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Maximization of Safety Lifetime of Critical Infrastructure Network with Cascading Effects Considering Climate-Weather Change Influence

Keywords

critical infrastructure, operation process, climate-weather change process, critical state, critical infrastructure safety lifetime maximization

Abstract
In the paper optimization of operation process and maximization of safety lifetimes for interconnected and interdependent critical infrastructure (CI) networks at variable operation conditions related to the climate-weather change are proposed. A multistate series network with assets dependent according to local load sharing (LLS) rule is analyzed and optimization of operation and safety of CI network with the LLS rule is introduced. For such CI network, the optimal transient probabilities of CI network operation process at operation states related to climate-weather change that maximize the mean value of CI network safety lifetimes are found. Finally, the optimal safety and resilience indicators of CI network are presented.

1. Introduction

To tie the investigation of the critical infrastructure safety together with the investigation of its operation the semi-Markov process models can be used to describe this critical infrastructure operation processes related to the climate-weather change [Klabjan, Adelman, 2006], [Kołowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011, 2012a-b, 2014]. The models of these two processes, under the assumption on the critical infrastructure structure multistate model [Xue, 1985], [Xue, Yang, 1995] can be used to construct the general safety model of the multistate critical infrastructure changing its safety structure and its components safety parameters during variable operation process [Kołowrocki, Soszyńska-Budny, 2011, 2012a-b] and at different climate-weather states of the critical infrastructure operating area [Kołowrocki, et al., 2017b-c]. Further, using this general model and the linear programming [Klabjan, Adelman, 2006], [Kołowrocki, Soszyńska-Budny, 2011], it is possible to find optimal values of the limit transient probabilities of the critical infrastructure operation process that maximize the unconditional CI network lifetimes in the safety state subsets. In a special case the optimal values of limit transient probabilities of CI operation process related to the climate-weather change, to maximize CI network lifetime in the set of safety states not worse than a critical safety state, are determined. Then, the CI network optimal safety and resilience indicators can be determined. Namely, the CI network optimal unconditional safety function, the optimal critical infrastructure risk function and the optimal moment when the risk exceeds a permitted level, the optimal intensities of degradation, the optimal coefficients of the operation process related to the climate-weather change impact on the CI network intensities of degradation and the optimal indicator of CI network resilience to operation process related to climate-weather change impact, can be estimated.

2. Critical Infrastructure Operation Process Related to Climate-Weather Change Process

We consider, similarly as in [Kołowrocki, et al., 2017c], the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t),\ t \in <0,\infty)$, in a various way at this process states $zc_{bl},\ b=1,2,...,v,\ l=1,2,...,w$. We assume that the changes of the states of operation process related to the climate-weather change process $ZC(t),\ t \in <0,\infty)$, at the critical infrastructure operating area have an influence on the critical infrastructure safety structure and on the safety of the critical infrastructure assets A_i, $i = 1,2,...,n.$

We assume, as in [Kołowrocki, et al., 2017b-c], that the critical infrastructure during its operation process is taking $\nu, \nu \in N$, different operation states $z_1, z_2, \ldots, z_\nu$. We define the critical infrastructure operation process $Z(t)$, $t \in < 0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_\nu\}$. These operation states may take into account actual load or demand and can influence on lifetimes of CI assets in the safety state subsets and way they decrease through various models of dependency. The vector of limit values of transient probabilities of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,...,\nu$, is defined in [Kołowrocki, et al., 2017c].

Moreover, as in [Kołowrocki, et al., 2017b-c], we assume that the climate-weather change process $C(t)$, $t \in < 0,+\infty)$, at the critical infrastructure operating area is taking w, $w \in N$, different climate-weather states c_1, c_2,…, c_w. Climate-weather conditions can have also influence on CI network safety including cascading effect and models of failure dependency between assets and subnetworks of CI network. The vector of limit values of transient probabilities of the climate-weather change process $C(t)$ at the particular climate-weather states c_l, $l = 1,2,...,w$, is defined in [Kołowrocki, et al., 2017c].

Then, the joint process of critical infrastructure operation process and climate-weather change process called the critical infrastructure operation process related to climate-weather change is proposed and it is marked by $ZC(t)$, $t \in < 0,+\infty)$. Further, we assume that it can take $\nu w, \nu, w \in N$, different operation states related to the climate-weather change $zc_{11}, zc_{12}, \ldots, zc_{\nu w}$. We assume that the critical infrastructure operation process related to climate-weather change $ZC(t)$, at the moment $t \in < 0,+\infty)$, is at the state zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, if and only if at that moment, the operation process $Z(t)$ is at the operation states z_b, $b = 1,2,...,\nu$, and the climate-weather change process $C(t)$ is at the climate-weather state c_l, $l = 1,2,...,w$, can be expressed as follows:

$$(ZC(t) = zc_{bl}) \Leftrightarrow (Z(t) = z_b \cap C(t) = c_l),$$
$$t \in < 0,+\infty). \quad (1)$$

The transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, are defined in [Kołowrocki, et al., 2017c]:

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}),\ t \in < 0,+\infty). \quad (2)$$

Further, the limit values of the transient probabilities of the critical infrastructure operation process related to climate-weather change process $ZC(t)$ at the operation states zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, are given by

$$pq_{bl} = \lim_{t \to \infty} pq_{bl}(t), \quad (3)$$

and in case when the processes $Z(t)$ and $C(t)$ are independent, they can be found in [Kołowrocki, et al., 2017b]

$$pq_{bl} = p_b q_l,\ b = 1,2,...,\nu,\ l = 1,2,...,w, \quad (4)$$

where $p_b, b = 1,2,...,\nu$, are the limit transient probabilities of the operation process $Z(t)$ at the particular operation startes z_b, $b = 1,2,...,\nu$, and q_l, $l = 1,2,...,w$, are the limit transient probabilities of the climate-weather change process $C(t)$ at the particular climate-weather states c_l, $l = 1,2,...,w.$

Other interesting characteristics of the critical infrastructure operation process $ZC_{bl}(t)$ are its total sojourn times $\hat{\theta}\hat{C}_{bl}$ at the particular operation states $zc_{bl}, b = 1,2,...,\nu$, $l = 1,2,...,w$, during the fixed sufficiently large critical infrastructure operation time θ. They have approximately normal distributions with the expected values given by

$$\hat{M}\hat{N}_{bl} = E[\hat{\theta}\hat{C}_{bl}] = pq_{bl}\theta, \quad (5)$$

where pq_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, are defined by (3) and given by (4) in the case the processes $Z(t)$ and $C(t)$ are independent.

3. LLS Model of Dependency Related to CI Operation and Climate-Weather Change Processes

In local load sharing (LLS) model of dependency for a multistate CI series network, described in [Blokus-Roszkowska, Kołowrocki, 2017a-c] and [Kołowrocki, et al., 2017a], the coefficients of the network load growth may be defined differently in various operation and climate-weather condions. In LLS rule, we assume that after the departure of asset A_j, $j = 1,\ldots,n$, in the network from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, the lifetimes of remaining assets A_i, $i = 1,\ldots,n$, $i \neq j$, in the safety state subsets decrease dependently on the coefficients of the network load growth concerned with the distance from the asset A_j. Thus, we assume that these coefficients of the network load growth in LLS rule can take different values or formulae at various operation states related to the climate-weather change zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$. For example, if we denote the load on CI at the state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, by L_{bl}, and maximal load on CI by L_{MAX}, the coefficients of the network load growth in LLS rule can be determined from following formula

$$q^{(bl)}(\upsilon, d_{ij}) = [1 - \frac{L_{bl}}{L_{MAX}}] \cdot q(\upsilon, d_{ij}), i = 1,\ldots,n,$$
$$j = 1,\ldots,n, \ \upsilon = u, u-1,\ldots,1, \ u = 1,2,\ldots,z-1, \quad (6)$$

where $L_{bl} \le L_{MAX}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, and the coefficients of the network load growth $q^{(bl)}(\upsilon, d_{ij})$, $0 < q^{(bl)}(\upsilon, d_{ij}) \le 1$, $i = 1,\ldots,n$, $j = 1,\ldots,n$, and $q^{(bl)}(\upsilon, 0) = 1$ for $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z-1$, are non-increasing functions of assets' distance $d_{ij} = |i - j|$ from the asset that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. The distance between network assets can be interpreted in the metric sense as well as in the sense of relationships in the functioning of the network assets.

We denote by $E[T_i^{(bl)}(u)]$ and $E[T_{i/j}^{(bl)}(u)]$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,n$, $u = 1,2,\ldots,z$, the mean values of assets' lifetimes $T_i^{(bl)}(u)$ and $T_{i/j}^{(bl)}(u)$, respectively, before and after departure of one fixed asset A_j, $j = 1,\ldots,n$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the operation state related to the climate-weather change zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$. With this notation, in considered local load sharing rule, the mean values of assets lifetimes in the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, at particular state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are decreasing according to the following formula:

$$E[T_{i/j}^{(bl)}(\upsilon)] = q^{(bl)}(\upsilon, d_{ij}) \cdot E[T_i^{(bl)}(\upsilon)],$$
$$i = 1,\ldots,n, j = 1,\ldots,n. \quad (7)$$

In different states not only different values of coefficients of the network load growth can be assumed, but in special cases the different models of dependency between assets and subnetworks can be adopted. We can also assume that in some CI operation states related to the climate-weather change cascading effect can be observed, while in others no dependency between assets or subnetworks are assumed. Then, $q^{(bl)}(\upsilon, d_{ij}) = 1$, for some states $b \in \{1,2,\ldots,v\}$, $l \in \{1,2,\ldots,w\}$, and for the other coefficients are given by (6).

Further, we define the safety function of an asset A_i, $i = 1,\ldots,n$, after departure of the asset A_j, $j = 1,2,\ldots,n$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, assuming that CI network is at the state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$,

$$[S_{i/j}(t,\cdot)]^{(bl)} = [1, [S_{i/j}(t,1)]^{(bl)}, \ldots, [S_{i/j}(t,z)]^{(bl)}],$$
$$t \ge 0, \ i = 1,\ldots,n, j = 1,\ldots,n, \quad (8)$$

with the coordinates given by

$$[S_{i/j}(t,\upsilon)]^{(bl)} = P(T_{i/j}^{(bl)}(\upsilon) > t), \ t \ge 0,$$
$$\upsilon = u, u-1,\ldots,1, \ u = 1,2,\ldots,z-1, \quad (9)$$

$$[S_{i/j}(t,\upsilon)]^{(bl)} = P(T_i^{(bl)}(\upsilon) > t) = [S_i(t,\upsilon)]^{(bl)},$$
$$\upsilon = u+1,\ldots,z, \ u = 1,2,\ldots,z-1. \quad (10)$$

Then, the conditional safety function of a multistate series CI network with assets dependent according to LLS rule impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in \langle 0, \infty)$, is given by the vector

$$[\boldsymbol{S}_{LLS}^4(t,\cdot)]^{(bl)} = [1, [\boldsymbol{S}_{LLS}^4(t,1)]^{(bl)}, \ldots, [\boldsymbol{S}_{LLS}^4(t,z)]^{(bl)}],$$
$$t \ge 0, \ b = 1,2,\ldots,v, \ l = 1,2,\ldots,w, \quad (11)$$

with the coordinates

$$[\boldsymbol{S}_{LLS}^4(t,u)]^{(bl)} = \prod_{i=1}^{n}[S_i(t,u+1)]^{(bl)}$$
$$+ \int_0^t \sum_{j=1}^{n}[[\tilde{f}_j(a,u+1)]]^{(bl)} \cdot \prod_{\substack{i=1 \\ i\neq j}}^{n}[S_i(a,u+1)]^{(bl)}$$
$$\cdot [S_j(a,u)]^{(bl)} \cdot \prod_{i=1}^{n}[S_{i/j}(t-a,u)]^{(bl)}]da, \quad (12)$$

for $u = 1,2,\ldots,z-1$,

$$[\boldsymbol{S}^4_{LLS}(t,z)]^{(bl)} = \prod_{i=1}^{n}[S_i\ (t,z)]^{(bl)}, \qquad (13)$$

where:

$[S_i\ (t,u+1)]^{(bl)}$ – the conditional safety function coordinate of an asset A_i, $i = 1,\ldots,n$, at the state zc_{bl}, $b=1,2,...,v,\ l=1,2,...,w,$

$[\tilde{f}_j\ (t,u+1)]^{(bl)}$ – the conditional density function coordinate of an asset A_j, $j = 1,\ldots,n$, at the state zc_{bl}, $b=1,2,...,v,\ l=1,2,...,w,$ corresponding to the distribution function $[\tilde{F}_j\ (t,u+1)]^{(bl)}$, given by

$$[\tilde{F}_j\ (t,u+1)]^{(bl)} = 1 - \frac{[S_j\,(t,u+1)]^{(bl)}}{[S_j\,(t,u)]^{(bl)}},$$
$$u = 1,2,\ldots,z-1,\ t \ge 0, \qquad (14)$$

$[S_j\,(t,u)]^{(bl)}$ – the conditional safety function coordinate of an asset A_j, $j = 1,\ldots,n$, at the state zc_{bl}, $b=1,2,...,v,\ l=1,2,...,w,$

$[S_{i/j}(t,u)]^{(bl)}$ – the conditional safety function coordinate of an asset A_i, $i = 1,\ldots,n$, at the state zc_{bl}, $b=1,2,...,v,\ l=1,2,...,w,$ after departure from the safety state subset $\{u+1,\ldots,z\}$, $u = 1,2,\ldots,z-1$, by the asset A_j, $j = 1,\ldots,n$, such that

$$[S_{i/j}(t-a,u)]^{(bl)} = \frac{[S_{i/j}(t,u)]^{(bl)}}{[S_i\ (a,u)]^{(bl)}},$$
$$u = 1,2,\ldots,z-1,\ \ 0 < a < t,\ \ t \ge 0. \qquad (15)$$

Then, the unconditional safety function of the multistate series CI network with assets dependent according to LLS rule impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, is given by the vector

$$\boldsymbol{S}^4_{LLS}(t,\cdot) = [1, \boldsymbol{S}^4_{LLS}(t,1),\ldots,\boldsymbol{S}^4_{LLS}(t,z)],\ t \ge 0, \qquad (16)$$

where its coordinates can be determined from following formula

$$\boldsymbol{S}^4_{LLS}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\boldsymbol{S}^4_{LLS}(t,u)]^{(bl)},\ \ t \ge 0,$$
$$u = 1,2,...,z, \qquad (17)$$

and $[\boldsymbol{S}^4_{LLS}(t,u)]^{(bl)}$, $b=1,2,...,v,\ \ l=1,2,...,w,$ are the coordinates of the conditional safety function of a series CI network with assets dependent according to LLS rule impacted by the operation process related to the climate-weather change process, defined by (12)-(13) and pq_{bl}, $b=1,2,...,v,\ l=1,2,...,w,$ are the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, at the critical infrastructure operating area limit transient probabilities at the states zc_{bl}, $b=1,2,...,v,$ $l=1,2,...,w,$ defined by (3) and in [Kołowrocki, et al., 2017b].

Similarly, the unconditional safety function of CIs and CI networks with other safety structures, presented in [Kołowrocki, et al., 2017a], and impacted by the operation process related to the climate-weather change process, can be found. Namely, the unconditional safety function of the multistate series-parallel and series-"*m* out of *k*" CI network with assets dependent according to LLS rule, impacted by the operation process related to the climate-weather change process, can be determined.

4. Optimization of Operation and Safety of CI Network with the LLS Rule

4.1. Optimal Transient Probabilities of CI Network Operation Process at Operation States Related to Climate-Weather Change Process

Taking into account (17), it is natural to assume that the CI network operation process has a significant influence on the CI network safety.

This influence is also clearly expressed in the equation for the mean lifetime of the critical infrastructure in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,...,z$, given by

$$\boldsymbol{\mu}^4_{LLS}(u) = \int_0^\infty [\boldsymbol{S}^4_{LLS}(t,u)]dt \ \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\mu^4_{LLS}(u)]^{(bl)},$$
$$u = 1,2,...,z,$$

where $[\boldsymbol{\mu}^4_{LLS}(u)]^{(bl)}$ are the mean values of the CI network conditional lifetimes $[T^4_{LLS}(u)]^{(bl)}$ in the safety state subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ at the CI operation states related to the climate-weather change zc_{bl}, $b=1,2,...,v,$ $l=1,2,...,w,$ assuming LLS model of dependency, given by

$$[\mu^4_{LLS}(u)]^{(bl)} = \int_0^\infty [\boldsymbol{S}^4_{LLS}(t,u)]^{(bl)}dt,\ \ u=1,2,...,z,$$
$$b=1,2,...,v,\ \ l=1,2,...,w.$$

From the linear equation, expressed in (18), we can see that the mean value of the critical infrastructure unconditional lifetime $\boldsymbol{\mu}^4_{LLS}(u)$ is determined by the limit values of transient probabilities pq_{bl}, $b=1,2,...,v,\ l=1,2,...,w,$ of the CI operation process related to the climate-weather change given by (3) and $[\boldsymbol{\mu}^4_{LLS}(u)]^{(bl)}$ are the mean values of the CI network conditional lifetimes in the safety state subset $\{u,u+1,...,z\},\ u=1,2,...,z,$ at the CI operation states related to the climate-weather change zc_{bl}, $b=1,2,...,v,\ l=1,2,...,w,$ given by (19).

Therefore, the critical infrastructure lifetime optimization approach based on the linear programming [Klabjan, Adelman, 2006], [Kołowrocki, Soszyńska-Budny, 2011] can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}q_{bl}$, $b=1,2,...,v,$ $l=1,2,...,w,$ of the limit transient probabilities pq_{bl} of the critical infrastructure operation process related to the climate-weather change at the states zc_{bl}, $b=1,2,...,v,\ l=1,2,...,w,$ to maximize the mean value $\boldsymbol{\mu}^4_{LLS}(u)$ of the unconditional CI network lifetimes in the safety state subsets $\{u,u+1,...,z\},$ $u=1,2,...,z,$ under the assumption that the mean values $[\boldsymbol{\mu}^4_{LLS}(u)]^{(bl)}$ of the CI network conditional lifetimes in the safety state subsets $\{u,u+1,...,z\},$ $u=1,2,...,z,$ are fixed. As a special case of the above formulated critical infrastructure lifetime optimization problem, if r, $r=1,2,...,z,$ is a critical safety state of CI network, we want to find the optimal values $\dot{p}q_{bl}$, $b=1,2,...,v,\ l=1,2,...,w,$ of the critical infrastructure operation process limit transient probabilities pq_{bl} at the operation states zc_{bl}, $b=1,2,...,v,\ l=1,2,...,w,$ related to the climate-weather change, to maximize the mean value $\boldsymbol{\mu}^4_{LLS}(r)$ of the unconditional CI network lifetimes in the safety state subset $\{r,r+1,...,z\},$ under the assumption that the mean values $[\boldsymbol{\mu}^4_{LLS}(r)]^{(bl)}$, $b=1,2,...,v,\ l=1,2,...,w,$ of the CI network conditional lifetimes in the safety state subset $\{r,r+1,...,z\}$ are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\boldsymbol{\mu}^4_{LLS}(r) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\mu^4_{LLS}(r)]^{(bl)}, \qquad (20)$$

for a fixed $r \in \{1,2,...,z\}$ and with the following bound constraints

$$\breve{p}q_{bl} \le pq_{bl} \le \hat{p}q_{bl},\ b=1,2,...,v,\ l=1,2,...,w,$$

$$\sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}=1,$$

where

$$[\boldsymbol{\mu}^4_{LLS}(r)]^{(bl)},\ [\boldsymbol{\mu}^4_{LLS}(r)]^{(bl)} \ge 0,\ b=1,2,...,v,$$
$$l=1,2,...,w,$$

are fixed mean values of the critical infrastructure conditional lifetimes in the safety state subset $\{r,r+1,...,z\},$ assuming LLS model of dependency, and

$$\breve{p}q_{bl},\ 0 \le \breve{p}q_{bl} \le 1 \text{ and } \hat{p}q_{bl},\ 0 \le \hat{p}q_{bl} \le 1,$$
$$\breve{p}q_{bl} \le \hat{p}q_{bl},\ b=1,2,...,v,\ l=1,2,...,w,$$

are lower and upper bounds of the transient probabilities pq_{bl}, $b=1,2,...,v,$ $l=1,2,...,w,$ respectively.

Now, we can obtain the optimal solution of the formulated by (20)-(24) the linear programming problem, i.e. we can find the optimal values $\dot{p}q_{bl}$, $b=1,2,...,v,$ $l=1,2,...,w,$ of the limit transient probabilities pq_{bl} that maximize the objective function given by (20). The procedure of finding the optimal values $\dot{p}q_{bl}$, $b=1,2,...,v,\ l=1,2,...,w,$ is the same as the procedure presented in Section 5.2.1 in [Kołowrocki, et al., 2017c].

Further, we obtain the maximum value of the mean lifetime in the safety state subset $\{r,r+1,...,z\},$ of CI network with LLS rule, defined by the linear form (20), giving its maximum value in the following form

$$\dot{\boldsymbol{\mu}}^4_{LLS}(r) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl}[\mu^4_{LLS}(r)]^{(bl)}$$

for a fixed $r \in \{1,2,...,z\}$.

4.2. Critical Infrastructure Optimal Safety and Resilience Indicators

From the expression (25) for the maximum mean value $\dot{\mu}^4_{LLS}(r)$ of the CI network unconditional lifetime in the safety state subset $\{r,r+1,...,z\},$ replacing in it the critical safety state r by the safety state u, $u=1,2,...,z,$ we obtain the corresponding optimal solutions for the mean values of the CI network unconditional lifetimes in the safety state subsets $\{u, u+1, ..., z\}$, assuming LLS model of dependency, of the form

$$\dot{\mu}^4_{LLS}(u) = \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_b[\mu^4_{LLS}(u)]^{(bl)}, \ u = 1,2,...,z. \qquad (26)$$

Further, according to (17), the corresponding optimal unconditional safety function of the multistate series CI network with assets dependent according to LLS rule is the vector

$$\dot{\boldsymbol{S}}^4_{LLS}(t,\cdot) = [1, \dot{\boldsymbol{S}}^4_{LLS}(t,1), \ldots, \dot{\boldsymbol{S}}^4_{LLS}(t,z)], \ t \geq 0 \qquad (27)$$

with the coordinates given by

$$\dot{\boldsymbol{S}}^4_{LLS}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl}[\boldsymbol{S}^4_{LLS}(t,u)]^{(bl)}, \quad u = 1,2,...,z. \qquad (28)$$

By applying (7.23) from [Kołowrocki, et al., 2017b], the corresponding optimal values of the variances of the CI network unconditional lifetimes in the critical infrastructure safety state subsets are

$$[\dot{\sigma}^4_{LLS}(u)]^2 = 2\int_0^\infty t \ \dot{\boldsymbol{S}}^4_{LLS}(t,u)dt - [\dot{\mu}^4_{LLS}(u)]^2, \quad u = 1,2,...,z, \qquad (29)$$

where $\dot{\mu}^4_{LLS}(u)$ is given by (26) and $\dot{\boldsymbol{S}}^4_{LLS}(t,u)$ is given by (28).
And, by (7.25) from [Kołowrocki, et al., 2017b], the optimal solutions for the mean values of the CI network unconditional lifetimes in the particular safety states, assuming LLS model of dependency, are

$$\dot{\bar{\mu}}^4_{LLS}(u) = \dot{\mu}^4_{LLS}(u) - \dot{\mu}^4_{LLS}(u+1), \ u = 1,...,z-1, \quad \dot{\bar{\mu}}^4_{LLS}(z) = \dot{\mu}^4_{LLS}(z). \qquad (30)$$

Moreover, considering (7.7) and (7.12) from [Kołowrocki, et al., 2017b], the corresponding optimal critical infrastructure risk function and the optimal moment when the risk exceeds a permitted level δ, respectively are given by

$$\dot{\boldsymbol{r}}^4_{LLS}(t) = 1 - \dot{\boldsymbol{S}}^4_{LLS}(t,r), \ t \geq 0, \qquad (31)$$

and

$$\dot{\tau}^4_{LLS} = \dot{\boldsymbol{r}}^{4 \ -1}_{LLS}(\delta),$$

where $\dot{\boldsymbol{S}}^4_{LLS}(t,r)$ is given by (28) for $u = r$ and $\dot{\boldsymbol{r}}^{4 \ -1}_{LLS}(t)$, if it exists, is the inverse function of the optimal risk function $\dot{\boldsymbol{r}}^4_{LLS}(t)$.
The optimal intensities of degradation / the optimal intensities of departure from the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, of the CI network with LLS rule, impacted by the operation process related to the climate-weather change, i.e. the coordinates of the vector

$$\dot{\boldsymbol{\lambda}}^4_{LLS}(t,\cdot) = [0, \dot{\boldsymbol{\lambda}}^4_{LLS}(t,1), \ldots, \dot{\boldsymbol{\lambda}}^4_{LLS}(t,z)], \quad t \in <0,+\infty), \qquad (33)$$

are given by

$$\dot{\boldsymbol{\lambda}}^4_{LLS}(t,u) = \frac{-\frac{d\dot{\boldsymbol{S}}^4_{LLS}(t,u)}{dt}}{\dot{\boldsymbol{S}}^4_{LLS}(t,u)}, \ t \in <0,+\infty), \quad u = 1,2,...,z. \qquad (34)$$

The optimal coefficients of the operation process related to the climate-weather change impact on the CI network intensities of degradation / the coefficients of the operation process related to the climate-weather change impact on CI network intensities of departure from the safety state subset $\{u, u+1, ..., z\}$, i.e. the coordinates of the vector are given by

$$\dot{\boldsymbol{\rho}}^4_{LLS}(t,\cdot) = [0, \dot{\boldsymbol{\rho}}^4_{LLS}(t,1), \ldots, \dot{\boldsymbol{\rho}}^4_{LLS}(t,z)], \quad t \in <0,+\infty), \qquad (35)$$

where

$$\dot{\boldsymbol{\lambda}}^4_{LLS}(t,u) = \dot{\boldsymbol{\rho}}^4_{LLS}(t,u) \cdot \boldsymbol{\lambda}^0_{LLS}(t,u), \ u = 1,2,...,z, \qquad (36)$$

i.e.

$$\dot{\boldsymbol{\rho}}^4_{LLS}(t,u) = \frac{\dot{\boldsymbol{\lambda}}^4_{LLS}(t,u)}{\boldsymbol{\lambda}^0_{LLS}(t,u)}, \ u = 1,2,...,z, \qquad (37)$$

and $\boldsymbol{\lambda}^0_{LLS}(t,u)$, $t \in <0,+\infty)$, $u = 1,2,...,z$, are the intensities of degradation of the CI network with LLS rule without of the operation process related to the climate-weather change impact, i.e. the coordinate of the vector

$$\dot{\boldsymbol{\lambda}}^0_{LLS}(t,\cdot) = [0, \dot{\boldsymbol{\lambda}}^0_{LLS}(t,1), \dots, \dot{\boldsymbol{\lambda}}^0_{LLS}(t,z)],$$
$$t \in <0,+\infty), \qquad (38)$$

and $\dot{\boldsymbol{\lambda}}^4_{LLS}(t,u)$, $u = 1,2,...,z$, are given by (34).

The optimal indicator of CI network resilience to operation process related to climate-weather change impact (ResI4) is given by

$$\dot{\boldsymbol{RI}}^4_{LLS}(t,r) = \frac{1}{\dot{\boldsymbol{\rho}}^4_{LLS}(t,r)}, \; t \in <0,+\infty), \qquad (39)$$

where $\dot{\boldsymbol{\rho}}^4_{LLS}(t,r)$, $t \in <0,+\infty)$, is the optimal coefficients of operation process related to climate-weather change impact on the CI network intensities of degradation, assuming LLS model of dependency, given by (37) for $u = r$.

4.3. Optimal Sojourn Times of CI Network Operation Process at Operation States Related to Climate-Weather Change Process

The optimal mean values of the total critical infrastructure operation process sojourn times $\hat{\theta}_b$ at the particular operation states z_b, $b = 1,2,...,v$, during the fixed critical infrastructure operation time θ, can help in planning more reliable and safe operation process of the critical infrastructures [Kołowrocki, Soszyńska-Budny, 2011]

$$\dot{\hat{M}}_b = \dot{E}[\hat{\theta}_b] = \dot{p}_b\theta, \; b = 1,2,...,v. \qquad (40)$$

Similarly, replacing in the formula (5) the transient probabilities pq_{bl} at the operation states z_{bl} by their optimal values $\dot{p}q_{bl}$, we get the optimal mean values of the total CI network operation process sojourn times $\hat{\theta}\hat{C}_{bl}$ at the particular operation states related to climate-weather change zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, during the fixed critical infrastructure operation time θ, given by

$$\dot{\hat{M}}\hat{N}_{bl} = \dot{E}[\hat{\theta}\hat{C}_{bl}] = \dot{p}q_{bl}\theta, \; b = 1,2,...,v,$$
$$l = 1,2,...,w. \qquad (41)$$

The knowledge of the optimal values $\dot{M}_b$ of the mean values of the unconditional sojourn times, the optimal values $\dot{M}_{bl}$ of the mean values of the conditional sojourn times at the operation states, the optimal mean values $\dot{\hat{M}}_b$ of the total sojourn times at the particular operation states z_b, $b = 1,2,...,v$, and the optimal mean values $\dot{\hat{M}}\hat{N}_{bl}$ of the total sojourn times at the particular operation states related to climate-weather change zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, during the fixed CI network operation time may by the basis for changing the critical infrastructures operation processes in order to ensure these critical infrastructures operation more safe. Their knowledge may also be useful in these critical infrastructures operation cost analysis.

5. Conclusion

In this paper optimization of operation and safety of CI networks with cascading effects is presented. More exactly, a multistate series network with assets dependent according to local load sharing (LLS) rule is described and its optimal safety and resilience indicators are determined. Similarly as in Section 4.2, CIs and CI networks with other safety structures and models of dependencies can be considered [Kołowrocki, et al., 2017a, c]. Then, the optimal unconditional safety function and other safety and resilience indicators of the multistate parallel, "*m* out of *n*", parallel-series and "*m* out of *l*"-series CI networks with assets dependent according to equal load sharing (ELS) rule, impacted by the operation process related to the climate-weather change process, can be determined. Considering cascading effects in networks with more complex structures we can proceed with parallel-series and "*m* out of *l*"-series CI networks assuming the dependence between their parallel, respectively "*m* out of *l*", subnetworks according to the LLS rule and the dependence between their assets in subnetworks according to the ELS rule, by constructing a mixed load sharing (MLS) model of dependency [Blokus-Roszkowska, Kołowrocki, 2017a-b], [Kołowrocki, et al., 2017a]. Namely, for such CI networks the optimal unconditional safety function, the optimal critical infrastructure risk function and the optimal moment when the risk exceeds a permitted level, the optimal intensities of degradation, the optimal coefficients of the operation process related to the climate-weather change impact on the CI network intensities of degradation and the optimal indicator of CI network resilience to operation process related to climate-weather change impact can be estimated [Kołowrocki, et al., 2017c].

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