

*chatter, grinding, vibration,  
contact stiffness,  
regeneration*

Rodney ENTWISTLE<sup>1</sup>  
Brian STONE<sup>2</sup>

## **FUNDAMENTAL ISSUES IN SELF-EXCITED CHATTER IN GRINDING**

The modelling of chatter in grinding is more complex than for metal cutting. This is because the number of parameters that influence the onset of chatter in grinding is daunting. Also, unlike metal cutting, the growth of chatter in grinding may take a significant time and so growth rates are also important. Initially the modelling of grinding chatter was simply an extension of that already developed for metal cutting. However this was soon found to be inadequate and the models were increased in complexity to include improved grinding force models, the contact stiffness of the wheel and regeneration of surface waves on both the work and wheel. Some solutions to chatter in grinding were also proposed. Most notably these included the use of varying speed and flexible grinding wheels. This position paper re-visits the almost universal assumption that grinding chatter is always regenerative. It is shown that a grinding force model for oscillating conditions, that has been experimentally confirmed, indicates that both torsional vibration and non-regeneration need to be considered. The consequences for current methods of chatter elimination are discussed.

### **1. INTRODUCTION**

Chatter in grinding is particularly unwanted as the surface finish immediately shows the presence of vibration. As a result the avoidance of chatter is of paramount importance and has been the subject of considerable research. An extensive survey of the research into chatter in grinding is given by Inasaki *et al.* [1] and need not be repeated in this paper. However it needs to be stated that much of the more recent research into chatter in grinding has aimed to develop more and more accurate models. These have rarely resulted in methods of stopping chatter and because of their complexity they have often proved to be a barrier to the discovery of novel solutions to chatter problems. There remain many possible areas of research into chatter in grinding that do not immediately give any expectation of providing improvements to chatter performance in grinding. As a result, with chatter avoidance in view, a review of models that have led to methods of avoiding chatter will be presented. The implicit assumptions used in these models will then be critically assessed.

---

<sup>1</sup> Curtin University, Western Australia

<sup>2</sup> The University of Western Australia

## 1.1. HISTORICAL DEVELOPMENT OF CHATTER THEORY IN GRINDING

It is informative to trace the development of the theory of chatter in grinding and even to go one stage further back and start with chatter in metal cutting as it shows how some of the early work has been forgotten or considered irrelevant. The publication most often referenced in the early days of investigating chatter was that of Arnold [2], who in 1945 devised some experiments to investigate chatter in metal cutting. Unfortunately he chose an unrealistic set up that involved turning with a tool that had an excessive overhang that would not have been used by skilled machinists, see Appendix A. This is a recurring danger for researchers who have little practical experience. It appears that the current wisdom of that time was that the variation of the cutting force with surface speed was the cause of chatter and a model of chatter with such a cutting force was developed. This is described in Appendix A as it will become significant later in this paper.

It was soon found that, in practice, chatter was usually the result of regeneration of waves left on the machined surface. Hahn [3] appears to be the first to have used the term regeneration but it was Tlustý and Spacek [4], who first developed a regenerative model of chatter. This is summarised in Appendix B. Their theory used a simple cutting force model not dependent on speed ( $F = Rb\delta$ , where  $b$  is the width of cut,  $\delta$  the depth of cut and  $R$  is called the cutting force coefficient) and predicted what was important regarding the machine response, viz., the maximum negative in phase component of the chatter receptance. Further developments by Tobias and his co-workers [5-7] resulted in the concept of stability lobes and Gurney and Tobias [8] produced an extremely useful graphical method for determining stability boundaries. At the same time the cutting force model became more complex and included penetration effects as discussed by Ito [9].

Initially the modelling of chatter in grinding used an extension of the models developed for metal cutting. Thus, for example, Inasaki [10] had either the work machining the grinding wheel or vice versa. In either case the metal cutting model of regenerative chatter in turning was applied.

The major advance in the modelling of chatter in grinding was made by Snoeys and Brown [11]. They developed a block diagram (Fig. 1) for the grinding process (and hence the characteristic equation), that included both work and wheel regenerative paths, the machine dynamic response, the contact stiffness and wave filtering - all of which are now considered to be of fundamental importance to grinding chatter. Further the stability boundary was, as with metal cutting, predicted to be determined by the location of the most negative real part of the machine response, the same as developed for metal cutting by Tlustý and Spacek [4].

To avoid confusion it is important to note that in the block diagram of Snoeys and Brown [11] the relationship between the force and the depth of wear (i.e. cut) is in terms of a cutting stiffness, for example  $k_w$ . If the simple cutting force model of Tlustý [4] is reconsidered,  $F = Rb\delta$ , then  $F = k_w\delta$  so that  $k_w = Rb$ , the cutting force coefficient times the width of cut. The results obtained using the block diagram model and extensive experimental results showed that, as had been observed experimentally, precession of the surface waves on the grinding wheel occurred. That is the circumferential position of the

waves moved slowly (0.1-0.2 rpm) around the periphery. It was also noted that as the amplitudes of the surface waves increased there would be some interference causing filtering, i.e. the amplitude of the waves would be attenuated by the contact zone. It was concluded, “...the finite length of contact is one of the most important features of the proposed model leading to a significant reduction of the upcome of instability in low workspeed grinding work.”

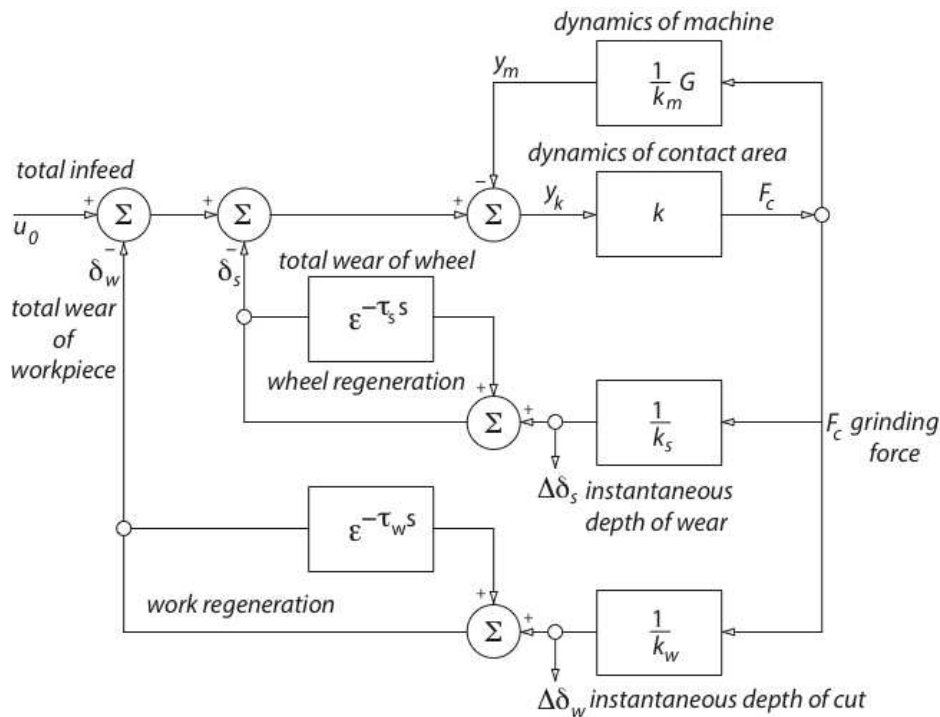


Fig. 1. Block diagram for chatter in grinding, after Snoeys and Brown [11]

From the comprehensive nature of the model and the detailed tabulation of measurements of the parameters involved, using their model Snoeys and Brown were able to show that most grinding took place under unstable conditions and hence the growth rate became important, “... a great deal of grinding work is performed under unstable work conditions because most grinding wheel widths are larger than the 5mm critical wheel width.” A detailed study of the growth rate was added by solving for the complex roots of the characteristic equation via a computer algorithm. One addition to their approach was the addition of ‘an overlap factor’ in the model, that provided for the partial work regeneration, that occurs when roll grinding.

Thus Entwistle [12] concluded in his doctoral thesis that, “It is considered that by about the year 1970 the fundamental parameters causing and influencing workpiece and grinding wheel regenerative chatter had been identified and adequate analysis techniques had been devised to model the essential dynamics of the system. Over the following decades many further developments were achieved but none discredited the fundamental understandings that had been achieved.” This is confirmed if current publications on

grinding chatter are examined, particularly the review paper of Inasaki [1]. However Entwistle was about to propose that there was more to chatter in grinding than had previously been suggested. He was connected to a research group, at the University of Western Australia, that was involved in extensive research into torsional vibration in rotating systems. For his doctoral work Entwistle investigated the possibility of torsional vibration in grinding chatter as the grinding force would inevitably produce a torque on both work and grinding wheel. It appeared that all previous work had assumed that both the work and wheel speeds were constant under chatter conditions. The consequence of his work was that measurements of grinding forces under oscillating conditions were attempted by Drew *et al.* [13], Qureshi *et al.* [14],[15] and Qureshi [16] in order to validate the grinding force model used by Entwistle. It is appropriate to consider this grinding force model used for modelling grinding chatter and the experimental basis for it.

## 2. FORCES IN GRINDING UNDER OSCILLATING CONDITIONS

There appears to be little published work on the experimental investigation of oscillating forces in grinding and their relationship to varying chip thickness, work speed or wheel speed. The extensive survey paper of Inasaki *et al.* [1] has no reference to any such measurements. Typically a force equation is assumed, often based on one proposed or confirmed for steady state grinding, i.e. no vibration. Thus Entwistle [12],[17] started from the equation used for predicting the grinding force under non-vibratory conditions presented by Chiu and Malkin [18]. With the ploughing and sliding forces omitted the tangential and normal components are,

$$P_t = \frac{u_{ch} V_w \delta_w b}{V_g} \text{ and } P_n = k_1 P_t \quad (1)$$

where:

- $P_t$  is the tangential component of the cutting force.
- $P_n$  is the normal/radial component of force.
- $u_{ch}$  is the specific chip formation energy, energy per unit volume.
- $V_w$  is the surface speed of the work.
- $\delta_w$  is the un-deformed chip thickness of the work.
- $b$  is the width of cut, i.e. the width of the contact between wheel and work.
- $V_g$  is the wheel surface speed.
- $k_1$  is a constant.

To be used in a model of chatter it needs to be assumed that the oscillating force would act in the same direction as the mean force, i.e.  $k_1$  applies to both the oscillating as well as the steady case and hence  $P_n = k_1 P_t$ . Also if the equation applies to the time varying situation it may be differentiated to give,

$$\frac{dP_t}{dt} = u_{ch} b \left( \frac{V_w}{V_g} \frac{d\delta_w}{dt} + \frac{\delta_w}{V_g} \frac{dV_w}{dt} - \frac{\delta_w V_w}{V_g^2} \frac{dV_g}{dt} \right) \quad (2)$$

It should be noted that, although equation (1) omitted the ploughing and sliding forces, equation (2) will not be affected if the omitted forces are constant under oscillating conditions. It appears that the only attempts at confirming if equation (2) is valid in practice were made by a collaboration between the National University of Singapore and the University of Western Australia. As a result of developing a torsional vibrator [19], Drew and Stone were able to collaborate with Ong and Mannan [13] to experimentally determine  $u_{ch}$  for work speed variation alone, i.e. for the  $dV_w$  term, by superposing an oscillation on the work rotational speed. It was found that for the conditions tested  $u_{ch}$  was  $16.3 \text{ J/mm}^3$ . It is of interest to note that the value obtained when there was no oscillation present was  $39.1 \text{ J/mm}^3$ . The same authors also investigated oscillating the chip thickness on the work by using an out of balance mass on the grinding wheel. Some preliminary results were published [14] and the full set of results may be found in the Master's Thesis of Qureshi [16]. The average value of  $u_{ch}$  for chip thickness variation was found to be  $17 \text{ J/mm}^3$ . This compares with the average value of  $16.3 \text{ J/mm}^3$  found when the work speed was varied [13]. No attempt was made to investigate the effect of the variation of oscillating forces with oscillating wheel speed. Thus the complete force model has yet to be experimentally verified but it has been established that, under oscillating conditions, the force varies with both chip thickness and work rotational speed. The latter suggests that torsional vibration of the work may result in or modify chatter in grinding. Entwistle [12] investigated the effects of torsional vibration of both work and wheel and a summary of his findings follows.

### 3. CHATTER IN GRINDING INCLUDING TORSIONAL EFFECTS

Using the force model described above (equation (2)) Entwistle [12] developed the following equations of motion involving a single structural mode and torsional modes for both the work and wheel. The latter two were not considered to be coupled. Displacements in the directions of the three degrees of freedom were assigned the symbols  $u$ ,  $\theta_w$  and  $\theta_g$  (refer to Fig. 2 which defines the coordinate system). The time varying parameters in the model are:

- $u(t)$  is the displacement from the mean position of the centre of the work in the mode direction.
- $x_2(t)$  is the displacement of the grinding wheel surface from the position without vibration.
- $x_3(t)$  is the displacement of the work surface from the position without vibration.
- $\theta_w(t)$  is the angular displacement of the work relative to the position if the work rotational speed were constant.
- $\theta_g(t)$  is the angular displacement of the grinding wheel relative to the position if the work rotational speed were constant.
- $P_n(t)$  is the contact force normal to the work surface.

- $P_t(t)$  is the contact force tangential to the work surface.

$x_2, x_3$  and the component of  $u$  in the direction of  $x_3$  are related by the geometric compatibility constraint:

$$u \cos \phi + x_3 + x_2 = 0 \quad (3)$$

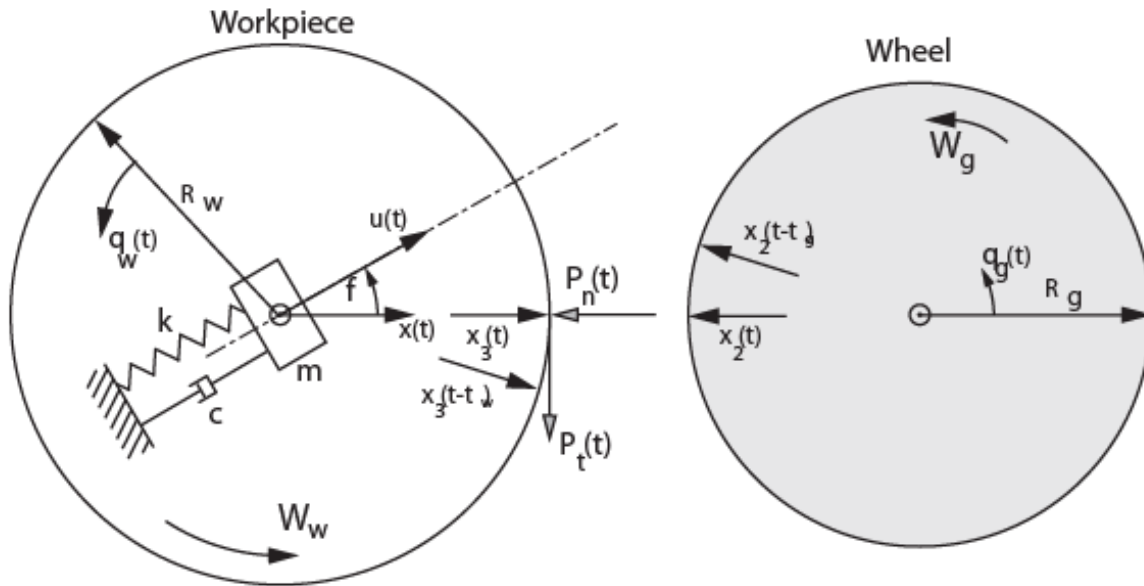


Fig. 2. Notation used for forces and displacements in grinding [12]

It should be noted that it was assumed that the contact between the work and wheel could be considered as a line, i.e. the effects of a contact zone were not considered. Also to keep the number of variables within limits the contact stiffness between the work and wheel was not included. The significance of these assumptions will be considered later.

The ratio of the volume of material worn from the work and the grinding wheel during a grinding operation is defined as the grinding ratio and was denoted by  $G$ . This parameter is constant under only limited conditions but was assumed constant and applicable to oscillating conditions.

In cylindrical grinding operations the work is traversed along the grinding wheel with an in-feed being applied during the direction reversal at the end of each stroke. When the traverse during one revolution of the work equals the width of the grinding wheel, the cutting zone does not overlap that of one revolution earlier. At smaller rates of traverse, some overlap will occur. In the limiting case, called plunge grinding, no traverse is used. This model will permit overlap factors ( $\mu$ ) between zero, representing traverse or roll grinding without any overlap, and one, which represents plunge grinding. The factor  $\mu$  can also be used as an approximate model of wave attenuation caused by the contact zone.

If the rotational periods of the grinding wheel and work are denoted by  $\tau_g$  and  $\tau_w$  respectively, a relationship can be found between the grinding ratio, overlap factor, work surface profile and the grinding wheel surface profile.

$$G = \frac{\bar{V}_w [\mu x_3(t - \tau_w) - x_3(t)]}{\bar{V}_g [x_2(t - \tau_g) - x_2(t)]} \quad (4)$$

The over-bars represent the mean or bulk values,  $\tau$  are rotational periods and the instantaneous surface speeds and work depth of cut are:

$$V_w = \bar{V}_w + \dot{u} \sin \phi + \dot{\theta}_w R_w, \quad V_g = \bar{V}_g + \dot{\theta}_g R_g, \quad \delta_w = \bar{\delta}_w + \mu x_3(t - \tau_w) - x_3(t) \quad (5,6,7)$$

where the over-dot represents differentiation with respect to time. It is important to note that the  $\dot{u} \sin \phi$  is only included in equation (5) and not (6). This is the velocity arising from the oscillation of the structural mode. The effect on the work is to increase the metal removed and hence is significant. The effect on the wheel speed was considered to not be significant. The tangential component of the contact force is  $P_t(t)$ , so that the component of the contact force in the 'u' direction is:

$$P_u = -P_t(k_1 \cos \phi + \sin \phi) \quad (8)$$

The negative sign recognises that the component of the contact force acting *on* the work will act in the opposite direction to the positive 'u' coordinate.

The well-known second order differential equation governing a spring-mass-viscous damper system is applied to each of the three single degree of freedom sub-systems without further elaboration. In the case of the structural mode in the 'u' direction, the equation is:

$$\ddot{u} + 2\zeta \omega_N \dot{u} + \omega_N^2 u = \frac{P_u}{m} \quad (9)$$

where  $\omega_N = \sqrt{k/m}$  is the undamped natural frequency and  $\zeta = c/(2\sqrt{mk})$  is the damping ratio.  $k$ ,  $c$ , and  $m$  are the stiffness, damping coefficient and modal mass as shown in Fig. 2. It proved convenient to define the natural frequencies of the two torsional systems as a proportion of the structural undamped natural frequency, viz.,  $\omega_{Nw} = F_w \omega_N$  and  $\omega_{Ng} = F_g \omega_N$ , and the damping ratios of the work and wheel torsional degrees of freedom as  $\zeta_w$  and  $\zeta_g$  respectively. The differential equations governing the oscillation of these two torsional systems can then be written as:

$$\ddot{\theta}_w + 2\zeta_w F_w \omega_N \dot{\theta}_w + F_w^2 \omega_N^2 \theta_w = \frac{-P_t R_w}{I_w} \quad (10)$$

$$\ddot{\theta}_g + 2\zeta_g F_g \omega_N \dot{\theta}_g + F_g^2 \omega_N^2 \theta_g = \frac{-P_t R_g}{I_g} \quad (11)$$

where  $I_w$  and  $I_g$  are the torsional system moments of inertia,  $R_w$  and  $R_g$  are the radii of the work and wheel and  $P_t$  is the tangential oscillating component of the force as described in equation (2). The negative signs on the applied torque recognise that they act in the opposite direction to the positive  $\theta$  coordinate. Equations (2) and (3-11) formed the mathematical model used by Entwistle [12]. His thesis contains theoretical investigations of the main parameters and where appropriate his results were found to agree with those presented in earlier work. However, of greater interest is to highlight what are regarded as novel findings. The solution to the equations was assumed to comprise an oscillation with an exponential growth or decay. Thus for example it was assumed that  $u(t) = Ue^{(\sigma+i\omega)t}$  and so a positive value of  $\sigma$  would indicate chatter was present as the amplitude of vibration increases with time. The boundary of stability is also found when  $\sigma = 0$ .

A set of the solutions to the equations, that highlight some interesting features, is shown in Fig. 3. These results include the effects of the transverse mode and the torsional mode involving the grinding wheel. The results were non-dimensionalised to be more widely applicable. Thus,  $W = \omega/\omega_N$ ,  $T_w = \omega_N\tau_w$  and  $B = (bu_{ch}R_w\omega_N)/(k\bar{\Omega}_gR_g)$  where  $\bar{\Omega}_g$  is the mean grinding wheel rotational speed. Fig. 3 illustrates the complexity of the results and needs some explanation. The colour scheme allows the frequency of vibration to be included, dots and circles indicate (as best can be determined within an algorithm) work piece and wheel regeneration respectively. As it was necessary to use log scales, negative growth, i.e. decay is shown as a separate plot. It is important to note that there are several curves involving regeneration, each involving different numbers of waves on the work and/or wheel. These curves have different widths at which  $\sigma$  is zero, i.e. at the boundary of stability. As in earlier modelling [20] that used a simple force model, the growth rates for regenerative chatter start negative, become positive, have a peak value and then reduce with increasing width of cut. It is conventional to present the results as a stability chart showing the width of cut (i.e. the width of the wheel) at the stability boundary against rotational speed. Fig. 4 shows the stability chart for the same conditions as for Fig. 3 with width against work speed, again using non-dimensional parameters.

Fig. 4 shows stability boundaries (these sampled curves are actually continuous) for any mode of self-excited vibration predicted by the model with frequencies,  $W$ , below twice the structural natural frequency. Since it is generally accepted that regenerative chatter occurs at frequencies  $W > 1$ , a sharp colour change has been used in the coloured frequency legend at  $W = 1$ . In this example, all unstable modes occur with  $W > 1$ . The curved 'loops' are work piece regeneration modes while the straight lines of negative slope are wheel regeneration modes. Clearly they interact under certain parameter combinations. The governing (least stable) mode is discussed below.

For all of the regenerative solutions resulting from the transverse mode, it was found that the response characteristic that determined the stability boundary was the most negative real part of the machine response. However, when torsional effects were included it was also predicted that torsional vibration of the work could result in improved stability. Fig.5 shows a typical stability chart with both the transverse mode and the work torsional mode active. It is evident that at low work speed the effects of the torsional mode are to reduce and even prevent chatter.



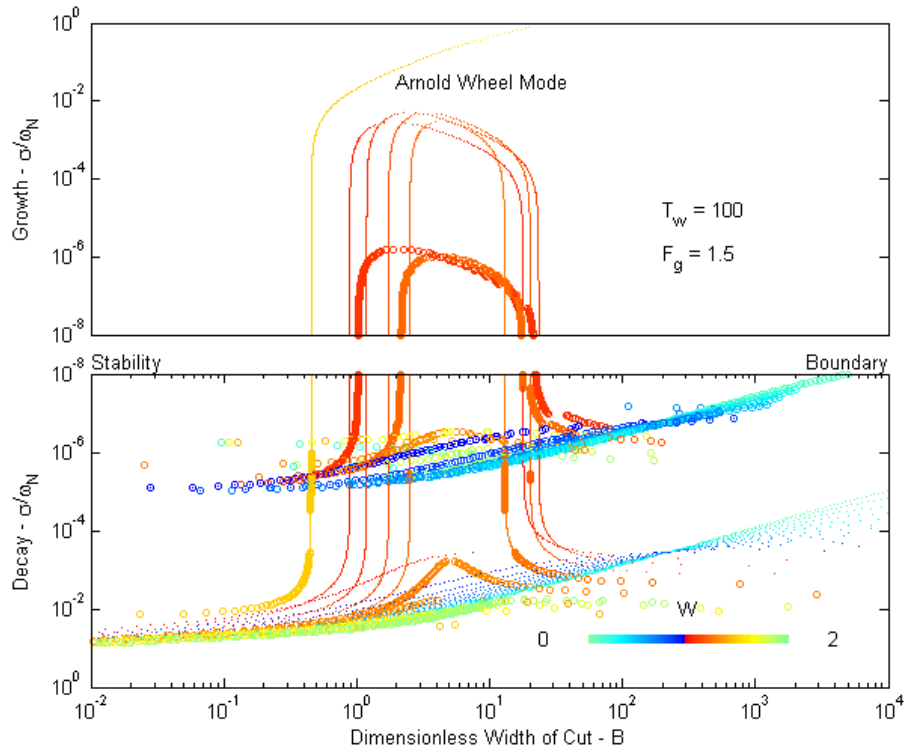


Fig. 3. Growth rates against non-dimensional width of cut, after Entwistle [12]

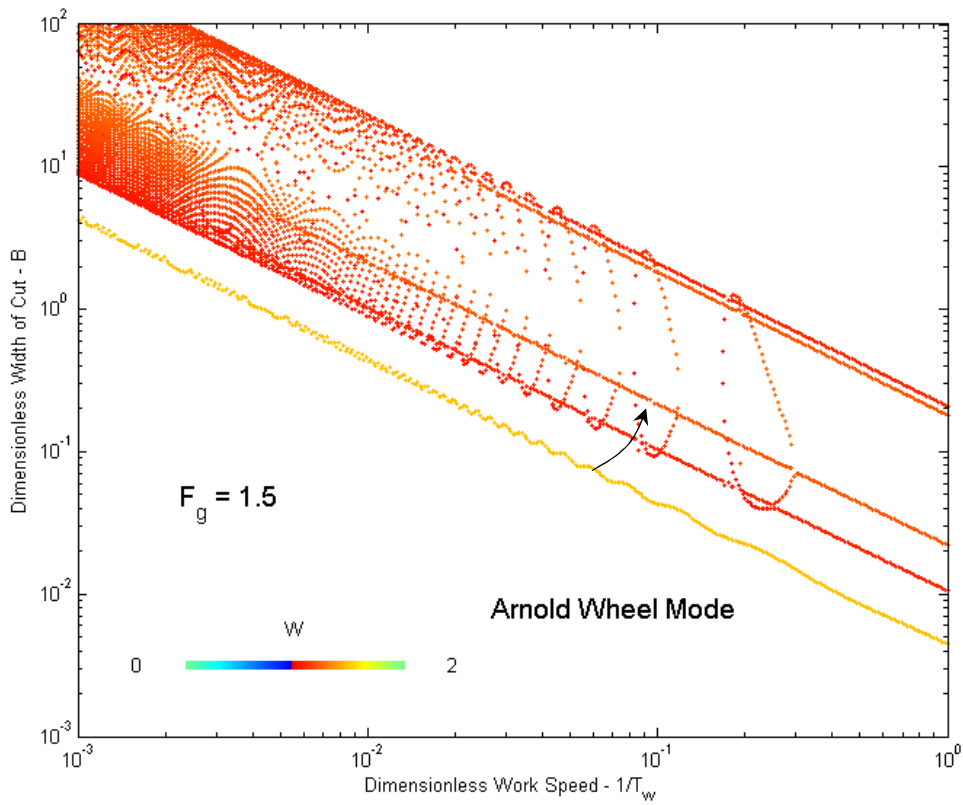


Fig. 4. Non-dimensional width at stability boundary against non-dimensional work speed, after Entwistle [12]

The results that have been described are all theoretical. As it appeared that no researcher had measured torsional vibration during chatter, Entwistle [12] devised an experiment aimed at determining if the torsional characteristics of the work system could affect chatter. In his experiments he ensured that as far as possible the only parameters that were changed involved torsion of the work. His tests involved plunge grinding and measuring the transverse and torsional spectra after grinding for 1000 secs with a feed rate of approximately  $7.5\mu\text{m}/\text{rev}$ . The grinding wheel speed was 3870 rev/min. Some of his results are shown in Fig. 6 where the size of the symbols indicates the amplitude of vibration. It is clear that torsion of the work affected chatter and needs to be considered further. However there is another prediction shown in Figs. 3 and 4 that may have even greater practical significance.

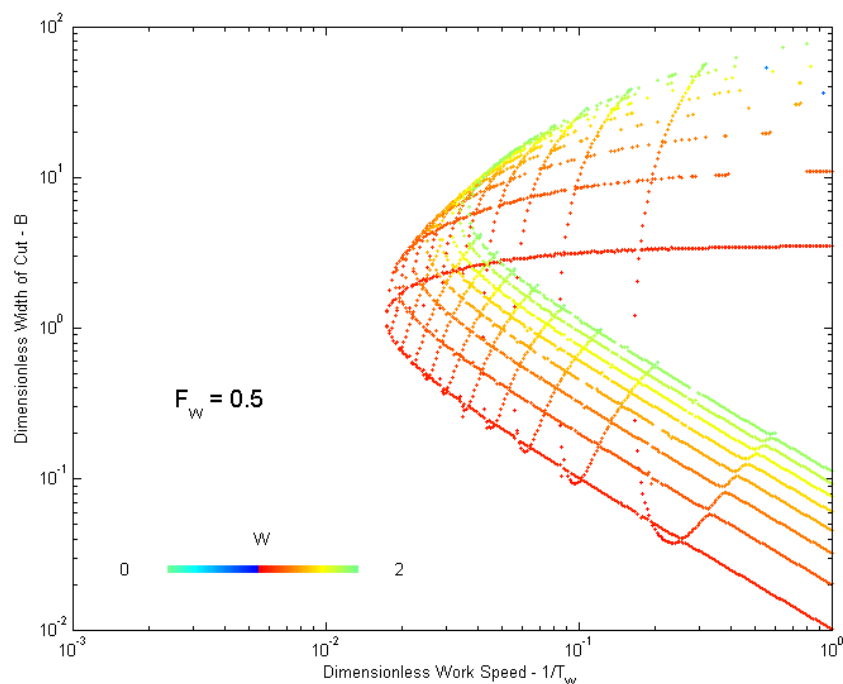


Fig. 5. Non-dimensional width at stability boundary against non-dimensional work speed, after Entwistle [12]

Figs. 3 and 4 show a solution that does not appear to have been considered previously. This is labelled 'Arnold Wheel Mode' in both graphs. As will be shown later this is non-regenerative (hence labelled Arnold) and may occur at widths smaller than those for regenerative chatter. Further, and of great concern, is the prediction that the growth rate has no peak but increases with increasing width. That Arnold [2] should appear again after so many years is of interest. It appears that once regenerative chatter in metal cutting was discovered and modelled it proved to be the form of chatter that occurred most frequently. As a result Arnold chatter was no longer considered to be significant and was largely forgotten. Then as grinding was investigated, chatter was only deemed to be important when surface waves were found on the work and/or the wheel and these were the result of regeneration. Thus regenerative chatter was the model developed for grinding chatter.

However Entwistle's modelling showed that it was possible, in theory, to have non-regenerative chatter in grinding of the same form as Arnold-type chatter in metal turning. The likelihood of this being possible and significant in practice needs to be considered.

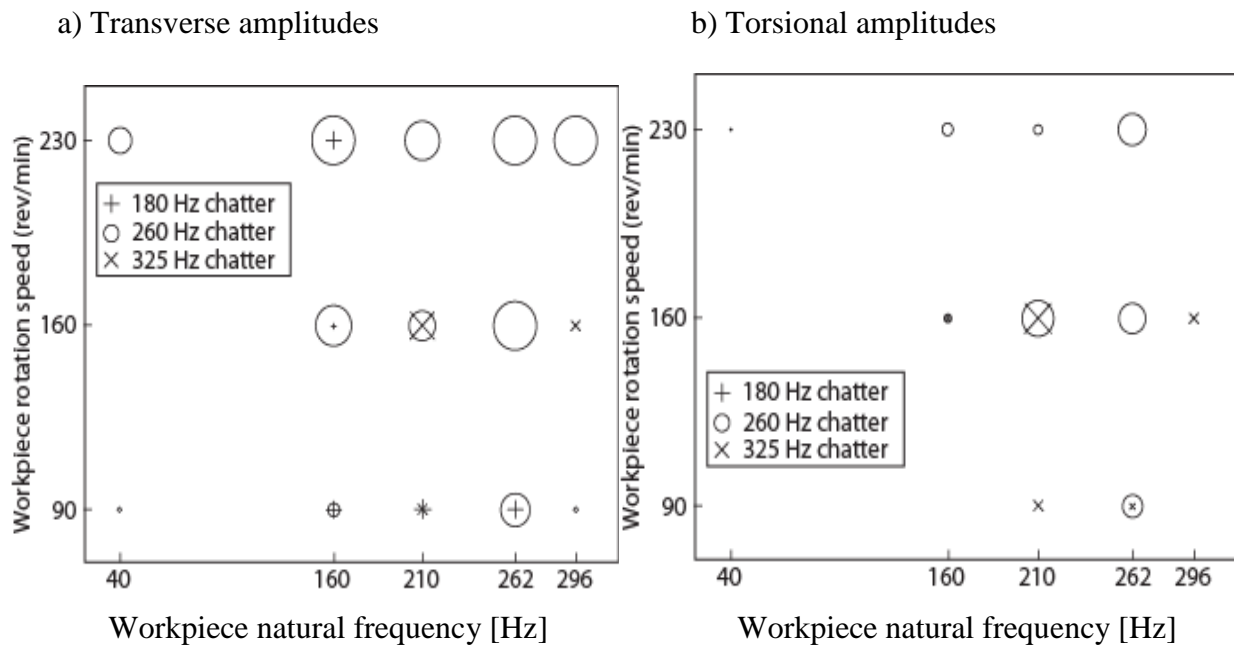


Fig. 6. Experimental results for various torsional natural frequencies, after Entwistle [12]

#### 4. NON-REGENERATIVE CHATTER IN GRINDING

The possibility of non-regenerative chatter in grinding has been raised by the results described above. The cause of such chatter would be the variation of the grinding force with speed. This could be the rotational speed of the wheel or of the work. There is also the possibility that as the transverse vibration of an inclined structural mode may affect the speed that non-regenerative chatter may arise from this cause. Each of the possibilities needs to be considered. Simple models are presented that highlight possible effects. These models involve major assumptions that will be considered later.

##### 4.1. GRINDING WHEEL TORSION

A simple model of Arnold type, non-regenerative chatter in metal cutting is given in Appendix A. This type of chatter occurs when the cutting force varies inversely with surface speed, i.e. there is a negative slope on the force/surface speed graph. It follows from the force model for grinding, that if a graph of force against wheel surface speed is drawn, with all other variables constant, it will also have a negative slope since from equation (1).

$$P_t = \frac{u_{ch} \bar{V}_w \bar{\delta}_w b}{\bar{V}_g} \quad \text{and hence} \quad \frac{dP_t}{dV_g} = -\frac{u_{ch} \bar{V}_w \bar{\delta}_w b}{\bar{V}_g^2} \quad (12)$$

It is thus possible that non-regenerative chatter may arise where a wheel is not torsionally rigid. For simplicity assume that only variation of wheel speed occurs and the other parameters remain constant. For small amplitude oscillations, i.e. at the onset of chatter, we may linearize the model as Arnold did. We have defined  $\theta_g$  as the angular displacement of the grinding wheel relative to its position if the work rotational speed was constant. The oscillating component of the angular velocity of the wheel is therefore  $\dot{\theta}_g$  and the oscillating component of the surface speed is  $\dot{\theta}_g R_g$ . The oscillating force is then from (12) given by,

$$P_t = -\frac{u_{ch} \bar{V}_w \bar{\delta}_w b}{\bar{V}_g^2} \dot{\theta}_g R_g$$

and substituting in equation (11)  $\ddot{\theta}_g + 2\zeta_g F_g \omega_N \dot{\theta}_g + F_g^2 \omega_N^2 \theta_g = \frac{R_g}{I_g} \frac{u_{ch} \bar{V}_w \bar{\delta}_w b}{\bar{V}_g^2} \dot{\theta}_g R_g$

and rearranging 
$$\ddot{\theta}_g + \left( 2\zeta_g F_g \omega_N - \frac{u_{ch} \bar{V}_w \bar{\delta}_w b R_g^2}{\bar{V}_g^2 I_g} \right) \dot{\theta}_g + F_g^2 \omega_N^2 \theta_g = 0 \quad (13)$$

As with metal cutting, grinding will become unstable when the effective damping becomes negative. The limiting value is when

$$\text{so } 2\zeta_g F_g \omega_N - \frac{u_{ch} \bar{V}_w \bar{\delta}_w b_{lim} R_g^2}{\bar{V}_g^2 I_g} = 0 \quad \text{that } b_{lim} = \frac{2\zeta_g F_g \omega_N \bar{V}_g^2 I_g}{u_{ch} \bar{V}_w \bar{\delta}_w R_g^2}$$

This equation may be simplified by substituting from the previous definitions, so that

$$b_{lim} = \frac{2\zeta_g F_g \omega_N \bar{\Omega}_g^2 I_g}{u_{ch} \bar{V}_w \bar{\delta}_w} = \frac{2c_g \omega_{Ng} \bar{\Omega}_g^2 I_g}{2\sqrt{I_g k_g} u_{ch} \bar{V}_w \bar{\delta}_w} = \frac{c_g \sqrt{k_g / I_g} \bar{\Omega}_g^2 I_g}{\sqrt{I_g k_g} u_{ch} \bar{V}_w \bar{\delta}_w} = \frac{c_g \bar{\Omega}_g^2}{u_{ch} \bar{V}_w \bar{\delta}_w} \quad (14)$$

This result has not been found in any previous literature and the implications for chatter in grinding are very significant. It is known that the levels of damping in systems undergoing torsional vibration may be very small so that the limiting width may also be small. At the boundary of stability the frequency of vibration, from equation (13), will be  $F_g \omega_N = \omega_{Ng}$  i.e. the natural frequency of the torsional system. This will often be high and there are many reports of unexplained high frequency chatter in grinding.

## 4.2. WORK TORSION

This is unlikely in practice, but if the work rotation is reversed so that it is opposite to that of the wheel then the equation of motion for work torsion (10), when the surface speed is varying, noting the change of sign on the right hand side, becomes

$$\ddot{\theta}_w + 2\zeta_w F_w \omega_N \dot{\theta}_w + F_w^2 \omega_N^2 \theta_w = \frac{P_t R_w}{I_w} = \frac{u_{ch} V_w R_w \bar{\delta}_w b}{\bar{V}_g I_w}$$

and the surface speed is given  $V_w = \bar{V}_w + \dot{\theta}_w R_w$  so that

$$\ddot{\theta}_w + 2\zeta_w F_w \omega_N \dot{\theta}_w + F_w^2 \omega_N^2 \theta_w = \frac{u_{ch} R_w \bar{\delta}_w b}{\bar{V}_g I_w} (\bar{V}_w + \dot{\theta}_w R_w)$$

and rearranging  $\ddot{\theta}_w + \dot{\theta}_w \left( 2\zeta_w F_w \omega_N - \frac{u_{ch} R_w \bar{\delta}_w b}{\bar{V}_g I_w} R_w \right) + F_w^2 \omega_N^2 \theta_w = \frac{u_{ch} \bar{V}_w R_w \bar{\delta}_w b}{\bar{V}_g I_w}$

The right hand side controls the mean values. The left hand side can indicate negative damping and instability will occur when

$$2\zeta_w F_w \omega_N - \frac{u_{ch} R_w \bar{\delta}_w b_{lim}}{\bar{V}_g I_w} R_w = 0$$

and rearranging and substituting from the previous definitions

$$b_{lim} = \frac{2\zeta_w F_w \omega_N \bar{V}_g I_w}{u_{ch} R_w^2 \bar{\delta}_w} = \frac{2c_w \omega_{Nw} \bar{V}_g I_w}{2\sqrt{I_w k_w} u_{ch} R_w^2 \bar{\delta}_w} = \frac{c_w \sqrt{k_w / I_w} \bar{V}_g I_w}{\sqrt{I_w k_w} u_{ch} R_w^2 \bar{\delta}_w} = \frac{c_w \bar{V}_g}{u_{ch} R_w^2 \bar{\delta}_w} \quad (15)$$

As with the wheel torsion case the frequency will be at the torsional natural frequency. The level of damping may be very small as this is often the case for torsion and so non-regenerative chatter may occur for small widths. However, as noted above it is not good practice to have work and wheel rotating in opposite directions.

## 4.3. TRANSVERSE VIBRATION

As noted previously it is possible to have a change in oscillating force caused by the velocity component of the structural vibration in the tangential direction. For this case the equation of motion is

$$m \frac{d^2 u(t)}{dt^2} + c \frac{du(t)}{dt} + ku(t) = -P_n \cos \phi - P_t \sin \phi \quad \text{and} \quad P_t = \frac{u_{ch} V_w \bar{\delta}_w b}{V_g} \quad \text{where} \quad P_n = k_t P_t$$

$$m \frac{d^2 u(t)}{dt^2} + c \frac{du(t)}{dt} + ku(t) = -\frac{u_{ch} V_w \delta_w b}{V_g} (k_1 \cos \phi + \sin \phi) = -\frac{u_{ch} V_w \delta_w b}{V_g} \varepsilon \quad (16)$$

where  $\varepsilon = k_1 \cos \phi + \sin \phi$ . Assuming exponential growth with oscillation,  $u(t) = Ue^{(\sigma+i\omega)t}$  so that

$$\begin{aligned} \frac{du(t)}{dt} &= \dot{u} = (\sigma + i\omega)Ue^{(\sigma+i\omega)t} & V_w(t) &= \bar{V}_w + \dot{u} \sin \phi \\ \delta_w(t) &= \bar{\delta}_w + \delta_w e^{(\sigma+i\omega)t} - \mu \delta_w e^{(\sigma+i\omega)(t-\tau_w)} & V_g(t) &= \bar{V}_g \end{aligned}$$

Now if only vibration about the mean values is considered

$$d\left(\frac{V_w \delta_w}{V_g}\right) = \frac{\bar{V}_w}{V_g} d\delta_w + \frac{\bar{\delta}_w}{V_g} dV_w \quad (17)$$

and the varying components of the relevant parameters are  $dV_w = (\sigma + i\omega)Ue^{(\sigma+i\omega)t} \sin \phi$  and for regeneration only on the work  $d\delta_w = \delta_w e^{(\sigma+i\omega)t} - \mu \delta_w e^{i(\sigma+i\omega)(t-\tau_w)}$  and  $\delta_w = U \cos \phi$ . So that substituting in (17)

$$d\left(\frac{V_w \delta_w}{V_g}\right) = \frac{\bar{V}_w}{V_g} (Ue^{(\sigma+i\omega)t} - \mu Ue^{i(\sigma+i\omega)(t-\tau_w)}) \cos \phi + \frac{\bar{\delta}_w}{V_g} (\sigma + i\omega)Ue^{(\sigma+i\omega)t} \sin \phi$$

Finally substituting all the above in equation (16)

$$\begin{aligned} m \frac{d^2 (Ue^{i(\sigma+i\omega)t})}{dt^2} + c \frac{d(Ue^{(\sigma+i\omega)t})}{dt} + k(Ue^{(\sigma+i\omega)t}) = \\ -u_{ch} b \varepsilon \left( \frac{\bar{V}_w}{V_g} U (e^{(\sigma+i\omega)t} - \mu e^{(\sigma+i\omega)(t-\tau_w)}) \cos \phi + \frac{\bar{\delta}_w}{V_g} (\sigma + i\omega) U e^{(\sigma+i\omega)t} \sin \phi \right) \end{aligned}$$

and after some manipulation and simplification

$$\begin{aligned} m(\sigma + i\omega)^2 + (\sigma + i\omega)c + k = \\ -\frac{u_{ch} b \varepsilon}{V_g} (R_w \Omega_w \cos \phi (1 - \mu e^{-\sigma \tau_w} \cos \omega \tau_w + i \mu e^{-\sigma \tau_w} \sin \omega \tau_w) + \bar{\delta}_w (\sigma + i\omega) \sin \phi) \\ \left( \frac{\sigma^2}{\omega_n^2} + i2 \frac{\omega}{\omega_n} \frac{\sigma}{\omega_n} - \frac{\omega^2}{\omega_n^2} \right) + \left( \frac{\sigma}{\omega_n} + i \frac{\omega}{\omega_n} \right) 2\zeta + 1 = \\ -\frac{u_{ch} b R_w \omega_n \varepsilon}{k V_g} \left( \frac{\Omega_w}{\omega_n} \cos \phi \left( 1 - \mu e^{-\frac{\sigma}{\omega_n} \omega_n \tau_w} \cos \left( \frac{\omega}{\omega_n} \omega_n \tau_w \right) + i \mu e^{-\frac{\sigma}{\omega_n} \omega_n \tau_w} \sin \left( \frac{\omega}{\omega_n} \omega_n \tau_w \right) \right) + \frac{\bar{\delta}_w}{R_w} \left( \frac{\sigma}{\omega_n} + i \frac{\omega}{\omega_n} \right) \sin \phi \right) \end{aligned}$$

$$1 + \left( \frac{\sigma^2}{\omega_n^2} + i2 \frac{\omega}{\omega_n} \frac{\sigma}{\omega_n} - \frac{\omega^2}{\omega_n^2} \right) + \left( \frac{\sigma}{\omega_n} + i \frac{\omega}{\omega_n} \right) 2\zeta =$$

$$- \frac{u_{\text{ch}} b R_w \omega_n \varepsilon}{k V_g} \left( \frac{1}{\omega_n \tau_w} \cos \phi \left( 1 - \mu e^{-\frac{\sigma}{\omega_n} \omega_n \tau_w} \cos \left( \frac{\omega}{\omega_n} \omega_n \tau_w \right) + i \mu e^{-\frac{\sigma}{\omega_n} \omega_n \tau_w} \sin \left( \frac{\omega}{\omega_n} \omega_n \tau_w \right) \right) + \frac{\bar{\delta}_w}{R_w} \left( \frac{\sigma}{\omega_n} + i \frac{\omega}{\omega_n} \right) \sin \phi \right)$$

Applying the dimensionless parameters  $B = \frac{b u_{\text{ch}} R_w \omega_N}{k \Omega_g R_g} = \frac{b u_{\text{ch}} R_w \omega_N}{k V_g}$ ;  $W = \frac{\omega}{\omega_N}$ ;  $S = \frac{\sigma}{\omega_N}$  and

$$T_w = \omega_n \tau_w$$

$$1 + (S^2 + i2WS - W^2) + (S + iW)2\zeta =$$

$$- B \varepsilon \left( \frac{1}{T_w} \cos \phi \left( 1 - \mu e^{-ST_w} \cos(WT_w) + i \mu e^{-ST_w} \sin(WT_w) \right) + \frac{\bar{\delta}_w}{R_w} (S + iW) \sin \phi \right)$$

Equating real and imaginary parts

$$1 + (S^2 - W^2) + 2\zeta S = -B \varepsilon \left( \frac{1}{T_w} \cos \phi \left( 1 - \mu e^{-ST_w} \cos(WT_w) \right) + \frac{\bar{\delta}_w}{R_w} S \sin \phi \right) \quad (18)$$

and

$$2WS + 2\zeta W = -B \varepsilon \left( \frac{1}{T_w} \cos \phi \mu e^{-ST_w} \sin(WT_w) + \frac{\bar{\delta}_w}{R_w} W \sin \phi \right) \quad (19)$$

For the case of  $S = 0$  (the stability boundary) and  $\mu = 0$  (no overlap) equation (19) gives,

$$2\zeta = -B \varepsilon \frac{\bar{\delta}_w}{R_w} \sin \phi \text{ and substituting, } B = \frac{b_{\text{lim}} u_{\text{ch}} R_w \omega_N}{k V_g} = -\frac{2\zeta R_w}{\varepsilon \bar{\delta}_w \sin \phi} \text{ thus } b_{\text{lim}} = -\frac{c V_g}{u_{\text{ch}} \bar{\delta}_w \varepsilon \sin \phi}$$

For  $b_{\text{lim}}$  to be positive it is required that  $\varepsilon \sin \phi$  be negative. Further, for practical values of the variables the predicted limiting width of cut will be large so this form of non-regenerative chatter is unlikely to be a problem, especially when compared with regenerative chatter when there is overlap. Equations (18) and (19) were solved numerically using a computer program in order to confirm this. Fig. 7 shows some typical results of growth rate against width, both non-dimensionalised. When there is no overlap  $\mu = 0$  the curve is a straight line for non-regenerative chatter. For a small overlap  $\mu = 0.01$  the straight line is modified and the regenerative curves for different numbers of waves start to appear. For greater overlaps the regenerative curves become the dominant ones and the non-regenerative curve disappears.

Following the normal practice, the damping in the models described has been assumed to be viscous. This historically has been assumed because the mathematics for any non-steady state vibration becomes far more complex with non-viscous damping. However the assumption of viscous damping results in more energy dissipation at high frequencies than, for example, hysteretic damping, which is not frequency dependent. For steady state vibration the relationship between a viscous damping coefficient and a hysteretic coefficient

is  $\omega c = h$ . If the three models presented above were to use hysteretic damping by substituting  $c = h/\omega$ , the predicted limiting widths for non-regenerative chatter become,

$$b_{\text{lim}} = \frac{h_g \bar{\Omega}_g^2}{u_{\text{ch}} \omega_c \bar{V}_w \bar{\delta}_w} \text{ for torsion alone of the grinding wheel}$$

$$b_{\text{lim}} = \frac{h_w \bar{V}_g}{u_{\text{ch}} \omega_c R_w^2 \bar{\delta}_w} \text{ for torsion alone of the work and}$$

$$b_{\text{lim}} = -\frac{hV_g}{u_{\text{ch}} \omega_c \bar{\delta}_w \varepsilon \sin \phi} \text{ for transverse alone where } \omega_c \text{ is the chatter frequency.}$$

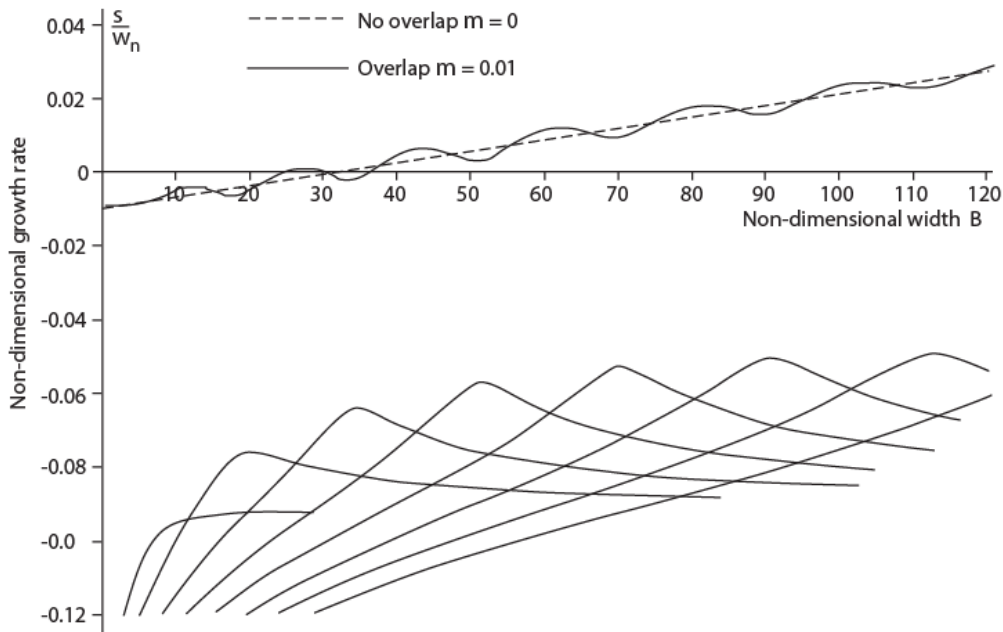


Fig. 7. Growth rates against width, both non-dimensionalised

Thus high frequency chatter is the most likely form of non-regenerative chatter to be found in practice as it is predicted to have a reduced limiting width.

The models presented above have involved major assumptions in order to highlight possible forms of non-regenerative chatter. The major omission has been that of the contact stiffness of the wheel on the work. This contact stiffness is significant when it comes to the prevention of chatter and is considered in the next section. In this section it has been assumed that only one mode of vibration will be significant, i.e. the various modes of vibration will not be active at the same time. This assumption will be discussed later.



## 5. SUPPRESSION OF CHATTER IN GRINDING

A review of methods that have aimed at avoiding chatter in grinding is given by Inasaki *et al.* [1]. They discuss,

- (1) *Modification of grinding conditions.*
- (2) *Increase in the dynamic stiffness of the mechanical system.*
  1. *Increase in the static stiffness.*
  2. *Decrease in the orientation factor.*
  3. *Increase in the damping.*
- (3) *Shifting the vector locus of the dynamic compliance to the positive real part.*
- (4) *Disturbing regenerative effects.*

The solutions in (1) and (2) above are generally well known but are not always possible on the shop floor. Modifications to the machine are generally impracticable in the typical workplace. Also solution (4) is best known when attempts are made to continuously vary the rotational speed of either the work or wheel [21]. This may be effective for regenerative chatter, as waves left on the surface do not reinforce the existing vibration as a phase shift occurs. However, there are major difficulties with this approach not the least that it is not a simple matter to introduce continuously varying speed. Also surface finish and depth of cut vary with speed so that the effects are seen on the finished work. By far the most effective and simple solution to preventing chatter has been the use of increased flexibility. This is achieved by using ‘softer’ wheels and also specially designed flexible wheels.

The work of Snoeys and Brown [11], Entwistle[12] and many others has confirmed that for regenerative chatter in grinding the machine characteristic that is significant is the maximum negative inphase component of the chatter receptance. This was shown to be the case for metal cutting (Appendix B). It is possible to reduce this by the use of flexibility.

### 5.1 CONTACT STIFFNESS

As the grits in grinding wheels are held in a non-rigid bond material they deflect under the influence of the grinding force. The deflection is complex but is often modelled as a stiffness  $k_c b$  between the wheel and work ( $k_c$  has units of force per unit contact area). To minimise the maths it is helpful to consider the envelope of the stability boundary, i.e. ignore the stability lobes as seen in Fig. 4. For metal cutting see Appendix B

$$b_{\text{lim}} = \frac{-1}{2RG_r(\omega)} \quad (\text{B.6})$$

where  $G_r(\omega)$  is the in-phase (real) part of the machine response. It can shown that the maximum negative in-phase component of the response of a single mode system is given by  $G_r(\omega) = -m/(c(2\sqrt{mk} + c))$  and the contact stiffness is added in series to this so that,

$$G_R(\omega) = \frac{1}{k_c b} - \frac{m}{c(2\sqrt{mk} + c)} \quad (20)$$

and for regeneration on the work alone using the force model of equation (1)  $R = k_1 u_{ch} V_w / V_g$

Thus substituting in equation (B.6)

$$b = \frac{-V_g}{2k_1 u_{ch} V_w \left[ \frac{1}{k_c b} - \frac{m}{c(2\sqrt{mk} + c)} \right]} \quad \text{and rearranging} \quad b_{lim} = \frac{c(2\sqrt{mk} + c)}{m} \left[ \frac{1}{k_c} + \frac{V_g}{2k_1 u_{ch} V_w} \right] \quad (21)$$

It is evident that the smaller the value of  $k_c$  the greater the value of  $b_{lim}$  so that chatter becomes less likely. A similar result may be obtained for regeneration on the wheel. Thus one of the simplest ways to avoid chatter is to use soft wheels. i.e. just change the grinding wheel. However if this is not possible then the solution proposed by Sexton *et al.* [22],[23] is to use a ‘flexible’ grinding wheel.

## 5.2. FLEXIBLE GRINDING WHEELS

It is important to note that flexible grinding wheels that prevent chatter must have the radial flexibility as close to the rim of the wheel as possible. Then there is another stiffness  $k_f$  in series with the contact stiffness so that

$$G_R(\omega) = \frac{1}{k_f} + \frac{1}{k_c b} - \frac{m}{c(2\sqrt{mk} + c)} \quad (22)$$

and hence it may be shown [24] that

$$b_{lim} = \frac{\frac{1}{k_c} + \frac{V_g}{2k_1 u_{ch} V_w}}{\left[ \frac{m}{c(2\sqrt{mk} + c)} - \frac{1}{k_f} \right]} \quad (23)$$

It is evident that the smaller the value of  $k_f$  the greater the value of  $b_{lim}$  so that chatter becomes less likely. Initially Sexton [22] produced a wheel with an outer rim, that included a 3 mm layer of CBN grits. The rim was mounted on rubber pads and the number used allowed the value of the flexibility to be easily adjusted. The final responses for the machine and wheel for both a conventional wheel and flexible wheel are shown in Fig. 8. It may be seen that the original response is moved in the positive real direction as predicted by equation (22). However another mode of vibration is introduced that involves the rim vibrating on the mounts. This mode has a large negative real part at 530Hz.

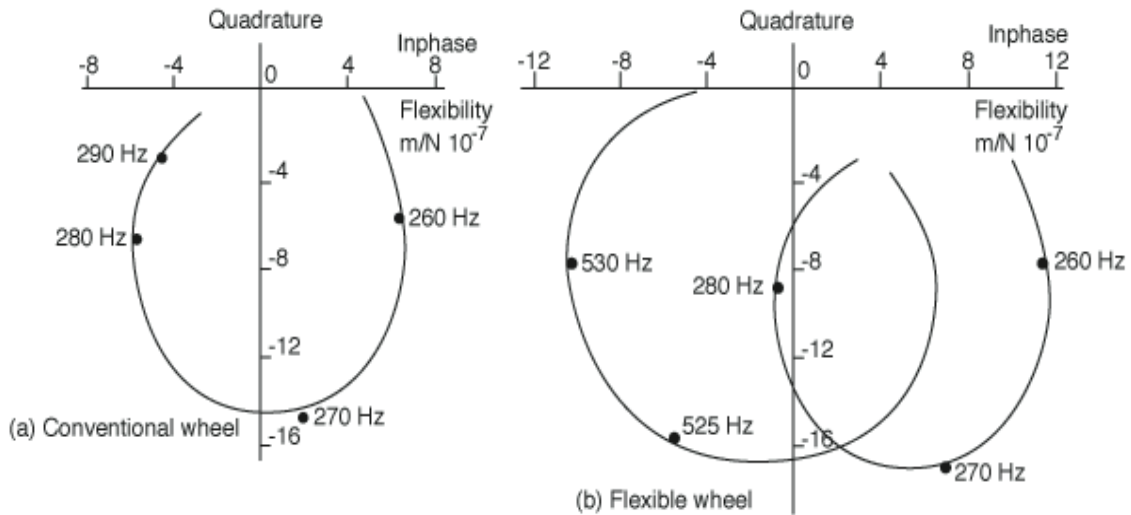


Fig. 8. Machine/wheel response: (a) conventional and (b) flexible grinding wheel

Sexton found that he could not get chatter with his flexible wheel. The wheel became rounder and rounder even after 12 hours of grinding. The reason postulated for no chatter at 530 Hz with the flexible wheel was that at high frequency the contact zone attenuates the amplitudes of regenerative surface waves. After the success of the development wheel Sexton [23] investigated alternative means of introducing flexibility into the wheel. He found that the foam metal Retimet could be used as the hub material. This introduced the desired flexibility and there was no associated mode involving vibration of a rim, as there was not one. It is surprising that such wheels have not been more extensively used as they do not require any modification of the machine.

There is however one reported grinding operation where it is predicted that a flexible wheel would not improve chatter performance. Pearce and Stone [25],[26] modelled centreless grinding with and without flexible wheels. They showed that geometric instability was simply low frequency chatter. They also showed that since surface waves on the work would interact with the wheel, the regulating wheel and the support plate the use of a flexible wheel would not be beneficial. This raises the question of whether flexible wheels would improve the possible non-regenerative forms of chatter considered in this paper. The improvement obtained from flexible wheels has been established for regenerative chatter but it is not self-evident that they would work for non-regenerative chatter in grinding.

## 6. CONCLUDING REMARKS

Several major questions have been raised in this paper. Some result from the assumptions that have been made and others from types of chatter in grinding that do not appear to have been considered previously.

1. All the models described have assumed that the onset of chatter arises from very small disturbances that grow. If these do not grow then chatter is prevented. Thus solutions

to chatter that do not involve non-linearities have been described. Where experimental evidence is available the predicted prevention of chatter has been found to be successful using these small amplitude models. The use of non-linear models is relevant for the conditions after chatter has commenced but may have limited amplitudes.

2. The grinding force model for oscillating conditions, that has been used, has limited experimental validation. It has been confirmed for oscillating work speed and chip thickness with freshly dressed wheels. There has been no confirmation of the model with respect to oscillating wheel speed and the effective grinding ratio for oscillating conditions. Further work is required.

3. The assumption of line contact is clearly questionable as there is a cutting zone and filtering is known to occur. Such filtering at high frequency was proposed for explaining why Sexton's flexible wheel did not chatter as a result of the higher frequency mode. If this high frequency filtering is operative then regenerative chatter at high frequencies will be limited. As a result high frequency non-regenerative could be predominant. This is particularly the case for grinding wheel torsional chatter.

4. The three possible causes of non-regenerative chatter all depend on the grinding force model being correct. Initial modelling of the form that involves the transverse mode has shown that non-regenerative and regenerative may 'couple' when regeneration occurs. It is thus an area worthy of further research to investigate conditions when both may occur at the same time.

5. The three forms of non-regenerative chatter that have been considered are not all equally likely. However if regeneration on the wheel is limited because of say filtering, then torsional vibration of the wheel that may involve very limited damping can give rise to non-regenerative chatter.

6. The predicted variation of growth rates with width show significant differences between regenerative and non-regenerative chatter. Regenerative chatter has small growth rates that do not increase linearly with width, but rather peak and fall away with increasing width. The modelling of the possible non-regenerative chatter conditions considered in this paper all indicate growth rates the continue to increase with width. It is thus possible that non-regenerative chatter will appear more rapidly than any regenerative chatter that is present.

7. The effect of contact stiffness and flexible wheels has been shown to improve regenerative chatter performance. It is not immediately apparent that they will have the same effect on non-regenerative chatter. This needs further investigation.

8. Single modes that are not coupled have been modelled. Real machines are far more complex and so modelling of real machines is an extremely complex task. However it is considered that real improvements in chatter performance may be achieved using solutions that are predicted from simpler models.

Finally the greatest need is for more experimental work in the area of chatter in grinding. There are great challenges because of the daunting number of variables. The possibility of work torsional vibration improving regenerative chatter should be investigated further and experimental measurements need to be made of the torsional vibration present.

## REFERENCES

- [1] INASAKI I., KARPUSCHEWSKI B., LEE H.S., 2001, *Grinding chatter – origin and suppression*, CIRP Annals - Manufacturing Technology, 50/2, 515-534.
- [2] ARNOLD R.N., 1946, *The mechanism of tool vibration in the cutting of steel*, Proc. I.Mech.E, 154.
- [3] HAHN R.S., 1952, *Metal-cutting chatter and its elimination*, ASME, 1073-1080.
- [4] TLUSTY J., SPACEK L., 1954, *Self-excited vibrations in machine tools*. Czech. Naklad. CSAV, Prague.
- [5] TOBIAS S.A., FISHWICK W., 1956, *The vibrations of radial drilling machines under test and working conditions*, Proceedings, I. Mech. E 170, 232-247.
- [6] TOBIAS S.A., FISHWICK W., 1957, *The chatter of lathe tools under orthogonal cutting conditions*, TRANS. ASME, 80, 1079-1088.
- [7] TOBIAS S.A., 1959, *The vibrations of vertical milling machines under test and working conditions*, Proc. Inst. Mech. Engrs, 173/18, 474–510.
- [8] GURNEY J.P., TOBIAS S.A., 1961, *A graphical method for the determination of the dynamic study of machine tools*, Int. J. Mach. Tool Des. Res., 1, 148–156.
- [9] ITO Y., 2013, *fundamental issues of self-excited chatter vibration in turning*, Special Issue within Journal of Machine Engineering (to be published).
- [10] INASAKI I., 1977, *Selbsterregte Ratterschwingungen beim Schleifen, Methoden zu ihrer Unterdrückung*, Werkstatt und Betrieb 110/8, 521-524.
- [11] SNOEYS R., BROWN D., 1969, *Dominating parameters in grinding wheel and workpiece regenerative chatter*, Proceedings of the 10th International Machine Tool Design and Research Conference, 325-348.
- [12] ENTWISTLE R.D., 1997, *Torsional compliance and chatter in grinding*, PhD Thesis, The University of Western Australia.
- [13] DREW S.J., MANNAN, M. A., ONG, K. L. and STONE, B. J., 2001, *The measurement of forces in grinding in the presence of vibration*. International Journal of Machine Tools and Manufacture, 41, 509-520.
- [14] QURESHI R.A., MANNAN, M. A., DREW, S. J. and STONE, B. J., 2002, *A comparison of grinding forces arising from oscillating workpiece speed and chip thickness*. Transactions of the North American Manufacturing Research Institution of SME, XXX, 169-176.
- [15] QURESHI R.A., MANNAN M.A., DREW S.J., STONE B.J., 2013, *Experimental investigation of grinding forces resulting from oscillating chip thickness*. Journal of Machine Engineering.
- [16] QURESHI R.A., 2002, *Experimental investigation of dynamic forces due to chip thickness variation and workpiece rotational speed variation in plunge grinding*, Masters Thesis, The National University of Singapore.
- [17] ENTWISTLE R.D., STONE B.J., 1996, *Torsional compliance in grinding chatter*, Proc. Eleventh Annual Meeting of The American Society for Precision Engineering, 9/14 November, 420-423.
- [18] CHIU N., MALKIN S. 1993, *Computer simulation for cylindrical plunge grinding*, CIRP Annals - Manufacturing Technology, 34/1/1, 383-387.
- [19] drew s.j., stone b j., 1997, *torsional (rotational) vibration - excitation of small rotating machines*, journal of sound and vibration, journal of sound and vibration, 201/4, 437-463.
- [20] BAYLIS R.J., STONE B.J., 1989, *The Effect of Grinding Wheel Flexibility on Chatter*, CIRP Annals - Manufacturing Technology, 38/1, 307-310.
- [21] HOSHI T., MATSUMOTO S., MITSUI S., HORIUCHI O., KOUMOTO Y., 1986, *Suppression of wheel regenerative grinding vibration by alternating wheel speed*, JSPE, 52-10,1802-1807.
- [22] SEXTON J.S., HOWES T.D., STONE B.J., 1982, *The use of increased wheel flexibility to improve chatter performance in grinding*. Proc. I. Mech. E., 196/25, 291-300.
- [23] SEXTON J.S., STONE B.J., 1981, *The development of an ultra-hard abrasive grinding wheel which suppresses chatter*. Annals of CIRP, 30/1/81, 215-218.
- [24] STONE B.J., 2013/14. *Chatter and machine tools*. Book to be published by Springer.
- [25] PEARCE T.R.A., STONE B. J., 2011, *Unstable vibration in centreless grinding: Part A. Geometric instability or chatter*. Proceedings of IMechE Part B, Journal of Engineering Manufacture, 225/8, 1227-1243.
- [26] PEARCE T.R.A., STONE B.J., 2011, *Unstable vibration in centreless grinding: Part B. Graphical method*. Proceedings of IMechE Part B, Journal of Engineering Manufacture, 225/8, 1245-1254.

## APPENDIX A – ARNOLD (OR TYPE B) CHATTER FOR TURNING

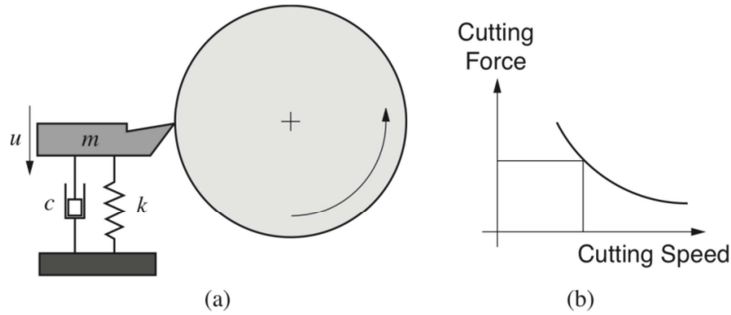


Fig. A.1. Condition at the stability boundary in metal cutting (with no regeneration)

At high rates of change of the surface speed the cutting force is assumed to vary with speed in the manner shown in Fig. A1(b), i.e. the force reduces with increasing surface speed. To illustrate the mechanism of Arnold chatter, a simple and approximate model of the cutting force component in the tangential direction may be assumed to be of the form,  $F = b\delta(R_o - \beta v)$ , where  $b$  is the width of cut,  $\delta$  the depth of cut (hence  $b\delta$  is the area of the undeformed chip cross-section),  $v$  is the instantaneous surface speed of the work relative to the tool and  $R_o$  and  $\beta$  are positive constants depending on numerous factors such as work material, condition and the geometry of the cutting edge etc. The term in parentheses is the straight line tangent to the cutting force curve about the mean operating condition. Note that with the coordinate directions chosen, a positive tool velocity increases the cutting speed and hence reduces the cutting force.

If the tool moves up and down there is, for small initial amplitudes, a negligible change in depth of cut so that if  $x(t)$  represents the displacement of the tool in the direction of the force the tangential cutting force is given by,

$$F = b\delta(R_o - \beta \frac{dx(t)}{dt}) \quad (A1)$$

For a one degree-of-freedom the equation of motion is

$$m \frac{d^2x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) = F = -b\delta(R_o - \beta \frac{dx(t)}{dt}) \quad (A2)$$

The constant force term  $b\delta R_o$  is ignored as it causes no oscillation and the equation of motion becomes,

$$m \frac{d^2x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) = b\delta\beta \frac{dx(t)}{dt}$$

and rearranging

$$m \frac{d^2x(t)}{dt^2} + (c - b\delta\beta) \frac{dx(t)}{dt} + kx(t) = 0 \quad (A3)$$

This is unstable when the coefficient of the velocity term becomes negative (i.e. equivalent to negative damping). Thus unstable vibration occurs when

$$(c - b\delta\beta) \leq 0 \quad \text{or} \quad b\delta\beta > c \quad (\text{A4})$$

The onset of chatter depends on both the width and depth of the cut, the original damping in the tool ( $c$ ) and the factor  $\beta$  in the assumed force equation.

## APPENDIX B – STABILITY BOUNDARY FOR REGENERATIVE CHATTER

The simplest model for the force in metal cutting has the force proportional to the undeformed chip thickness ( $\delta$ ) so that  $F = Rb\delta$  where  $b$  is the width of cut and  $R$  is called the cutting force coefficient. If it is assumed that at the boundary of stability the vibration is sinusoidal with a constant amplitude (Fig. B.1) The resultant force, when all the components are included, is from Fig. B.1 where  $\delta$  is the feed per revolution, given by

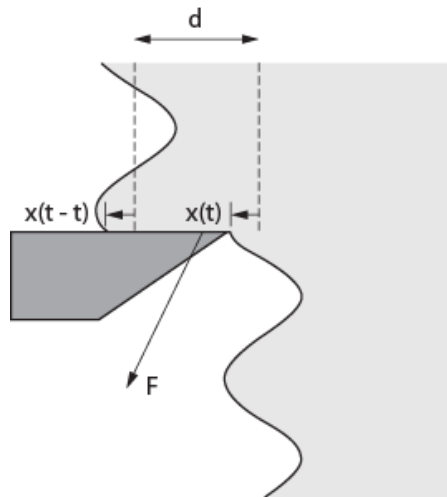


Fig. B.1. Condition at the stability boundary in metal cutting

$$F = Rb\delta = Rb[\bar{\delta} - x(t) + x(t - \tau)] \quad (\text{B.1})$$

The machine response that is of interest is the relative deflection,  $x(t)$ , between the tool and work in the chip thickness direction when there are equal and opposite oscillating forces, i.e. without the mean force  $Rb\bar{\delta}$ ,  $-Rb[x(t) - x(t - \tau)]$ , in the cutting force direction. This response is often called the chatter receptance of the machine. This is a function of frequency -  $G(\omega)$  - and can be represented by its real and imaginary parts  $G_R(\omega)$  and  $G_I(\omega)$  so that.

$$G(\omega) = G_R(\omega) + iG_I(\omega) \quad (\text{B.2})$$

If the assumed vibration is to continue steadily (stability boundary) then the oscillating cutting force must act on the structure to maintain it. Thus by definition the response is given by

$$x(t) = G(\omega)F_{\text{oscillating}} = G(\omega)\{Rb[-x(t) + x(t - \tau)]\} \quad (\text{B.3})$$

Rearranging and substituting for  $G(\omega)$  from (B.2)

$$\frac{x(t)}{x(t - \tau)} = \frac{G_R(\omega) + iG_I(\omega)}{\frac{1}{Rb} + G_R(\omega) + iG_I(\omega)} \quad (\text{B.4})$$

Thusty [4] now notes that at the boundary of stability the magnitude of  $x(t)$  and  $x(t - \tau)$  will be the same so that,

$$\frac{|x(t)|}{|x(t - \tau)|} = \left| \frac{G_R(\omega) + iG_I(\omega)}{\frac{1}{Rb} + G_R(\omega) + iG_I(\omega)} \right| = \frac{\sqrt{G_R^2(\omega) + G_I^2(\omega)}}{\sqrt{\left(\frac{1}{Rb} + G_R(\omega)\right)^2 + G_I^2(\omega)}} = 1 \quad (\text{B.5})$$

Squaring both sides and rearranging

$$G_R^2(\omega) = \left(\frac{1}{Rb} + G_R(\omega)\right)^2 = \left(\frac{1}{Rb}\right)^2 + 2\left(\frac{1}{Rb}\right)G_R(\omega) + G_R^2(\omega)$$

so that  $\frac{1}{Rb} = -2G_R(\omega)$  and we have that the width of cut at the stability boundary is given by

$$b = \frac{-1}{2RG_R(\omega)} \quad (\text{B.6})$$

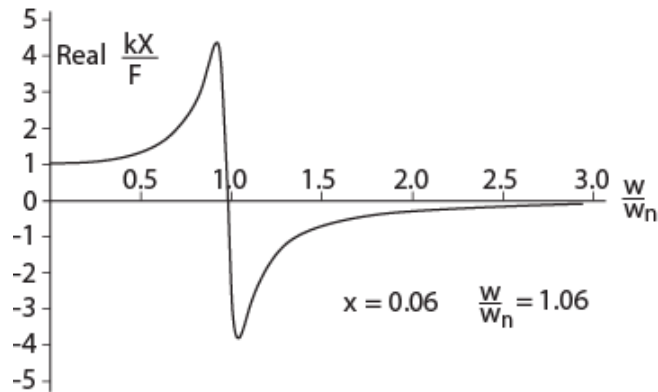


Fig. B.2. Non-dimensional plot of the real part of the response of a spring/mass system with damping



Further, when cutting, the width of cut is positive and so the minimum value of  $b$  (usually denoted as  $b_{\text{lim}}$ ) is determined by the maximum negative value of  $G_R(\omega)$  - commonly termed the maximum negative in-phase component of the chatter receptance,  $G_{R,\text{max}}(\omega)$ . This is usually found to occur above the undamped natural frequency as shown in Fig. B.2. For the example shown it occurs at 1.06 times the undamped natural frequency.