ABOUT ONE METHOD OF MATHEMATICAL MODELLING OF HUMAN VISION FUNCTIONS

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Abstract. The comparison method, widely used in the colorimetric experiments, is applied to simulation of human vision functions. To solve such class of problems the mathematical model is obtained with the help of special type predicates.

Key words: visual system, colorimetric experiment, null-organ method, mathematical model of the vision organ.

Fig. 1. Human eye

INTRODUCTION

The visual system of a man is one of the most perfect systems created by evolution in the process of a long phylogenesis. Visual perception is a complex neurophysiological process formed under the action of photic stimuli on the retina photoreceptors. The spectral sensitivity of an eye coincides with the solar spectrum distribution curve maximum. The maximal sensitivity of an eye at light is observed to more long wavelength beams (the area of the green-orange part of the spectrum), the maximal sensitivity of an eye at half-lights is observed to more short-wave beams (the area of the blue-violet part of the spectrum). In the genegal understanding the form of an eye resembles an incorrect sphere because of some prolated fore-part, fig. 1.
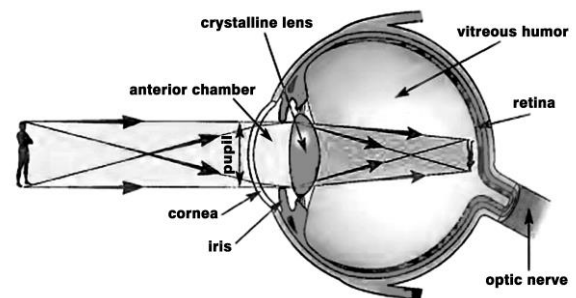
There is the pupil inside this sphere directly in the iris. As it is, as a matter of fact, a hole, it seems black, as behind it there is a dark interior of the eye. The iris has a form of the disk with a hole (pupil) at the centre. The iris of each man is pigmented in a particular colour: shades of grey or brown. The crystalline lens is located behind the iris. It has a fotm of a convexo-convex lens. The crystalline lens actively participates in accommodating the eye to the external conditions. The outside shell of the eye consists of sclera (protein) and cornea. Sclera envelops the whole eyeball and is a peculiar housing, fulfilling the function of protection and providing persistence of the form of the eye. The spherical form of a human eyeball reminds by its functional capabilities the camera with a field of view of 160° across the width and 135° throughout the height. The front

convex and transparent part of the outside shell is named a a cornea, which represents an objective lens. Between the iridescent shell and cornea there is " a chamber fluid", which is the lens, as well as the crystalline lens. The back internal surface of the eye is"lined" with a retina, which is formed of millions of photosensitive cells. The retina is a receiver of light pulses, due to its functioning we can see this or that object

THE HUMAN EYE FUNCTIONS

At bright rays the iris is expanding, and the pupil is narrowing. In the dark everything happens vice versa. Having passed through the pupil, the rays are refracted by the crystalline lens, the the crystalline lens form can vary depending on the distance between a subject and the human eye. If the subject is located close to the eye, the crystalline lens becomes thicker, and if the subject is located far then the crystalline lens becomes thinner. Then the light stikes the retina, where the photosensitive cells convert it to nervous impulse through the composite chemical processes. This impulse is transmitted by the optica nerve in that part of the brain which is responsible for vision, where it is processed, then a visual pattern of the considered subject is reconstructed. A part of nerve fibers of the optic nerve catches red light, another one catches green light and the third optic nerve catches blue light. The brain processes all information, and as a result the man sees colour.

According to anatomy the eyeball in the section on the vertical plane of symmetry looks [9] like as it is shown in fig. 2
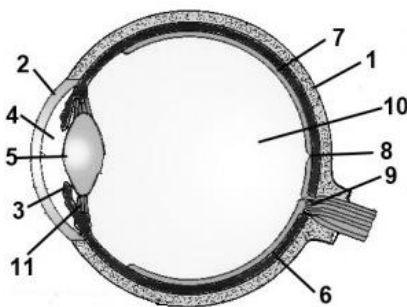

Fig. 2 Structure of the human eye

1 – is the sclera –the dense white color shell of the eye, which protects it against damages; 2 – is the cornea – the front, transparent, strongly curved window, refracting light. Light through the cornea strikes the eye before it is partially overlapped by the color and opaque surfaces of the iris; 3 – is the iridescent shell (or iris), a thin opaque shell in the form of the disk with a hole in the center (pupil), which contains colorants (add color to eyes) and muscle making it possible to change a size of the pupil; 4 – is the pupil, a hole in the center of the iridescent shell, which diameter varies from 1 up to 8 mms, depending on lighting. At poor lighting the pupil is expending to increase energy, which strikes the retina. At normal lighting the pupil is narrowing to resist spherical aberrations in the eyeball and to receive sharper image. The refracting force (analogue of the focal length) of the eye generally depends on refraction at the interface and is will regulated by the change of the crystalline lens form, and also holds the object strictly in focal point. An adult normal person has a focal length variation in the limits of 15 – 17mm; 5 – is the crystalline lens, a biological lens, which is located behind the pupil (iris) and performs an important role of light refraction and accommodation; 6 – is the choroid shell, which enriches the retina with nutrient materials; 7 – is the retina, highly differentiated nervous tissue with photoreceptors perceiving adequate light rays with a wavelength from 380 - 770 nm, i.e. from violet up to red. There are approximately one hundred twenty million rods and seven million cones in the human eye. The rods are sensitive photoreceptors, which are capable to react to one photon, though they give not enough information on a spatial arrangement of the object, as they are bound to the same neuron on the retina. The cones become active at more high level of illumination and the signal, given by every cone, is decrypted by several neurons, that has an effect of a high-resolution in this area; 8 – is the yellow spot (or macula), the point of the highest visual acuity on the retina, which is located opposite to the pupil (there are only cones in the macula). 9 – is the area of the disk of a visual nerve, which is a "blind zone" of the fundus of the eye, as it contains not photoreceptors but the nervous fibers, which depart from the cells of the retina. This is the point of the visual nerve exit from the eyeball; 10 – is the vitreous humor, which fills 2/3 of the eye sizes and provides shape and density for it; 11 – is the cylindrical body, which is, on the one hand, the gland for ultrafiltration of the intraocular fluid, on the other hand it is an accommodation muscle providing conditions for near and far clear vision.

Perception of the color spectrum diversity of the surrounding world is realized by the cones – the retina cells. Three types of color-perceiving elements (photosensitive colorants) are included in them, each of them perceives only one of three primary colors - red, green or violet. All remaining colors and shades can be obtained using different versions of these colors mixing. In the process of color perception it happens due to that the visible

part of the spectrum of luminous radiation includes waves of different length. The long-wave radiations (552-557 nm) act on the red color- perceiving element, MF band radiation (530 nm) acts on green color- perceiving element, short-wave radiation (426 nm) acts on violet color- perceiving element. (Fig.3).
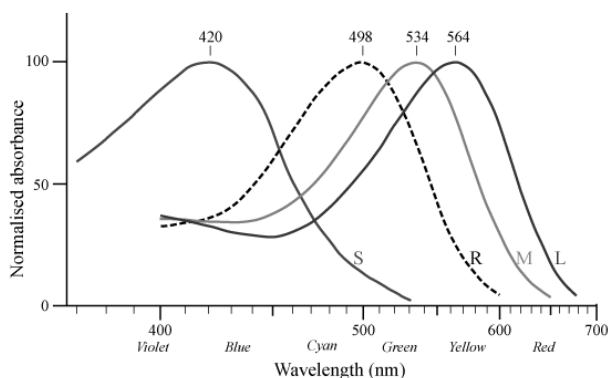

Fig. 3 - Sensitivity of three types of cones and rods (dashed line) to radiation of different wavelength

The pure colors shades differ depending on the intensity of the action: at long wavelength action the vary from the purple color up to the orange one, at MF band action - from the emerald color up to the yellow one, at short-wave action - from the blue color up to the violet one. The luminous flux, containing radiations from different length waves, causes different excitation of all three color perceiving units unequal on intensity, due to that a full color image is shaped in the visual centers of the cerebral cortex. The cerebral cortex synthesizes these excitations in uniform resulting color of the subject according to the laws of optical colors mixing, and the analysis and synthesis of the color perceiving happen permanently and simultaneously. The color perceiving is a function of the retina cones, a papilla-macular bundle of the optic nerve and cork vision centers of the cerebrum.

Relevance. At present a great attention is given to the problems, bound with learning psyxhphysical phenomenae, thus the object of research are as follows: human sensations; physical processes acting on our organa sensoria and causing sensations; ratio, which link sensations to subjects of the external world, appropriate to them. The area of a science investigating conversions of the information by the organa sensoria is called psychophysics. Psychophysics has numerous practical and technical applications. The cybernetics, systems engineering, computer technology, automation, light engineering, engineering of cinema and television and many other areas of practical activity of a man rests on

the outcomes of its researches. The mathematical description of sensations of a man sets before the researchers the task on development of the mathematical apparatus. The vision psychophysics classical task consists in learning the link between light radiation, i.e. visual patterns and characteristics of visual imoges (saturation, colour tone etc.) The main tool of the colorimetry, the science about colour measurements, is the colours matching method [10].

SETTING OF THE RESEARCH PROBLEM

It is offered to utillize a special case of the method of matching, namely, the null - organ method, where the mathematical apparatus of predicates of a special sort is used, and the functional space will be utillized as entry spectrums of light radiations. According to this method [1, 5, 8], two small fields having common boundary, fig. 4, light radiations described by spectrums $b'(\lambda)$ and $b''(\lambda)$, respectively, are shown to an observer. The observer perceives these radiations as two adjoining colour spots.
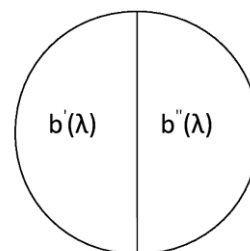

Fig. 4 - Light radiations described by spectrums $b'(\lambda)$ and $b''(\lambda)$ presented to an observer on two small fields having a common boundary

It is required from him to give an answer to the problem, whether the colours of the fields of matching coincide with each other or not. The answer formation is greatly facilitated bythe fact that the boundary between colour spots fades in case of the colours coincidence. Thus, the observer, as a matter of fact, makes a decision on coincidence or distinction of colours with the help of a very thin indicator - absence or presence of a visible boundary between fields of matching. Such fact testifies to a sharp response of the method of matching. If a pair of identical radiations is given on fields of matching $(b'(\lambda), b''(\lambda))$, the observer will register an equality of colours. But if a pair of radiations $(b(\lambda), 1,01 \cdot b(\lambda))$ is presented, i.e. if we increase the energy level of radiation only by 1 % without change of a spectral distribution of light on the right-hand field, the observer with a normal

vision will fix the colours difference clearly. It is found out, that using the method of matching it is possible to distinguish many millions of light radiations by colour. At first sight it may seem, that the colours are bound one-to-one to the light radiations generating them, and, consequently. the observer registering equality or inequality of colours thus finds out coincidence or difference of the appropriate light radiations.

However, there is a set of light radiations (completely different in the spectrum and in power, they are called metameric ones), which are purely not distinguishable in colour for eyes. It follows, that there are much less different colours, than different light radiations. The organ of vision reacts with the same colour to huge number of different light radiations. Thus, an eye, forming a colour, thus groups light radiations in some classes. Moreover, it is found out, that different observers classify light radiations not absolutely equally. Therefore light radiations, seen by one observer as monochrome ones, will look, as a rule, unequal in colour for other observer. It follows from these facts, that each man represents a special object for colorimetric inspection. Moreover, it appears, that the same observer can react differently in colorimetric experiences in different periods of the life. It means, that the parameters of the visual system of a man vary, evolve with time. Despite of these circumstances and that in colorimetric experiences it is necessary to deal with subjective sensations of the observer and with his subjectively formed solution about equality or inequality of colours, nevertheless these experiences are quite objective and can be classified as clearly objective physical experiments. The outcome of the colorimetric experiments does not depend at all on the obsrver's desire. Though the observer can arbitrary invent the answer or make an error when choosing an exact answer (for example at diverting attention during matching colours), but the researcher has all possibilities to detect such answers and to reject them, just as during processing outcomes of physical experiments it is possible to reveal and to eliminate gross errors of the experimenter. The observer in the colorimetric experiment operates quite machinelike: the repeated presentation reduces the same couple of light radiations results in the answer (if, certainly, to not expand carrying out of the experiment for many years, when the observer becomes the other). But in the special cases, namely, when the colours are on the boundary between equality and inequality, the element of chance in the answer is observed. But the same element of chance occurs in any physical experiment when it is necessary to work on a limit of possibilities of measuring instruments. In these cases the accuracy of the outcomes of physical experiments usually are increased at the expense of multiple repetitions of the same tests with the subsequent statistical processing of the outcomes of experiments. The same statistical processing of the subjects' answers is possible in colorimetric experimentse as well. The accuracy, achievable in the colorimetric experiments, is 2 - 3 signs, and at deep statistical processing four signs can be reached. Such an accuracy is at the level of the rather perfect physical experiment accuracy. It follows from the above From all said the output is those: in colorimetric experiments we have that, essentially amazing, case, when the subjective sensations of a man and his subjective actions, when matching the colours, are successfully researched with quite objective physical methods. In other words, the colorimetric experiments demonstrate a key possibility of objective learning of the subjective states of a man, give a specific precedent of such learning. This conclusion is a great responsibility, as it is possible to extract a number of far-reaching outputs from it. Really, if it so, then there is no insurmoumtable gapbetween the objective physical world and subjective world of a man. Thus, the concepts expressed by the words "objective" and "subjective" logically do not eliminate each other, and the second is absorbed by the first. It means also, that the subjective states can be studied quite objectively with physical methods only. In connection with such radical conclusions, the thesis about the possibility of successful objective learning of some subjective states of a man with the colorimetric experiments, on which these conclusions are grounded, should be subjected to a careful check.

The purpose of the work is to build a mathematical model of the organ of vision of a man on the basis of the matching method, which is described with the help of predicates, as well as executions of the predicates properties and numerical implementation of the tasks with the given method using a PC.

MATERIAL AND OUTCOMES OF RESEARCHES

Objectively registereg behaviour of a man is rseally studied in colorimetric experiements. An observer acts in them as a certain "black box" with two inputs and one outpu, fig. 5. The light radiations, characterized by the spectrums $b'(\lambda)$ and $b''(\lambda)$, arrive to the input of the "black box".
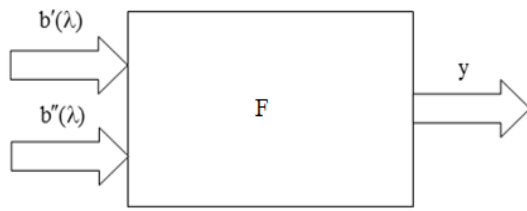
Fig. 5 - Man's vision organ as black box

Fig. 6 - Block diagram of the man's vision organ

From the mathematical point of view these spectrums represent some functions of the real argument λ, set at the interval $[\lambda_1, \lambda_2]$, with material values $b'(\lambda)$ and $b''(\lambda)$. The binary signal $y \in \{0,1\}$ is formed at the output of the "black box". Its value 1 will be interpreted as the answer "yes" of the observer, which means equality of colours on fields of matching, so the value 0 will mean inequality of colours. Thus, the observer implements some predicate by his behaviour**:**

$$y = F(b'(\lambda), b''(\lambda)), \qquad (1)$$

and the properties of this predicate are studied in the colorimetric experiments. Both the input signals $b'(\lambda)$ and $b''(\lambda)$, and output signal y can be registered by physical instruments and, consequently, give the quite objective information for establishment of the predicate F type. But still there is no place for subjective states of the observer in all this; not a single word is said about colours of visual sensations and about the operation of colours matching realized by consciousness of the observer.

But, using the subjective experience as a basis, we can state, that:

1) when the observer forms a signal $y = 1$, the colours of his sensations really are equal.

2) in this case the observer is really compares colours among themselves by some condition of his consciousness and comes to the conclusion about their equality.

Nevertheless, we can not be sure in the validity of these two statements by means of objective observations. How is it possible to deal with this opposition? It would lose force, if it was possible to us, outgoing only from is objective observable properties of predicate F, to prove somehow, that the signal converter presented in fig. 5, can be shown as the scheme in fig. 6.
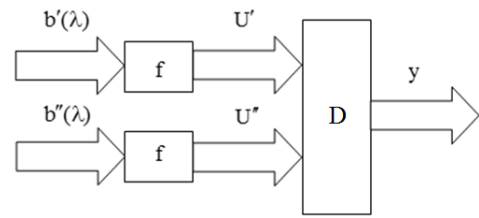
Here signals:

$$U' = (U_1', U_2', U_3') \text{ and } U'' = (U_1'', U_2'', U_3'') \quad (2)$$

three-dimensional vectors with real components U_1', U_2', U_3' and U_1'', U_2'', U_3'', computed with the formulas:

$$U_1' = \int_{\lambda_1}^{\lambda_2} b'(\lambda) K_1(\lambda) d\lambda,$$

$$U_2' = \int_{\lambda_1}^{\lambda_2} b'(\lambda) K_2(\lambda) d\lambda, \qquad (3)$$

$$U_3' = \int_{\lambda_1}^{\lambda_2} b'(\lambda) K_3(\lambda) d\lambda,$$

and

$$U_1'' = \int_{\lambda_1}^{\lambda_2} b''(\lambda) K_1(\lambda) d\lambda,$$

$$U_2'' = \int_{\lambda_1}^{\lambda_2} b''(\lambda) K_2(\lambda) d\lambda, \qquad (4)$$

$$U_3'' = \int_{\lambda_1}^{\lambda_2} b''(\lambda) K_3(\lambda) d\lambda,$$

The formulas (3) and (4) mathematically describe the kind of the functions $U_1' = f(b'(\lambda))$ and $U_1'' = f(b''(\lambda))$. The character D marks predicate of equalities defined as follows:

$$D(U', U'') = \begin{cases} 1, if \ U' = U'' \\ 0, if \ U' \neq U'' \end{cases}, \qquad (5)$$

The presented predicate F is easily interpreted in the psychological terms. The signals U' and U'' can be understood as colours of the fields of

matchings subjectively experienced by the observer. The function f is interpreted as a conversion of light radiation into a colour of visual sensation producible by the visual system of a man. Predicate D will be interpreted as the colours matching operation of the fields of matching realized by consciousness of the observer. If it would be possible to prove, that the predicate (1) can be represented as relations (2)-(5), it would give us the right to state, that:

1) the signals U' and U'' can be adopted as the mathematical description of colours on the fields of matching,

2) the function f can be adopted as the mathematical description of conversion of light radiation acting on the eye retina, in the colour of visual sensation originating in consciousness of the observer.

As a result the task of the logical substantiation with the objective methods of the mathematical model of colour vision (3)-(4), offered by Maxwell, would be completely solved. The described approach, however, also can be subjected criticism. The objection is that at this approach only the proof of the possibility to represent the signal convertion in the visual system as the block diagram shown in fig. 5 is taken into account. It is necessary to prove the necessity of such structure. According to this point of view it is necessary to prove, that the visual analyzer really has anatomic and physiological structures computing in the process of view the values of integrals (3)-(4), and that the colour of visual sensations actually is the triple of numerical codes materially introduced as some physical - chemical process. It is possible to reply to this objection as follows. No doubt, it would be very tempting to receive not only the functional, but also structural description of the visual analyzer. But derivation of mathematical dependences desciding only a way of the visual system functioning, it's also too much. Even in physics in most cases they restrict themselves to the functional (phenomenological) description of processes. Celestial mechanics, nuclear physics and many other important sections of physics go almost exclusively on this way. If we want to restrict ourselves to the functional part of the problem, then inevitabily it will have to content content only with possible mathematical models of the investigated processes. Thus, all possible identical formulas, distinguishing among themselves on structure, describing the same function should be considered equivalent. None of these formulas should be preferred at the functional approach, no matter how much they may differed from each other in their structure.

The choice of kinds of the investigating predicates is defined by practice and it is expedient to study converters most frequently met in the real life. One of such converters is the linear mapping

$$F : H \rightarrow R^n,$$

where: H - is Hilbert space of the input signals, R^n - is n-dimensional Euclidean space. The real systems, as a rule, as input signals have any functional dependencies and have properties of linearity. Thus, more often, the input signals themselves form a linear space, and the introduction on it of a scalar product makes it possible to describe linear functionals by easy way.

By virtue of the known Riesz theorem of the general view of the linear functional [7] each linear continuous functional $\alpha(x)$ on $L_2[\alpha, b]$ is set by the formula:

$$\alpha(x) = \int_\alpha^b g(t)x(t)dt, \ g(t) \in L_2 , \qquad (6)$$

Hereinafter the following statement will be utilised.

Lemma 1. The linear continuous functionals $\alpha_i(x)(i = 1,...,k)$ of the kind of (6) are linearly independent whenand only when the appropriate functions $g_i(x)$ are linearly independent.

The proof. If $\alpha_1 g_1(x) + ... + \alpha_n g_n(x)$ - is a function equal to null almost everywhere on L_2, then by virtue of (6) $\alpha_1 f_1(x) + ... + \alpha_n f_n(x) = 0$. Vice versa, suppose, that $\alpha_1 f_1(x) + ... + \alpha_n f_n(x) = 0$. Then from (6) we obtained $\int_\alpha^b (\alpha_1 g_1(t) + ... + \alpha_n g_n(t))\alpha(t)dt = 0$ for all $x(t) \in L_2$. But it is known, that any function $y(x) \in L_2$, orthogonal to all functions from L_2, is equal to null almost everywhere. **The lemma is proved.**

Utillizing the obtained above auxiliary results let us study now special predicates $T(x, y)$. Let us introduce the predicate of equality:

$$D(x, y) = \begin{cases} 0, if \ (x \neq y) \\ 1, if \ (x = y) \end{cases}, \ x, y \in L_2, \ (7)$$

Let $A : L_2 \rightarrow H$ - be the continuous linear

operator [2,4] mapping L_2 on a finite-dimensional vector space H above the field of real numbers. Main result of the present work is the axiomatic characterization of predicates $T(x, y)$ of the kind:

$$T(x, y) = D(A(x), A(y)), \qquad (8)$$

specified on the Cartesion square. First of all, let us note that if predicate $T(x, y)$ is presented as (8), it will also appear as:

$$T(x, y) = D(Px, Py), \qquad (9)$$

where P - is the operator of designing L_2 on some subspace H of the spaces L_2. Really, let S_1 - be the core of mapping $A: S_1 = \{x \in L_2, A(x) = 0\}$. As A - is the continuous operator, S_1 - is the closed subspace of the space L_2. According to the main theorem of homomorphic vector spaces the quotient space $L_2 \mid S_1$ is isomorphous to the space H. By virtue of S_1 closeness the orthogonal decomposition takes place:

$$L_2 = S_1 + S_2, \qquad (10)$$

where $L_2 \mid S_1 \cong S_2 \cong H$. In view of (9), for any unit $x \in L_2$ one significant decomposition takes place $x = x_1 + x_2$, where $x_1 \in S_1, x_2 \in S_2$, and $x_2 = P(x)$, where $P: L_2 \to S_2$ - is the operator of designing L_2 on S_2. The isomorphism between the spaces S_2 and H is set by the formula $P(x) \leftrightarrow A(x)$ $(x \in L_2)$ and consequently:

$$T(x, y) = D(A(x), A(y)) = D(P(x), P(y)), (11)$$

Quod erat demonstrandum (Q.E.D.). From (11) it follows that for any $x \in L_2$ $T(x, (P(x)) = 1$, and if $T(x, y) = 1$ and $y \in S_2$, then $y = P(x)$.

Now proceed to the axiomatic characterization of predicates of the kind of (8). As the relation of the equality is a reflexive one, symmetric and transitive, the predicate $T(x, y)$ defines the relation with such properties:

1) $T(x, x) = 1$,

2) if $T(x, y) = 1$, then $T(y, x) = 1$,

3) if $T(x, y) = 1$ and $T(y, z) = 1$, then $T(x, z) = 1$.

The remaining properties of the predicate $T(x, y)$ should take into account linearity and continuity of the operator A, and also finite dimensionality of the space $H = A(L_2)$.

Theorem 1. Predicate $T(x, y)$, set on $L_2 \times L_2$, then and then will be presented as (8), when $T(x, y)$ obeys to the properties 1), 2), 3), and also the following three properties:

4) if $T(x_1, y_1) = 1$ and $T(x_2, y_2) = 1$, then $T(x_1 + x_2, y_1 + y_2) = 1$,

5) there is such a finite system of vectors: $l_1, ..., l_n \in L_2$, that for each vector $x \in L_2$ there will be the unique n - dimensional vector $(\alpha_1, ..., \alpha_n)$, for which

$$T(x_1 \alpha_1 l_1 + ... + x_n \alpha_n l_n) = 1,$$

6) the functionals $\alpha_i(x)$ $(i = 1, ..., n) = 1$ are continuous on L_2.

The proof. Let us define, first of all, the necessity of conditions of the theorem. Let the predicate $T(x, y)$ be set by the formula (8). Then, evidently, the properties 1), 2), 3) are fulfilled.

If $T(x_1, y_1) = 1$, $T(x_2, y_2) = 1$, then by virtue of (8):

$A(x_1) = A(y_1)$, $A(x_2) = A(y_2)$, and then

$$A(x_1) + A(x_2) = A(y_1) + A(y_2)$$

or $A(x_1 + x_2) = A(y_1 + y_2)$, i.e.

$$T(x_1 + x_2, y_1 + y_2) = D(A(x_1 + x_2), A(y_1 + y_2)) = 1.$$

This proves the property 4). Let us present the predicate $T(x, y)$ as (11) $T(x, y) = D(Px, Py)$, where $P: L_2 \to H$ - is the operator of designing L_2 on some finite-dimensional subspace $H \subset L_2$.

Let $l_1, ..., l_n$ - be the basis of the subspace H and $P(x) = (\alpha_1(x)l_1 + ... + \alpha_n(x)l_n)$.

Then $T(x, \alpha_1(x)l_1 + ... + \alpha_n(x)l_n) = 1$, from $T(x, j_1 l_1 + ... + j_n l_n) = 1$ it follows, that $j_1 = \alpha_1, .., j_n = \alpha_n$ and also $\alpha_1(x), .., \alpha_n(x)$ - are the continuous functionals on L_2, the operator of designing $L_2 \to H$ is continuous. Thus the properties 5), 6) of the predicate $T(x, y)$ are defined and the necessity of the theorem conditions is proved. Let us prove their sufficiency. Let $T(x, y)$ on $L_2 \times L_2$ satisfies the properties 1) ÷ 6).

Then the vectors $l_1,...,l_n$ - are linearly independent. Really, if for $(j_1,...,j_k) \neq (0,...,0)$ $j_1 l_1 + ... + j_n l_n = 0$, then by virtue of 1), $T(0,0) = 1$, $T(0, j_1 l_1 + ... + \alpha_n l_n) = 1$, and this equality contradicts the axiom 5). Let us denote by H the subspace of the space L_2, spanned by the vectors $l_1,...,l_n$ and let us assume, by virtue of 5), $A(x) = \alpha_1(x)l_1 + ... + \alpha_n(x)l_n$. We have:

$$T(x, A(x)) = 1, \qquad (12)$$

Then A - is the linear operator from L_2 on H, as from $T(x, A(x)) = 1$, $T(y, A(y)) = 1$ on the basis of 4) is obtained $T(x + y, A(x) + A(y)) = 1$, that by virtue of (12) gives $A(x + y) = A(x) + A(y)$. The continuity of the operator A follows from continuity of the functionals $\alpha_i(x)$ $(i = 1,...,n)$. From equalities $T(x, y) = 1$, $T(x, A(x)) = 1, T(y, A(y)) = 1$, by virtue of 1) ÷ 3), the formula follows:

$$T(x, y) = T(A(x), A(y)), \qquad (13)$$

Further:

$$T(A(x), A(y)) = D(A(x), A(y)), \qquad (14)$$

Really, $T(\alpha_1 l_1 + ... + \alpha_n l_n, j_1 l_1 + ... + j_n l_n) = 1$, where $(\alpha_1,...,\alpha_n) \neq (j_1,...,j_n)$, then comparison of this equality with the equality $T(\alpha_1 l_1 + ... + \alpha_n l_n, \alpha_1 l_1 + ... + \alpha_n l_n) = 1$ contradicts the axiom 5). The equality (14) completes the proof of sufficiency of the theorem 1 conditions.

The remark. It is possible to write the equality (14) as:

$$T(x, y) = D(U(x), U(y)), \qquad (15)$$

where $U(x) = (\alpha_1(x),...,\alpha_n(x))$. On the Riesz theorem the functionals $\alpha_i(x)$ are set by the equalities:

$$\alpha_i(x) = \int_{\alpha}^{b} g_i(t)x(t)dt, \ (i = 1,...,n), \quad (16)$$

Thus, on **lemma 1** the linear independence of the functionals $\alpha_1(x),...,\alpha_n(x)$ are equivalent to the linear independence of functions $g_1(x),...,g_n(x) \in L_2$ (i.e.of the appropriate classes of functions). Therefore **theorem 1** admits the following equivalent statement.

Theorem 1. Predicate $T(x, y)$ on $L_2 \times L_2$ then and only then can be presented as (15), (16) with linearly independent functionals $g_1(x),...,g_n(x)$, when it obeys to properties 1) ÷ 6).

The proof. If the properties 1) ÷ 6) are fulfilled, then, as it is shown, the equalities (15), (16) with linearly independent $g_1(x),...,g_n(x) \in L_2$ take place. Vise versa, if the equalities (15), (16) with linearly independent $g_1(x),...,g_n(x) \in L_2$ take place, the mapping $A(x) = (\alpha_1(x),...,\alpha_n(x))$ is the linear continuous operator from L_2 on n - dimendional vector space

$$H = \{(\alpha_1(x),...,\alpha_n(x))\},$$

and

$$T(x, y) = D(A(x), A(y)).$$

Then by virtue of theorem 1 the axioms 1) ÷ 6) are fulfilled. The theorem is proved.

In the specific case numerical mathematical methods [3, 6,] can be used for constructing a model of the vision on the basis of the obtained dependences.

CONCLUSION

1. The feasibility of the "method of matching " application to simulation of functions of human vision is realized.

2. The mathematical model for solution of the considered tasks is descibed with the help of the theory of predicates of a special form.

3. The properties of predicates are proved.

4. The theorem is proved, that the predicate: $T(x, y) = D(A(x), A(y))$, if all six conditions are feasible.

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