,8$, are fixed in [7].

Further, according to (13) and (32)-(33), the corresponding optimal variances and standard deviations of the system unconditional lifetime in the system reliability state subsets are

$$\dot{\sigma}^2(1) = 2\int_0^\infty t \, \dot{\boldsymbol{R}}_3(t,1)dt - [\dot{\mu}(1)]^2 \cong 0.084,$$

$$\dot{\sigma}(1) \cong 0.289, \quad (79)$$

$$\dot{\sigma}^2(2) = 2\int_0^\infty t \, \dot{\boldsymbol{R}}_3(t,2)dt - [\dot{\mu}(2)]^2 \cong 0.056,$$

$$\dot{\sigma}(2) \cong 0.237, \quad (80)$$

where $\dot{\boldsymbol{R}}_3(t,1)$, $\dot{\boldsymbol{R}}_3(t,2)$ are given by (77)-(78) and $\dot{\mu}(1)$, $\dot{\mu}(2)$ are given by (73)-(74).

If the critical safety state is $r = 1$, then the optimal system risk function, according to (7) and (37), is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}_3(t,1) \text{ for } t \geq 0, \quad (81)$$

where $\dot{\boldsymbol{R}}_3(t,1)$ is given by (77).

Hence and considering (38), the moment when the optimal system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.19 \text{ years}. \quad (82)$$

Replacing $\mu(r)$ by $\dot{\mu}(1)$ given by (73) and $\sigma(r)$ by $\dot{\sigma}(1)$ given by (79) in the expressions for the renewal systems characteristics pointed in *Theorem 1* and *Theorem 2*, we get their corresponding optimal values pointed below.

Under the assumption that the oil pipeline system is repaired after its failure and that the time of the system renovation is ignored, we obtain the following optimal results:

i) the distribution of the time $S_N(1)$ until the *Nth* exceeding of reliability critical state 1 of this system, for sufficiently large *N*, has approximately normal distribution $N(0.274N, 0.289\sqrt{N})$, i.e.,

$$F^{(N)}(t,1) = P(S_N(1) < t) \cong F_{N(0,1)}\left(\frac{t - 0.274N}{0289\sqrt{N}}\right),$$

$$t \in (-\infty, \infty),$$

ii) the expected value and the variance of the time $S_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$$E[S_N(1)] = 0.274N, \quad D[S_N(1)] = 0.084N,$$

iii) the distribution of the number $N(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0$, for sufficiently large *t*, is approximately of the form

$$P(N(t,1) = N) \cong F_{N(0,1)}\left(\frac{0.274N - t}{0.552\sqrt{t}}\right)$$

$$- F_{N(0,1)}(\frac{0.274(N+1)-t}{0.552\sqrt{t}}), \quad N = 0,1,2,\ldots,$$

iv) the expected value and the variance of the number $N(t,1)$ of exceeding the reliability critical state 1 of this system at the moment $t, t \geq 0,$ for sufficiently large *t*, approximately take respectively forms

$$H(t,1) = 3.649t, \quad D(t,1) = 4.06t.$$

Under the assumption that the oil pipeline system is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value $\mu_0(1) = 0.005$ and the standard deviation $\sigma_0(1) = 0.005,$ we obtain the following optimal results:

i) the distribution function of the time $\overline{\overline{S}}_N(1)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(0.279N, 0.289\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)}(t,1) = P(\overline{\overline{S}}_N(1) < t) \cong F_{N(0,1)}(\frac{t - 0.279N}{0.289\sqrt{N}}),$$

$$t \in (-\infty, \infty), \; N = 1,2,\ldots,$$

ii) the expected value and the variance of the time $\overline{\overline{S}}_N(1)$ until the *Nth* system's renovation take respectively forms

$$E[\overline{\overline{S}}_N(1)] \cong 0.279N, \; D[\overline{\overline{S}}_N(1)] \cong 0.084N,$$

iii) the distribution function of the time $\overline{S}_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system takes form

$$\overline{F}^{(N)}(t,1) = P(\overline{S}_N(1) < t)$$

$$= F_{N(0,1)}(\frac{t - 0.279N + 0.005}{\sqrt{0.084N - 0.000025}}),$$

$$t \in (-\infty, \infty), \; N = 1,2,\ldots,$$

iv) the expected value and the variance of the time $\overline{S}_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$$E[\overline{S}_N(1)] \cong 0.274N + 0.005(N-1),$$

$$D[\overline{S}_N(1)] \cong 0.08352N + 0.000025(N-1),$$

v) the distribution of the number $\overline{\overline{N}}(t,1)$ of system's renovations up to the moment $t, t \geq 0,$ is of the form

$$P(\overline{\overline{N}}(t,1) = N) \cong F_{N(0,1)}(\frac{0.279N - t}{0.549\sqrt{t}}$$

$$- F_{N(0,1)}(\frac{0.279(N+1)-t}{0.549\sqrt{t}}, \; N = 1,2,\ldots,$$

vi) the expected value and the variance of the number $\overline{\overline{N}}(t,1)$ of system's renovations up to the moment $t, t \geq 0,$ take respectively forms

$$\overline{\overline{H}}(t,1) \cong 3.584t, \; \overline{\overline{D}}(t,1) \cong 3.868t,$$

vii) the distribution of the number $\overline{N}(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0,$ is of the form

$$P(\overline{N}(t,1) = N) \cong F_{N(0,1)}(\frac{0.279N - t - 0.005}{0.549\sqrt{t + 0.005}})$$

$$- F_{N(0,1)}(\frac{0.279(N+1) - t - 0005}{0.49\sqrt{t+0.005}}), \; N = 1,2,\ldots,$$

viii) the expected value and the variance of the number $\overline{N}(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0,$ are respectively given by

$$\overline{H}(t,1) \cong \frac{t + 0.005}{0.279}, \; \overline{D}(t,1) \cong 3.868(t + 0.005),$$

ix) the availability coefficient of the system at the moment *t* is given by the formula

$$A(t,1) \cong 0.982, \quad t \geq 0,$$

x) the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ is given by the formula

$$A(t,\tau,1) \cong 3.584\int_{\tau}^{\infty} \overline{\boldsymbol{R}}_3(t,1)dt, \quad t \geq 0, \quad \tau > 0.$$

To obtain the optimal mean sojourn times in the particular operation states maximizing the mean lifetime of the port oil piping transportation system we substitute the optimal limit transient probabilities $\dot{p}_b$ determined by (72) and probabilities π_b determined by (45) into the system of equation (40) and we get its following form

$$\begin{aligned}
&-0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.01634\dot{M}_6 \\
&+ 0.18705\dot{M}_7 + 0.00645\dot{M}_8 = 0 \\
&0.0198\dot{M}_1 + 0.0058\dot{M}_5 + 0.0019\dot{M}_6 \\
&+ 0.02175\dot{M}_7 + 0.00075\dot{M}_8 = 0 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 \\
&+ 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 \\
&+ 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0 \\
&0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.00304\dot{M}_6 + \\
&0.0348\dot{M}_7 + 0.0012\dot{M}_8 = 0 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 - 0.03762\dot{M}_6 + \\
&0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0 \\
&0.1584\dot{M}_1 + 0.0464\dot{M}_5 + 0.0152\dot{M}_6 \\
&- 0.261\dot{M}_7 + 0.006\dot{M}_8 = 0 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 \\
&+ 0.00435\dot{M}_7 - 0.01485\dot{M}_8 = 0.
\end{aligned} \tag{83}$$

Since the above system is homogeneous then it has nonzero solutions when the determinant of the system equations main matrix is equal to zero, i.e. if its rank is less than 8. Moreover, in this case the solutions are ambiguous.

Since the second equation multiplied by five gives the third equation and the third and fourth equations are identical, then after omitting two of them (the second and the third ones), we have

$$\begin{aligned}
&-0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.01634\dot{M}_6 \\
&+ 0.18705\dot{M}_7 + 0.00645\dot{M}_8 = 0 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 \\
&+ 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0 \\
&0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.00304\dot{M}_6 \\
&+ 0.0348\dot{M}_7 + 0.0012\dot{M}_8 = 0 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 - 0.03762\dot{M}_6 \\
&+ 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0 \\
&0.1584\dot{M}_1 + 0.0464\dot{M}_5 + 0.0152\dot{M}_6 \\
&- 0.261\dot{M}_7 + 0.006\dot{M}_8 = 0 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 \\
&+ 0.00435\dot{M}_7 - 0.01485\dot{M}_8 = 0.
\end{aligned} \tag{84}$$

As we are looking for nonzero solutions, we omit the second equation and we get

$$\begin{aligned}
&-0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.01634\dot{M}_6 \\
&+ 0.18705\dot{M}_7 + 0.00645\dot{M}_8 = 0 \\
&0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.00304\dot{M}_6 \\
&+ 0.0348\dot{M}_7 + 0.0012\dot{M}_8 = 0 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 - 0.03762\dot{M}_6 \\
&+ 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0 \\
&0.1584\dot{M}_1 + 0.0464\dot{M}_5 + 0.0152\dot{M}_6 \\
&- 0.261\dot{M}_7 + 0.006\dot{M}_8 = 0 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 \\
&+ 0.00435\dot{M}_7 - 0.01485\dot{M}_8 = 0.
\end{aligned} \tag{85}$$

From the above we get nonzero solutions in case when the rank of the main matrix is not greater than 4. In our case, since the above system of equations is satisfied by any values of $\dot{M}_2$, $\dot{M}_3$ and $\dot{M}_4$, than after considering expert opinions, it is sensible to assume

$$\dot{M}_2 \cong 480, \ \dot{M}_3 \cong 1440, \ \dot{M}_4 \cong 480, \tag{86}$$

and in order to get 4 nonzero solutions of the system of equations (85) to fix one of the remaining unknown variables for instance, according to (44), assuming

$$\dot{M}_6 \cong 360. \tag{87}$$

After this the system of equations (85) takes the form

$$\begin{aligned}
&-0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.18705\dot{M}_7 \\
&+ 0.00645\dot{M}_8 = -5.8824 \\
&0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.0348\dot{M}_7 \\
&+ 0.0012\dot{M}_8 = -1.0944 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00435\dot{M}_7 \\
&+ 0.00015\dot{M}_8 = 13.5432 \\
&0.1584\dot{M}_1 + 0.0464\dot{M}_5 - 0.261\dot{M}_7 \\
&+ 0.006\dot{M}_8 = -5.472 \\
&0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00435\dot{M}_7 \\
&- 0.01485\dot{M}_8 = -0.1368.
\end{aligned} \tag{88}$$

Next, after subtracting the third equation from the fifth equation, we get

$$-0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.18705\dot{M}_7 + 0.00645\dot{M}_8 = -5.8824$$
$$0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.0348\dot{M}_7 + 0.0012\dot{M}_8 = -1.0944$$
$$0.1584\dot{M}_1 + 0.0464\dot{M}_5 - 0.261\dot{M}_7 + 0.006\dot{M}_8 = -5.472$$
$$-0.015\dot{M}_8 = -13.68. \quad (89)$$

The solutions of the above system of equations are

$$\dot{M}_1 \cong 330,\ \dot{M}_5 \cong 210,\ \dot{M}_7 \cong 280,\ \dot{M}_8 = 912. \quad (90)$$

Hence and considering (86) and (87), we get the following final solution of the equation (83)

$$\dot{M}_1 \cong 330,\ \dot{M}_2 \cong 480,\ \dot{M}_3 \cong 1440,\ \dot{M}_4 \cong 480,\ \dot{M}_5 \cong 210,\ \dot{M}_6 = 360,\ \dot{M}_7 \cong 280,\ \dot{M}_8 = 912. \quad (91)$$

Now, substituting in (41) the above mean values $\dot{M}_b$ of the system unconditional sojourn times in the particular operation states and the known probabilities p_{bl} of the system operation process transitions between the operation states given in the matrix (42), we may look for the optimal values $\dot{M}_{bl}$ of the mean values of the system conditional sojourn times in the particular operation states that maximizing the mean lifetime of the port oil piping transportation system in the reliability states subset $\{1,2\}$. The optimal values $\dot{M}_{bl},\ b,l = 1,2,...,8,\ b \neq l,$ should to satisfy the following obtained this way system of equations

$$0.06\dot{M}_{15} + 0.06\dot{M}_{16} + 0.86\dot{M}_{17} + 0.02\dot{M}_{18} = 330$$
$$p_{21}M_{21} + p_{22}M_{22} + p_{23}M_{23} + p_{24}M_{24} + p_{25}M_{25} + p_{26}M_{26} + p_{27}M_{27} + p_{28}M_{28} = 480$$
$$p_{31}M_{31} + p_{32}M_{32} + p_{33}M_{33} + p_{34}M_{34} + p_{35}M_{35} + p_{36}M_{36} + p_{37}M_{37} + p_{38}M_{38} = 1440$$
$$p_{41}M_{41} + p_{42}M_{42} + p_{43}M_{43} + p_{44}M_{44} + p_{45}M_{45} + p_{46}M_{46} + p_{47}M_{47} + p_{48}M_{48} = 480$$
$$0.125\dot{M}_{51} + 0.125\dot{M}_{56} + 0.687\dot{M}_{57} + 0.063\dot{M}_{58} = 210$$
$$0.4\dot{M}_{61} + 0.6\dot{M}_{65} = 360$$
$$0.82\dot{M}_{71} - 0.16\dot{M}_{75} + 0.02\dot{M}_{78} = 280$$
$$0.67\dot{M}_{81} + 0.33\dot{M}_{87} = 912.$$

Unfortunately, the solution of the above system of equations are ambiguous

7. Conclusion

The joint general model of reliability and availability of complex technical systems in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to system reliability and availability analysis constructed in the paper Part 1 was applied to reliability evaluation of the port oil piping transportation system. The main reliability and availability characteristics were evaluated and maximized after its operation process optimization.

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