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ADVANCED DETERMINATION OF SAFETY FACTOR OF FATIQUE STRENGTH OF COILED SPRINGS

Coiled compression springs are widely used in suspension systems of vehicles, light weight trailers, trains etc and their task is to minimize dynamic loading effect to vehicles body or frame in different road condition over many years. In these systems, springs are exposed to large number of cycles of loading and determination of their durability and reliability quantities is important. Consequently, to increase the reliability of these systems, a more accurate determination of the values of the endurance limits of springs is necessary. Usually, in many strength problems, the major components of stress are static, with less accurately alternating stresses superposed. Most failures of springs are originated with stress of alternating type. The solution of problem is accomplished due to acting shear and axial stresses in plane of cross-section of spring wire.
By existing theory, when springs are loaded by non-steady loading, then to consider of unlimited number of combinations of amplitude and mean stresses, the line of failure of the material must be used. Since experimental data for the line of failure of material are usually not at hand, it is customary to make a conservative approximation, that it is a straight line. However, use this straight line may give considerable mistakes on determination of the limit range stresses. To avoid fore aft mentioned mistakes, the method of approximation of the failure curve line by the line with the properly chosen parameters for the coiled compression and tension springs is proposed in this paper. Theory to use proper parameters in approximation of failure curve and calculation example with results is given as well.

1. INTRODUCTION

Extensive research over the last century has given a partial understanding of the basic mechanism associated with fatigue failures. To completely understand the mechanism of fatigue failures, a derivation of complete theory of plasticity is necessary. As far as developed mathematical theory of plasticity is too complicated for use in practice, the data collected from experimental tests is an only tool for engineering purposes.

Many references for example [1], [2], [3] contain a summary of current knowledge and applicable theory, as far as it applies to engineering practice. By theory, to calculate a safety factor of fatigue strength of coiled springs at arbitrary values of mean and amplitude stresses, the whole diagram of endurance limit stresses must be available. Available experimental data for springs are very limited and often it is impossible to find the values of limited stresses for different springs. In case of limited data, an approximation of real

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endurance limit stress curve is necessary.

As it is known, there exist many methods of approximation of curve of endurance limit stresses. The curve of endurance limit stresses is replaced by straight line or lines, which passes some characteristic points specific for used approximation method. More widely used methods of approximation are known as Serensen-Kinasoshvili method [3], which is represented in Fig.1 and method of Rabinovich [2], which is represented in Fig.2. Diagrams of limited stresses are represented by curved lines in $\tau_m - \tau_a$ coordinates and they are known as Haigh's diagrams.

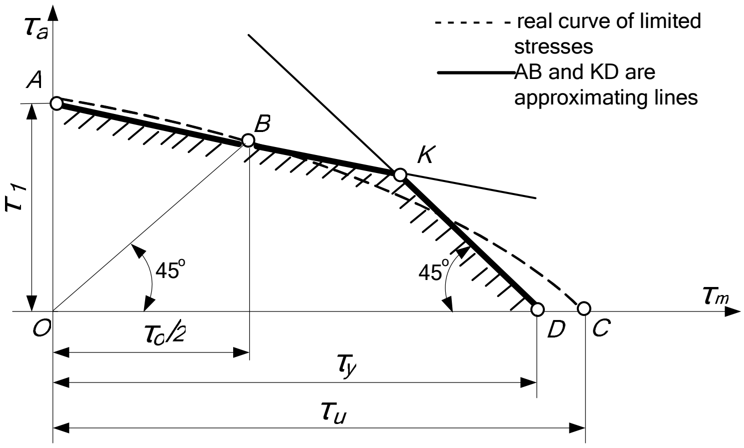

Fig. 1. Approximation by method of Serensen-Kinasoshvili

Stresses indicated in diagram of Fig. 1 are:

τ_u – ultimate shearing stress,

τ_y – yield shearing stress,

τ_a – amplitude of cycled shearing stress,

τ_{-1} – endurance limit shearing stress of symmetrical cycle,

τ_0 – endurance limit shearing stress of pulsating cycle,

τ_m – mean shear stress.

Serensen and Kinasoshvili proposed for approximation of real curve of limited stresses, to replace it by straight line, which connects points A and C in Fig.1. As it is known, for determination of these values of shearing stresses, it is necessary to carry out two series of experiments on fatigue. One of these series of experiments must be carried out with symmetrical cycle of stress, which corresponds to points A and another one with pulsating cycle of stress, which corresponds to point B. Determined lines between points AK and KD represent then approximated values of real endurance limit stress curve. As it is seen from the diagram represented in Fig.1, the deviation of approximated lines compared to real curve of limit stresses, gives higher values of safety factor between points A and B and lower values of safety factor between points B and K.

Rabinovich [2] has shown in his studies, that fatigue tests provided for steels with axial or shearing stresses, give stress values of points A and B, when straight line, which passes them intersect reference line of mean stress τ_m at point L. The value of point L is approximately 1200 MPa. The method of Rabinovich is represented in Fig.2. Method of Rabinovich allows determining approximated endurance limit stress curve by points A and L. For determination of values of stress of point A, fatigue tests with symmetrical cycle of stresses are necessary only.

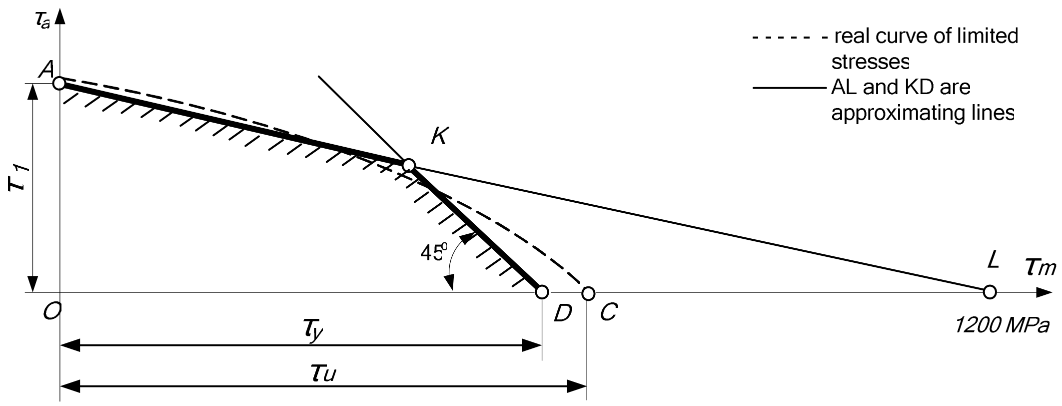

Fig. 2. Approximation by method of Rabinovich

As it is seen from diagram of Fig. 2, a calculation of safety factors by method of Rabinovich leads to higher values of them in area between points A and K and lower values between points D and K, as it was by method of Serensen-Kinasoshvili.

Method of Rabinovich is widely used for practical engineering purposes as it allows approximating an endurance limit stress diagrams with use of short time and limited tests. Method of Serensen-Kinasoshvili is used more for thorough analysis and in case of new materials, when the value of mean stress at point L is unknown.

In this paper, a method of approximation of endurance limit stress curve by curve with properly chosen parameters is proposed. For approximation of endurance limit stress curve it is assumed that helix angle of compression-tension coiled springs is $\alpha \leq 12^\circ$.

2. APPROXIMATION OF DIAGRAMM OF LIMITED SHEAR STRESS

Let the diagram of limited shear stresses in co-ordinates $\tau_m - \tau_a$ is approximated by equation

$$\tau_a = \tau_{-1} - \alpha\tau_m - \frac{\beta}{\tau_u}\tau_m^2, \tag{1}$$

From Fig.1 it is seen, that endurance limit stress curve is naturally at point *A*, when the pulsation cycle of loading takes place. For determination of factors α and β , it is assumed, that the endurance limit stress curve passes the point B ($\tau_0/2$, $\tau_0/2$) and *C* (τ_u, 0) for pulsating cycle of loading and constant load, accordingly. Substituting these coordinates in Eq.1 gives

$$\frac{\tau_0}{2}=\tau_{-1}-\alpha\frac{\tau_0}{2}\frac{\beta}{\tau_u}\left(\frac{\tau_0}{2}\right)^2 \quad and \quad 0=\tau_{-1}-\alpha\tau_u - \beta\tau_u\,, \tag{2}$$

Solving these equations, we can obtain factors α and β as follows

$$\alpha=\frac{4\dfrac{\tau_{-1}}{\tau_u}-2\dfrac{\tau_0}{\tau_u}-\left(\dfrac{\tau_0}{\tau_u}\right)^2\cdot\dfrac{\tau_{-1}}{\tau_u}}{\dfrac{\tau_0}{\tau_u}\left(2-\dfrac{\tau_0}{\tau_u}\right)} \quad and \quad \beta=\frac{2\left(\dfrac{\tau_0}{\tau_u}+\dfrac{\tau_{-1}}{\tau_u}\cdot\dfrac{\tau_0}{\tau_u}-2\dfrac{\tau_{-1}}{\tau_u}\right)}{\dfrac{\tau_0}{\tau_u}\left(2-\dfrac{\tau_0}{\tau_u}\right)}, \tag{3}$$

From Fig.3, the equation of yield line *DK* can be expressed as

$$\tau_a+\tau_m=\tau_y\,, \tag{4}$$

After exception of τ_y from Eq.1 and Eq.4, following equation can be obtained

$$\tau_m^2-\frac{(1-\alpha)\tau_u}{\beta}\tau_m+\frac{\left(\tau_y-\tau_{-1}\right)\tau_u}{\beta}=0\,, \tag{5}$$

and from Eq.5, the abscissa of point K from Fig.3 can be found as

$$OK=\left[\frac{1-\alpha}{2\beta}-\sqrt{\left(\frac{1-\alpha}{2\beta}\right)^2-\frac{1}{\beta}\left(\frac{\tau_y}{\tau_u}-\frac{\tau_{-1}}{\tau_u}\right)}\right]\tau_u\,, \tag{6}$$

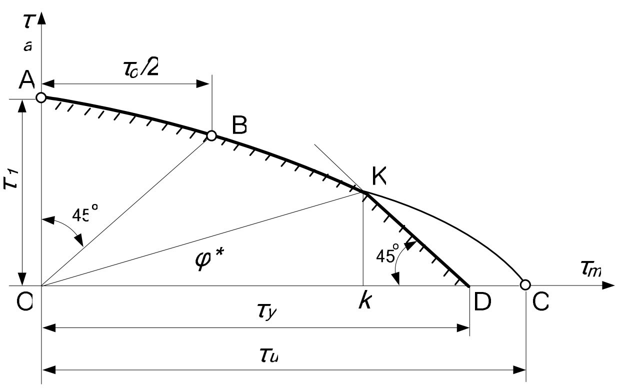

Fig. 3. Approximation curve of endurance limit stresses

Line *OK* is determined by angle φ^*

$$\tan\phi^* = \left(\frac{\dfrac{\tau_{-1}}{\tau_u}}{\dfrac{1-\alpha}{2\beta} - \sqrt{\left(\dfrac{1-\alpha}{2\beta}\right)^2 - \dfrac{1}{\beta}\left(\dfrac{\tau_y}{\tau_u} - \dfrac{\tau_{-1}}{\tau_u}\right)}} \right) - 1 \tag{7}$$

The line *OK* separates the area of the cycles of loading limited by yielding ($\varphi \le \varphi^*$) from area of cycles of loading limited by fatigue limit ($\varphi \ge \varphi^*$).

3. DETERMINATION OF SAFETY FACTOR ON FATIQUE

When $\varphi \le \varphi^*$, then usually safety factor is calculated using yield limit stress values. However, if $\varphi \ge \varphi^*$, for calculation of safety factor, Eq.1 is to be used. Let us suppose, that we have cycled loading, where the point *M* represented in Fig.4 with coordinates (τ_m, τ_a) corresponds to the working stresses. The limit cycle, as the cycle of point M will be marked by point *L*.

From the diagram represented in Fig.4, the safety factor *FS* can be determined as

$$FS_\tau = \frac{OL}{OM} = \frac{Ol}{Om} \tag{8}$$

The equation of the straight line *OL* is

$$\tau_a = \tau_m \, tg\,\phi \tag{9}$$

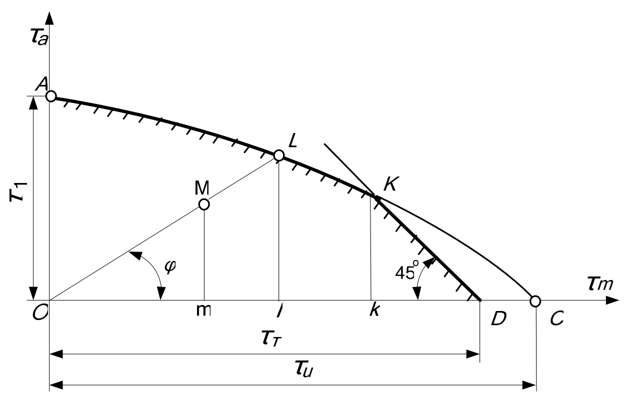

Fig. 4. Diagram for determination of safety factor

Substituting Eq.9 into Eq.1 and following procedure from Eq.1-5, the equation of approximated curve line of limit stresses can be expressed as

$$\tau_m^2 + \frac{\alpha + \tan\phi}{\beta}\tau_u \tau_m - \frac{\tau_u \tau_{-1}}{\beta} = 0 \tag{10}$$

Then the abscissa of the point L has the value

$$Ol = \left[-\frac{\alpha + \tan\phi}{2\beta} + \sqrt{\left(\frac{\alpha + \tan\phi}{2\beta}\right)^2 + \frac{1}{\beta}\frac{\tau_{-1}}{\tau_u}} \right]\tau_u \tag{11}$$

and the equation for safety factor gives

$$FS_\tau = \left[-\frac{\alpha + \tan\phi}{2\beta} + \sqrt{\left(\frac{\alpha + \tan\phi}{2\beta}\right)^2 + \frac{1}{\beta}\frac{\tau_{-1}}{\tau_u}} \right]\frac{\tau_u}{\tau_m} \tag{12}$$

4. SAMPLE OF PROBLEM

The equations for stress and deformation of a closely coiled helical spring are derived from the corresponding equations for the torsion of a round bar. However, it is assumed that the helix angle of spring is $\alpha \le 10^0 ... 12^0$. When a helix angle $\alpha > 12^0$, then it is necessary to take into account bending stresses.

Let the coiled spring is made of chromium-vanadium steel having the following properties:

- shear strength, τ_u = 1200 MPa
- yield strength in shear, τ_y = 950 Mpa
- endurance limit stress for pulsating shearing, τ_0 = 550 Mpa
- endurance limit stress for symmetrically cycled shearing, $\tau_{-1} = 0.6\tau_0$ = 330 Mpa
- shear modulus of elasticity, $G = 0.8 \cdot 10^5$ Mpa.

The spring is made of the wire having round cross-section. The wire diameter d = 7 mm, the external diameter D_e = 60 mm, the number of spring coils I = 6, the initial spring deflection f_1 = 3,8 mm, the working spring deflection h = 28 mm. It is necessary to determine the safety factor FS_τ.

<u>*Solution*</u>

The mean spring diameter

$$D = D_e - d = 60 - 7 = 53\ mm. \tag{13}$$

Coefficient of A.M. Wahl [4] taking into consideration the curvature of the spring coil

$$k = \left(\frac{4D/d - 1}{4D/d - 4} + \frac{0.615}{D/d} \right) = \left(\frac{4 \cdot 53/7 - 1}{4 \cdot 53/7 - 4} + \frac{0.615}{53/7} \right) = 1.195 . \tag{14}$$

Rigidity of the spring (it is equal to the force for spring deformation on one unit).

$$c = \frac{G d^4}{8 D^3 i} = \frac{0.8 \cdot 10^5 \cdot 7^4}{8 \cdot 53^3 \cdot 6} = 26.88 \quad N / mm \tag{15}$$

The force for initial deflection of the spring

$$F_{min} = c \cdot f_1 = 26.88 \cdot 3.8 = 102.1 \, N \tag{16}$$

The maximal deflection of the spring

$$f_2 = f_1 + h = 3.8 + 28 = 31.8 \, mm \tag{17}$$

The maximal force on the spring

$$F_{max} = c \cdot f_2 = 26.88 \cdot 31.8 = 854.7 \, N \tag{18}$$

The minimal and maximal stresses

$$\tau_{\min} = k \frac{8 F_{\min} D}{\pi d^3} = 1.195 \frac{8 \cdot 102.1 \cdot 53}{3.14 \cdot 7^3} = 48.0 \ MPa \tag{19}$$

$$\tau_{\max} = k \frac{8 F_{\max} D}{\pi d^3} = 1.195 \frac{8 \cdot 854.7 \cdot 53}{3.14 \cdot 7^3} = 402.0 \ MPa \tag{20}$$

Then

$$\tau_m = \frac{\tau_{\max} + \tau_{\min}}{2} = \frac{402.0 + 48.0}{2} = 225.0 \ MPa \tag{21}$$

$$\tau_a = \frac{\tau_{\max} - \tau_{\min}}{2} = \frac{402.0 - 48.0}{2} = 177.0 \ MPa \tag{22}$$

The coiled springs usually have no stress concentrations, and then characteristic of the loading cycle is given as

$$\tan \phi = \frac{\tau_a}{\tau_B} = \frac{177.0}{225.0} = 0.787 . \tag{23}$$

The value of $\tan \varphi$ must be compared with the value of $\tan \varphi^*$ by Eq.4. For comparison it is necessary to determine the factors α and β by Eq. 2

$$\alpha = \frac{4\dfrac{\tau_{-1}}{\tau_u} - 2\dfrac{\tau_0}{\tau_u} - \left(\dfrac{\tau_0}{\tau_u}\right)^2 \cdot \dfrac{\tau_{-1}}{\tau_u}}{\dfrac{\tau_0}{\tau_u}\left(2 - \dfrac{\tau_0}{\tau_u}\right)} = \frac{4\dfrac{330}{1200} - 2\dfrac{550}{1200} - \left(\dfrac{550}{1200}\right)^2 \cdot \dfrac{330}{1200}}{\dfrac{550}{1200}\left(2 - \dfrac{550}{1200}\right)} = 0.176, \tag{24}$$

$$\beta = \frac{2\left(\dfrac{\tau_0}{\tau_u} + \dfrac{\tau_{-1}}{\tau_u} \cdot \dfrac{\tau_0}{\tau_u} - 2\dfrac{\tau_{-1}}{\tau_u}\right)}{\dfrac{\tau_0}{\tau_u}\left(2 - \dfrac{\tau_0}{\tau_u}\right)} = \frac{2\left(\dfrac{550}{1200} + \dfrac{330}{1200} \cdot \dfrac{550}{1200} - 2\dfrac{330}{1200}\right)}{\dfrac{550}{1200}\left(2 - \dfrac{550}{1200}\right)} = 0.097. \tag{25}$$

When the values α and β are substituted into Eq. (4) the $\tan\varphi^*$ is equal to

$$\begin{aligned} \tan\varphi^* &= \frac{1}{\dfrac{1-\alpha}{2\beta} - \sqrt{\left(\dfrac{1-\alpha}{2\beta}\right)^2 - \dfrac{1}{\beta}\left(\dfrac{\tau_T}{\tau_u} - \dfrac{\tau_{-1}}{\tau_u}\right)}} \cdot \frac{\tau_{-1}}{\tau_u} - 1 = \\ &= \frac{1}{\dfrac{1-0,176}{2\cdot 0,097} - \sqrt{\left(\dfrac{1-0,176}{2\cdot 0,097}\right)^2 - \dfrac{1}{0,097}\left(\dfrac{950}{1200} - \dfrac{330}{1200}\right)}} \cdot \frac{330}{1200} - 1 = 0,16 \end{aligned} \tag{26}$$

As it is seen, $\varphi > \varphi^*$ and the safety factor is to be determined by Eq.12

$$\begin{aligned} FS_\tau &= \left[-\frac{\alpha + \tan\phi}{2\beta} + \sqrt{\left(\frac{\alpha + \tan\phi}{2\beta}\right)^2 + \frac{1}{\beta}\frac{\tau_{-1}}{\tau_u}}\right]\frac{\tau_u}{\tau_m} = \\ &= \left[-\frac{0,176 + 0,79}{2\cdot 0,097} + \sqrt{\left(\frac{0,176 + 0.79}{2\cdot 0,097}\right)^2 + \frac{1}{0,097}\frac{330}{1200}}\right]\frac{1200}{222,2} = 1,46 \end{aligned} \tag{27}$$

Such a value of safety factor is satisfactory; even, it is close to limit value.

5. DEPENDENCE OF SAFETY FACTOR FROM MECHANICAL PROPERTIES AND CYCLE CHARACTERISTICS

Let us consider the influence of mechanical properties of material and coefficient of cycle asymmetry of variable loading on safety factor of fatigue. For comparison are used the different methods of approximation of real diagram of limit stresses. In the first case, the calculation is carried out by method provided in present work. Another two calculations are carried out by methods of Serensen-Kinosashvili [3] and Rabinovich [2].

In the case of approximation by method of Serensen-Kinosashvili, it is necessary to use three points from real diagram of limit stresses. These points are indicated in Fig. 1 as points A, B and D of real curve. Safety factor is expressed by [3] as

$$FS_\tau = \frac{\tau_{-1}}{\tau_a + \beta \cdot \tau_m}, \tag{28}$$

where

$$\beta = \frac{\tau_{-1} - 0.5\tau_0}{0.5\tau_0} \tag{29}$$

In case of approximation by method of Rabinovich [2], a simpler diagram with two points only is used. These points are indicated in Fig. 1 as points A and C. Safety factor will be found by Eq. 13, but factor β is expressed as

$$\beta = \frac{\sigma_{-1}}{\sigma_b} \tag{30}$$

The calculation results are given in Fig. 5 and Fig. 6. The error of calculation by Serensen leads to increased FS_τ as compared with present method. Calculation results by method of Rabinovich leads to decreased values of FS_τ.

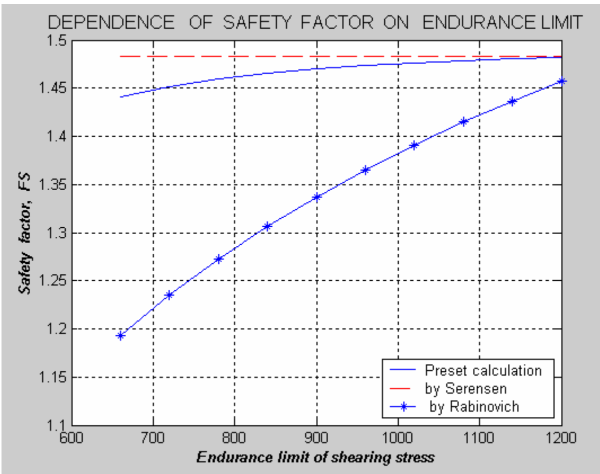

Fig. 5. Influence of endurance limit shear stress values on safety factor

From Fig. 5 and Fig. 6, it is seen, that with increasing of ultimate shear stress values τ_b, the difference in calculation results decreasing accordingly. When an ultimate shear stress value of spring is equal to $\tau_b = 1200\,MPa$ or has higher values, then difference between calculation results is infinitesimal.

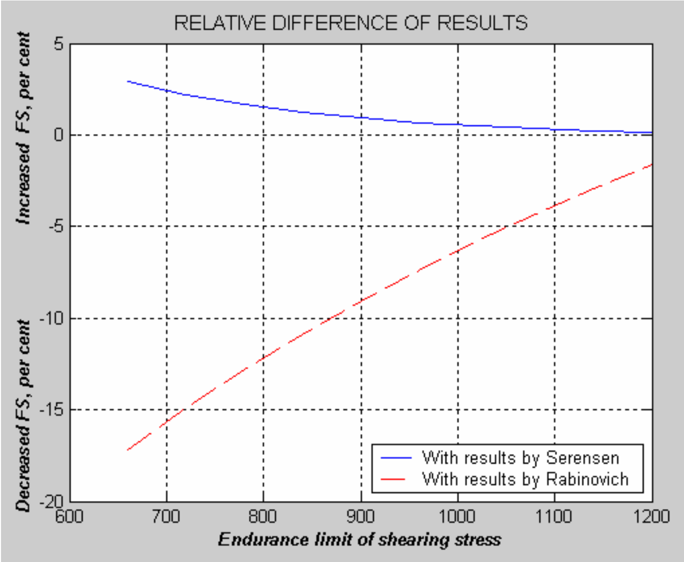

Fig. 6. Relative difference of calculated safety factor values depending on endurance limits shear stress values

Calculation results by method of [2] and [3] are more influenced by asymmetry factor

$$r = \frac{\tau_{\min}}{\tau_{\max}} \tag{31}$$

When a value of asymmetry factor r is increasing and sign of stresses is a constant, then difference between calculation results is increasing considerably. Difference between calculation results in case of various asymmetry factor are presented in Fig. 7 and Fig. 8.

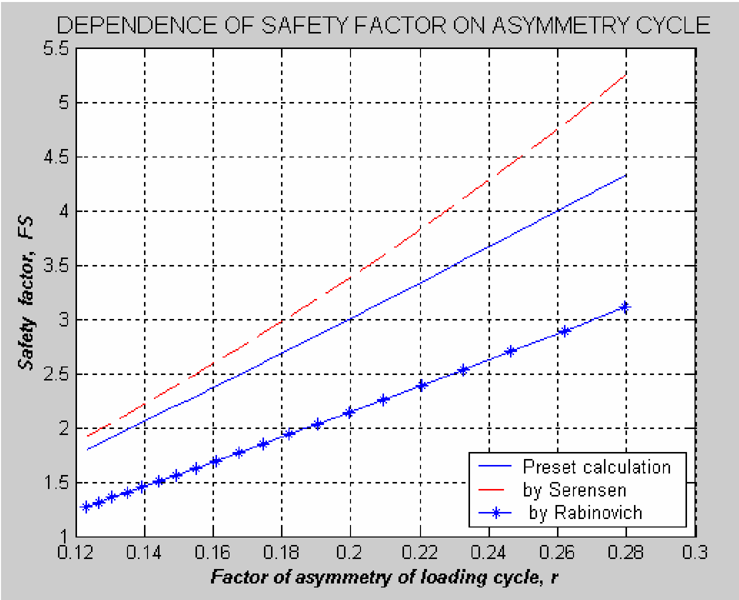

Fig. 7. Influence of factor *r* on calculation of safety factor

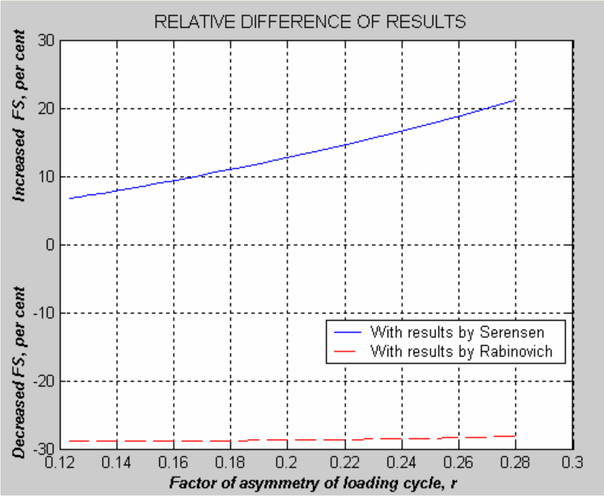

Fig. 8. Relative difference of calculated safety factor values depending on different factors *r*

6. CONCLUSION

In this paper an advanced method for determination of safety factor of fatigue strength of coiled springs under axial forces on tension and compression has been proposed. Theory to use this method and calculation examples has been included as well. Calculation results were compared with existing methods and it has been proved that proposed method is suitable to use for approximation of real curve of endurance limit shear stress curve and on basis of it to calculate a safety factor of fatigue strength.

Even, the theory provided fore aft has been developed for springs with helix angle $\alpha \leq 10...12^{\circ}$; it can be extended to coiled springs with helix angle of $\alpha \geq 12^{\circ}$. In this case, for determination of a safety factor, bending stresses in wire of coiled springs must be included in calculation. Procedure to calculate of safety factor of bending stresses is same as was provided in previous theory; only shear stress values τ must be replaced by values of bending stresses σ. Total safety factor FS is expressed then as

$$\frac{1}{FS^2} = \frac{1}{FS_{\sigma}^2} + \frac{1}{FS_{\tau}^2} \tag{32}$$

Theory provided in this paper will be proved by experiments in future

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