GENERAL ANALYSIS OF BROADBAND SIGNAL SPATIAL FILTRATION

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The majority of underwater acoustic systems use narrow band sounding signals. Consequently, there is extensive knowledge on how the relevant spatial filters should be analysed and designed. The difficulty begins when the sounding signal has a broad spectrum. Applying the results of narrow band signal analysis to broadband signals leads to serious errors. The same is true for spatial filtration methods when applied to narrow band signals.

The article presents a general method for analysing spatial filters designed for broadband signals. It also gives an assessment of the errors caused by applying spatial filtration methods designed for narrow band signals.

INTRODUCTION

The majority of today's multi-beam sonars use narrow band sounding signals. The theory and methods for designing relevant spatial filters is well researched, [4]. The most frequent filtration methods are those in the frequency domain or spatial spectrum estimation. To simplify, the echo signal is treated as sinusoidal which is followed by signal delay compensation. For a very narrow echo signal spectrum, envelope and beam pattern distortions are minor and acceptable in practice.

In an effort to improve the range, resolution and signal to noise and reverberation ratio, some sonars use broadband signals, usually ones with linear frequency modulation. When phase compensation is carried out for central frequency only, the result is a significantly deteriorated signal envelope at receiver output and beam width exceeding the limits. As a result, phase compensation must cover all frequencies of the spectrum. To do it digitally would require very powerful computers, much more powerful than necessary for narrow band spatial filters. Consequently, there is a need for more cost effective and simplified algorithms for signal processing. This sets the context for assessing the negative effects of using simplified methods. These problems are discussed in the article.

1. ANALYTICAL DESCRIPTION OF THE BROADBAND SPATIAL FILTER.

Let us assume that a broadband signal *s(t)*,with a limited spectrum width is incident on a linear array, as illustrated in Fig.1.

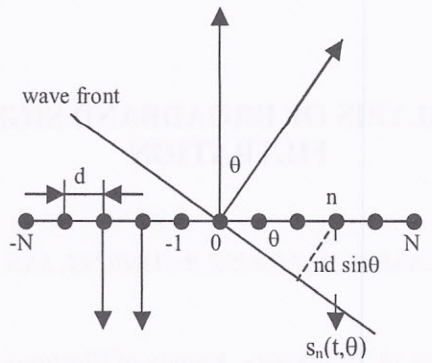

Fig. 1. Linear receiving array.

The signal at the output of the array's *n*-th element has this form:

$$s_n(t,\theta)=S_0 s[t+\tau_n(\theta)], \qquad (1)$$

where for the velocity of acoustic wave *c*, the delay is equal to $\tau_n(\theta)=(nd/c)sin\theta$. We assume that signal *s(t)* is a copy of the sounding signal, i.e. it is known in the receiver. What is unknown is the signal amplitude designated as S_0.

To produce a beam deflected by θ_k the delay must be compensated and all signals added up. At summator output we get:

$$s(t,\theta,\theta_k)=S_0\sum_{-N}^{N} s[t+\tau_n(\theta)-\tau_n(\theta_k)], \qquad (2)$$

where $\tau_n(\theta_k)=(nd/c)sin\theta_k$.

The signal's Fourier transform can be written as:

$$S(j\omega,\theta,\theta_k)=S_0 S(j\omega)\sum_{-N}^{N} exp[j(\omega d/c)n(sin\theta-sin\theta_k], \qquad (3)$$

where: $S(j\omega,\theta,\theta_k)=\Im\{s(t,\theta,\theta_k)\}$ and $S(j\omega)=\Im\{s(t)\}$.

The sum in the above formula is the sum of a geometric series which can be easily computed. The result is:

$$S(j\omega,\theta,\theta_k)=MS_0 S(j\omega)\frac{sin[M(\omega d/2c)(sin\theta-sin\theta_k)]}{M\,sin[(\omega d/2c)(sin\theta-sin\theta_k)]}, \qquad (4)$$

where $M=2N+1$ is the number of antenna elements.

For each spectrum component of the signal received the fractional expression in the above formula describes the beam pattern. All patterns are described with the same function, but the width of the main lobe, the position of side lobes and any grating lobes depends on the frequency of the spectrum component. The "amplitude" of the beam pattern for a given frequency is proportional to the value of the spectrum module $S(j\omega)$ in that frequency.

In broadband sonars it is not the adder output signal that is studied, but the one at the detector output. The detector is usually a filter matched to the sounding signal. The filter is usually realized in the frequency domain and its transfer function $K(j\omega)$ is equal to:

$$K(j\omega) = S^*(j\omega)\,. \tag{5}$$

Because the amplitude of the signal received is unknown, it cannot be made part of the filter transfer function.

The signal spectrum at the matched filter output $X(j\omega)$ is equal to the product of signal spectrum at the adder output and the transfer function $K(j\omega)$, and we get:

$$X(j\omega,\theta,\theta_k) = S_0 M\,|S(j\omega)|^2\,\frac{sin[M(\omega d/2c)(sin\theta - sin\theta_k)]}{M\,sin[(\omega d/2c)(sin\theta - sin\theta_k)]}\,. \tag{6}$$

The signal at the matched filter output is the inverse Fourier transform of spectrum $X(j\omega,\theta,\theta_k)$. The signal can be written as the convolution of two functions of time, that is:

$$x(t,\theta,\theta_k) = S_0 M \Im^{-1}\{|S(j\omega)|^2\} * \Im^{-1}\{\frac{sin[M(\omega d/2c)(sin\theta - sin\theta_k)]}{M\,sin[(\omega d/2c)(sin\theta - sin\theta_k)]}\}\,. \tag{7}$$

As you know, the inverse Fourier transform of energy density spectrum is equal to the auto-correlation function $r_{ss}(t)$ of the signal. The inverse Fourier transform of the fractional function in equation (7) is equal to Dirac's pulse series whose number is equal to M, and the spacing on the time scale is $(d/c)(sin\theta\text{-}sin\theta_k)$. The signal at the matched filter output can be written as:

$$x(t,\theta,\theta_k) = S_0 r_{ss}(t) * \sum_{n=-N}^{N} \delta[t - (nd/c)(sin\theta - sin\theta_k)]\,. \tag{8}$$

Because convolution is commutative to adding, there is a simpler method to write the above formula:

$$x(t,\theta,\theta_k) = S_0 \sum_{n=-N}^{N} r_{ss}[t - (nd/c)(sin\theta - sin\theta_k)]\,. \tag{9}$$

As you can see, the signal at the filter output is the sum of auto-correlation functions of the signal received, shifted on the time axis. When the angle of wave incidence is equal to the assumed beam deflection angle, the output signal is the sum of M unshifted autocorrelation functions, i.e. it is the highest. Such a signal is described with the auto-correlation function of the sounding signal. Signal duration at the matched filter output is approximately equal to the reverse of its spectrum width. Signal maximum occurs in the maximum of the auto-correlation function and is equal to the energy of the signal received. For the remaining angles of incidence, the value of the signal is lower. In addition, the signal is extended compared to the auto-correlation function.

2. SPATIAL FILTERING FOR LOW BAND SIGNALS

Using the general relations derived above, let us now consider the operation of the spatial filter when receiving a low band signal. Let us assume, that the spectrum of a low band signal is rectangular and symmetric from $-fg$ to $+fg$. The auto-correlation function of the signal received is equal to:

$$r_{ss}(t)=\Im^{-1}\{|S(j\omega)|=\Im^{-1}\{|\Pi(\omega/2\omega_g)|=2f_g\frac{sin(2\pi f_g t)}{2\pi f_g t}. \qquad (10)$$

We substitute the auto-correlation function to formula (9) and get:

$$x(t,\theta,\theta_k)=2f_gS_0\sum_{n=-N}^{N}\frac{sin\{2\pi f_g[t-(nd/c)(sin\theta-sin\theta_k)]\}}{2\pi f_g[t-(nd/c)(sin\theta-sin\theta_k)]}. \qquad (11)$$

We can prove that signal maximum occurs at moment $t=0$, so we get:

$$x_{max}(\theta,\theta_k)=2f_gS_0\sum_{n=-N}^{N}\frac{sin[2\pi f_g(nd/c)(sin\theta-sin\theta_k)]}{2\pi f_g(nd/c)(sin\theta-sin\theta_k)}. \qquad (12)$$

Space d between adjacent antenna elements should be equal to at least half the length of the highest frequency wave. This criterion helps us to simplify the above formula to get a formula as follows:

$$x_{max}(\theta,\theta_k)=2f_gS_0\sum_{n=-N}^{N}\frac{sin[\pi n(sin\theta-sin\theta_k)]}{\pi n(sin\theta-sin\theta_k)}. \qquad (13)$$

Signal maximum is always present for angle $\theta=\theta_k$, which is in accordance with the principle of spatial filter operation. The value of the maximum is:

$$x_{max}(\theta=\theta_k)=2Mf_gS_0. \qquad (14)$$

As expected, it is proportional to the number of antenna elements, the width of the band occupied by the signal and its amplitude. After normalisation of relation (13) with regard to signal maximal value, we get the beam pattern for the signal in question:

$$b(\theta,\theta_k)=\frac{1}{M}\sum_{n=-N}^{N}\frac{sin[\pi n(sin\theta-sin\theta_k)]}{\pi n(sin\theta-sin\theta_k)}. \qquad (15)$$

Fig.2 shows the computed beam pattern of an antenna consisting of M=9 elements under the assumption that the beam deflection angle is $\theta_k=0^0$ and $\theta_k=30^0$. The same figure shows beam patterns of the sinusoidal signal with frequency f_g. As you can see, the beam patterns of the low band signal are wider than those of the sinusoidal signal with a frequency equal to the highest frequency of the low band signal spectrum. You can also see, that there are no distinct side lobes and that the beam value decreases almost monotonically as the observation angle increases. The width of the main lobe is almost doubled compared to the main lobe of a comparable beam pattern, and it is exactly the same width as the main lobe of the sinusoidal signal beam pattern where frequency is f_g and the antenna consists of M=5 elements. The diagrams in Fig. 3 explain that. The increased width of the main lobe for low band signal reception is the result of broad beam patterns of low frequency spectrum components.

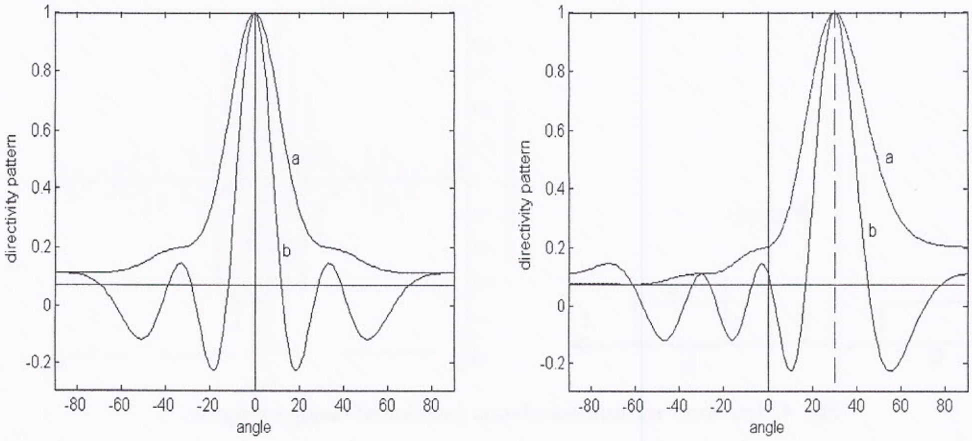

Fig. 2. Beam patterns for low band signals (a- *real beam pattern*, b- *beam pattern for frequency* f_g).

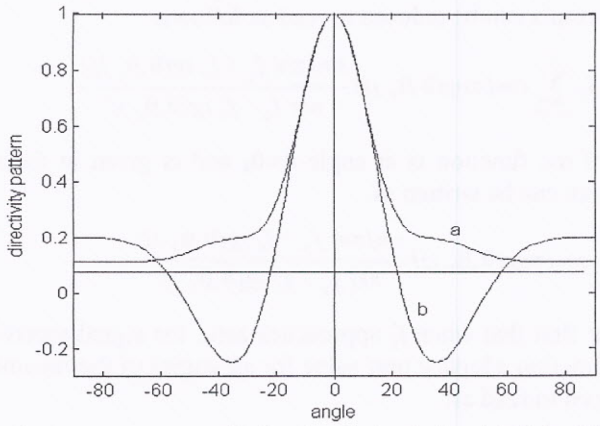

Fig. 3. Beam patterns for low band signals (a - $M=9$, b - $M=5$).

3. SPATIAL FILTERING FOR BANDPASS SIGNALS

Underwater acoustics often uses bandpass signals with a relatively wide spectrum, e.g. signals with linear frequency modulation. The spectrum of these signals is rectangular-like with frequencies around f_0 and $-f_0$. The spectrum of the signal in question and its auto-correlation function are shown in Fig.4. The auto-correlation function is determined from formula (10) using the known theorem about the frequency domain shift. The signal at the matched filter output has the following form:

$$x(t,\theta,\theta_k) = 2f_g S_0 \sum_{n=-N}^{N} cos\{\omega_0[t-(nd/c)g(\theta,\theta_k)]\}\frac{sin\{\omega_g[t-(nd/c)g(\theta,\theta_k)]\}}{\omega_g[t-(nd/c)g(\theta,\theta_k)]}, \quad (16)$$

where, to shorten the notation, the following are given as $g(\theta,\theta_k)=sin\theta-sin\theta_k$ and $\omega_0=2\pi f_0$ and $\omega_g=2\pi f_g$.

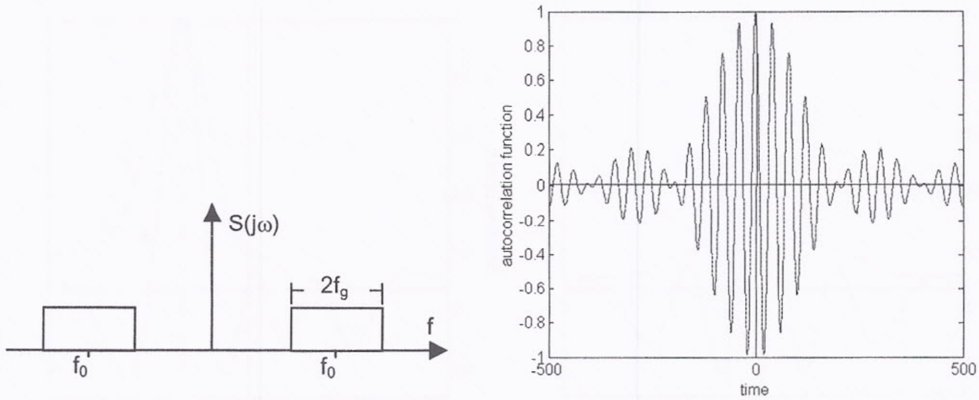

Fig. 4. Spectrum and autocorrelation function of bandpass signal.

As previously, signal maximum occurs at moment t=0. If we assume that the spacing between antenna elements is equal to half the wave length for the medium frequency of the spectrum, the above formula can be reduced to read as follows:

$$x_{max}(\theta,\theta_k) = 2 f_g S_0 \sum_{n=-N}^{N} cos\{\pi n g(\theta,\theta_k)]\} \frac{sin\{\pi n(f_g / f_0) g(\theta,\theta_k)]\}}{\pi n(f_g / f_0) g(\theta,\theta_k)]} . \quad (17)$$

The maximum of the function is at angle $\theta=\theta_k$ and is given in formula (14). Consequently, the beam pattern can be written as:

$$b(\theta,\theta_k) = \frac{1}{M} \sum_{n=-N}^{N} cos\{\pi n g(\theta,\theta_k)]\} \frac{sin\{\pi n(f_g / f_0) g(\theta,\theta_k)]\}}{\pi n(f_g / f_0) g(\theta,\theta_k)]} . \quad (18)$$

Let us observe at first that when f_g approaches zero, the signal received becomes sinusoidal. A *sinx/x* type function adopts a unit value for all angles of the incoming wave, and the above formula is reduced to read as:

$$b(\theta,\theta_k) = \frac{1}{M} \sum_{n=-N}^{N} cos\{\pi n g(\theta,\theta_k)]\} = \frac{1}{M} \sum_{n=-N}^{N} exp\{ j\pi n g(\theta,\theta_k)]\} . \quad (19)$$

The beam pattern is described with the sum of the power series and is equal to:

$$b(\theta,\theta_k) = \frac{sin[\pi(M/2)(sin\theta - sin\theta_k)]}{sin[\pi(1/2)(sin\theta - sin\theta_k)]} . \quad (20)$$

As expected, it is equal to the beam pattern of the spatial filter in question for a sinusoidal signal with frequency f_0.

In practice, the width of the beam (apart from the level of side lobes) is critical. The question arises – how does the width of the signal's spectrum affect the width? Formula (18) suggests that in neighbourhood of angle θ_k the fractional expression assumes values close to unity. This means that beam width is practically equal to the width of the sinusoidal signal beam with frequency f_0. The width increases as the beam deflection angle increases, as was the case with narrow band signals.

There are significant differences between the level of side lobes for broadband signals and narrow band signals. This is illustrated in Fig. 5.

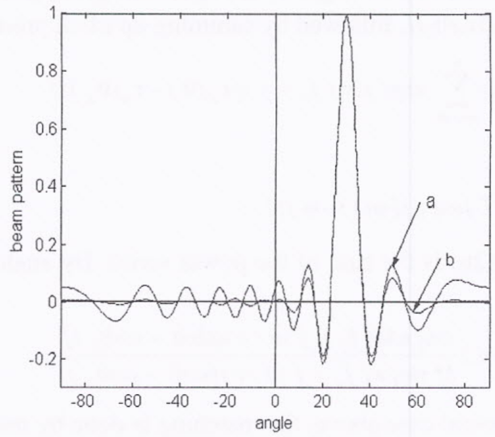

Fig. 5. Beam pattern for bandpass signals
(a - *narrowband signal*, b- *broadband signal, M=19,* f_g/f_o=0.2).

4. BROADBAND SPATIAL FILTER WITH QUADRATURE DETECTION

Broadband spatial filtration can be carried out using quadrature detection. We will demonstrate whether, and if so, to what extent, quadrature detection has an effect on the properties of the spatial filter.

Let the broadband signal have this form:

$$s(t)=S_0\Pi(t/T)\sin[2\pi f_0 t+\varphi(t)+\psi], \tag{21}$$

where $\Pi(t/T)$ means a rectangular envelope with duration *T*.

The signal at the output of the *n*-th antenna element is delayed and based on formula (1) it can be written as:

$$s_n(t,\theta)=S_0\Pi\{(t+\tau_n(\theta))/T\}\sin\{2\pi f_0[t+\tau_n(\theta)]+\varphi[t+\tau_n(\theta)]+\psi\}. \tag{22}$$

As you know, in quadrature detection the narrow band signal is multiplied by $\sin(2\pi f_0 t)$ and $\cos(2\pi f_0 t)$ and undergoes low band filtration. At the output of low pass filters we get:

$$\begin{aligned} x_n(t,\theta)&=S_0\Pi\{(t+\tau_n(\theta))/T\}\cos\{2\pi f_0\tau_n(\theta)+\varphi[t+\tau_n(\theta)]+\psi\}, \\ y_n(t,\theta)&=S_0\Pi\{(t+\tau_n(\theta))/T\}\sin\{2\pi f_0\tau_n(\theta)+\varphi[t+\tau_n(\theta)]+\psi\}. \end{aligned} \tag{23}$$

The signals can be treated as real and imaginary components of the complex signal $z_n(t)=x_n(t)+jy_n(t)$. The complex signal takes this form:

$$z_n(t,\theta)=S_0\Pi\{(t+\tau_n(\theta))/T\}\exp[j2\pi f_0\tau_n(\theta)]\cdot\exp\{j\varphi[t+\tau_n(\theta)]+j\psi\}. \tag{24}$$

The signal's Fourier transform can be written as:

$$Z_n(j\omega,\theta)=S_0\exp[j2\pi(f_0+f)\tau_n(\theta)]\cdot\Im\{\Pi(t/T)\exp\{j[\varphi(t)+\psi]\}\}. \tag{25}$$

In the above formula, the Fourier transform describes the spectrum of the low band signal which we get following quadrature detection of signal *s(t)*. The spectrum is identical for all signals and does not depend on the wave incidence angle.

To produce a beam deflected by angle θ_k the spectrum given in formula (25) should be multiplied by $exp[-j2\pi(f_0+f)\tau_n(\theta_k)]$, followed by summing up of all products. We get:

$$Z_n(j\omega,\theta,\theta_k) = S_l(j\omega)\sum_{n=-N}^{N} exp\{j2\pi(f_0+f)[\tau_n(\theta)-\tau_n(\theta_k)]\}\cdot \quad (26)$$

where

$$S_{lp}(j\omega) = S_0\Im\{\Pi(t/T)exp\{j[\varphi(t)+\psi]\}\}. \quad (27)$$

The sum in formula (26) is the sum of the power series. By analogy to formula (4), we get:

$$S(j\omega,\theta,\theta_k) = MS_{lp}(j\omega)\frac{sin[\pi M(f_0+f)d/c)(sin\theta - sin\theta_k)]}{M\,sin[\pi(f_0+f)d/c)(sin\theta - sin\theta_k)]}. \quad (28)$$

By analogy to the general case above, the matching is done by multiplying the spectrum by $K(j\omega)=S_{lb}*(j\omega)/S_0$. We get:

$$X(j\omega,\theta,\theta_k) = \frac{M}{S_0}|S_{lp}(j\omega)|^2\frac{sin[\pi M(f_0+f)d/c)(sin\theta - sin\theta_k)]}{M\,sin[\pi(f_0+f)d/c)(sin\theta - sin\theta_k)]}. \quad (29)$$

As expected, for a sinusoidal signal with frequency f_0 and amplitude S_0 we get:

$$X(j\omega_0,\theta,\theta_k) = MS_0\frac{sin[\pi M(f_0 d/c)(sin\theta - sin\theta_k)]}{M\,sin[\pi(f_0 d/c)(sin\theta - sin\theta_k)]}. \quad (30)$$

The spectrum consists of one line whose value is proportional to the beam pattern $b(\theta,\theta_k)$, which for $d/\lambda_0=0.5$ is described with formula (20). For angle $\theta=\theta_k$ we get the square of the module of the low band signal spectrum whose inverse Fourier transform is equal to the auto-correlation function.

If we assume that the spectrum of the low band signal is rectangular and limited by frequency f_g, the signal at the matched filter output is equal to the inverse Fourier transform of its spectrum and can be written as:

$$s(t,\theta,\theta_k) = 2f_gS_0\sum_{n=-N}^{N} exp[j2\pi f_0(nd/c)g(\theta,\theta_k)]\frac{sin\{2\pi f_g[t-(nd/c)g(\theta,\theta_k)]\}}{2\pi f_g[t-(nd/c)g(\theta,\theta_k)]}. \quad (31)$$

Signal maximum is at moment $t=0$ and is equal to $2f_gS_0M$. The beam pattern is described with the following formula:

$$b(\theta,\theta_k) = \frac{1}{M}\sum_{n=-N}^{N} exp[j2\pi f_0(nd/c)g(\theta,\theta_k)]\frac{sin\{2\pi f_g(nd/c)g(\theta,\theta_k)]\}}{2\pi f_g(nd/c)g(\theta,\theta_k)]}. \quad (32)$$

When $d/c=1/2f_0$, the above formula takes a form similar to formula (18), because a sinx/x type function is even and $e^{jnx}+e^{-jnx}=2cos(nx)$. This means that quadrature detection has no effect on the properties of the spatial filter during reception of broadband signals.

5. SIMPLIFIED SPATIAL FILTERING METHOD.

To digitally carry out the spatial filter, the spectra of signals received must be multiplied by complex numbers described with the formula:

$$w_{nkf}=exp[-j2\pi(f_0+f)\tau_n(\theta_k)]. \quad (33)$$

The number of the multiplications is equal to $L=MKF$, where M is the number of antenna elements, K – the number of beams, and F – the number of spectrum lines. Although number L can be reduced four times using beam and spectrum symmetry, it is usually very big. It can be reduced using fast Fourier transformation FFT to estimate the spatial spectrum. The problems are extensively discussed in separate papers [1], [2], [3]. In this paper we will demonstrate the effects of using a simplified algorithm in which phase compensation is done on the medium frequency f_0 of the spectrum of the signal received.

Where quadrature detection is used, the spectrum of the low band signal is multiplied by $w_{nk}=exp[-j2\pi f_0\tau_n(\theta_k)]$, and the result is formula (3) in the following form:

$$Z_n(j\omega,\theta)=S_l(j\omega)\sum_{n=-N}^{N} exp\{j2\pi f_0[\tau_n(\theta)-\tau_n(\theta_k)]\}exp[j2\pi f\tau_n(\theta)]\cdot \qquad (34)$$

When we fulfil the same conditions as in the previous section and repeat the same computations, the result is the following beam pattern relation:

$$b(\theta,\theta_k)=\frac{1}{M}\sum_{n=-N}^{N} exp[j2\pi f_0(nd/c)g(\theta,\theta_k)]\frac{sin[2\pi f_g(nd/c)sin\theta]}{2\pi f_g(nd/c)sin\theta}. \qquad (35)$$

The above analysis shows that the beam pattern is less different from the ideal one when the values of a *sinx/x* type function in the above formula are closer to unity. In formula (32) that condition was satisfied for angles θ close to the beam deflection angle θ_k, because function $g(\theta,\theta_k)$ adopted values close to zero. In formula (35), on the other hand, the values of a *sinx/x* type function depend directly on angle θ, which has an adverse effect in particular for bigger beam deflection angles.

To demonstrate the effects of beam deflection and signal spectrum width, let us write formula (35) as follows:

$$b(\theta,\theta_k)=\frac{1}{M}\sum_{n=-N}^{N} exp[j\pi ng(\theta,\theta_k)]\frac{sin[\pi n(f_g/f_0)sin\theta]}{2\pi n(f_g/f_0)sin\theta}, \qquad (36)$$

in which the assumption is that $d/c=1/2f_0$.

Fig.6A shows the beam patterns for M=31, $\theta_k=0^0$ and $\theta_k=30^0$, $f_g/f_0=0.1$. To compare, in the same Figure you can see the beam patterns of a full delay compensation filter and a sinusoidal signal filter with frequency f_0. The beam width has clearly increased compared to the other patterns and the shape of the main lobe has suffered. Fig.6B shows analogous beam patterns for a smaller number of antenna elements. You can see, that a reduction in the number of elements has improved the beam pattern. The shapes of the main lobes are practically identical with only a certain increase in the width, when the filter phase compensation is incomplete. Using this as the basis, we can propose a hypothesis that there are certain boundary values of the number of M elements, beam deflection angles θ_k and relative spectrum width (f_g/f_0) for which the increase in beam width does not exceed the assumed value. The hypothesis is confirmed in the diagram in Fig.7, showing the percentage increase in beam width in the function of parameter M. The diagram helps to establish the maximal number of antenna elements and maximal beam deflection angle for which the increase in beam width is still acceptable.

By using the above relations, we can significantly reduce the number of arithmetic operations for digital spatial filters. The reduction is made possible by multiplying complex signals $z_n(t,\theta)$ by numbers $w_{nk}=exp[-j2\pi f_0\tau n(\theta_k)]$ before Fourier transforms are computed. Number of multiplications was limited to $L=MK$, making it F times smaller than the number of multiplications required for complete phase compensation filters. The Fourier transform must now be determined for matched filtration only.

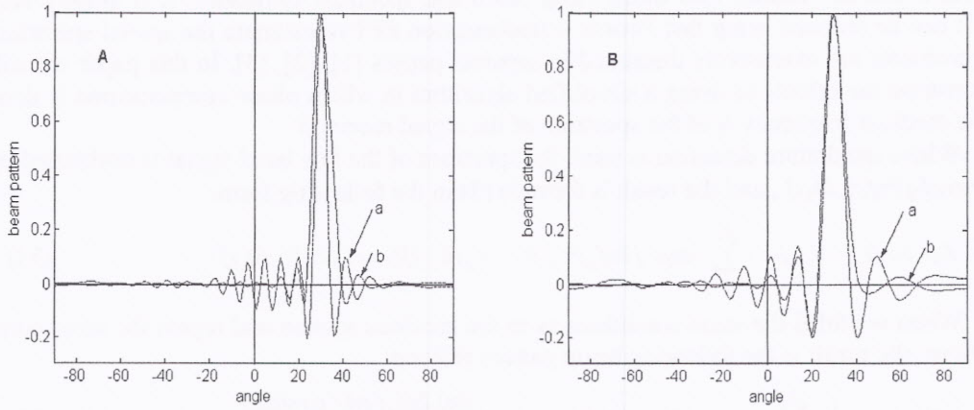

Fig. 6. Beam patterns for broad band signals (A – $M=31, f_g/f_o=0.15$, B - $M=31, f_g/f_o=0.15$; a-*full compensation of phase*, b - *limited compensation of phase*).

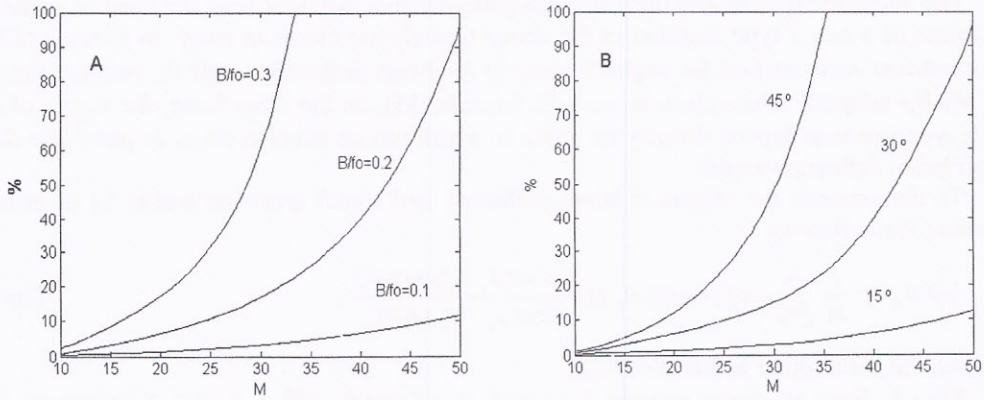

Fig. 7. Percentage increase in beam width (A – $\theta_k=30^0$, $B=2f_g$; B - $f_g/f_o=0.2$, $\theta_k=15^0, 30^0, 45^0$;).

Summing up, the results of the analysis may come as an attractive alternative and help significantly simplify the spatial filter and reduce its costs. As demonstrated, this is a viable option and filter quality is compromised to a small extent only. Unfortunately, to do that all the above criteria must be met, i.e. the number of antenna elements and beam deflection angle.

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