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THE MODELING OF INFLUENCE OF TAXATION ON THE DYNAMICS OF INVESTMENT IN THE CONDITIONS OF MARKET SATURATION WITH NONLINEARITY PARAMETERS

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Summary. The formation of a competitive economic structure of the society requires the mobilization of all national resources, including investment. Therefore, at the stage of the transformational development of the Ukrainian economy, the problem of stimulating investment activity by the state is particularly important.

The purpose of the article is to determine the main directions of improving the tax regulation of investment processes in the modern economy and the tax instruments for regulating investment processes in Ukraine by identifying the economic and mathematical dependence of tax influences on the dynamics of investment processes.

Keywords: mathematical model, investment, tax instruments, investment processes

1. INTRODUCTION

In modern models of economic dynamics, which reflect not only the dependence of the changing states on time, but also their interconnection in time, the differential equations are widely used [3]. The dynamic models in the economy make possible a qualitative (sometimes even quantitative) description of the transition between stationary states and identifying the main parameters that govern these processes. In the general case, different variants of transient processes are possible under the influence of external factors. There are two types of such external influences: parametric and power. Parametric influence occurs due to the changes in control parameters and have been considered before. Power influence is the result of changing the dynamic variables, while maintaining the unchanged parameters of the model. The example of a dynamic model of this type in the economy may be an investment project [2].

Let the investment project have a one-dimensional accumulation function and is described by the differential equation of the form:

$$\frac{dx}{dt} = p(x)x \ , \tag{1}$$

where $p(x) = x$.

Let us investigate the dynamics of investment, using additional parameters. Consider the case when the expression for the demand curve has the form:

$$p(x) = \alpha - \beta x^{\gamma} \ , \tag{2}$$

where $\alpha = \beta - x^{\gamma}$ – controlling parameter,

β – the parameter of the "saturation of the market", which characterizes the "speed" of demand disappearance.

γ – the parameter that characterizes the level of non-linearity in the disappearance of the demand for goods.

Equations (1) and (2) describe the growth (decrease) of the accumulation function in the investment project, in the conditions of the gradual saturation of the market [7].

2. MATHEMATICAL MODEL

2.1. There is no taxation and the initial capital is invested in the project

1. Let us consider the case when there is no taxation and the initial capital is invested in the project $x = x_0$ when $t = 0$. Taking into account (2) equation (1) will be

$$\frac{dx}{dt} = (\alpha - \beta x^{\gamma})x \tag{3}$$

Equation (3) has an exact analytical solution that is obtained by separating the variables.

$$\int \frac{dx}{(\alpha - \beta x^{\gamma})^{x}} = \int dt$$

We can put it as:

$$\frac{1}{\alpha}\int \frac{\beta x^{\gamma-1}dx}{\alpha - \beta x^{\gamma}} + \frac{1}{\alpha}\int \frac{dx}{x} = \int dt \text{, further } \frac{\beta}{\alpha\gamma}\int \frac{d(x^{\gamma})}{\alpha - \beta x^{\gamma}} + \frac{1}{\alpha}\int \frac{dx}{x} = \int dt$$

After integrating, we get the following relationship:

$$-\frac{1}{\alpha\gamma}\ln(\alpha - \beta x^{\gamma}) + \frac{1}{\alpha}\ln x = t + \overset{\approx}{c}$$

When $x(0) = x_0$ – initial condition

$$\ln \frac{x}{(\alpha - \beta x^{\gamma})^{1/\gamma}} = \alpha t + c \quad (c = \alpha \overset{\approx}{c}).$$

Then
$$c = \ln \frac{x_0}{(\alpha - \beta x_0^{\gamma})^{1/\gamma}}.$$

Hence, taking into account the initial condition, the solution (3)

$$\ln \frac{x(\alpha - \beta x_0^{\gamma})^{1/\gamma}}{x_0(\alpha - \beta x^{\gamma})^{1/\gamma}} = \alpha t \quad \text{or} \quad \frac{x(\alpha - \beta x_0^{\gamma})^{1/\gamma}}{x_0(\alpha - \beta x^{\gamma})^{1/\gamma}} = e^{\alpha t} \tag{4}$$

From expression (4) we get dependence $x(t)$ in explicit form

$$x(t) = \frac{\alpha^{1/\gamma} x_0 e^{\alpha t}}{(\alpha - \beta x_0^{\gamma} + \beta x_0^{\gamma} e^{\alpha \gamma t})^{1/\gamma}} \tag{5}$$

The stationary value is obtained from (5) if we find the boundary from $x(t)$ when $t \to \infty$, namely: $\lim_{t\to\infty} x(t) = x_c = (\frac{\alpha}{\beta})^{1/\gamma}$

Let us convert the expression (5) to a form more convenient for the analysis. To do this, the numerator and denominator will be divided into $\beta x_0^{\gamma} e^{\alpha\gamma t}$. We get

$$x(t) = \left(\frac{x_c^{\gamma}}{((\frac{x_c}{x_0})^{\gamma} - 1)e^{-\alpha\gamma t} + 1} \right)^{1/\gamma} \tag{6}$$

In this case, the initial condition is the initial capital invested in the project. Characteristic form of the curve described by equation (6) is schematically depicted in Figure 1 [7].

Along with the initial and stationary values of functions $x(t)$, another important characteristic is the point of overlap. It is found in accordance with the general rules in the conditions when the second derivative equals zero.

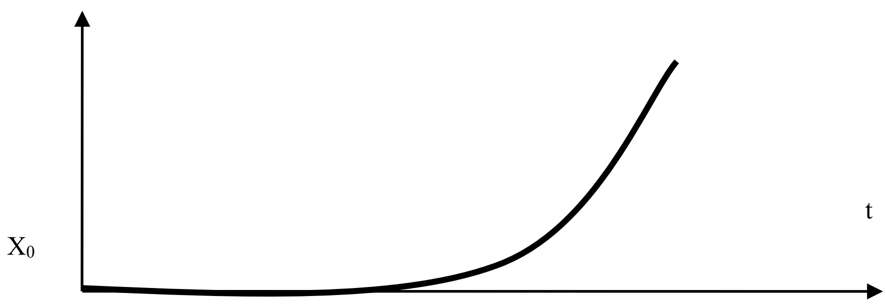

Fig. 1. The growth of the accumulation function in the investment project, in the conditions of the gradual saturation of the market

Having differentiated (3), we get

$$\frac{d^2x}{dt^2} = (\alpha - \beta(1+\gamma)x^{\gamma})\frac{dx}{dt} = 0$$

Hence, for the point of overlap, we get:

$$\alpha - \beta(1+\gamma)x_{\Pi}{}^{\gamma} = 0$$

$$x_{\Pi} = \left(\frac{\alpha}{\beta(1+\gamma)}\right)^{1/\gamma} = \frac{x_c}{(1+\gamma)^{1/\gamma}} \tag{7}$$

This is the point of change in the growth dynamics of the accumulation function. That is, to this level the function grew faster, and after – more slowly, and then the growth will go to zero. From the economic point of view, this means that the profitability of the investment project begins to decrease. So, at this stage, the investor should withdraw some of the funds and transfer them to another project.

The time of reaching $x(t)$ some level we can get from formula (6):

$$t_{\eta} = \frac{1}{2}\ln\left[\left(\left(\frac{x_c}{x_0}\right)^{\gamma} - 1\right)\times\eta\right]^{1/\gamma},$$

where $\eta = \frac{x^{\gamma}}{x_c^{\gamma} - x^{\gamma}}$

Namely, for reaching the point of overlap $x_{\Pi} = \frac{x_c}{(1+\gamma)^{1/\gamma}}$ when $\eta = \frac{1}{\gamma}$, time equals to $t_{\Pi} = \frac{1}{\alpha}\ln\left[\left(\left(\frac{x_c}{x_0}\right)^{\gamma} - 1\right)\times\frac{1}{\gamma}\right]^{1/\gamma}$. This model has demonstrated the dynamic process of investment in the conditions of market saturation, taking into account the nonlinearity parameter when the demand for the goods disappears.

2.2. There is no taxation and the initial capital is invested in the project

2.2.1. Constant tax δ

With such a tax, the difference between the factors of income growth and expenditure per time unit will be $\alpha - \delta$. In this case, the solution of the original equation (5) remains unchanged, and the formulas of the stationary value and the time of reaching a certain level will be written as follows:

$$x_c = \left(\frac{\alpha - \delta}{\beta}\right)^{1/\gamma}$$

$$t_{\eta} = \frac{1}{\alpha - \delta}\ln\left[\left(\left(\frac{x_c}{x_0}\right)^{\gamma} - 1\right)\times\eta\right]^{1/\gamma} \tag{a.1}$$

Thus, a constant tax leads to the disappearance of the marginal value of capital – x_c and to the increase of the characteristic level of reaching capital at a certain level – t_η. Formula (a.1) demonstrates that the time of growth of the accumulation function increases with the increase of taxes. It can be concluded that tax deductions in the form of constant taxes reduce the intensity of the growth of the accumulation function.

2.2.2. Proportional tax

When using the proportional tax in this model, the input equation (3) will be written as follows. The controlling parameter α is substituted by the expression $(1-\mu)\alpha$, where $(0\langle\mu\langle 1)$ is the rate of proportional tax. Then

$$\frac{dx}{dt} = (1-\mu)\alpha x - \beta x^{1+\gamma} \tag{9}$$

The solution of this equation has the same form as (5). In this case, the marginal value of capital will be equal to

$$x_c = \left(\frac{(1-\mu)\alpha}{\beta}\right)^{1/\gamma}$$

And the time of achieving by the accumulation function of a certain level with proportional taxation will be determined as follows:

$$t_\eta = \frac{1}{(1-\mu)} \ln\left[\left(\left(\frac{x_c}{x_0}\right)^\gamma - 1\right) \times \eta\right]^{1/\gamma}$$

2.2.3. Progressive tax

Let us assume that the rate of progressive tax depends on the accumulation function and increases nonlinearly, so instead of the tax rate μ in equation (3) we will have kx^γ.

Then, this equation will be as follows

$$\frac{dx}{dt} = (1-kx^\gamma)\alpha x - \beta x^{1+\gamma} \text{ or } \frac{dx}{dt} = \alpha x - \xi x^{1+\gamma}, \text{ where } \xi = \beta + k\alpha \tag{10}$$

In this case, the dynamic equation becomes similar to equation (3). The stationary solution (10) is found from the condition $\frac{dx}{dt} = 0$, or $(\alpha - \xi x^\gamma)x = 0$

Hence, $x_c = \left(\frac{\alpha}{\beta - k\alpha}\right)^{1/\gamma}$ (Fig. 2.)

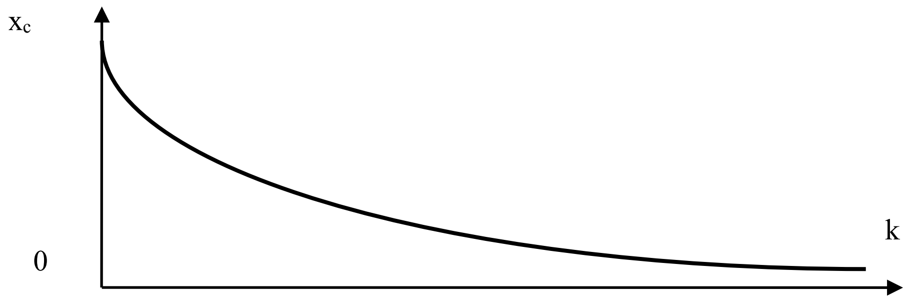

Fig.2. The dynamics of accumulation function in progressive taxation

2.2.4. Regressive tax

Let us assume that the tax rate is reduced and depends on the accumulation function nonlinearly, and in such a way that instead of the tax rate μ in equation (9) we will have $-kx^{\gamma}$. Then

$$\frac{dx}{dt} = (1 + kx^{\gamma})\alpha x - \beta x^{1+\gamma} \text{ or } \frac{dx}{dt} = \alpha x - \lambda x^{1+\gamma}, \text{ where } \lambda = \beta - k\alpha$$

The stationary solution will be recorded as follows: $x_c = \left(\frac{\alpha}{\beta - k\alpha}\right)^{\frac{1}{\gamma}}$, $\beta - k\alpha \rangle 0$

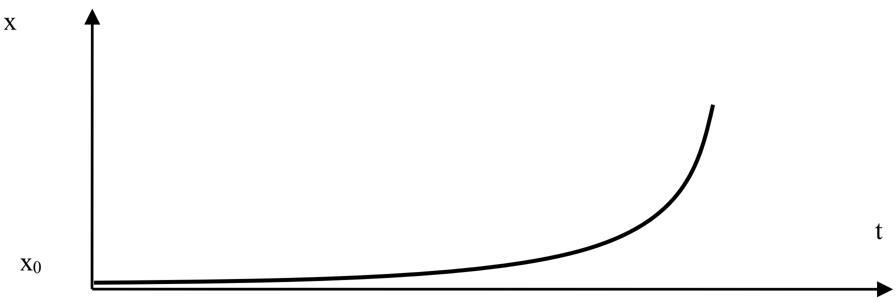

Fig. 3. The dependence of the accumulation function on time with a regressive tax

In this case, there is a very rapid growth of x_c in the so-called "exacerbation mode", which means the mode of ultra-fast development of the dynamic process, when the state of the system varies indefinitely in the course of a short period, which is called aggravation.

Thus, a regressive proportional tax contributes to the very rapid growth of the accumulation function of investments and can be an effective tool to support a priority investment project.

3. CONCLUSIONS

Investment activity is one of the main components of the economy of any country. Without effective investment activity, the dynamic development of economic processes and the stable growth of the welfare of citizens cannot be achieved. Therefore, its conditions must be based on mutually beneficial cooperation between the state and the investor, first of all in the support of the state in the form of various incentive methods. One of the methods of creating a favorable investment climate in the country is tax incentives.

Permanent and proportional taxes reduce the marginal (stationary) value of capital in an investment project and increase the characteristic time to achieve this value. Such a pattern is characteristic for the production of goods in the conditions of both unsaturated and saturated markets.

Progressive proportional taxes cause qualitative changes in the nature of the growth of the accumulation function in the investment project. The dynamic equation of this process, instead of the original linear (continuous growth mode), becomes nonlinear (similar to the saturation regime of the market). As a result, provided that there are sufficiently large time intervals from the beginning of the investment process, the capital accumulation function in this project asymptotically approaches the boundary stationary level.

Under the condition of progressive rates of proportional taxes, the investment processes with a steady growth rate and investment processes in the production of goods in the market saturation mode have two stages of increasing the accumulation function. At the initial stage, there is an accelerated growth of the accumulation function and with time and approaching to its steady-state value, its growth slows down. Such a change of a mode occurs at the conditional point of an overlap of the growth curve of the accumulation function, in which it acquires half its limit value. This allows us to propose an optimal investor's strategy for its further participation in this project: at the moment of reaching the point of overlap, it is possible to withdraw the accumulated capital when it reaches half the limit value of capital in this project, and then part of the accumulated funds can be invested in another or the same investment project.

The incentive factor for investment development are the regressive proportional tax rates. In this case, the rapid growth of capital accumulation function of investment projects in the so-called regime with aggravation, which is understood as a mode of rapid development of a dynamic process in which the variable state of the system increases indefinitely in a short period, known as the exacerbation. So, the regressive tax contributes to the very rapid growth in investment and accumulation functions can be an effective tool for the support of a priority project.

The growth of the function of accumulation of investments in the mode of saturation of the market and the constant receipt of financial flows with constant intensity during the operation of the investment project is possible and provided that there is no initial investment in this project. In such conditions, the accumulation function grows rapidly at the beginning of the process, and then its growth decelerates and it asymptotically approaches its stationary value, which, from an economic point of view, corresponds to the limit value of capital in this project.

In the case of permanent withdrawal of finance from an investment project, the function of capital accumulation in it decreases. Consequently, constant financial flows of stable intensity play the role of effective controlling parameters in the dynamics of investments.

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MODELOWANIE WPŁYWU OPODATKOWANIA NA DYNAMIKĘ INWESTYCJI W WARUNKACH NASYCENIA RYNKU Z PARAMETRAMI NIELINIOWOŚCI

Streszczenie

Artykuł jest poświęcony wyznaczeniu głównych kierunków modelowania regulacji podatkowej procesów inwestycyjnych we współczesnej gospodarce oraz instrumentów podatkowych regulowania procesów inwestycyjnych w Ukrainie za pomocą ujawnienia ekonomicznej i matematycznej zależności wpływu podatków na dynamikę procesów inwestycyjnych. Zaznaczono, że podatki stałe i proporcjonalne zmniejszają graniczne (stacjonarne) znaczenie kapitału w projekcie inwestycyjnym i zwiększają charakterystyczny czas dla osiągnięcia tego znaczenia. Taka prawidłowość jest charakterystyczną dla produkcji towarów w reżimach rynku nienasyconego i nasyconego. Pokazano, że progresywne proporcjonalne podatki wywołują zmiany jakościowe charakteru wzrostu funkcji gromadzenia w projekcie inwestycyjnym. Równanie dynamiczne tego procesu zamiast wyjściowego liniowego (reżim wzrostu stałego), staje się nieliniowym (podobnym do reżimu nasycania rynku). W wyniku tego pod warunkiem dostatecznie dużych interwałów czasowych z początku procesu inwestowania, funkcja akumulacji kapitału w danym projekcie asymptotycznie dążą do granicznego poziomu stacjonarnego.

Udowodniono, że czynnikiem pobudzającym dla rozwoju inwestycji są stawki regresywne podatku proporcjonalnego. W tym przypadku obserwuje się szybki wzrost funkcji akumulacji kapitału w projekcie inwestycyjnym w tak zwanym reżimie z zaostrzeniem, który rozumieją jako reżim z bardzo szybkim rozwojem procesu dynamicznego, podczas którego zmienna stanu systemu nieograniczonie rośnie w krótkim okresie, znanym jak czas zaostrzenia. Więc, podatek regresywny sprzyja bardzo szybkiemu wzrostowi funkcji gromadzenia inwestycji i może być instrumentem efektywnym dla poparcia projektu charakteru priorytetowego.

Jednocześnie wzrost funkcji gromadzenia inwestycji w reżimie nasycania rynku i nadejściu stałym przepływów finansowych ze intensywnością stałą w trakcie funkcjonowania projektu inwestycyjnego jest możliwy również pod warunkiem braku początkowego inwestowania kapitału w dany projekt. W danych warunkach funkcja gromadzenia szybko rośnie na początku procesu, a potem jej wzrost zwalnia się i ona asymptotycznie przybliża się do swojego znaczenia stacjonarnego, co z ekonomicznego punktu widzenia odpowiada granicznemu znaczeniu kapitału w danym projekcie.

W razie stałego wyłączenia finansów z projektu inwestycyjnego funkcja gromadzenia kapitału w nim spada się. Więc, stałe przepływy finansowe stabilnej intensywności odgrywają rolę efektywnych parametrów rządzących w dynamice inwestycji.

Słowa kluczowe: inwestycje, podatki, funkcja gromadzenia, projekt inwestycyjny