

Z-NUMBERS BASED MODELING OF GROUP DECISION MAKING FOR SUPPLIER SELECTION IN MANUFACTURING SYSTEMS

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Abstract. *The health of the supply chain, the company's performance, and the quality of the production as well as the success of the entire enterprise, directly depends on the reliability of the company's existing suppliers. Processing enterprises that depend on suppliers are trying to find the best option that will satisfy all customer requirements. With high-quality and inexpensive raw materials, the products produced by the enterprise will largely determine its economic indicators such as revenue, profit, and profitability. Therefore, this enterprise is especially faced with the issue of choosing the most appropriate supplier of resources. Basically, for processing enterprises it is very important to consider the parameters such as quality of incoming materials, terms of supply of raw materials, price of received raw materials, terms of contracts. The challenge in determining of supplier is how to choose reliable suppliers that can maintain supply chain continuity in an environment of ever-increasing instability and uncertainty. For this purpose, a methodology for selecting suppliers using Z-numbers was proposed. Using fuzzy Z numbers in supplier selection, decision-makers can assign values to various criteria in a way that reflects both the uncertainty and the confidence associated with those values. This can lead to more nuanced and robust supplier selection processes, considering a wider range of factors and uncertainties.*

Keywords: supplier selection, multi-criteria decision making, fuzzy group decision making, fuzzy numbers, Z-numbers

OPARTE NA LICZBACH Z MODELOWANIE GRUPOWEGO PODEJMOWANIA DECYZJI DOTYCZĄCYCH WYBORU DOSTAWCÓW W SYSTEMACH PRODUKCYJNYCH

Streszczenie. *Zdrowie łańcucha dostaw, wydajność firmy i jakość produkcji, a także sukces całego przedsiębiorstwa, zależą bezpośrednio od niezawodności obecnych dostawców firmy. Przedsiębiorstwa przetwórcze zależne od dostawców starają się znaleźć najlepszą opcję, która spełni wszystkie wymagania klientów. Dzięki wysokiej jakości i niedrogim surowcom produkty wytwarzane przez przedsiębiorstwo będą w dużej mierze determinować jego wskaźniki ekonomiczne, takie jak przychody, zyski i rentowność. W związku z tym przedsiębiorstwo stoi przed szczególnym wyzwaniem, jakim jest wybór najbardziej odpowiedniego dostawcy zasobów. Zasadniczo dla przedsiębiorstw przetwórczych bardzo ważne jest uwzględnienie takich parametrów, jak jakość przychożących materiałów, warunki dostaw surowców, cena otrzymanych surowców, warunki umów. Wyzwaniem przy określaniu dostawcy jest wybór wiarygodnych dostawców, którzy mogą utrzymać ciągłość łańcucha dostaw w środowisku stale rosnącej niestabilności i niepewności. W tym celu zaproponowano metodologię wyboru dostawców przy użyciu liczb Z. Wykorzystując rozmyte liczby Z w wyborze dostawców, decydenci mogą przypisywać wartości do różnych kryteriów w sposób, który odzwierciedla zarówno niepewność, jak i zaufanie związane z tymi wartościami. Może to prowadzić do bardziej dopracowanych i solidnych procesów wyboru dostawców, biorąc pod uwagę szerszy zakres czynników i niepewności.*

Słowa kluczowe: wybór dostawcy, wielokryterialne podejmowanie decyzji, rozmyte grupowe podejmowanie decyzji, liczby rozmyte, liczby Z

Introduction

The development of any company and the level of its competitiveness largely depends on how well the management of production resources is organized. Recent innovations, such as mass production, customized production, and the widespread use of external sources of supply by enterprises, are forcing companies to find new flexible methods of purchasing management. In cost management, supply is undoubtedly the most important part of a company's economy, since the cost of purchasing raw materials, supplies, and components can amount to up to a third of the cost of goods. The main goal of procurement management is to meet the enterprise's needs for material resources with the highest possible efficiency. Effective sourcing decisions are fundamental to creating a sustainable supply basis for any organization. The decision to pass a special order to a supplier depends on multiple attributes. The art of good buying is to justify your decision whenever possible. Typically, the buyer's decision relies on his evaluation of the supplier's opportunities to satisfy attributes of delivery reliability, flexibility, quality, cost, delivery conditions, size, and service. Some of the very significant features of a supplier that indicate its ability to meet these criteria are the company's business, industrial power and infrastructure, financial situation, establishment and control, image, compliance with mainly received norms, rate of collaboration, labor relations and location. The ability to respond to a company's needs separates a good supplier from an average one. Determining the best supplier can be regarded as decisions made under vagueness circumstances. Supplier selection decisions are often made under conditions of uncertainty due to various factors that may impact the supply chain and business operations. Fluctuations in market conditions, such as changes in demand, currency exchange rates, raw material prices, and geopolitical events, can

create uncertainty in supplier selection decisions. These factors can affect the availability, cost, and reliability of suppliers. Uncertainties related to supply chain disruptions, such as natural disasters, political instability, transportation delays, and labor strikes, can disrupt the flow of materials and components from suppliers. This uncertainty requires businesses to select suppliers that are resilient and capable of mitigating risks. Despite thorough evaluations and assessments, there may still be uncertainty regarding the quality and performance of selected suppliers. Variability in product quality, consistency, and adherence to specifications can impact manufacturing processes and product outcomes. Economic uncertainties, market fluctuations, and financial instability can affect the financial health of suppliers. Selecting financially stable suppliers reduces the risk of disruptions due to bankruptcy, insolvency, or inability to fulfill orders. Rapid technological advancements and innovations can introduce uncertainty into supplier selection decisions. Businesses must assess suppliers' capabilities to adopt and integrate new technologies into their manufacturing processes to remain competitive. Compliance with evolving regulatory requirements and standards can introduce uncertainty into supplier selection decisions. Suppliers must demonstrate their ability to meet regulatory obligations, such as environmental regulations, safety standards, and product certifications. Given these uncertainties, businesses must employ strategies to manage and mitigate risks in supplier selection decisions. This may include conducting thorough risk assessments, diversifying supplier bases, establishing contingency plans, fostering supplier relationships, and implementing robust monitoring and performance management systems.

Group decision-making in uncertainty conditions involves making decisions when there are incomplete, ambiguous, or conflicting pieces of information. Uncertainty can arise from



various sources, such as limited data availability, unpredictable external factors, or divergent opinions among group members. In uncertain conditions, it's essential for the group to gather as much relevant information as possible. This may involve conducting research, collecting data, consulting experts, and considering various perspectives. Analyzing the available information helps the group understand the nature and extent of uncertainty surrounding the decision. The group assesses the potential risks and uncertainties associated with each decision alternative. This involves identifying potential outcomes, estimating their likelihood and impact, and evaluating the level of uncertainty associated with each option. Techniques such as risk analysis, scenario planning, and sensitivity analysis can help quantify and manage uncertainty. In uncertain conditions, diverse perspectives and expertise within the group become particularly valuable. Encouraging open discussion and soliciting input from different stakeholders can help uncover blind spots, challenge assumptions, and generate creative solutions. Group members with different backgrounds, experiences, and expertise can offer unique insights and approaches to address uncertainty. Flexibility is key when making decisions in uncertain conditions. The group may need to adapt its decision-making process based on the evolving nature of uncertainty, new information, or changing circumstances. This may involve revisiting assumptions, adjusting decision criteria, or considering additional alternatives as new information emerges. Utilizing decision-making tools and techniques specifically designed for handling uncertainty can enhance the group's ability to make informed decisions. For example, probabilistic modeling, decision trees, and fuzzy logic can help quantify and manage uncertainty, providing a structured framework for decision-making. Given the inherent unpredictability of uncertain conditions, the group should develop contingency plans to mitigate potential risks and uncertainties. These plans outline alternative courses of action to be implemented if certain events or outcomes occur. Contingency planning helps the group prepare for unforeseen circumstances and maintain agility in responding to changes. Decision-making in uncertainty often requires an iterative approach, where decisions are revisited and refined over time as new information becomes available or as the situation evolves.

Ultimately, navigating supplier selection decisions under conditions of uncertainty requires a proactive and adaptive approach to ensure the resilience and competitiveness of the supply chain. Z-number modeling in group decision making for supplier selection is a technique used to handle uncertainty and imprecision in decision-making processes, particularly in the context of selecting suppliers for manufacturing organization. This method extends the traditional decision-making models by allowing decision-makers to express their preferences and judgments in a more flexible and nuanced manner. Identification of criteria is important for supplier selection. These criteria could include factors such as price, quality, reliability, delivery time, and reputation. Instead of using crisp numerical values to represent the performance of each supplier on each criterion, decision-makers use Z-numbers. Z-numbers are an advanced tool for representing uncertainty and imprecision in decision-making processes, providing decision-makers with a more comprehensive framework for expressing preferences and making informed decisions in complex and uncertain environments. Once Z-numbers are assigned to each supplier on each criterion, they can be aggregated using fuzzy aggregation methods to determine an overall assessment of each supplier's suitability. This aggregation process considers the uncertainty and imprecision inherent in the Z-numbers to provide a more realistic representation of each supplier's performance. In many cases, supplier selection involves multiple decision-makers with different preferences and expertise. Z-number modelling allows for the integration of these diverse perspectives into

the decision-making process. Decision-makers can express their preferences using Z-numbers, and these preferences can be aggregated to reach a consensus on the best supplier to select. Z-number modeling also facilitates sensitivity analysis, allowing decision-makers to assess the robustness of their decisions to changes in the criteria weights or the underlying Z-numbers.

Z-number modeling offers a more flexible and robust approach to group decision making in supplier selection, enabling decision-makers to better cope with the inherent uncertainties and imprecisions in real-world decision environments. Ultimately, navigating supplier selection decisions under conditions of uncertainty requires a proactive and adaptive approach to ensure the resilience and competitiveness of the supply chain.

The structure of the paper is designed as follows. Section 1 is a literature review. Section 2 represents the main definitions of fuzzy numbers and Z-numbers that are used in problems. In section 3, is offered methodology with Z numbers that is applied for determining best supplier for manufacturing systems (section 4). Conclusion section of paper shows the basic outcomes in this article.

1. Literature review

Vagueness gets integrated with the inputs that are deliberated for the supplier selection process. Uncertainty in supplier selection arises from the complex interplay of various factors, and effectively managing it requires thorough research, risk assessment, and strategic decision-making to mitigate potential risks and optimize supplier relationships. Methodology fuzzy multicriteria decision making is accepted one of the best ways in supplier selection process under uncertainty conditions. Multicriteria decision-making attributes to making decisions in the existence of multiple and customarily antagonistic criteria. Fuzzy decision-making is utilized in conditions uncertain and imperfect data available for the solution. Fuzzy multicriteria decision-making is very favored methodology that is employed by the explorers in the literature. Fuzzy multicriteria decision making ensures powerful decision making when choice of alternatives is largely versatile. Fuzzy Multi-Criteria Decision Making (MCDM) is a decision-making methodology that deals with situations where there are multiple criteria or factors to consider, and these criteria are inherently uncertain, imprecise, or vague.

Fuzzy MCDM techniques are particularly useful when decision-makers need to handle ambiguity and fuzziness in the decision-making process. Singh et al. studied how to optimize the purchasing value of the product employing TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) methodology [14]. Fuzzy TOPSIS provides a systematic approach for supplier selection in uncertain and ambiguous environments, allowing decision-makers to consider both quantitative and qualitative factors while handling imprecise data effectively. Fuzzy TOPSIS facilitates the identification of trade-offs among competing criteria. By evaluating alternatives based on their proximity to the ideal and negative ideal solutions, decision-makers can assess the compromise solutions that best balance conflicting objectives. This methodology is employed for ranking supplier alternatives based on their overall performance. By considering the relative proximity of alternatives to the ideal solution, decision-makers can identify the most favorable suppliers that best meet their criteria and objectives. Utilizing multi-criteria decision-making technique fuzzy TOPSIS, organizations can effectively handle the uncertainty and subjectivity inherent in supplier selection processes, leading to more informed and rational decisions that align with their strategic goals and objectives [4]. Chan and Kumar applied fuzzy AHP (Analytic Hierarchy Process) for globally choice of supplier [2]. Applying fuzzy AHP for selecting suppliers globally involves

considering the complexities of international supply chains, diverse cultural contexts, and varying regulatory environments. Supplier selection criteria are often interrelated, and the importance of one criterion may depend on others. Fuzzy AHP captures these interdependencies by structuring the decision problem hierarchically and allowing decision-makers to assess the relative importance of criteria through pairwise comparisons. Fuzzy AHP includes mechanisms for assessing the consistency of decision-makers' judgments, ensuring the reliability of the decision process. Consistency checks help detect any inconsistencies in pairwise comparisons and prompt decision-makers to revise their judgments if necessary. Chang et al. used fuzzy DEMATEL (Decision Making Trial and Evaluation Laboratory) methodology for revealing correlation between different attributes that affect each other in the supplier selection [3]. Fuzzy DEMATEL provides a structured and systematic approach for analyzing the complex relationships between criteria in supplier selection processes. By considering the interdependencies between criteria and identifying key factors that drive decision-making, organizations can make more informed and robust supplier selection decisions. This methodology can effectively analyze large datasets and identify the underlying structure of the decision problem, making it suitable for supplier selection scenarios with diverse and interconnected criteria. Soroor et al. offered a hybrid methodology for supplier selection employing fuzzy logic, Analytic Hierarchy Process (AHP), and Quality Function Deployment (QFD) [15]. By integrating fuzzy logic, AHP, and QFD, organizations can develop a robust and comprehensive approach to supplier selection that considers both customer requirements and internal objectives while addressing uncertainty and complexity in decision-making. Wang offered a methodology employing fuzzy logic and GA (Genetic Algorithm) to evaluate suppliers for parts replacement and to vary the form of the product [17]. By combining fuzzy logic for representing subjective evaluations with genetic algorithms for optimizing supplier selection decisions, organizations can develop a sophisticated and adaptive approach that considers both qualitative assessments and quantitative objectives while addressing uncertainty and complexity in supplier evaluation and selection processes. Kahraman and Kaya utilized process capability index as a model for the supplier selection [8]. Using process capability indices as a tool for the process of supplier selection can be valuable, especially when considering the capability of potential suppliers to consistently meet quality requirements. Incorporating process capability indices into the supplier selection process, organizations can make more informed decisions about potential suppliers' ability to meet quality requirements consistently. This can help mitigate risks related to quality issues and contribute to the overall effectiveness and efficiency of the supply chain. Kar employed FAHP and fuzzy GP (Goal Programming) hybrid methodology for supplier selection. Judgments of groups experts were placed in have been put to a concerted precedence, which is integrated with fuzzy GP [9]. By integrating FAHP and fuzzy goal programming, organizations can develop a comprehensive approach for supplier selection that considers both the relative importance of criteria and the attainment of fuzzy objectives. This approach enables decision-makers to make informed decisions in complex and uncertain environments while balancing multiple conflicting objectives. Supplier selection problems are usually very complex, due to various criteria affect to decision-making process. Kontis et al. survey multi-criteria decision-making methodology based on DEA (Data Envelopment Analysis) [13]. DEA is a non-parametric method used to evaluate the relative efficiency of decision-making units based on their input-output relationships. While DEA is commonly used in performance evaluation and benchmarking, it can also be applied to supplier selection processes to identify the most

efficient suppliers. By applying DEA in supplier selection, organizations can identify and select suppliers that offer the best value in terms of efficiency and performance, leading to improved operational effectiveness and cost savings. DEA provides a quantitative and objective method for evaluating suppliers, which can enhance the transparency and credibility of the supplier selection process. Khurram surveys the different approaches for solution selecting best supplier [12]. Karsak and Dursun totalize the various attributes, the different conditions of supplier selection and the existing approaches to solution of the problem [10]. Wang applied 2-tuple fuzzy linguistic representation model for defining the general supplier performance with dynamic supply behaviors [18]. Using a 2-tuple fuzzy linguistic representation model, organizations can capture and analyze subjective assessments of supplier performance in a structured and systematic manner. This approach allows decision-makers to consider both quantitative data and qualitative judgments effectively, leading to more informed and robust supplier selection decisions. Chen and Wang procured a combined VIKOR methodology and fuzzy logic for defining the best supplier and compromise decision from different possible suppliers in information technology outsourcing project [5]. The VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) method is a multi-criteria decision-making technique used for ranking alternatives with conflicting criteria. Integrating VIKOR into a fuzzy environment for supplier selection involves considering both quantitative and qualitative criteria while accounting for uncertainty and imprecision. Fuzzy VIKOR is robust to fuzzy data and variations in decision criteria. It can handle fuzzy input data, such as fuzzy ratings or fuzzy evaluations of supplier performance, ensuring reliable decision outcomes even when dealing with incomplete or uncertain information. Fuzzy VIKOR offers flexibility and adaptability to different decision-making contexts and problem domains. It can be customized to incorporate specific criteria, preferences, and constraints relevant to supplier selection processes in various industries and organizational settings. Kavita and Kumar expanded TOPSIS methodology for interval-valued intuitionistic fuzzy numbers [11]. Extended TOPSIS for interval-valued intuitionistic fuzzy (IVIF) data is a method used in decision-making processes where criteria are represented as intervals and membership degrees are associated with the degrees of truth and falsity, as well as the degrees of uncertainty. Vinodh et al. employed fuzzy Analytic Network Process (ANP) for supplier selection and introduced research in electronics switches production industry [16]. Employing Fuzzy ANP for supplier selection, organizations can systematically evaluate potential suppliers while considering the complex interactions and dependencies between evaluation criteria. This approach enables decision-makers to make more informed and robust supplier selection decisions in uncertain and dynamic environments. Fuzzy ANP provides a comprehensive, flexible, and transparent approach to supplier selection that considers complex relationships, uncertainty, and multiple criteria simultaneously. It empowers decision-makers to make more informed and robust supplier selection decisions that align with organizational goals and stakeholder preferences. The sustainability attributes are elaborated in the estimation process. Chu and Varma proposed a hierarchical multicriteria decision-making (MCDM) model under uncertainty conditions to estimate and choose suppliers [7].

Developing an MCDM model under a fuzzy environment for supplier evaluation and selection, organizations can systematically assess potential suppliers while considering uncertainty and imprecision in decision-making. This approach enables decision-makers to make more informed and robust supplier selection decisions that align with organizational goals and stakeholder preferences.

2. Theoretical framework

Definition 1. A triangular fuzzy number [19] is determined as $\tilde{A} = (a, m, b)$. The graphical representation of triangular fuzzy numbers typically involves a triangular-shaped membership function on a graph. Triangular fuzzy numbers are characterized by three parameters: the lower bound (a), the core value (b), and the upper bound (c). The membership function assigns a degree of membership (μ) to each value within the range defined by these parameters. The graphical form of triangular fuzzy numbers is represented in figure 1.

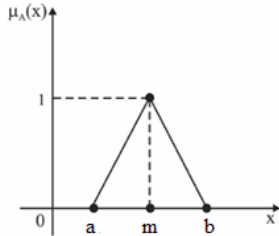


Fig. 1. Triangular fuzzy numbers

Membership function $\mu_{\tilde{a}}(x)$ of triangular fuzzy number $\tilde{A} = (a, m, b)$ is represented as equation (1).

$$\mu_A(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{m-a}, & a < x < m \\ \frac{x-m}{b-m}, & m < x < b \\ 0, & x \geq b \end{cases} \quad (1)$$

where, a, m, b are real numbers and $a < m < b$.

Definition 2. Z-number is an ordered pair of fuzzy numbers denoted as $Z = (A, B)$ [20]. The first component \tilde{A} , a restriction on the values, is a real-valued uncertain variable. The second component \tilde{B} , is a measure of reliability for the first component.

Definition 3. Converting Z-number to regular fuzzy number. Let, for $Z = (A, B)$ number $\{\tilde{A} = (x, u_{\tilde{A}}) | x \in [0, 1]\}$ and $\{\tilde{B} = (x, u_{\tilde{B}}) | x \in [0, 1]\}$, where $u_{\tilde{A}}$ and $u_{\tilde{B}}$ are triangular membership functions. For transforming Z-number into fuzzy number the second reliability part of Z-number converted into a crisp number by using equation (2) and weight of the reliability part added to the restriction part.

$$\alpha = \frac{\int x \mu_{\beta} dx}{\int \mu_{\beta} dx} \quad (2)$$

Weighted Z-number can be determined as equation (3).

$$\tilde{Z}^{\alpha} = \left\{ (X, \mu_{\tilde{A}^{\alpha}}) | \mu_{\tilde{A}^{\alpha}}(X), X \in [0, 1] \right\} \quad (3)$$

Definition 4. Defuzzification [6] is process of converting fuzzy set representations, which express uncertainty or imprecision using linguistic variables, into crisp, non-fuzzy values that can be easily understood and used in decision-making. This process is essential in fuzzy logic systems to derive actionable outcomes from fuzzy input data. When dealing with fuzzy numbers, which represent fuzzy quantities or values, defuzzification involves determining a single, representative value (crisp value) that best captures the fuzzy information conveyed by the fuzzy number. One method of defuzzification fuzzy number represented in equation 4.

$$v = \frac{a + m + b}{3} \quad (4)$$

3. Research methodology

Research methodology for supplier selection involves outlining the approach and techniques used to gather, analyze, and interpret data related to the process of selecting suppliers for a particular product. The methodology typically includes research design, identification of criteria, collection of data, analysis of data, supplier evaluation, decision-making regarding supplier selection based on the findings of the research and analysis. In this research Z-numbers based modeling is used for group decision making of supplier selection in manufacturing systems.

3.1. Research design

Assume that $A = \{A_1, A_2, \dots, A_n\}$ is a set of alternatives and $C = \{C_1, C_2, \dots, C_m\}$ is a set of criteria. Every criterion, $C_j, j = 1, \dots, m$ is defined by weight W_j determined by decision maker. As we deal with Z-information value group decision environment, the characteristic of the alternative $A_i (j = 1, \dots, n)$, on criteria $C_j (j = 1, \dots, m)$ is described by the equation (5)

$$A_i = \left\{ \left(Z(A_{i1}, B_{i1}), Z(A_{i2}, B_{i2}), \dots, Z(A_{ij}, B_{ij}), Z(A_{im}, B_{im}) \right) \right\} \quad (5)$$

where, $Z(A_{ij}, B_{ij})$ is estimation of an alternative A_i with respect to criterion C_j . The value of criteria and weights of criteria are defined by decision makers and are uncertain and characterized with partial reliability [1]. In this case, the weights $W_j, j = 1, \dots, m$ are determined using equation (6).

$$W_j = \left\{ Z(A_j^w, B_j^w) \right\}, j = 1, \dots, m \quad (6)$$

where A_j^w is value of weight of j -th criterion, B_j^w is reliability of this value. A decision matrix is a tool used to systematically evaluate and prioritize alternatives based on multiple criteria or factors. It helps decision-makers make informed choices by structuring the decision-making process and providing a transparent framework for comparing options.

Decision matrix is represented as in table 1.

Table 1. Decision matrix with Z-numbers

		C_1	C_1	...	C_1
$D_{m \times n} =$	A_1	$Z(A_{11}, B_{11})$	$Z(A_{11}, B_{11})$...	$Z(A_{11}, B_{11})$
	A_2	$Z(A_{11}, B_{11})$	$Z(A_{11}, B_{11})$...	$Z(A_{11}, B_{11})$
	\vdots	\vdots	\vdots	\vdots	\vdots
	A_n	$Z(A_{11}, B_{11})$	$Z(A_{11}, B_{11})$...	$Z(A_{11}, B_{11})$

3.2. Participants

We consider multi-attribute decision making for supplier selection in manufacturing systems selection. Let's say we have three potential suppliers (Supplier A, Supplier B, and Supplier C) and three criteria to evaluate them on reliability, quality, flexibility factors. Criteria C_1 is reliability of suppliers that consider factors such as on-time delivery performance, lead times, and capacity to fulfill orders reliably, criteria C_2 is quality of suppliers that assess the supplier's quality management systems, track record, and ability to meet quality standards consistently, and criteria C_3 is flexibility of suppliers that assess the supplier's ability to adapt to changes in demand, product specifications,

or other variables within the production process. The reliability of suppliers in manufacturing is crucial for the efficiency and quality of production processes. Main factors that influence supplier reliability are communication and transparency, financial stability, supplier relationships, etc. Establishing and maintaining reliable supplier relationships requires careful evaluation, communication, and collaboration to mitigate risks and ensure smooth operations in manufacturing. The quality of suppliers in manufacturing is paramount to the success and reputation of any production process and directly influences product quality, production efficiency, risk management, and customer satisfaction. Selecting and maintaining relationships with high-quality suppliers is essential for the success and competitiveness of manufacturing operations. The flexibility of suppliers in manufacturing refers to their ability to adapt to changes in demand, product specifications, and other variables within the production process. Supplier flexibility is essential for enabling manufacturers to respond effectively to market dynamics, customer demands, and unforeseen disruptions. Flexible suppliers contribute to supply chain agility, resilience, and innovation, ultimately enhancing competitiveness and long-term success in manufacturing.

4. Solution of problem

Supplier selection in manufacturing requires careful consideration of factors specific to the industry's demands for reliability, quality, and flexibility. Components of Z-numbers are represented by triangle fuzzy numbers. The significance weights of different criteria and the grades of qualitative determined by linguistic terms. Linguistic terms for significance weight can vary depending on the context of the decision-making process and the preferences of the decision-makers. These terms are used to express the relative significance of criteria in a qualitative manner. Linguistic terms that represented by triangular fuzzy numbers for the significance weight of each attribute are shown below.

Equally significant (E) – (1, 1, 1;1), Very less significant (VLS) – (1, 2, 3;1). Less significant (LS) – (2, 3, 4;1), Ordinary significant (OS) – (3, 4, 5;1), Significant (S) – (4, 5, 6;1), Much significant (MS) – (5, 6, 7;1), Very much significant (VMS) – (6, 7, 8;1), Absolutely significant (AS) – (7, 8, 9;1), Extremely significant (SI) – (8, 9, 10;1)

Reliability scale with its corresponding triangular Z-fuzzy numbers is represented below.

Absolutely reliable – (AR) (1, 1, 1;1), Powerful reliable (PR) – (0.7, 0.8, 0.9;1), Very much reliable (VMR) – (0.6, 0.7, 0.8;1), Much reliable (MR) – (0.5, 0.6, 0.7;1), Reliable (R) – (0.4, 0.5, 0.6;1), Less reliable (LR) – (0.3, 0.4, 0.5;1), Very less reliable (VLR) – (0.2, 0.3, 0.4;1), High unreliable (HU) – (0.1, 0.2, 0.3;1), Absolutely unreliable (AU) – (0, 0.1, 0.2;1).

It's important to ensure that the linguistic terms used for significance weight are clearly defined and understood by all decision makers. The relative significance of these terms should be consistent with the objectives and priorities of the decision-makers.

Basic steps of group decision making by using Z-numbers are represented below.

Step 1: Construction a decision matrix using linguistic terms based on the opinions of the three decision makers (table 2).

Constructing a decision matrix using linguistic terms involves assigning qualitative descriptors to criteria and options rather than numerical values.

Step 2. Constructing the aggregated decision matrix that is represented in table 2. Constructing the aggregated decision matrix involves combining the linguistic assessments from the decision matrix to determine an overall ranking of the options based on their performance across all criteria.

Table 2. The values of alternatives and criteria weights based on decision makers

	C_1	C_2	C_3
A	(4,5,6; 0.5,0.6,0.7)	(5,6,7; 0.4,0.5,0.6)	(3,4,5; 0.6,0.7,0.8)
	(5,6,7; 0.4,0.5,0.6)	(4,5,6; 0.3,0.4,0.5)	(6,7,8; 0.5,0.6,0.7)
	(3,4,5; 0.5,0.6,0.7)	(6,7,8; 0.4,0.5,0.6)	(5,6,7; 0.3,0.4,0.5)
	C_1	C_2	C_3
B	(5,6,7; 0.4,0.5,0.6)	(4,5,6; 0.5,0.6,0.7)	(3,4,5; 0.6,0.7,0.8)
	(3,4,5; 0.5,0.6,0.7)	(5,6,7; 0.4,0.5,0.6)	(6,7,8; 0.4,0.5,0.6)
	(4,5,6; 0.3,0.4,0.5)	(6,7,8; 0.3,0.4,0.5)	(5,6,7; 0.6,0.7,0.8)
	C_1	C_2	C_3
C	(3,4,5; 0.6,0.7,0.8)	(5,6,7; 0.4,0.5,0.6)	(6,7,8; 0.3,0.4,0.5)
	(4,5,6; 0.5,0.6,0.7)	(6,7,8; 0.5,0.6,0.7)	(4,5,6; 0.5,0.6,0.7)
	(5,6,7; 0.4,0.5,0.6)	(6,7,8; 0.4,0.5,0.6)	(4,5,6; 0.3,0.4,0.5)

Table 3. The aggregated decision matrix

	C_1	C_2	C_3
A	(12,15,18; 0.57,0.61,0.7)	(14, 17,20; 0.56,0.6,0.65)	(12,15,18; 0.57,0.61,0.66)
B	(12, 15, 18; 0.56,0.59,0.65)	(15,18,21; 0.54,0.57,0.61)	(15,18,21; 0.56,0.59,0.6)
C	(12, 15, 18; 0.54,0.57 0.61)	(18,21,24; 0.53,0.56,0.59)	(14,17,20; 0.53,0.56,0.6)

Step 3. Transforming fuzzy Z-numbers into fuzzy numbers involves refining the representation of uncertainty by transforming the fuzzy Z-numbers, which already express imprecision or uncertainty, into more granular fuzzy sets or fuzzy numbers. At first, we should convert reliability part of each Z-number of matrixes into crisp number by using equation 7.

$$\alpha = \frac{\int x\mu_\beta dx}{\int \mu_\beta dx} \tag{7}$$

For example, reliability part of Z-number of A / C_1 is (0.57, 0.61, 0.7). So, $\alpha = \frac{\int x\mu_\beta dx}{\int \mu_\beta dx} = 0.65$. Next, we add the weight

of the reliability to the constraint: $\tilde{Z}^\alpha = (12, 15, 18; 0.65)$. Then, we transform the weighted Z-number into regular fuzzy number.

$$\tilde{Z}' = (\sqrt{65} \times 12, \sqrt{65} \times 15, \sqrt{65} \times 18) = (9.7, 12.1, 14.5)$$

Repeating the same procedure for all Z-numbers we determine the global weights of the main criteria that are shown in table 4.

Table 4. Global weights of the main criteria

	C_1	C_2	C_3
A	(9.7,12.1,14.5)	(10.8,13.1,15.4)	(9.2,11.5,13.8)
B	(9.2,11.5,13.8)	(11.3,13.5,15.7)	(11.7,14,16.4)
C	(9,11.25,13.5)	(13.5,15.8,18)	(10.5,12.8,15)

Step 4. Determining weighted matrix. Determining a weighted matrix involves assigning weights to the criteria used in decision-making to reflect their relative importance. Weighted matrices are commonly used in decision analysis and multi-criteria decision-making methods to prioritize alternatives based on their performance across different criteria.

Assume, weights of criteria C_1 , C_2 and C_3 are given with Z numbers:

$$w_{C_1} = (4, 5, 6; 0.5, 0.6, 0.7)$$

$$w_{C_2} = (5, 6, 7; 0.4, 0.5, 0.6)$$

$$w_{C_3} = (6, 7, 8; 0.3, 0.4, 0.5)$$

After converting weights of criteria with Z-numbers into fuzzy numbers we obtain the next weights of criteria.

$$w_{C_1} = (0.29, 0.3, 0.31)$$

$$w_{C_2} = (0.33, 0.34, 0.36)$$

$$w_{C_3} = (0.34, 0.35, 0.37)$$

Decision matrix with alternatives and criteria weights represented in table 5.

Table 5. Decision matrix with alternatives and criteria weights

	C_1	C_2	C_3
	(0.29,0.3,0.31)	(0.33,0.34,0.36)	(0.34,0.35,0.37)
A	(9.7,12.1,14.5)	(10.8,13.1,15.4)	(9.2,11.5,13.8)
B	(9.2,11.5,13.8)	(11.3,13.5,15.7)	(11.7,14,16.4)
C	(9,11.25,13.5)	(13.5,15.8,18)	(10.5,12.8,15)

Creating a weighted decision matrix involves assigning weights to the criteria used in decision-making to reflect their relative importance and then evaluating alternatives based on these weighted criteria. Weighted decision matrix represented in table 6.

Table 6. Weighted decision matrix

	C_1	C_2	C_3
A	(2.81,3.63,4.9)	(3.56,4.45,5.54)	(3.13,4.03,4.2)
B	(2.67,3.45,4.8)	(3.73,4.59,5.65)	(3.98,4.9,6.07)
C	(2.61,3.38,4.9)	(4.45,5.37,6.48)	(3.57,4.48,5.5)

Step 5. Defuzzification of fuzzy numbers. Defuzzification is an essential step in fuzzy logic systems, as it converts fuzzy outputs into actionable information that can be used to control systems, make decisions, or provide recommendations. Defuzzification is the process of converting fuzzy sets or fuzzy numbers into crisp values, allowing for easier interpretation or further analysis. There are several methods of defuzzification, each with its own approach to summarizing the information contained within the fuzzy set or fuzzy number. By using centroid method defuzzification function fuzzy numbers transformed into crisp numbers that represented in table 7.

Step 6. Ranking alternatives.

Table 7. Defuzzified decision matrix

	C_1	C_2	C_3
A	3.64	4.51	3.81
B	3.47	4.62	5
C	3.39	5.43	4.53

Table 8. Ranking alternatives

	A	B	C
C_1	3.64	3.47	3.39
C_2	4.51	4.62	5.43
C_3	3.81	5	4.53
	11.96	13.09	13.35

Comparison of alternatives represent that alternative C is best alternative and $C > B > A$.

5. Discussion and conclusion

Group decision-making in supplier selection involves a collaborative process where a team or committee collectively evaluates and chooses suppliers for organization. This process is crucial because selecting the right suppliers can significantly impact the success and efficiency of operations. The group starts by defining the criteria that are important for selecting suppliers. These criteria may include factors such as price, quality, reliability, delivery time, geographical location, financial stability, sustainability practices, and reputation. Each criterion should be clear, measurable, and relevant to the organization's needs. The group researches and identifies potential suppliers who meet the criteria established in the first step. This may involve gathering information through market research, supplier databases, industry contacts, and referrals. Once potential suppliers are identified, the group evaluates their profiles against the selection criteria. This evaluation may include reviewing supplier capabilities, conducting site visits, assessing financial health, examining past performance, and checking references. After evaluating supplier profiles, the group ranks the suppliers based on their suitability and alignment with the selection criteria. This ranking helps prioritize suppliers and identify top contenders for further consideration. The group engages in discussions to review the ranked suppliers, share insights, address any concerns or discrepancies, and ultimately reach a consensus on the best suppliers to select. This discussion may involve weighing the pros and cons of each supplier, considering trade-offs, and reconciling conflicting preferences among group members. Once consensus is reached, the group may enter negotiations with the selected suppliers to finalize terms, contracts, and agreements. Negotiations may involve price negotiations, service level agreements, delivery terms, payment terms, and other contractual terms. After selecting suppliers, the group continues to monitor their performance, track key performance indicators, and evaluate their ongoing suitability. This allows the organization to adjust as needed, address any issues that arise, and continuously improve its supplier relationships. Throughout the supplier selection process, effective communication, collaboration, and transparency are essential for ensuring that diverse perspectives are considered, and decisions are made in the best interest of the organization.

The conclusion of a supplier selection process typically involves summarizing the key findings, outlining the decisions made, and providing recommendations for moving forward. In concluding a supplier selection process in uncertainty, it's crucial to acknowledge the challenges posed by uncertain conditions and emphasize the strategies employed to mitigate risks and make informed decisions. By addressing uncertainty head-on and demonstrating resilience in the face of challenges, the conclusion of the supplier selection process can instill confidence and inspire trust among stakeholders in the organization's ability to thrive in dynamic and uncertain environments. Applying Z-numbers to supplier selection involves integrating fuzzy logic with Z-number theory to handle uncertainty and imprecision in supplier evaluation and selection processes. Z-number theory deals with uncertainty in a more structured manner. In fuzzy Z-numbers, each element has two real numbers representing the lower and upper bounds of its membership degree. By applying fuzzy Z-numbers to supplier selection, organizations can make more informed decisions in the presence of uncertainty and imprecision, leading to better supplier relationships and improved overall performance.

In this article supplier selection for manufacturing systems under Z-information problem is characterized by different criteria such as, reliability, quality, and flexibility. For decision making on supplier selection is applied fuzzy aggregation technique under Z-information accepting high uncertainty as appropriate for the problem under consideration. In this article three possible

alternatives (A, B, C) and three criteria (reliability, quality, and flexibility) is used for determining best supplier for manufacturing. By applying offered Z-number modeling was identified that C is best supplier for selection.

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