INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2023, VOL. 69, NO. 4, PP. 675–681
Manuscript received July 20, 2023; revised October, 2023. DOI: 10.24425/ijet.2023.147687

Transient Processes Associated with Turning an Antenna On and Off

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***Abstract*—The paper discusses the phenomena accompanying switching the sinusoidal excitation of an antenna on and off when the antenna is excited by a train of sinusoids containing several to several hundred periods. Transient phenomena are presented against the background of the resonant properties of the antenna. The processes of turning the antenna on and off take place under different conditions and therefore are different. When the antenna is switched on, the transient processes are determined by the antenna properties and the excitation properties. When the antenna is switched off, excitation is no longer present, and the properties of the antenna determine the transient process. We define a new measure of time: the effective light meter.**

***Keywords*—effective light meter; pulse excitation; transient processes in antennas**

I. Introduction

ANTENNAS are basic devices in communication systems. Antennas represent the beginning of a receiving channel and the end of a transmission channel. Such antennas are increasingly functioning through pulses.

Antennas designed for pulse operation can be divided into three main categories: antennas radiating a single pulse, antennas radiating a repetition of single pulses and antennas radiating a train of sinusoids, with several to several dozen sinusoidal periods [1].

In the case of antennas radiating single pulses, the designer focuses on sending energy through as narrow as possible of a beam over the distance needed. Importantly, this type of antenna usually exhibits a short pulse rise time and sends most of the energy in the intended direction; the behaviour of the antenna after pulse radiation is less important [2], [3].

If the energy transmitted in one pulse is too low, the energy can be increased by quickly repeating the pulses. This repetition places additional requirements on the antenna. It is also important that the antenna quickly returns to its original state after radiating a single pulse and is ready to transmit the next pulse. If, after sending the first pulse, the antenna continues to radiate, this would affect the radiation of the next pulse or pulses [4].

Antennas radiating a train of sinusoids seem to be the easiest to design [5]. An antenna radiating a train of sinusoids usually sends a sinusoidal wave with a dozen to several hundred periods in a pack.

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It is usually assumed that after the excitation is applied to the input terminals, the antenna starts working without transient processes. This assumption is incorrect because it takes time for the antenna to reach a steady state. The time to set these parameters is described later for the example of switching the voltage exciting the terminals of a simple dipole on and off. The changes in the input current of such an antenna are presented.

The concept of effective light meter is introduced. An effective light meter is a specific measure of the time in which phenomena occur in an antenna. The effect of the antenna resonance location and excitation frequency on the phenomenon of current stabilization in an antenna is also described.

II. Dipole antenna excited with pulse train

A. Dipole model

A well-known dipole was chosen from the literature as an example [6]. The phenomena occurring in a linear dipole are so well known that interpretation of the related phenomena is not difficult.

A simple dipole of total length $L = 1\,\text{m}$ is center fed by a voltage source. The dipole is a cylinder with a circular cross-section and a diameter of $d = 0.014044\,\text{m}$. The geometry of the antenna is shown in Fig. 1. Note that the size of the antenna is physically realizable.

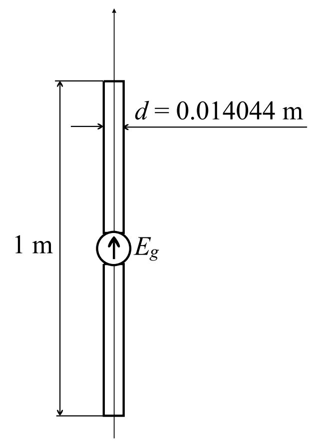

Fig. 1. Model of a symmetrical dipole 1 m long and with a diameter of 0.014044 m.

For a dipole of this length and diameter, the slenderness parameter Ω introduced by King [6]

$$\Omega = 2 \ln \frac{2L}{d} \tag{1}$$

is approximately 10, which means that the dipole is thin but on the equity limit of the thin-wire model.

The ideal dipole is infinitely thin and made of a perfect conductor.

Under such conditions, the current wave flowing along the antenna cable flows at almost light speed c in free space. The wavelength λ in this situation is calculated as

$$\lambda = \frac{c}{f} \tag{2}$$

where f is the frequency of antenna excitation.

The first (current) resonance of an infinitely thin antenna appears when a standing wave of a length of $\lambda/2$ is formed along the dipole, which with a dipole length of 1 m gives a frequency of 150 MHz.

If the antenna is not infinitely thin, then the current wave along the antenna slows as the thickness of the antenna increases.Thus, we can no longer use formula (2) to calculate the wavelength. In expression (2), if the frequency does not change, then when the speed of the wave reduces from c to some real speed v, the wavelength decreases:

$$\lambda = \frac{v}{f} \tag{3}$$

and as a consequence, the resonance length $\lambda/2$ decreases, and then, the resonance frequency decreases.

If we want to work in the first resonance with a given antenna length of 1 m, we need to reduce the input working frequency [7].

Thus, as the thickness of the antenna increases, the frequency of the first resonance decreases; this also applies to subsequent resonances.

As the antenna slenderness decreases - i.e., the thickness increases - the first resonance frequency decreases. This phenomenon is described as dipole shortening and is taken into account through the dipole shortening factor in the design. This phenomenon is caused by the decreasing speed of the current flowing along the dipole.

There are various reasons for the slower wave speed. If the conductor is not a perfect conductor, the speed decreases with increasing electric and magnetic permeability. As the thickness of the conductor increases, the effect of the skin effect also appears. Increasing the thickness of the conductor causes the current to flow not only along the side of the cylindrical conductor, but also along the ends of the conductor, which in turn extends the current path.

For thin copper conductors, the speed reduction is usually around 4-6%, but this is true for dipoles operating near resonances. For such dipoles, some textbooks provide formulas to calculate the antenna shortening factors [10].

The analysed antenna is thick in the sense discussed above, and a significant reduction in the first resonance frequency is to be expected.

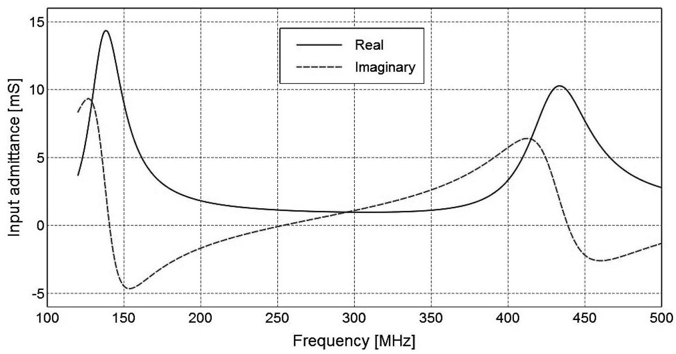

Fig. 2. Input admittance of the dipole from Fig. 1; the dipole 1 m long and 0.014044 m in diameter is excited at its center.

Fig. 2 shows the antenna input admittance calculated in the 500 MHz band. In our case, the admittance is calculated from the Hallén integral equation formulated in the frequency domain assuming the thin-wire approximation. The equation is solved by the method of moments with sinusoidal base functions in subdomains. The antenna model is excited by a delta-gap voltage generator. A program by the authors is used for the calculations.

In resonance, the imaginary component of input admittance disappears, and at the same time, the real component increases. From the course of the input admittance shown in Fig. 2, we see that the first resonance occurs at approximately 140 MHz (half-wave dipole) and the second occurs at approximately 440 MHz (one and a half wave-dipole).

Based on relationship (2), it can be calculated that for an infinitely thin antenna, the first two resonances mentioned above occur at 150 MHz and 450 MHz, respectively. The reduction in the resonance frequencies due to the thickness of the dipole is therefore significant.

Fig. 2 shows only the first two resonances. Of course, there are more resonances, and they repeat every $280 - 300$ MHz. However, the role of these subsequent resonances is less significant in the phenomena discussed here.

The data of the first two resonances already show that the first resonance plays a most important role in shaping the antenna behaviour.

The first resonance is very sharp, and both parts of the admittance, $Y_{in} = G_{in} + jB_{in}$, rapidly change around the first resonance frequency. The input conductance changes G_{in} and input susceptibility B_{in} near the second resonance are not so rapid, and this tendency of diminishing changes is maintained in the case of subsequent resonances.

The changes in the real and imaginary parts of the admittance near a resonance are related to the quality factor Q associated with a given resonance. Faster admittance changes around a resonance correspond to greater values of quality factor associated with the resonance. The greater the quality factor is, the greater the capacity to accumulate energy. From Fig. 2, we conclude that the quality factor - and thus the ability to accumulate energy around the resonance frequency - is greatest for the first resonance f_{r1}, equal to approximately 140 MHz. For the second resonance f_{r2}, the quality factor is smaller, and this tendency to reduce the quality factor continues for subsequent resonances.

Additionally, we note where the resonance susceptance goes through zero B_{in}, changing the character from capacitive to inductive. At the same time, the conductance G_{in} reaches the highest local value. Therefore, in a narrow resonance environment, the admittance can be considered real. These conditions are very favorable for the energy conversion of the antenna because with the real nature of the admittance of the antenna, all the power at the input terminals is active power.

As we reach the end of the description of the operation of the antenna model, note that the presented model describes the work of the dipole in transmission mode, i.e., the voltage signal of the generator connected to the input terminals is known and defined. However, because the considered model meets the assumptions of the Rayleigh-Helmholtz principle of reciprocity generalized by Carson [7], the described behaviour of input admittance is also valid for the antenna receiving mode, in which the antenna is excited by the electromagnetic wave falling from infinity onto the antenna, which causes, in turn, the current flow in the antenna and the electromotive force at the antenna terminals.

B. Narrowband Antenna Excitation

With such antenna behavior, we can work in several ways, primarily depending on whether the excitation is narrowband or broadband. Note that at the very beginning, the use of the concepts of narrowband and broadband makes sense for a given object (here, the antenna) and a given excitation.

For the purposes of this study, we adopt the following definition: a signal is called a narrowband signal if the object parameters in the excitation band change negligibly, in other words, if the object's parameters in the excitation band can be considered constant.

The above definition differs from the commonly used definitions of narrowband and broadband. In the definition presented here, we neither use absolute bandwidth nor relative bandwidth. Only the variability of the object parameters in the analyzed band is important. If this variability negligibly affects the analysis, then we are talking about a narrow band.

With this definition of narrowband, let us look again at Fig. 2.

Working with a dipole in resonance means high efficiency of energy conversion and actual input admittance. However, this type of work is only appropriate when working with narrowband excitation. As the excitation band widens, the admittance begins to change rapidly.

The frequency range between resonances is well suited for working with signals with a wider band. In the approximately 200 MHz to 370 MHz band, the dipole conductance changes minimally. Although the antenna susceptance changes more in the discussed band, it passes through zero at approximately 250 MHz. This condition is sometimes called antiresonance: we have the compensated susceptance, but the conductance reaches small values. As a result, a wide work band requires an appropriate matching system.Subsection text here.

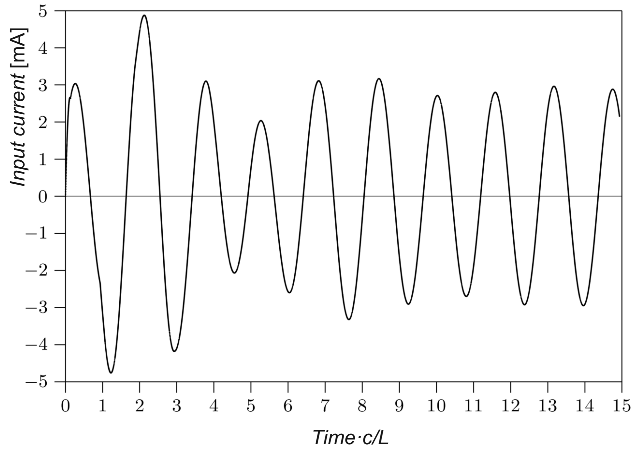

Fig. 3. Input current of a symmetrical dipole 1 m long and 0.014044 m in diameter excited at $t = 0$ with a sinusoidal voltage of 1 V amplitude and 190 MHz frequency.

C. Turning on the Transmission Antenna

Knowing the behaviour of the antenna model, we can consider what happens to the current flowing along the antenna when the power is turned on.

At the moment $t = 0$, we apply an input signal with a voltage of 1 V and a frequency of 190 MHz to the antenna terminals in such a way that at this moment, the signal has a zero phase. In response to the excitation, a current flows in the antenna. In our case, the current is calculated from the Hallén equation formulated in the time domain. The marching-on-in-time method can be applied. The Hallén equation is solved in space by the method of moments with sinusoidal base functions in subdomains. The antenna model is excited by a delta-gap voltage generator. A program by the authors is used for the calculations. The current waveform at the antenna terminals is shown in Fig. 3.

In Fig. 3 and the following figures, the time axis is traditionally scaled in light meters, i.e., normalized in such a way that the axis unit corresponds to the time needed for the current wave to flow from the centre of the dipole to the end and back, assuming that the wave is moving at a speed of c, i.e., as in a free space. Note the unit of measure on the horizontal axis of this figure. We calculate this measure by multiplying the time unit by the speed and dividing by the antenna length. As a result, the horizontal axis unit is dimensionless and corresponds to the time needed for the wave moving at the speed of light to travel the length of $L = 1$ m of the dipole. We call this unit of time a light meter [9]. We define this unit by analogy with the light year, which is a unit of length.

Note that the use of the time measure described above serves to better visualize phenomena only when the wave is moving at a speed of c. However, in our case, the antenna is not an infinitely thin wire and is not perfectly conductive, so the current waves move more slowly than those in free space; therefore, we propose a new measure of time.

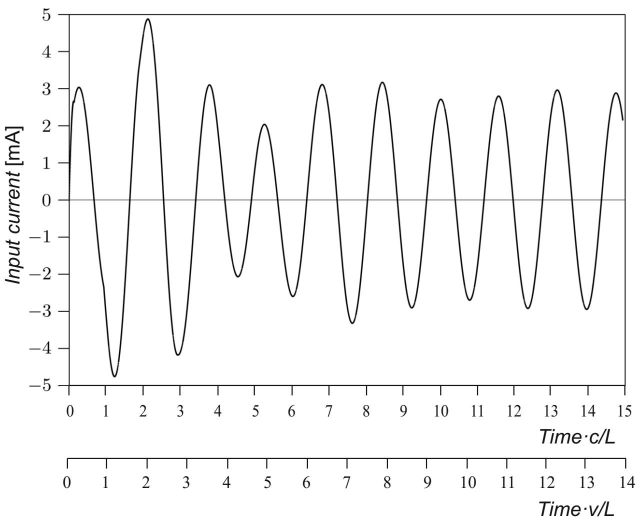

Fig. 4. The input current of the symmetrical dipole (Fig. 3) with the time axis scaled in light meters (upper axis) and effective light meters (lower axis).

The effective light meter is the time the wave needs to travel one meter in a given medium. An effective light meter is a unit of time relative to a given medium.

To calculate one effective light meter in the model under consideration, we first need to calculate the speed of the electromagnetic disturbance in the antenna. Based on the first resonance location $f_{r1} = 140\,\text{MHz}$, we can easily calculate the speed v of the current wave in the antenna, that is, $v = 2.8 \cdot 10^8$ m/s. Now, in place of the speed c in Fig. 3, we substitute the effective speed v, which allows us to scale the process timeline.

The input current waveform with two time axes is shown in Fig. 4. The upper axis is scaled in light meters, while the lower axis is scaled in effective light meters. Because the current wave in the model dipole is approximately 15% smaller than that in free space, the unit of time on the lower axis is larger and corresponds to the time when moving at a speed of $v = 2.8 \cdot 10^8$ m/s. The current wave travels a distance equal to the length of the dipole.

The duration of the transient process is usually divided into early time and late time [8]. The early time lasts from the initiation of the phenomenon ($t = 0$) until the return of the wave reflected from the outermost heterogeneity of the structure. In the analysed case, the furthest heterogeneities of the structure - from the centre - are the antenna ends, and the early time lasts until $t = 1$ count in effective light meters.

The definition of early time introduced above is very significant. Note that in the first unit of time, the course of the phenomenon is identical for all dipoles with the same structure and longer than 1 m, even for infinitely long dipoles.

This happens because the early time of the input current is dominated by excitation and there is no interaction with the current reflected from the antenna ends. Of course, if we choose a more complex antenna model in which the heterogeneities of the structure are closer to the feed point, then the early time would be shorter.

We should also emphasize that the effective light meter does not depend on the frequency of the excitation but on the first resonance of the antenna. It follows that for frequencies much higher than the frequencies f_{r1} of the first resonance in the effective antenna, the early time can fit many periods of current at the excitation frequency. As a result, if the excitation is in the form of a train of sinusoids, the whole excitation can finish within the early time. In this case, the antenna parameters determined in the frequency domain for steady-state excitation with the same frequency as the frequency of the train are not significant to the induced input current.

Based on Fig. 4, we conclude that after the early time, which ends at the moment $t = 1$, despite the steady state excitation, the input current does not stabilize. The current value exhibits stabilized oscillations after approximately 20 time units.

Note that the stabilized oscillations seem to have a frequency of approximately 50 MHz (compare: Fig. 6).

Since the input current exhibits approximately three oscillations within five units of the time axis, the current reaches steady state after approximately 12 full oscillations. This means that a train of sinusoids consisting of less than 12 periods is not a steady state phenomenon in this case.

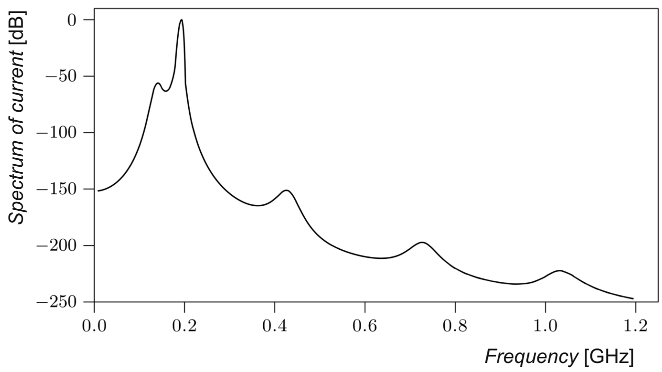

Fig. 5. Spectrum of the current from Fig. 3 normalized to its maximum value.

Fig. 5 shows the current spectrum from Fig. 3 calculated for 20 time units. The spectrum does not provide an explanation for the current behaviour. We can see that the energy of the phenomenon is definitely concentrated around the excitation frequency of 190 MHz. However, due to the logarithmic scale used, subsequent antenna resonances at approximately 140, 420, 720 and 1175 MHz become visible. Of course, the participation of these resonances - especially the last three - is lower by at least 150 dB, which is virtually zero.

The ordinary spectrum of the process does not contain information about the temporal distribution of energy. Certainly, more information about the spectral evolution of the process over time would be obtained from the use of some time-frequency representation of the signal. A properly selected and used joint representation gives spectacular results (see: [8]). The interested reader can easily find the available literature on this topic.

The spectrum image is affected by the first antenna resonance, which occurs at a frequency of $f_{r1} = 140\,\text{MHz}$. This resonance is close to the 190 MHz frequency of antenna

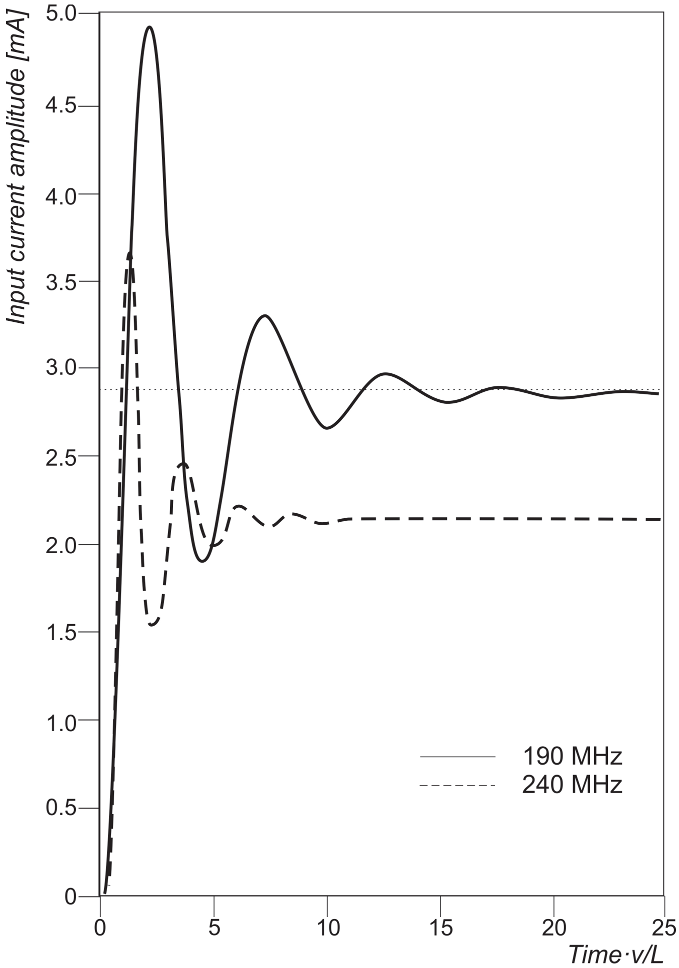

Fig. 6. Stabilization of the input current amplitude.

excitation. The energy stored at frequency f_{r1} is 55 dB less than the energy stored at the excitation frequency.

In the analysed case, the excitation frequency is chosen in such a way that it is slightly higher than the basic resonance frequency of the antenna.

With any selection of the excitation frequency, this frequency may be adjacent to other antenna resonance frequencies. Please note, however, that the subsequent resonance frequencies of the antennas are associated with increasingly lower quality factors. This means that the presence of higher resonances manifests even with larger differences between the resonance frequency and excitation frequency, while the importance of these resonances diminishes.

We know from experience that the spectrum of a sine wave with a fixed amplitude is the Dirac delta at the frequency of the sine wave. This means that the rest of the spectrum shown in Fig. 3 must exist at the beginning of the process. It is at this initial time that further antenna resonances are stimulated.

Since there is less quality factors associated with these further resonances, the influence of further resonances on the initial course of the process quickly decreases, and the process is shaped by the excitation frequency and the first resonance frequency. The stabilization of the current amplitude is a combination of two sinusoids with frequencies: excitation and first resonance. Hence the characteristic rumble with a frequency being the difference between the two frequencies mentioned above.

An illustration of this phenomenon is shown in Fig. 6, which shows the stabilization of the input current amplitude for two different excitation frequencies: 190 MHz and 240 MHz. The rumble at these frequencies is approximately 50 MHz and 100 MHz, respectively. Note that the excitation is a voltage with a constant frequency and constant amplitude of 1 V.

The figure in question does not leave any doubt that during the first dozen or several dozen units of time, the input current amplitude of the dipole changes, after which it reaches a steady state. However, until steady state is reached,the antenna cannot be considered to be a linear and steady state radiator.

From the data used for Fig. 6, we can count the oscillation frequency of the changes in the input current amplitude. These figure values oscillate at a frequency of approximately 50 MHz. We also know that the frequency difference between the excitation voltage (190 MHz) and the location of the first resonance (140 MHz) is also approximately 50 MHz.

D. Turning off the Transmission Antenna

The physical processes associated with turning off the transmission antenna seem slightly less complicated.

As an example, consider the same symmetrical dipole described in chapter II.A. Let us assume that the dipole works in the transmission mode and that to the input terminals - as before - voltage with an amplitude of 1 V and a frequency of 190 MHz is applied. In the twentieth time unit, when the current waveform is already established (Fig. 7), we turn off the excitation voltage ('out' in the figure).

After the excitation disappears, the input current oscillates to zero. The oscillations of this current are, however, no longer controlled by excitation and result from the properties of the antenna itself. In Fig. 7, it can be seen that after the excitation voltage of 190 MHz is turned off, the oscillations of the fading current increase the period. This period is now shaped by the first antenna resonance corresponding to the frequency of $f_{r1} = 140$ MHz.

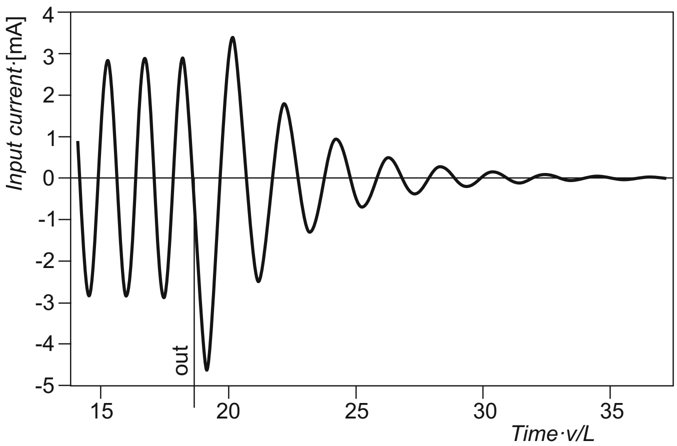

Fig. 7. Input current of a symmetrical dipole 1 m long excited by sinusoidal voltage of 1 V amplitude and 190 MHz frequency, turned off in 20 time units.

The distribution of the energy between resonances in the studied example is shown in Fig. 2. By far, most of the energy is stored around the first resonance at a frequency of approximately 140 MHz, and this energy is responsible for the fading oscillations of the input current in Fig. 7. Note that after switching off the dipole excitation at moment t = out, the process energy comes entirely from the energy stored in the near field.

When the input current oscillates after turning off the antenna, the current initially increases. The graph in Fig. 2 is helpful in explaining this phenomenon. As shown in Fig. 2, the suceptance for the 190 MHz frequency is negative, which means that the antenna susceptance is inductive. There is a certain energy associated with this susceptance. After the antenna is turned off, the current oscillates at the frequency of the first resonance of the antenna. In resonance, the input admittance is purely real and the susceptance drops to zero. The energy previously associated with inductive susceptance is no longer needed and now increases the real energy of the input current, in effect increasing the current.

The oscillation timeout does not exceed 20 time units.

For a more structurally complicated antenna with more resonances with a high quality factor, the image of fading vibrations after turning off the excitation is different. Of course, not all resonances equally affect the decreasing oscillations, and this happens, among other things, depending on the amount of energy accumulated around individual resonances and the quality factors associated with these resonances. However, it can generally be assumed that the highest oscillation energy is focused at the frequency of the first resonance (resonance of the highest quality factor), regardless of the frequency of the excitation.

III. Frequency and Time Domain

Electromagnetic phenomena associated with antennas are usually described in one of two domains: the frequency domain or the time domain. Frequency domain models are more common. This is because frequency modeling is mathematically simpler; function differentiation and integration $\sin(\omega t)$ come down to algebraic multiplication and the shifting of the function by a set angle.

However, using frequency models, we forget that all phenomena happen over time. As a result, we stop using any references to time. Differentiation or integration of functions $\sin(\omega t)$ is not about shifting by angle but about shifting along the time axis, which we always interpret - according to the principle of causality - as a delay.

In engineering, complex numbers and symbolic notation are used to model processes in the frequency domain. This record consists of focusing changes in time in the factor $e^{-j\omega t}$ or $e^{j\omega t}$; the first variant is more often used in circuit theory, while the second variant is more common in field theory.

Without going into the use of symbolic notation, we notice that by using this notation in linear process models in the frequency domain, we obtain equations in which exponential factors do not occur. This means that the equations describe process parameters that do not depend on time; in short - steady-state parameters. This is one of the main features of models in the frequency domain, and these models describe steady-state process parameters. It follows that the processes modeled in this way should exist for infinite amounts of time or at least long enough for the steady-state process parameters to stabilize.

Modeling in the frequency domain gives good results wherever steady-state process parameters are determined. Problems arise when considering questions about the beginning of the process or the end of the process or when there are elements in the system with nonsinusoidal variation in time.

A good example for discussing stationarity is the changes in antenna admittance in the analysed model. The input admittance is defined as the ratio of the symbolically recorded current to the symbolically recorded voltage. This admittance does not depend on time, although it can have different values for different frequencies. In the consideration presented here, we use the input current, which is a function of time and changes over time.

Models created in the time domain seem more natural. In the time domain, it is easy to create descriptions of the start of a process, sudden changes, and the disappearance of the process.

Some authors treat the time domain and the frequency domain as two equivalent possibilities of free choice to describe a phenomenon. The time domain and the frequency domain, however, have only a mathematical sense, are purely mathematical constructs and are related to the choice of a mathematical model describing the process under study. The process itself does not take place in either of these two domains because processes always happen over time - not in the time domain, but just over time.

Unfortunately, the concepts of time and the time domain are often equated and confused with each other (similar to the concept of frequency and the concept of the frequency domain).

It seems that the described results are a good example for making the differences visible.

IV. Conclusion

The main practical conclusion is that the processes of turning an antenna on and off take place under different conditions and therefore are different.

Classic antennas working in steady state do not cause major design problems because the ways to achieve the right characteristics and fit are well-researched issues.

The case of pulse antennas is different.

Given the nature of the phenomena, some groups of devices can be distinguished among pulse antennas. Antennas radiating single pulses are a fairly well-known category. Such antennas also include antennas that radiate repetitive pulses but in such a way that the processes excited by one pulse disappear before the appearance of the next pulse.

On the other hand, there are a few publications discussing the behaviour of antennas stimulated by a train of sinusoids containing several to several hundred periods. If such a sequence of periods lasts long enough, the processes run at a steady state, and the analysis of these processes can take place in a manner appropriate for the steady state.

When an antenna is turned on and off, the processes do not occur at steady state. We show that the processes of turning an antenna on and off take place under different conditions and therefore are different. When an antenna is switched on, the transient processes are determined by the the antenna properties and the excitation properties. When an antenna is switched off, excitation is no longer present, and the properties of the antenna determine the transient process.

This conclusion, in our opinion, seems to be particularly important when individual elements of antenna systems are often turned on and off. If the trains of sinusoids are too short or are repeated too quickly, overlapping transient processes can significantly change the behavior of the antenna and affect the operation of the generator.

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