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Experimental verification of theoretical calculations of the natural frequencies of axisymmetric vibrations of thin circular plates clamped at the edge

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ABSTRACT:

The paper presents an overview of methods of measuring the frequency of vibration. Harmonic or random excitations are commonly used methods for this purpose. The classic method of harmonic excitation involves the use of sinusoidally variable, frequency-modulated excitation. In the case of study of structural dynamics, a special significance deserves to be attached to the shock pulse method (SPM), in which short shocks function as a way of excitation of vibration, and the fact that this method does not require the use of complex measurement systems is one of its obvious advantages. The paper demonstrates that when measuring frequency of vibrations of thin plates and membranes with piezoelectric sensors, disregarding the influence of their weight, which may comprise from 0.05 to 0.03 in relation of weight of the plate, may result in considerable errors in the method, in the range of 5-10%. In such cases, negative approximations should be considered whose values in this study were calculated using analytical method of influence function and a numerical method of finite elements (FEM).

Weryfikacja doświadczalna obliczeń teoretycznych częstości własnych drgań osiowosymetrycznych cienkich płyt kołowych utwierdzonych na obwodzie

Słowa kluczowe: eksperyment, drgania osiowo symetryczne, płyta kołowa

STRESZCZENIE:

W pracy przedstawiono przegląd metod pomiaru częstości drgań. Powszechnie stosuje się metody wymuszenia harmonicznego lub losowego. Klasyczna metoda wymuszenia harmonicznego polega na stosowaniu sinusoidalnie zmiennego wymuszenia z modulowaną częstotliwością. W przypadku badania dynamiki konstrukcji na szczególną uwagę zasługuje metoda impulsowa, w której jako wymuszenie drgań stosuje się krótkotrwały udar. Jej niewątpliwą zaletą jest to, że nie wymaga ona stosowania złożonych układów pomiarowych. Uzasadniono, że w przypadku pomiaru częstości drgań cienkich płyt i membran czujnikami piezoelektrycznymi nieuwzględnienie wpływu ich masy, która może stanowić od 0,05 do 0,03 w stosunku do masy płyty, prowadzi do znacznych błędów metody, w granicach 5-10%. W takich przypadkach należy uwzględnić poprawki ujemne, które w niniejszej pracy zostały obliczone metodą analityczną funkcji wpływu oraz numeryczną elementów skończonych.

1. METHODS OF MEASURING FREE VIBRATIONS OF PLATES

The main parameters characterising the dynamics of the mechanical system are the frequency and mode of vibrations [1-3]. Commonly applied methods of determining these parameters include free vibrations, harmonic excitation and self-excited vibrations [4, 5]. In the studies of structural dynamics, a special attention should be paid to shock pulse method [6]. Vibration excitation using the method of free vibrations is carried out by hitting a plate with a rubber or lead mallet.

Mechanical vibrations excited in an object need to be converted into electrical vibrations that are transmitted via an amplifier to the vertical deflection plates of the oscilloscope. Horizontal deflection plates of the oscilloscope are supplied with the frequency-modulated voltage from a generator. As a result, the screen of oscilloscope shows forming Lissajous figures, the shape of which depends primarily on the ratio of the measured frequency and the waveform produced by the generator. When using the method of harmonic excitation, an exciter powered by a generator is applied (Fig. 1). By changing the frequency of the generator, the frequency can be equalled with plate's free vibrations [7].

Self-oscillation, however has not found wider application in practice as a method of measuring, and is primarily used to support the automatic vibration of a test piece, e.g., a turbine blade at its natural frequency [8]. By applying this method only free vibration modes are produced as the easiest to excite, and they usually are first mode tangential vibrations [9, 10]. The method can also be used for fatigue testing. A particular attention was paid in this paper to two of the most effective methods, i.e. the method of resonance vibration method and shock pulse method [11, 12].

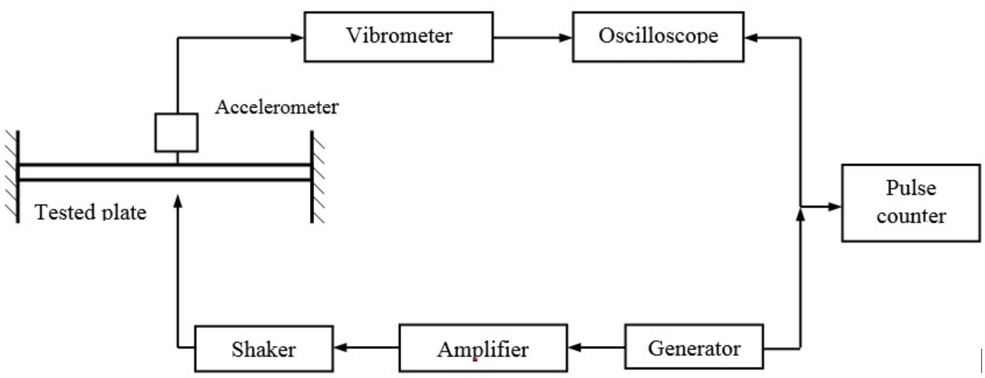

Figure 1 A block diagram of a test rig for analysing free vibrations of a plate with the use of resonance method [4]

2. TEST RIG USING SPM AND RESONANCE METHOD

Experimental determination of the free vibration frequency of tested plates was carried out by two methods, i.e. resonance and shock pulse. Schematic designs of test rigs and measuring systems, built in the Bialystok University of Technology, are shown in Figures 2-3.

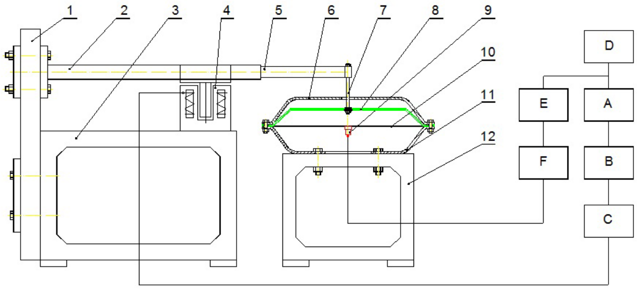

Figure 2 Schematic design of the test rig and measuring unit for measuring plate's free vibrations with the use of resonance method: 1 – cantilever, 2 – arm, 3 – casing, 4 – electromagnet, 5 – pipe, 6 – top cover, 7 – shaft, 8 – membrane, 9 – KD 35 accelerometer, 10 – test plate, 11 – bottom cover, 12 – casing, A – PM 5121 generator, B – amplifier, C – PR 9270 vibration exciter, D – pulse counter, E – PM 3206 oscilloscope, F – SM 231 vibrometer [4]

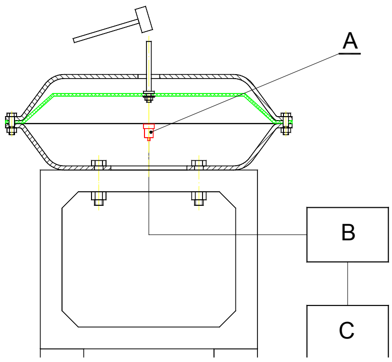

Figure 3 Schematic design of the test rig and measuring system for measuring plate's free vibrations with the use of SPM excitation

In both cases, the test plate (10) has been mounted between two circular covers (11 and 6) which are pressed against the plate by means of 20 M10 bolts. This has provided the conditions of a rigid clamping of the plate's edge. By cause of a rubber membrane (8) which has been placed between the top cover (6) and the plate, the vibration of plate is initiated by movement of air, enclosed in a sealed space between the plate and the membrane and caused by the membrane moved by a shaft (7). The vibration excitation system of a plate operating by means of resonance method comprises: generator (A), amplifier (B), resonant system (LC) with exciter (C) and the electromagnet (4). Electromagnetic jumper is connected to the arm (2) whose end is rigidly fixed to the cantilever (1). The arm is made out of a steel pipe of 42 mm in diameter. Vibrations of the electromagnetic jumper initiate lateral oscillation of the arm (2). At the end of the arm an aluminium tube is attached (5) which is connected to the shaft (7) of the membrane (8). In this way the vibrations of the arm are transferred to the membrane. Vibrations of tested plate (10) are processed by the accelerometer (9) and vibrometer (F) into an electric signal, which then is supplied to the vertical deflection plates of the oscilloscope (E). At the same time, horizontal deflection plates of the oscilloscope are supplied with the voltage from the generator (A). Since the frequencies of each waveform are the same, an ellipse appears on the screen of oscilloscope. Alteration of the frequency of the generator changes the frequency of excited vibrations of exciter-plate circuit. At the moment of equalization of the frequency of excited vibration with a frequency of free vibration of plate a resonance occurs, which evinces a rapid increase in the amplitude of the vibration of plate. Then, the greatest amplitude of vertical deflection appears on the oscilloscope screen. By slowly changing frequency of the generator in the resonance area, the phase shift between the exciting force and vibrating arm is altered. At the resonance the shift is 90°, which corresponds to the vertical position of the ellipse on the oscilloscope screen. Thus, when passing through the resonance a phenomenon of rolling ellipse occurs. After a precise capture of the moment of the resonance, frequency of free vibration of plate is read from the pulse counter (D). The search for those frequencies in case of the discussed system is conducted by manual tuning of the frequency of the generator to natural frequencies of plates. Vibrations of test plate in shock pulse excitation are raised by a pulse blow with a hammer equipped with a shock pulse metre. Impact energy causes the formation of free, short-term vibrations in the plate which, a result of dissipation of energy disappear very quickly. Frequencies of free vibrations are then determined on the basis of the recorded vibration waveform. In the present case, mechanical vibrations of the plate are processed by the accelerometer (9) and vibrometer (F) and recorded by the recorder (G), and then subjected to spectral analysis using appropriate software. The classic method of harmonic excitation involves the use of sinusoidally variable, frequency-modulated excitation. In the process of constructing a work station an impedance head is used. The fundamental issue here is the determination of point mechanical impedance using the formula [8]:

$$Z = \frac{|F| e^{i\omega t}}{|V| e^{i(\omega t + \varphi)}} \,. \qquad (1)$$

A given value of the amplitude of the vibration speed V(t) stabilises during the measurement, with the application of a feedback loop. A waveform of exciting force F(t) is recorded and the resulting minima of frequency characteristics indicate resonance frequencies. Typical measurement system, shown in Figure 2, comprises: vibration exciter, a sine wave generator and voltage amplifier in excitation circuit [9]. Impedance head used in the system allows the simultaneous measurement of the exciting force and acceleration in oscillating motion. The signal from the head is amplified and saved on a recorder.

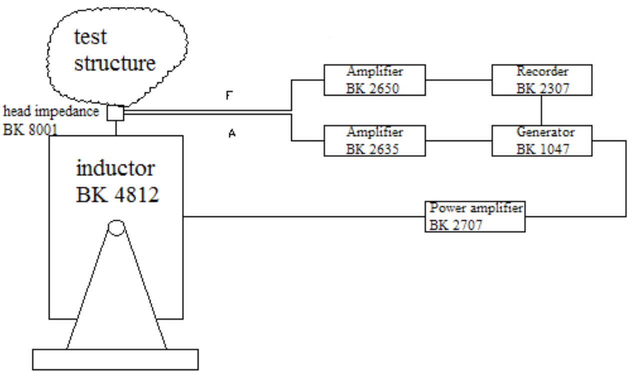

Figure 4 A schematic design of measuring system for testing vibrations of mechanical system forced with harmonic excitation with impedance head [12]

All devices, installed on both the station using resonance method and the method of harmonic excitation with impedance head come from the product range of a trade company Brüel-Kjaer: (Product Data of Brüel & Kjaer - Cada PC Modal Analysis Software - WT 9240 to WT 9245). Harmonic excitation method has been described above for its innovative solution in excitation technology. Examples of results from Uhl and Panuszka, illustrating the correctness of this method are shown in Table 1 [12].

Table 1 Tests and calculations results [Hz] for a circular plate, similar to the one tested at the University of Technology in Bialystok, presented by Uhl and Panuszka in [12]

Resonant frequency	Theoretical calculations	Harmonic excitation	Absolute error	Relative error
f_{01}	35.6	28.1	7.5	26.6
f_{02}	139.2	123.2	16	12.9
f_{03}	313.0	286.3	24.7	8.7
f_{04}	552.9	507.0	45.9	9.0

The error analysis demonstrates that along with increasing frequency, the absolute error also increases, whereas the relative error decreases. Harmonic excitation is quite simple and convenient method in practice of measurement. One of the major problems only is the necessity to use a high-power exciter. Subsequent statistical calculations give the variability coefficient of about 1%, which is indicative of a high reproducibility of the results.

3. EXAMPLES OF RESULTS OF STUDIES ON FREQUENCY OF AXISYMMETRIC VIBRATIONS OF THIN CIRCULAR PLATE CONDUCTED AT THE UNIVERSITY OF TECHNOLOGY IN BIALYSTOK

The study used a thin, uniform, circular plate made of steel sheet with a thickness of 0.7 mm.

Table 2 Dimensions and material of tested plate

No.	Dimension	Symbol	Value	Unit
1	Diameter	d	0.5	m
2	Thickness	h	0.0007	m
3	Density	ρ	7860	kg/m^3
4	Young modulus	E	2.1×10^{11}	Pa
5	Poisson distribution	ν	0.3	-

Examples of the results of experimental studies conducted by the author at the University of Technology in Bialystok are shown in Figures 5 ÷ 7. For a plate with concentrated mass in the form of 35 KD piezoelectric accelerometer amplitude-frequency characteristics were obtained. Frequencies have been marked on the abscissas, while the amplitudes of vibrations on the ordinate.

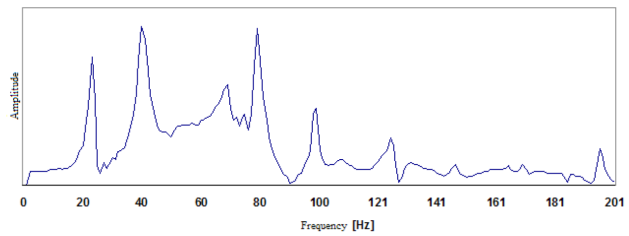

Figure 5 System response to shock pulse excitation

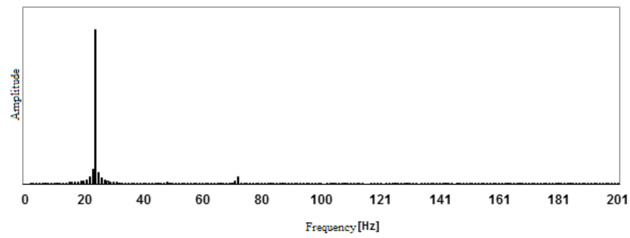

Figure 6 System response to harmonic excitation at a frequency of f=24[Hz]

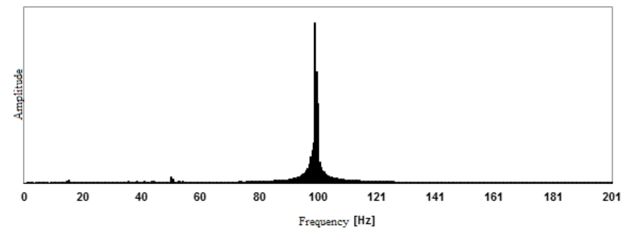

Figure 7 System response to harmonic excitation at a frequency of f=99[Hz]

4. EXAMPLES OF RESULTS OF CALCULATIONS FOR TESTED CIRCULAR PLATE WITH THE REGARD TO THE ACCELEROMETER WEIGHT CONDUCTED WITH THE ANALYTICAL METHOD OF INFLUENCE FUNCTION

The object of experimental research is the plate whose dimensions and material data (steel) are summarized in Table 2. The supplementary parameter: the additional mass concentrated in the middle of the plate: md = 30 [g] = 0.03 [kg]. The weight of accelerometer, centred in the middle of the symmetry of the plate has also been determined. Adopted mass of the plate M = 1.08 kg was based on the data presented in Table 1. Thus, the ratio of the mass of accelerometer to the weight of the plate is approx. 3/100 (0.03). The following examination concerns the influence of additional mass focused in the centre of symmetry, which is a weight of used piezoelectric accelerometer. In the case of thin circular plates discounting these mass results in significant measurement errors [13, 14]. According to Roberson [8], the frequency ω_{mo} of symmetrical vibrations is calculated from the formula:

$$\omega_{no} = \frac{4 \cdot \alpha_{no}}{d^2} \sqrt{\frac{D}{\rho \cdot h}} \,, \tag{2}$$

where: α_{no} – frequency coefficient, D – rigidity of plate, ρ – specific weight, h – thickness of plate, d – diameter of plate.

Calculations of plate's free vibrations were accomplished by using a formula (2) in the forms [4, 5]:

$$\varpi_{no} = a \cdot \alpha_{no} \,, \tag{3}$$

where:

$$a = \frac{4}{d^2} \sqrt{\frac{E \cdot h^2}{12(1-\nu^2) \cdot \rho}} \,. \tag{4}$$

In the first place, values of coefficient α_{no} from Table 3 were interpreted for the ratio of additional mass m_d and specific mass of the plate M:

$$\frac{m_d}{M} = 0 \,.$$

Further calculations were also carried out for the circular plate, taking into account the specific weight of the piezoelectric accelerometer used to measure vibration. Calculated values of parameters: a, ω1 and f1 for the circular plate without taking into account the additional concentrated mass were obtained based on the following data contained in Tables 3 and 4: a = 17.52 rad/s, ω_1 = 178.94 rad/s and f_1 = 28.48 Hz. Calculated values of the frequency of vibrations of the plate without additional concentrated mass are summarised in Table 3.

Table 3 Calculated values of vibrations of a plate without additional mass obtained through the method of influence function

Order of vibrations n	Frequency of vibrations f[Hz]
1	28.48
2	110.88
3	248.47
4	441.04

Frequencies of vibrations of circular plate were calculated taking into account the weight of the accelerometer, whose adopted mass m_d = 30 g was centered at the axis of the plate. The mass of the plate: M = 1.08 kg was adopted on the basis of data presented in Table 5. The mass ratio of the accelerometer and plate is then:

$$\frac{m_d}{M} = \frac{0.03}{1.08} = 0.028 \,. \tag{5}$$

Frequencies of free vibrations were calculated with the use of formula (3). According to Roberson, Table 3 does not include values of coefficient α_{no} for mass ratio

$$\frac{m_d}{M} = 0.028 \, . \qquad [8]$$

Therefore, approximation of these values has been conducted using quadratic function. Resulting values α_{no} have been summarised in Table 4.

Table 4 Values of coefficient α_{no} after approximation based on data in Table 3, after Roberson [8]

Concentrated mass - plate mass ratio m_d / M / Order of vibrations n=	0.028
	Values α_{no}
1	9.48
2	35.29
3	78.12
4	138.43

When referring to the data contained in Table 3, values of the parameters a, ω_1 and f_1 are obtained, taking into account the additional concentrated mass. These values have been calculated on the basis of formulas (3 and 4) and are as follows: a = 17.519 rad/s, ω_1 = 166.080 rad/s, and f_1 = 26.43 Hz. Calculated values of vibration frequency for a plate, taking into account concentrated mass of the accelerometer are summarised in Table 5.

Compiled calculated values of vibration frequency of the plate with a mass concentrated in the centre of symmetry of the plate and without the additional weight, presented in Tables 3, 4, show that a slight concentrated weight of the accelerometer significantly reduces the frequency of vibration.

5. EXAMINING INFLUENCE OF THE MASS OF PIEZOELECTRIC ACCELEROMETER ON THE VIBRATION FREQUENCIES OF A PLATE BY USING FINITE ELEMENTS METHOD

The calculation of the frequency and mode of free vibration of a plate by using finite element method (FEM) was performed with the assistance of the applications: MSC / Patran and MSC / Nastran [15, 16]. Considerations apply to the plate which is the object of experimental research. Its dimensions and material data (steel) are summarised in Table 2. The additional parameter: the additional weight concentrated in the middle of the plate: m_d = 30 [g] = 0.03 [kg].

Table 5 A summary of values of plate's free vibrations and modes calculated with FEM

Order of vibrations n	Frequency of vibrations f[Hz]		Mode
	Plate without concentrated mass	Plate with concentrated mass	
1	28.48	26.48	MODA 0.0
2	110.16	98.20	MODA 0.1
3	247.21	212.09	MODA 0.2
4	437.58	372.34	MODA 0.3

a) b)

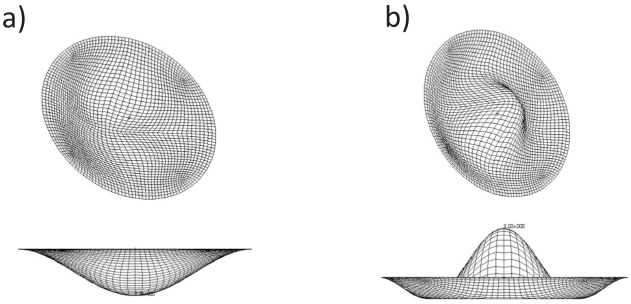

Figure 8 Modes of vibrations: a) mode 0.0; b) mode 0.1

6. COMPARISON OF THE RESULTS OF THEORETICAL CALCULATIONS AND EXPERIMENTAL MEASUREMENTS

Analysis of the results shows that the use of FEM for solving this type of problems is appropriate, which is further supported by a small absolute error (Δ_{max} = 3.53%) compared with the theoretical values. The influence of the mass is presented in the tables below.

Table 6 Results of theoretical calculations of subsequent frequencies of plates with and without additional mass based on Bessel function [4]

Resonance frequency	Theoretical calculations results, after Roberson [8]		Difference [Hz]	Difference [%]
	m_d=0	m_d=30 [g]		
$f_{0.0}$ [Hz]	28.48	26.43	2.05	7.19
$f_{0.1}$ [Hz]	110.88	98.38	12.49	11.27
$f_{0.2}$ [Hz]	248.47	217.82	30.65	12.33
$f_{0.3}$ [Hz]	441.04	385.97	55.09	12.49

Table 7 Results of theoretical calculations of subsequent frequencies of plates with and without additional mass based on FEM (Finite Elements Method)

Resonance frequency	FEM calculations results		Difference [Hz]	Difference [%]
	m_d=0	m_d=30 [g]		
$f_{0.0}$ [Hz]	28.48	26.48	1.99	7.00
$f_{0.1}$ [Hz]	110.16	98.20	11.96	10.86
$f_{0.2}$ [Hz]	247.21	212.09	35.12	14.21
$f_{0.3}$ [Hz]	437.58	372.34	65.24	14.91

Table 8 Comparison of the results of theoretical calculations and experimental measurements with the consideration of piezoelectric accelerometer mass (through shock pulse and resonance methods)

Resonance frequency	Roberson's precise solution [8]	FEM	Shock pulse method		Resonance method	
	f_t[Hz]	f[Hz]	f_u[Hz]	Δ[%]	f_r[Hz]	Δ[%]
$f_{0.0}$ [Hz]	26.43	26.48	23.11	14.37	24.11	8.78
$f_{0.1}$ [Hz]	98.38	98.20	98.96	0.58	99.46	1.09

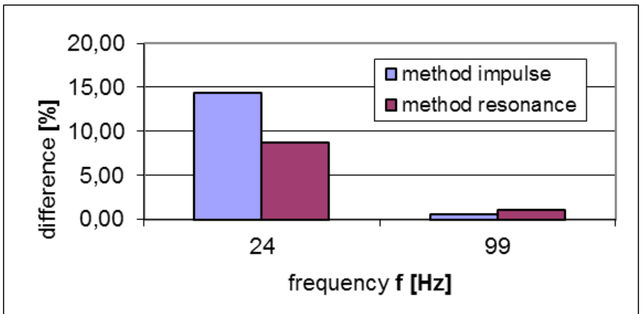

Figure 9 Difference in percentage for the first two frequencies obtained by using impulse method and resonance method

Figure 9 ilustrates a difference of the results between the experiment and theoretical calculations obtained through a precise method.

7. SUMMARY

Accurate determination of the impact of concentrated masses on the frequency of free vibrations of a plate belongs to the more complex problems of dynamics of plates. On the basis of the results of theoretical and experimental studies, obtained on the test facility built in the Bialystok University of Technology for the purpose of teaching and research (Fig. 2 and 3), it can be stated that the mass of piezoelectric accelerometer has a significant impact on the measurement of vibration of thin circular plates. Summary of calculated and experimental values of vibration frequencies show that the concentrated mass of piezoelectric accelerometer used in the test significantly affects the reduction of the actual frequency of vibrations. An analysis method based on the Bessel function and influence function proves that the concentrated mass decreases the first few frequency values by 7.19 ÷ 12.49% (Tab. 10). A numerical method of FEM indicates a decrease of frequencies due to the additional mass by 7 ÷ 14.91% (Tab. 8). The use of Finite Elements Method is, in this case purposeful and expedient, as it allows to take into account not only the weight of the sensor but also its geometry, as evidenced by a small absolute error in the calculation results compared with the precise values (Δ_{max} = 3.53 %).

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