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INTERVAL-VALUED INTUITIONISTIC FUZZY FAILURE MODES AND EFFECT ANALYSIS OF THE SYSTEM FAILURE RISK ESTIMATION

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Abstract

Among the risk assessment methods, failure modes and effects analysis (FMEA) is a popular, widely used engineering technique in many areas. It can be used to identify and eliminate known or potential failure modes to enhance reliability and safety of complex systems. In practice, risk estimations encounter difficulties connected with shortage of data. In such cases, we have to rely on subjective estimations made by persons with practical knowledge in the field of interest, i.e. experts. However, in some realistic situations, the decision makers might be unable to assign the exact values to the evaluation judgments due to his/her limited knowledge. In other words, there is a certain degree of hesitancy in human cognition and his/her judgment, who may have insufficient knowledge of the problem domain or uncertainty in assigning the evaluation values to the objects considered. In order to deal with ambiguity and uncertainty in the imperfect information, there have been recently proposed many various such theories as fuzzy sets, interval-valued fuzzy sets, type-2 fuzzy sets, hesitant sets, grey sets, rough sets and intuitionistic fuzzy sets. They have drawn more and more attention of scholars and been adopted in many applications This article addresses the Atanassov's interval-valued intuitionistic fuzzy sets and FMEA methods in the risk estimation of the system failures based on the expert judgments.

Keywords: *interval-valued intuitionistic fuzzy sets, failure modes and effects analysis, system failure risk estimation, expert judgment*

1. Introduction

Failure modes and effects analysis (FMEA) is an effective methodology to identify potential failure modes of a process or product, to assess the risk associated with failure modes and to determine the preventive actions to eliminate or reduce the risk of the respective failure modes. It is also intended to provide information for making risk management decisions. The traditional FMEA determines the risk priorities of failure modes using the so-called risk priority numbers (RPNs), which is defined as simple product of probabilities of the occurrence (O), severity (S) and detection (D) of the failure mode. Determination of these probabilities is in practice confronted with difficulties connected with shortage of data [3]. The traditional FMEA also seems to be inadequate explicitly to capture the important assessments for deriving the priorities. The shortcomings in practical implementation of FMEA pertain to its risk evaluation and prioritization issues, complexity and intricacy of use.

The main weakness of traditional FMEA is that the same risk priority numbers can be obtained from different combinations of the risk factors O, S and D. To overcome this issue, therefore, a wide variety of methods has been proposed in the literature to overcome the shortcomings and improve the effectiveness of the traditional FMEA. The risk evaluation methods can be divided into five groups [8]: artificial intelligence techniques, mathematical programming methods, multi-criteria decision-making (MCDM) methods, integrated approaches and other methods. Among these methods, the fuzzy theory is powerful tool for FMEA in dealing with the vagueness of human perception and recognition, making it more flexible in solving the real-life problems. Wang, Chin, Poon and Yang [14] have showed drawbacks and given significant criticisms for the traditional FMEA. Then, the authors proposed in [14] a new fuzzy FMEA, which allows the risk factors and their relative weights to be evaluated in linguistic forms. Abdelgawad and Fayek [1] pointed out several drawbacks of the traditional FMEA in applications and showed the capability of fuzzy FMEA to overcome these issues. Liu et al. [8] proposed a novel approach for FMEA based on combination weighting and fuzzy VIKOR (Visekriterijumska optimizacija i KOmpromisno Resenje) method to deal with the uncertainty and vagueness from humans' subjective perception and experience in risk evaluation process. To demonstrate potential applications, authors adopted the new fuzzy FMEA for analysing the risk of general anesthesia process. Besides, Zhou and Thai [17] applied grey theory and fuzzy theory in FMEA of the oil tanker equipment failure to show that the evaluation of failure modes by both fuzzy theory and grey theory are quite similar. The fuzzy set theory has been also applied to the decision-making problems [4, 6, 7] and preference relations [15].

Since the membership function of a fuzzy set is only single-valued function, it cannot be used to express the support and objection evidences simultaneously. In many practical situations, the decision makers may not be able to express their evaluations or preferences accurately due to the fact that they may not grasp sufficient knowledge on the domain considered, or they are unable discriminate explicitly the degree to which an alternative is better than others do. In other words, there is a certain degree of hesitation. In order to describe such situations and to model human's perception and cognition more comprehensively, Atanassov [2] extended Zadeh's fuzzy set to the intuitionistic fuzzy set (IFS), which is characterized by membership degree, non-membership degree, and hesitancy degree, which sum up to one. Afterwards, the IFSs have attracted increasingly scholars' attention and has been applied to many different fields, such as decision-making [15], IF cognitive maps [10], medical diagnosis [5], fault diagnosis [13] and pattern recognition [9, 14]. Xu and Liao [16] extended the classical AHP and the FAHP to the IF circumstances and developed the IF-AHP procedure for handling comprehensive multi-criteria decision-making problems.

In 1989, as a generalization of the intuitionistic fuzzy set (IFS), Atanassov and Gargov (1989) introduced the so-called interval-valued intuitionistic fuzzy set (IvIFS). The justification of this generalization is that the degrees of membership or non-membership in IFSs cannot be sometimes determined exactly as a number due to the increasing complexity of the real-life decision-making problems or the lack of precise information about the problem domains. Therefore, IvIFSs can be more suitable to represent ambiguous and uncertain decision information since their membership and non-membership degrees are represented by a whole interval of values. Nevertheless, this introduces an additional dimension of uncertainty to the intuitionistic fuzzy setting, making it more difficult in computation.

In this article, a new method for estimating the risk caused by system failures is proposed. The estimation is fully based on the expert judgments. Expert is assumed to be well acquainted with the subject he is expected to formulate his judgment on. Expert should also be capable of formulating his judgment. This is connected with level of his education and the language used in the elicitation process, particularly as regards the parameters the expert is expected to estimate. The experts, on the other hand, prefer to formulate their opinions in the linguistic categories. However, in some realistic situations, the decision makers might be reluctant or unable to assign the crisp evaluation values to the judgments due to his/her limited knowledge. Therefore, their practical knowledge may contain ambiguousness and uncertainty in some extent. This article presents an interval-valued intuitionistic fuzzy FMEA method in the subjective estimation of the system failure risk based on the expert judgments. The method presented has been developed with an intention of using it in the decision-making procedures in risk prediction during the system operation.

2. Basic concepts

2.1. Interval-valued intuitionistic fuzzy sets

Atanassov and Gargov (1989) generalized the concept of interval-valued fuzzy sets (IvFSs) and IFSs by introducing an interval-valued intuitionistic fuzzy set (IvIFS) A^I over a finite universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ as:

$$A^I = \{\langle x_i, M_{A^I}(x_i), N_{A^I}(x_i)\rangle | x_i \in X\}, \tag{1}$$

where $M_{A^I}(x_i) = [\mu^-_{A^I}(x_i), \mu^+_{A^I}(x_i)] \subset [0,1]$ denotes the interval membership degree and $N_{A^I}(x_i) = [\nu^-_{A^I}(x_i), \nu^+_{A^I}(x_i)] \subset [0,1]$ denotes the interval non-membership degree of an element x_i to IvIFS A^I, such that for every $x_i \in X$, $\mu^+_{A^I}(x_i) + \nu^+_{A^I}(x_i) \leq 1$ and

$$[\pi^-_{A^I}(x_i), \pi^+_{A^I}(x_i)] = [1 - \mu^+_{A^I}(x_i) - \nu^+_{A^I}(x_i), 1 - \mu^-_{A^I}(x_i) - \nu^-_{A^I}(x_i)], \tag{2}$$

where $[\pi^-_{A^I}(x_i), \pi^+_{A^I}(x_i)]$ denotes the interval hesitancy degree of x_i to A^I.
The complementary set A^I_C of A^I is defined as:

$$A^I_C = \{\langle x_i, N_{A^I}(x_i), M_{A^I}(x_i)\rangle | x_i \in X\}. \tag{3}$$

Definition 2.1. [9]. Let $\mu^I_A = [\mu^-_A, \mu^+_A]$ and $\mu^I_B = [\mu^-_B, \mu^+_B]$ be any two intervals in [0, 1], i.e. $[\mu^-_A, \mu^+_A] \subseteq [0,1]$ *and* $[\mu^-_B, \mu^+_B] \subseteq [0,1]$, relations between them are defined as follows:

$$\begin{cases} [\mu^-_A, \mu^+_A] \geq [\mu^-_B, \mu^+_B] \ iff \ \mu^-_A \geq \mu^-_B \ and \ \mu^+_A \geq \mu^+_B, \\ [\mu^-_A, \mu^+_A] \succcurlyeq [\mu^-_B, \mu^+_B] \ iff \ \mu^-_A \geq \mu^-_B \ and \ \mu^+_A \leq \mu^+_B, \\ [\mu^-_A, \mu^+_A] = [\mu^-_B, \mu^+_B] \ iff \ \mu^-_A = \mu^-_B \ and \ \mu^+_A = \mu^+_B, \end{cases} \tag{4}$$

where $\succcurlyeq$ denotes “preferred to”.

2.2. Interval-valued intuitionistic fuzzy operators

Let A and B be two singleton IvIFSs, called interval-valued intuitionistic fuzzy values (IvIFVs) and denoted by $A = \langle \mu_A, \nu_A \rangle$ and $B = \langle \mu_B, \nu_B \rangle$. The operations of addition $\oplus$ and multiplication $\otimes$ on IvIFVs were defined by Atanassov [2] as follows:

$$A \oplus B = \langle [\mu^-_A + \mu^-_B - \mu^-_A\mu^-_B, \mu^+_A + \mu^+_B - \mu^+_A\mu^+_B], [\nu^-_A\nu^-_B, \nu^+_A\nu^+_B]\rangle, \tag{5}$$

$$A \otimes B = \langle [\mu^-_A\mu^-_B, \mu^+_A\mu^+_B], [\nu^-_A + \nu^-_B - \nu^-_A\nu^-_B, \nu^+_A + \nu^+_B - \nu^+_A\nu^+_B]\rangle, \tag{6}$$

$$\lambda A = \langle [1 - (1 - \mu^-_A)^\lambda, 1 - (1 - \mu^+_A)^\lambda], [(\nu^-_A)^\lambda, (\nu^+_A)^\lambda]\rangle, (\lambda > 0), \tag{7}$$

$$A^\lambda = \langle [(\mu^-_A)^\lambda, (\mu^+_A)^\lambda], [1 - (1 - \nu^-_A)^\lambda, 1 - (1 - \nu^+_A)^\lambda]\rangle, (\lambda > 0). \tag{8}$$

The operations (5-8) are used to aggregate local criteria for solving MCDM problems in the interval-valued intuitionistic fuzzy (IvIF) setting. Let $A_1, \dots, A_m$ be IvIFVs representing the values of local criteria and $\lambda_1, \dots, \lambda_m$; $\sum_{k=1}^m \lambda_k = 1$ be their weights. Then intuitionistic fuzzy weighted arithmetic mean (IvIFWA) can be obtained using operations (5) and (7) as follows:

$$\begin{aligned} IvIFWA_\lambda(A_1, \dots, A_m) &= \lambda_1 A_1 \oplus \lambda_2 A_2 \oplus \dots \oplus \lambda_m A_m = \\ &= \langle [1 - \textstyle\prod_{k=1}^m (1 - \mu^-_{A_k})^{\lambda_k}, \ 1 - \prod_{k=1}^m (1 - \mu^+_{A_k})^{\lambda_k}], \ [\prod_{k=1}^m (\nu^-_{A_k})^{\lambda_k}, \ \prod_{k=1}^m (\nu^+_{A_k})^{\lambda_k}]\rangle. \end{aligned} \tag{9}$$

Intuitionistic fuzzy weighted geometric mean (IvIFWG) can be obtained using operations (6) and (8) as follows:

$$IvIFWG_{\lambda}(A_1, \dots, A_m) = \lambda_1 A_1 \otimes \lambda_2 A_2 \otimes \dots \otimes \lambda_m A_m =$$
$$= \langle [\prod_{k=1}^{m} (\mu_{A_k}^{-})^{\lambda_k}, \prod_{k=1}^{m} (\mu_{A_k}^{+})^{\lambda_k}], \left[1 - \prod_{k=1}^{m} (1 - \nu_{A_k}^{-})^{\lambda_k}, 1 - \prod_{k=1}^{m} (1 - \nu_{A_k}^{+})^{\lambda_k}\right] \rangle. \quad (10)$$

These aggregation operators provide IvIFVs, are most popular in the solution of decision-making problems in the intuitionistic fuzzy setting. The weighted arithmetic average operator emphasizes the group's influence, whereas the weighted geometric average operator emphasizes the individual influence. An important problem is the comparison of IvIFVs to choose the best alternative when the resulting evaluations of alternatives are expressed in IvIFVs. Various methods have been developed to compare IvIFVs, though they are rather of heuristic nature [11, 12]. Therefore, in the following, we propose a new, intuitive appealing measure of IvIFSs, which is suitable in IvIF FMEA based on expert judgments.

2.3. Interval-valued membership knowledge measure for IvIFVs

In order to rank the IvIFVs, we utilize the interval-valued membership knowledge measure $K_F(\alpha)$ proposed in [9], which is intuitively appealing and simple in computation. The uncertainty of imperfect information in term of IvIFSs implies an additional uncertainty in the measures on IvIFSs. Therefore, the new interval-valued knowledge measure of A^I is naturally defined as a closed interval of knowledge measures. Firstly, we recall definition of the membership knowledge measure K_F from [9].

Definition 2.2. Let *IFS(X)* denotes the family of all IFSs over the finite universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ and $A \in IFS(X)$ given by $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$. A map $K_F: IFS(X) \longrightarrow [0,1]$ is called the membership knowledge measure for IFSs and defined as:

$$K_F(A) = \begin{cases} \frac{1}{n\sqrt{2}} \sum_{i=1}^{n} \sqrt{\left(\mu_A(x_i)\right)^2 + \left(\nu_A(x_i)\right)^2 + \left(\mu_A(x_i) + \nu_A(x_i)\right)^2}, for\ \mu_A(x_i) \geq \nu_A(x_i), \\ -\frac{1}{n\sqrt{2}} \sum_{i=1}^{n} \sqrt{\left(\mu_A(x_i)\right)^2 + \left(\nu_A(x_i)\right)^2 + \left(\mu_A(x_i) + \nu_A(x_i)\right)^2}, for\ \mu_A(x_i) < \nu_A(x_i). \end{cases} \quad (11)$$

Definition 2.3. Let $IvIFS(X)$ be the family of IvIFSs over X, $int[0,1]$ all subsets of the unit interval [0,1] and an IvIFS $A^I \in IvIFS(X)$ is given by $A^I = \{\langle x_i, [\mu_A^-(x_i), \mu_A^+(x_i)], [\nu_A^-(x_i), \nu_A^+(x_i)]\rangle | x_i \in X\}$. A map $K_F^I: IvIFS(X) \longrightarrow int[0,1]$ is called the interval-valued knowledge measure for IvIFSs and defined as:

$$K_F^I(A^I) = [K_F(A^-), K_F(A^+)], \quad (12)$$

where:

$$A^- = \{\langle x_i, \mu_A^-(x_i), \nu_A^-(x_i)\rangle | x_i \in X\} \text{ and } A^+ = \{\langle x_i, \mu_A^+(x_i), \nu_A^+(x_i)\rangle | x_i \in X\}. \quad (13)$$

Interpretation of interval-valued knowledge measure for IvIFSs is that it provides not exact amount of knowledge but an interval of knowledge measures of IFSs belonging to the IvIFS A^I, from minimum $K_F(A^-)$ to maximum $K_F(A^+)$.

3. Methodology of the interval-valued intuitionistic fuzzy (IvIF) FMEA

3.1. Interval-valued intuitionistic fuzzy risk factors

Usually, the risk factors O, S and D are evaluated by experts in linguistic terms. The linguistic terms and their related IvIF numbers can be summarized as shown in Tab. 1-3. For example, experts revealed their opinions on the occurrence probability of the system failures in the form of linguistic values chosen from the given linguistic set (Tab. 1): very high (VH), high (H), moderate (M), low (L) and very low (VL).

Tab. 1. Interval-valued intuitionistic fuzzy ratings for probability of failure occurrence

Rating	Probability of occurrence	IvIF number
Very high (VH)	Failure is almost inevitable	<[0.7,0.8], [0.1, 0.2]>
High (H)	Repeated failures	<[0.5, 0.6], [0.3, 0.4]>
Moderate (M)	Occasional failures	<[0.4, 0.5], [0.4. 0.5]>
Low (L)	Relatively few failures	<[0.3, 0.4], [0.5, 0.6]>
Remote (R)	Failure is unlikely	<[0.1, 0.2], [0.7, 0.8]>

Tab. 2. Interval-valued intuitionistic fuzzy ratings for probability of failure severity

Rating	Severity of occurrence of a failure	IvIF number
Very high (VH)	Very high severity ranking with warning	<[0.7, 0.8], [0.1, 0.2]>
High (H)	System inoperable with destructive failure	<[0.6, 0.7], [0.2, 0.3]>
Moderately high (MH)	System inoperable with equipment damage	<[0.5, 0.6], [0.3, 0.4]>
Moderate (M)	System inoperable with minor damage	<[0.4, 0.5], [0.4. 0.5]>
Moderately low (ML)	System inoperable without damage	<[0.3, 0.4], [0.5, 0.6]>
Low (L)	System operable with some degradation of performance	<[0.2, 0.3], [0.6, 0.7]>
Very low (VL)	System operable with minimal interference	<[0.1, 0.2], [0.7, 0.8]>

Tab. 3. Interval-valued intuitionistic fuzzy ratings for probability of failure detection

Rating	Probability of occurrence	IvIF number
Very remote (VR)	Very remote chance	<[0.0, 0.1], [0.8, 0.9]>
Very low (VL)	Very low chance	<[0.1, 0.2], [0.7, 0.8]>
Low (L)	Low chance	<[0.2, 0.3], [0.6, 0.7]>
Moderately low (ML)	Moderately low chance	<[0.3, 0.4], [0.5, 0.6]>
Moderate (M)	Moderate chance	<[0.4, 0.5], [0.4. 0.5]>
Moderately high (MH)	Moderately high chance	<[0.5, 0.6], [0.3, 0.4]>
High (H)	High chance	<[0.6, 0.7], [0.2, 0.3]>
Very high (VH)	Very high chance	<[0.7, 0.8], [0.1, 0.2]>
Almost certain (AC)	Almost certainty	<[0.8,0.9], [0.0, 0.1]>

Suppose there are n failure modes $F_i, (i = 1, \dots, n)$ of the system, and m experts $E_j, (j = 1, \dots, m)$. Let $R_{ij}^o = \langle [\mu_{ij}^{o-}, \mu_{ij}^{o+}], [\nu_{ij}^{o-}, \nu_{ij}^{o+}] \rangle$, $R_{ij}^s = \langle [\mu_{ij}^{s-}, \mu_{ij}^{s+}], [\nu_{ij}^{s-}, \nu_{ij}^{s+}] \rangle$ and $R_{ij}^d = \langle [\mu_{ij}^{d-}, \mu_{ij}^{d+}], [\nu_{ij}^{d-}, \nu_{ij}^{d+}] \rangle$ be the IvIF ratings of F_i on the risk factors O, S and D; ω_o, ω_s and ω_d be the weights of the three risk factors, $\lambda_j, (j = 1, \dots, m)$ be the relative importance weights of the experts, $\sum_{j=1}^{m} \lambda_j = 1$.

Using the IvIFWA operator (9) we aggregate the IvIF ratings of all experts on failure mode F_i with respect to the risk factors O, S and D, respectively:

$$R_i^o = IvIFWA_\lambda(R_{i1}^o, R_{i2}^o, \dots, R_{im}^o) = \lambda_1 R_{i1}^o \oplus \dots \oplus \lambda_m R_{im}^o =$$
$$= \left\langle \left[1 - \prod_{j=1}^{m} \left(1 - \mu_{ij}^{o-}\right)^{\lambda_j}, 1 - \prod_{j=1}^{m} \left(1 - \mu_{ij}^{o+}\right)^{\lambda_j}\right], \left[\prod_{j=1}^{m} \left(\nu_{ij}^{o-}\right)^{\lambda_j}, \prod_{j=1}^{m} \left(\nu_{ij}^{o+}\right)^{\lambda_j}\right] \right\rangle, \quad (14)$$

$$R_i^s = IvIFWA_\lambda(R_{i1}^s, R_{i2}^s, \dots, R_{im}^s) = \lambda_1 R_{i1}^s \oplus \dots \oplus \lambda_m R_{im}^s =$$
$$= \langle \left[1 - \prod_{j=1}^m \left(1 - \mu_{ij}^{s-}\right)^{\lambda_j}, 1 - \prod_{j=1}^m \left(1 - \mu_{ij}^{s+}\right)^{\lambda_j}\right], \left[\prod_{j=1}^m \left(\nu_{ij}^{s-}\right)^{\lambda_j}, \prod_{j=1}^m \left(\nu_{ij}^{s+}\right)^{\lambda_j}\right] \rangle, \quad (15)$$

$$R_i^d = IvIFWA_\lambda(R_{i1}^d, R_{i2}^d, \dots, R_{im}^d) = \lambda_1 R_{i1}^d \oplus \dots \oplus \lambda_m R_{im}^d =$$
$$= \langle \left[1 - \prod_{j=1}^m \left(1 - \mu_{ij}^{d-}\right)^{\lambda_j}, 1 - \prod_{j=1}^m \left(1 - \mu_{ij}^{d+}\right)^{\lambda_j}\right], \left[\prod_{j=1}^m \left(\nu_{ij}^{d-}\right)^{\lambda_j}, \prod_{j=1}^m \left(\nu_{ij}^{d+}\right)^{\lambda_j}\right] \rangle. \quad (16)$$

3.2. Interval-valued intuitionistic fuzzy risk priority number (IvIFRPN)

The traditional FMEA defines RPNs as the simple product of O, S and D without considering their relative importance weights, whereas the IvIFRPN is defined as the IvIF weighted geometric mean of the three risk factors O, S and D. This overcomes the drawback that the three risk factors are treated equally in traditional FMEA and the ratings on failure modes must be exact values in the fuzzy FMEA. IvIFRPN of the failure mode F_i can be aggregated using the intuitionistic fuzzy weighted geometric (IvIFWG) operator (10) as follows:

$$IvIFRPN_i = \omega_o R_i^o \otimes \omega_s R_i^s \otimes \omega_d R_i^d =$$
$$= \langle \left[(\mu_i^{o-})^{\omega_o}.(\mu_i^{s-})^{\omega_s}.\left(\mu_i^{d-}\right)^{\omega_d}, (\mu_i^{o+})^{\omega_o}.(\mu_i^{s+})^{\omega_s}.\left(\mu_i^{d+}\right)^{\omega_d}\right], \quad (17)$$
$$\left[1 - (1 - \nu_i^{o-})^{\omega_o}.(1 - \nu_i^{s-})^{\omega_s}.\left(1 - \nu_i^{d-}\right)^{\omega_d}, 1 - (1 - \nu_i^{o+})^{\omega_o}.(1 - \nu_i^{s+})^{\omega_s}.\left(1 - \nu_i^{d+}\right)^{\omega_d}\right] \rangle$$

Using (12), the IvIF knowledge measure of the IvIFRPNs of failure modes F_i can be calculated. Because the resulting evaluations are intervals, they can be ranked by using relations (4). The ranking order of the IvIF knowledge measures represents the risk priority of potential causes. For the ship system failure analysis, failure mode with the biggest score function should be given the top priority.

4. Numerical application

To demonstrate the applicability of the proposed method, an example about tanker system failure from a global tanker ship management company is adopted from [17]. Assume that a FMEA team consisting of five experts identifies 17 potential system failure modes on tankers (Tab. 4) and needs to prioritize them in terms of their failure risks so that high risky failure modes can be corrected with top priorities. Experts evaluate the risk factors of failure modes as probability of their occurrence, severity and detect ability using the linguistic terms defined in Tab. 1-3. The five experts are assigned with the following relative weights: 0.15, 0.25, 0.25, 0.20 and 0.15. Aggregated IvIF ratings of all experts on failure mode F_i with respect to the risk factors O, S and D, respectively are computed from (9). For example, aggregated IvIF ratings of all experts on failure mode F_1 (auxiliary engine) with respect to the risk factors O, S and D, respectively are as follows:

$$R_1^o = \langle [0.327, 0.428], [0.47, 0.572] \rangle,$$

$$R_1^s = \langle [0.358, 0.459], [0.438, 0.541] \rangle,$$

$$R_1^d = \langle [0.42, 0.521], [0.377, 0.479] \rangle.$$

The weights of the risk factors O, S and D are assumed to be 0.40, 0.35 and 0.25. Based on (18), interval-valued intuitionistic fuzzy RPNs of the 17 failure modes and their interval-valued knowledge measures (IvIFKMs) can be calculated as shown in Tab. 4. The IvIFKMs of the obtained IvIFRPNs can be ranked using relations (4), which indicate the priority order of ship system failure modes (Tab. 4).

Tab. 4. Results of the proposed IvIF FMEA method in ranking risk priority order

	Failure modes	IvIF RPN				IvIF knowledge Measures (KM)		Ranking by IvIF KM	Ranking by [17]
		μ_A^-	μ_A^+	ν_A^-	ν_A^+	lower	upper		
1	Auxiliary engine	0.359	0.461	0.437	0.539	–0.691	–0.867	7	8
2	Auxiliary machinery	0.333	0.442	0.448	0.558	–0.679	–0.868	9	6
3	Boiler	0.348	0.449	0.449	0.551	–0.692	–0.868	8	7
4	Cargo pump	0.187	0.323	0.560	0.677	–0.673	–0.884	14	14
5	Cargo system	0.000	0.201	0.687	0.799	–0.687	–0.916	17	17
6	Deck machinery	0.463	0.568	0.325	0.432	0.686	0.869	3	3
7	Electrical system	0.322	0.431	0.463	0.569	–0.683	–0.869	10	10
8	Emergency system	0.252	0.360	0.535	0.640	–0.696	–0.877	12	12
9	Hull part	0.142	0.274	0.612	0.726	–0.694	–0.895	15	15
10	Hydraulic system	0.248	0.363	0.528	0.637	–0.686	–0.877	13	13
11	Inert gas system	0.317	0.423	0.472	0.577	–0.688	–0.869	11	11
12	Main engine	0.660	0.766	0.078	0.234	0.702	0.906	1	1
13	Monitoring system	0.402	0.487	0.392	0.513	0.684	0.866	4	4
14	Mooring	0.376	0.678	0.403	0.322	–0.675	–0.866	6	9
15	Navigation system	0.571	0.496	0.117	0.504	0.687	0.884	2	2
16	Piping system	0.392	0.231	0.399	0.769	–0.686	–0.866	5	5
17	Steering Gear	0.123	0.461	0.665	0.539	–0.734	–0.907	16	16

Table 4 shows also the comparison of the proposed method with fuzzy method [17] for the given example. The rankings of the tanker system failure modes made by both approaches are almost the same, i.e. the riskiest failure is F_{12} (main engine) and the least risky one is F_5 (cargo system). The ranking of other failure modes is also consistent, e.g. the five most risky failures and three least risky ones. There are some differences in the middle rankings between approaches due to different used methods. For example, the rank of F_1 (Auxiliary engine) is seventh by the IFRPN method, while it is eighth by the fuzzy method. Meanwhile, the rank of F_3 (boiler) is eighth by the IFRPN method, while it is seventh by the fuzzy method. As can be seen from Tab. 4, F_{12} (main engine) is apparently the failure mode with the maximum overall risk and should be given the top priority, followed by F_{15} (navigation system), F_6 (deck machinery), F_{13} (monitoring system) and F_{16} (piping system). The ranking can be used for the decision-making of managers, arranging the period inspections and maintenances of the equipment properly, which can improve the system reliability and safety.

5. Conclusions

In this article, the IvIF method has been proposed for the risk estimation of the system failures, which is based exclusively on the judgments elicited by experts. The obtained results show that the proposed method is powerful and useful in dealing with imprecise and uncertain data, which are available in such cases. Combining IvIFS and FMEA methods allows incorporating the hesitancy and limited knowledge of expert judgments. Compared with the traditional FMEA, the proposed method seems more useful and effective for risk evaluation. Compared with the fuzzy FMEA, the proposed method shows more practical and flexible in describing the real-life problems. The proposed method is particularly useful in the expert investigations. It is worth noticing that subjective investigation results may (but not necessarily) be charged with greater error than objective results acquired in real operational process. Therefore, the further researches should be focused on validation of the proposed method by the objective results.

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