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THERMAL PERFORMANCE OF POROUS FINS WITH TEMPERATURE-DEPENDENT HEAT GENERATION VIA THE HOMOTOPY PERTURBATION METHOD AND COLLOCATION METHOD

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Abstract. An analysis has been performed to study the problem of the thermal performance of a nonlinear problem of the porous fin with temperature-dependent internal heat generation. Highly accurate semi-analytical methods called the collocation method (CM) and the homotopy perturbation method (HPM) are introduced and then are used to obtain a nonlinear temperature distribution equation in a longitudinal porous fin. This study is performed using passage velocity from the Darcy's model to formulate the heat transfer equation through porous media. The heat generation is assumed to be a function of temperature. The effects of the natural convection parameter Nc, internal heat generation εg, porosity S_h and generation number G parameter on the dimensionless temperature distribution are discussed. Also, numerical calculations called the fourth order Runge-Kutta method were carried out for the various parameters entering into the problem for validation. Results reveal that analytical approaches are very effective and convenient. Also it is found that these methods can achieve more suitable results compared to numerical methods.

Keywords: *collocation method, homotopy perturbation method, porous fin, temperature-dependent heat generation*

1. Introduction

Fins are frequently used in many heat transfer applications to improve performance. On the other hand, for many years, high rate of heat transfer with reduced size and cost of fins is the main target for a number of engineering applications such as heat exchangers, economizers, super heaters, conventional furnaces, gas turbines, etc. Some engineering applications such as airplane and motorcycle also require a lighter fin with a higher rate of heat transfer. Increasing the heat transfer mainly depends on the heat transfer coefficient (h), the surface area available and the temperature difference between the surface and surrounding fluid. However, this requirement is often justified by the high cost of the high-thermal-conductivity metals, in which high thermal conductivity metals also have high cost. The fin is porous to allow the flow of infiltrating through it. Extensive research has been done in this area and many references are available, especially for heat transfer in porous fins [1-5]. Described below are a few papers relevant to the study described herein. Nonlinear problems and phenomena play an important role in applied mathematics, physics, engineering and other branches of science, especially some heat transfer equations. Except for a limited number of these problems, most of them do not have precise analytical solutions. Therefore, these nonlinear equations should be solved using approximation methods. Perturbation techniques are too strongly dependent upon the so-called “small parameters” [6]. Many other different methods have been introduced to solve a nonlinear equation such as the δ-expansion method [7], Adomian’s decomposition method [8], many homotopy perturbation method (HPM) [9-15], many variational iteration method (VIM) [16-25] and many collocation method [26-28].

In this work, we have applied the CM and the HPM to find the approximate solutions of nonlinear differential equations governing on porous fin with temperature-dependent internal heat generation. Results demonstrate that the proposed methods are simple and accurate compared with numerical method. It is found that these methods are powerful mathematical tools and that they can be applied to a large class of linear and nonlinear problems arising in different fields of science and engineering.

Nomenclature			
A	Section area of fin	c_p	Specific heat
X	Dimensional space coordinates	q^*	Heat generation
x	Horizontal direction	V_w	Velocity of fluid passing through the fin
h	Convection heat transfer coefficient	S_h	Porosity parameter
Kr	Thermal conductivity ratio	ε_g	Internal heat generation
k	Thermal conductivity	Nc	Natural convection parameter
q	Conducted heat	G	Generation number
p	Fin perimeter	T	Local fin temperature
CM	Collocation method	T_b	Fin base temperature
HPM	Homotopy perturbation method	**Greek symbols**	
NUM	Numerical method	β	Coefficient of volumetric thermal expansion
L	Length of the fin	θ	Dimensionless temperature
$Re(x)$	Residual function	ε	Fin surface emissivity (dimensionless)
$\tilde{u}$	Trial function	**Subscripts**	
ci	Constants	eff	Porous properties
Wi	Weight function	b	Conditions at the fin base

2. Analysis

As shown in Figure 1, a rectangular porous fin profile is considered. The dimensions of this fin are length L, with wand thickness t. The cross-section area of the fin is constant and the fin has a temperature-dependent internal heat generation.

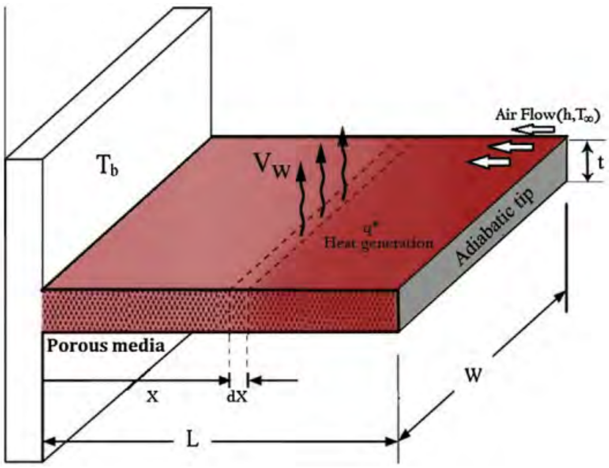

Fig. 1. Schematic diagram for the problem under consideration

Also, the heat loss from the tip of the fin compared with the top and bottom surfaces of the fin is assumed to be negligible. Since the transverse Biot number should be small for the fin to be effective [27], the temperature variation in the transverse direction is neglected. Thus heat conduction is assumed to occur solely in the longitudinal direction. The energy balance can be written as:

$$q(x)-q(x+\Delta x)+q^{*}A\Delta x=\dot{m}c_p\left[T(x)-T_\infty\right]+h(p.\Delta x)\left[T(x)-T_\infty\right] \tag{1}$$

The mass flow rate of the fluid passing through the porous material can be written as:

$$\dot{m}=\rho V_w \Delta x\, w \tag{2}$$

The value of V_w should be estimated from the consideration of the flow in the porous medium. From Darcy's model we have:

$$V_w=\frac{gk\beta}{\upsilon}\left[T_{(x)}-T_\infty\right] \tag{3}$$

Substitution of Equations (2) and (3) into Equation (1) yields:

$$\frac{q(x)-q(x+\Delta x)}{\Delta x}+q^{*}A=\frac{\rho c_p g k \beta w}{\Delta x}\left[T(x)-T_\infty\right]^2+hp\left[T(x)-T_\infty\right] \tag{4}$$

As $\Delta x \to 0$, Equation (4) becomes

$$\frac{dq}{dx} + q^* A = \frac{\rho c_p g k \beta w}{\Delta x}\left[T(x) - T_\infty\right]^2 + hp\left[T(x) - T_\infty\right] \tag{5}$$

Also from Fourier's law of conduction:

$$q = -k_{eff}\, A \frac{dT}{dx} \tag{6}$$

where A is the cross-sectional area of the fin $A = wt$ and k_{eff} is the effective thermal conductivity of the porous fin that can be obtained from the following equation [3]:

$$k_{eff} = \varphi k_f + (1-\varphi) k_s \tag{7}$$

where φ is the porosity of the porous fin. Substitution of Equation (6) into Equation (5) leads to:

$$\frac{d^2T}{dx^2} - \frac{\rho c_p g k \beta w}{t\, k_{eff}\, \upsilon}\left[T(x) - T_\infty\right]^2 + \frac{h\,p}{k_{eff}\, A}\left[T(x) - T_\infty\right] + \frac{q^*}{k_{eff}} = 0 \tag{8}$$

It is assumed that heat generation in the fin varies with temperature as Equation (9) [27]:

$$q^* = q_\infty^*\left[1 + \varepsilon(T - T_\infty)\right] \tag{9}$$

where q_∞^* is the internal heat generation at temperature T_∞. For simplifying the above equations, some dimensionless parameters are introduced as follows:

$$\begin{gathered} \theta = \frac{(T - T_\infty)}{(T_b - T_\infty)}, \quad X = \frac{x}{L}, \quad Nc^2 = \frac{hpL^2}{k_0 A}, \quad Sh = \frac{Da\, x\, Ra}{kr}\left(\frac{L}{t}\right)^2 \\ G = \frac{q_\infty^*}{h\,p(T_b - T_\infty)}, \quad \varepsilon g = \varepsilon(T_b - T_\infty) \end{gathered} \tag{10}$$

where Sh is a porous parameter that indicates the effect of the permeability of the porous medium as well as the buoyancy effect, so a higher value of Sh indicates higher permeability of the porous medium or higher buoyancy forces. Nc is a convection parameter that indicates the effect of surface convecting of the fin. Finally, Equation (8) can be rewritten as:

$$\frac{d^2\theta}{dX^2} - Nc^2\theta + Nc^2 G(1 + \varepsilon g\, \theta) - Sh\,\theta^2 = 0 \tag{11}$$

In this research, we study a finite-length fin with an insulated tip. For this case, the fin tip is insulated so that there will not be any heat transfer at the insulated tip and boundary condition will be

$$\theta(1)=1, \quad \theta'(0)=0 \tag{12}$$

3. Implantation of the analytical solution

3.1. Principles of collocation method (CM)

Suppose we have a differential operator D acting on a function u to produce a function p [28]:

$$D(u(x)) = p(x) \tag{13}$$

We wish to approximate u by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is:

$$u \cong \tilde{u} = \sum_{i=1}^{n} C_i \varphi_i \tag{14}$$

Now, when substituted into the differential operator, D, the result of the operations is not, in general, $p(x)$. Hence an error or residual will exist:

$$E(x) = R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \tag{15}$$

The notion in the collocation is to force the residual to zero in some average sense over the domain. That is:

$$\int_x R(x)\, W_i(x) = 0 \quad i = 1,2,...,n \tag{16}$$

where the number of weight functions W_i is exactly equal to the number of unknown constants C_i in $\tilde{u}$. The result is a set of n algebraic equations for the unknown constants C_i. For the collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is, $W_i(x) = \delta(x - x_i)$. The Dirac δ function has the property that:

$$\delta(x - x_i) = \begin{cases} 1 & \text{if } x = x_i \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

And residual function in Equation (15) must be zero at specific points.

Application

Consider the trial function as:

$$\theta(X) = 1 + c_1(1 - X^2) + c_2(1 - X^4) \tag{18}$$

which satisfies the boundary condition in Equation (12) and sets it into Equation (11), the residual function, $R(c_1, c_2, X)$, is found as:

$$\begin{aligned} R(c_1,c_2,X) &= -2Sh c_1 X^6 c_2 + 2Sh\, c_1 x X^2 c_2 + 2Sh\; c_1\; c_2 X^4 + Nc^2 G\; \varepsilon g\, c_1 - Sh\; c_2^2 - 2Sh\; c_2 \\ &+ Nc^2 G \varepsilon g c_2 - 2Sh c_1 c_2 - Sh c_2^2 X^8 + 2Sh c_2 x^4 - Sh + Nc^2 G - Nc^2 + Nc^2 c_2 X^4 - 2Sh c_1 \\ &+ 2Sh\, c_1 X^2 - Sh\; c_1^2 X^4 + 2Sh\; c_1^2\; X^2 - 2c_1 - Nc^2 G\; \varepsilon g\; c_1 X^2 - Nc^2 G\; \varepsilon g\; c_2 X^4 - Sh\, c_1^2 \\ &- 12 c_2 X^2 + 2Sh c_2^2 X^4 - Nc^2\; c_1 - Nc^2 c_2 + Nc^2 G\; \varepsilon g + Nc^2 c_1 X^2 = 0 \end{aligned} \tag{19}$$

On the other hand, the residual function must be close to zero. For reaching these important, two specific points in the domain $t \in [0,1]$ should be chosen. These points are:

$$X_1 = \frac{1}{3}, \quad X_2 = \frac{2}{3} \tag{20}$$

Finally, by substituting these points into the residual function, $R(c_1, c_2, X)$, a set of two equations and two unknown coefficients is obtained. After determining these unknown parameters (c_1, c_2), the temperature distribution equation will be determined. Using the collocation method, the temperature formulation is as follows: For example, when $Nc = 0.5$, $Sh = 0.3$, $\varepsilon g = 0.1$ and $G = 0.3$:

$$\begin{aligned} \theta(X) &= 0.82494745033772148088 + 0.16419328426849089835 X^2 \\ &+ 0.010859265393787620774 X^3 \end{aligned} \tag{21}$$

3.2. The homotopy perturbation method (HPM)

In this section, we will apply the HPM to a nonlinear ordinary differential Equation (11) with the boundary condition (12). According to the HPM, we can construct a homotopy of Equation (11) as described in the following [8]:

$$H(\theta, p) = (1-p)\left[\frac{d^2\theta}{dX^2}\right] + (1-p)\left[\frac{d^2\theta}{dX^2} - Nc^2\theta + Nc^2 G(1 + \varepsilon g\, \theta) - Sh\, \theta^2\right] \tag{22}$$

where $p \in [0,1]$ is an embedding parameter. For $p=0$ and $p=1$ we have

$$\theta(X,0)=\theta_0(X), \quad \theta(X,1)=\theta(X) \tag{23}$$

Note that when p increases from 0 to 1, $\theta(X,p)$ varies from $\theta_0(X)$ to $\theta(X)$. By substituting

$$\theta(X)=\theta_0(X)+p\,\theta_1(X)+p^3\,\theta_2(X)+p^3\,\theta_3(X)+\cdots=\sum_{i=0}^{n} p^i\,\theta_i(X) \quad g_0=0 \tag{24}$$

into Equation (8) and rearranging the result based on powers of p-terms, we have

$$\begin{aligned} p^0 \qquad & \theta_0''(X)=0 \\ & \theta_0(1)=1 \quad \theta_0'(0)=0 \end{aligned} \tag{25}$$

$$\begin{aligned} p^1 \qquad & \theta_1''(X)+Nc^2G\;\varepsilon g\;\theta_0(X)-Sh\;\theta_0(X)^2-Nc^2\,\theta_0(X)+Nc^2G=0 \\ & \theta_1(1)=0 \quad \theta_1'(0)=0 \end{aligned} \tag{26}$$

$$\begin{aligned} p^2 \qquad & \theta_2''(X)+Nc^2G\;\varepsilon g\;\theta_1(X)-2\,Sh\,\theta_0(X)\theta_1(X)-Nc^2\theta_1(X)=0 \\ & \theta_2(1)=0 \quad \theta_2'(0)=0 \end{aligned} \tag{27}$$

Solving Equations (25)-(27) with boundary conditions, we have for example:

$$\theta_0(X)=1 \tag{28}$$

$$\begin{aligned} \theta_1(X)=&\frac{1}{2}\left(-Nc^2\,G-Nc^2G\;\varepsilon g+Nc^2+Sh\right)X^2+\frac{1}{2}Nc^2G+\frac{1}{2}Nc^2G\;\varepsilon g \\ &-\frac{1}{2}Nc^2-\frac{1}{2}Sh \end{aligned} \tag{29}$$

$$\begin{aligned} \theta_2(X)=&\frac{1}{24}X^4\left(-2Sh-Nc^2+Nc^2G\,\varepsilon g\right)\left(Nc^2A+Nc^2G\,\varepsilon g-Nc^2-Sh\right) \\ &+\frac{1}{2}\Big(\frac{3}{2}Sh\,Nc^2G\,\varepsilon g+Sh\,Nc^2G-\frac{1}{2}Nc^4G^2\,\varepsilon g-\frac{1}{2}Nc^4\,G^2\,\varepsilon g^2+Nc^4\,G\,\varepsilon g \\ &-\frac{1}{2}Nc^4-\frac{3}{2}Sh\,Nc^2-Sh^2+\frac{1}{2}Nc^4\,G\Big)X^2-\frac{5}{8}Sh\,Nc^2\,G\,\varepsilon g-\frac{5}{12}Sh\,Nc^2\,G \\ &+\frac{5}{24}Nc^4\,G^2\varepsilon g+\frac{5}{24}Nc^4\,G^2\varepsilon g^2-\frac{5}{12}Nc^4\,G\,\varepsilon g+\frac{5}{8}Sh\,Nc^2-\frac{5}{24}Nc^4\,G+\frac{5}{12}Sh^2 \\ &+\frac{5}{24}Nc^4 \end{aligned} \tag{30}$$

The solution of this equation, when $p \to 1$, will be as follows:

$$\theta(X) = \sum_{i=0}^{n} \lim_{P \to 1} p^i \, \theta_i(X) \tag{31}$$

4. Results and discussion

In the present study analytical techniques called CM and HPM are applied to obtain an explicit analytic solution of the rectangular porous fin temperature dependent internal heat generation (Fig. 1). First, a comparison between the applied methods, obtained by the CM, HPM and numerical method for different values of active parameters is shown in Figures 4 to 6. The numerical solution is performed using the Maple 16.0, algebra package to solve the present case. The package uses a boundary value (*B-V*) problem procedure. The algorithm can be used to find moderate accuracy solutions for ODE boundary value problems and initial value problems, both with a global error bound. The method uses either Richardson extrapolation or deferred corrections with a base method of either the trapezoid or midpoint method. The trapezoid method is generally efficient for typical problems, but the midpoint method is able to handle the harmless of end-point singularities. The midpoint method, also known as the fourth-order Runge-Kutta-Fehlberg method, improves the Euler method by adding a midpoint in the step that increases the accuracy by one order. Thus, the midpoint method is used as a suitable numerical technique [29, 30]. In addition, the validity of the proposed methods is shown in Table 1. In this table, the % *Error* is defined as:

$$\%\,Error = \left| \theta(X)_{NUM} - \theta(X)_{Analytical} \right| \tag{32}$$

The results are proven to be precise and accurate in solving a wide range of mathematical and engineering problems, especially fluid mechanic cases.

This accuracy gives us high confidence about the validity of this problem and reveals an excellent agreement of engineering accuracy. This investigation is completed by depicting the effects of some important parameters the convection parameter, generation number, internal heat generation parameter and porous parameter evaluate how these parameters influence this temperature.

From a physical point of view, Figures 2 to 5 are prepared in order to see the effects of the *K* and *R* flow parameters on the temperature distribution. As can be seen, the effect of natural convective heat loss (*Nc*) on non-dimensional temperature is shown in Figure 2. However, these figures show that as the buoyancy effects become stronger, i.e., *Nc* increases, the local temperature in the fin decreases.

Table 1

The results of *HPM*, *CM* and numerical methods for $\theta(X)$ for Nc = 0.3, εg = 0.2, S_h = 0.1 and G = 0.4

X	*CM*	*HPM*	*NUM*	*Error* of *CM*	*Error* of *HPM*
0.00	0.934222193	0.934213444	0.934213428	8.76440E-06	1.5500E-08
0.05	0.934383116	0.934374229	0.934374223	8.89280E-06	6.6000E-09
0.10	0.934866626	0.934856727	0.934856715	9.91110E-06	1.2800E-08
0.15	0.935673835	0.935661244	0.935661229	1.26058E-05	1.5200E-08
0.20	0.936805856	0.936788323	0.936788309	1.75465E-05	1.4200E-08
0.25	0.938263800	0.93823873	0.938238716	2.50839E-05	1.4300E-08
0.30	0.940048779	0.940013444	0.940013429	3.53503E-05	1.4500E-08
0.35	0.942161907	0.942113664	0.942113650	4.82565E-05	1.3700E-08
0.40	0.944604294	0.944540815	0.944540802	6.34917E-05	1.3100E-08
0.45	0.947377052	0.947296545	0.947296532	8.05207E-05	1.2900E-08
0.50	0.950481295	0.950382725	0.950382714	9.85813E-05	1.1700E-08
0.55	0.953918134	0.953801462	0.953801451	0.000116683	1.1100E-08
0.60	0.957688681	0.957555090	0.957555079	0.000133601	1.0800E-08
0.65	0.961794048	0.961646179	0.961646169	0.000147878	9.6000E-09
0.70	0.966235347	0.966077540	0.966077531	0.000157816	8.3000E-09
0.75	0.971013691	0.970852227	0.970852218	0.000161473	8.3000E-09
0.80	0.976130192	0.975973539	0.975973531	0.000156660	7.5000E-09
0.85	0.981585961	0.981445028	0.981445023	0.000140937	4.7000E-09
0.90	0.987382110	0.987270513	0.987270505	0.000111605	8.7000E-09
0.95	0.993519753	0.993454067	0.993454050	6.57030E-05	1.7300E-08
1.00	1.000000000	1.000000000	1.000000000	0.000000000	0.00000000

On the other hand, the trend is opposite in the case of internal heat generation parameter as illustrated in Figure 3. The increase in the internal heat generation parameter indicating, the temperature profile reaches to the higher value.

Figure 4 shows the effect of porosity on temperature profiles. As seen, it is noticed that the tip temperature increases with the decrease of a porosity parameter. Hence, as the values of S_h increase, the fin cools down faster and quickly reaches the surrounding temperature.

Also, Figure 5 also allows us to see the effect of the e generation number on the temperature for the insulated tip case. As it can be seen, the local fin temperature increases as the parameters G increase.

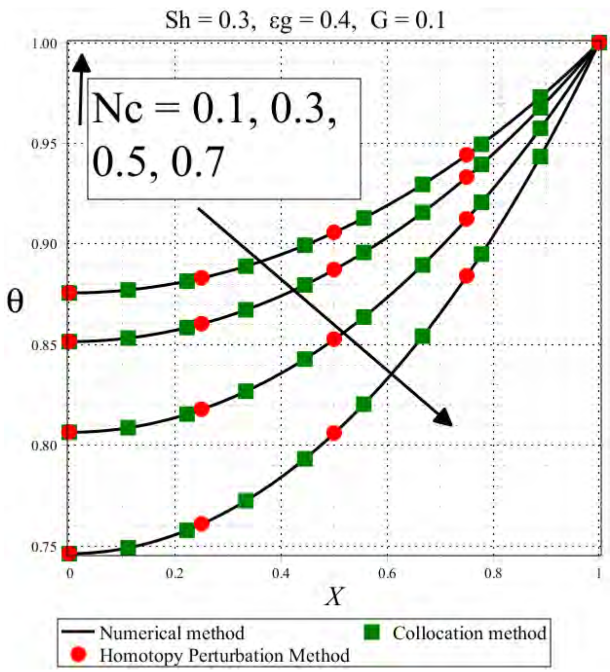

Fig. 2. Effect convective parameter and comparison of CM, HPM results with the numerical solution

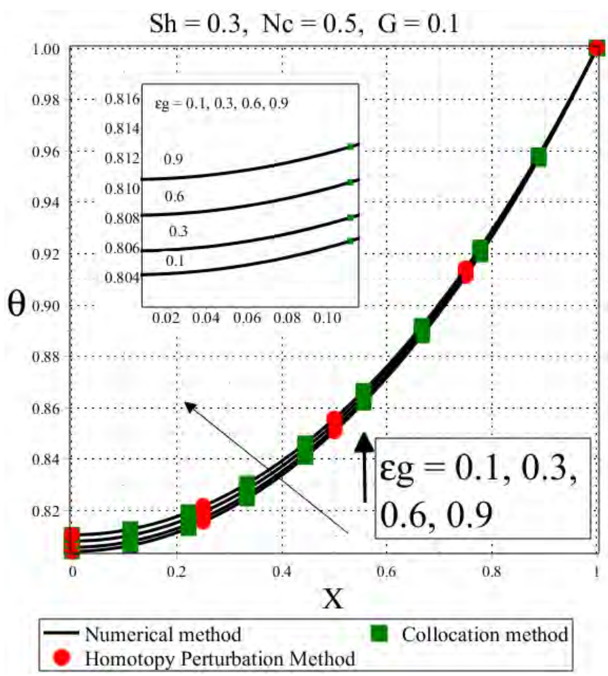

Fig. 3. Effect of internal heat generation parameter and comparison of CM, HPM results with the numerical solution

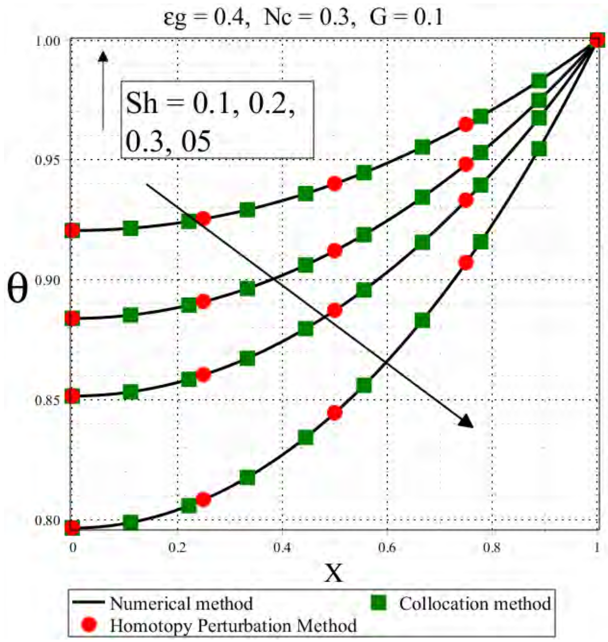

Fig. 4. Effect of porous parameter and comparison of CM, HPM results with the numerical solution

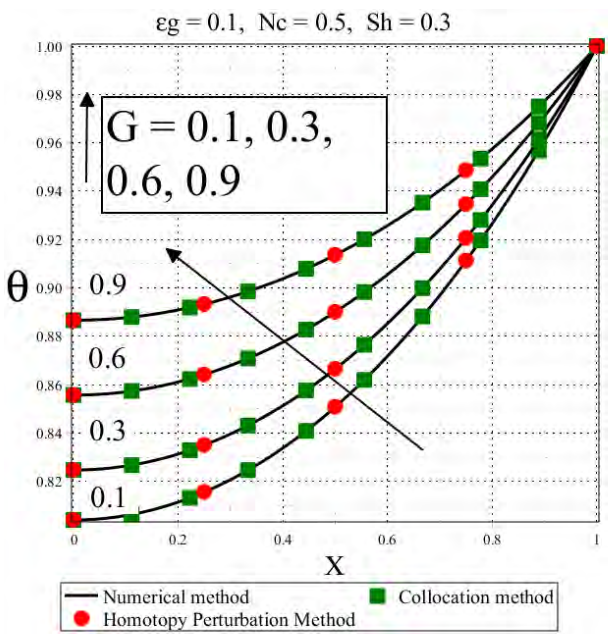

Fig. 5. Effect of generation number and comparison of CM, HPM results with the numerical solution

5. Conclusion

In this paper, an analytical analysis called the collocation method (CM) and the homotopy perturbation method (HPM) was presented to determine the temperature distribution in a porous fin with temperature-dependent of the internal heat generation. In order to derive the heat transfer equation, energy balance and the Darcy model are used. Also, the results obtained by the proposed method are validated by using a numeric scheme called the fourth order Runge-Kutta. The following important points can be concluded from the present study:
For assessment of the CM and HPM, the solution and numerical tool, Table 1 has been presented. Comparison of the analytical solution with the numerical outcomes shows that the proposed methods are a convenient and powerful method in the engineering problem.

It was also found that increasing S_h while increasing either Da or Ra increases the heat transfer from fin. In addition, as the buoyancy effects become stronger, i.e., Nc increases, the local temperature in the fin decreases.

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