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THE SELECTION OF PRELOAD IN ANGULAR CONTACT BALL BEARINGS ACCORDING TO THE CRITERION OF MOMENT OF FRICTION

DOBÓR NAPIĘCIA WSTĘPNEGO W ŁOŻYSKACH KULKOWYCH SKOŚNYCH WEDŁUG KRYTERIUM MOMENTU TARCIA

Key words: angular contact ball bearings, selection of preload, basic rating life, moment of friction.

Abstract The article **[L. 3]** presents a method of preload selection in angular contact ball bearings based on their durability. The preload can be expressed as a force or as a dislocation (distance). Depending on the regulation method, the preload is also indirectly connected with the moment of friction in a bearing.
The objective of the paper is to determine a maximum preload in angular contact ball bearings, based on the moment of friction in a system of bearings.

Słowa kluczowe: łożysko kulkowe skośne, napięcie wstępne, trwałość łożyskowania, moment tarcia.

Streszczenie W artykule **[L. 3]** przedstawiono metodę doboru napięcia wstępnego w łożyskach kulkowych skośnych, kierując się ich trwałością. Napięcie wstępne może być wyrażone jako siła lub jako przemieszczenie (odległość). Zależnie od metody regulacji, napięcie wstępne jest pośrednio związane także z momentem tarcia w łożysku. Celem tej pracy jest określenie maksymalnego napięcia wstępnego w łożyskach kulkowych skośnych, kierując się momentem tarcia występującym w układzie łożysk.

INTRODUCTION

Axial preload in single-row angular contact ball bearings, tapered roller bearings, as well as regular ball bearings is obtained by axial dislocation of one of the bearing rings in relation to the other at a distance corresponding to a required force of preload. According to the literature **[L. 8]**, there exist two basic methods of setting that differ in relation to rules on which they are based: individual setting (in a few presented variations) and system setting.

In the individual setting, a specific bearing arrangement is preloaded separately with the use of nuts, shims, setting spacers, etc. The procedures of measurement and control assure obtaining the maximum force of preload with the lowest possible deviation. There exist different methods, depending on the number of bearings that have to be measured **[L. 8]**:

- Setting through dislocation (necessary in order to achieve preload): The method is based on the interdependence between the strength of preload and elastic deformations in a preloaded system. The required preload can be determined on the basis of a graph of the interdependence of force and dislocation responding to the preload. The method is often used when elements of a bearing are preliminarily assembled.
- Setting with the use of a frictional moment: This method widely used in serial production, due to the speed and possibility of automation. Because there is a specific relation between the preload and the moment of friction in the bearing, it is possible to stop the setting when the moment of friction responding to the demanded preload is achieved (under the condition that the friction moment is continuously monitored). However, it has to be

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remembered that the moment of friction can differ in particular bearings, because it also depends on the used preservatives and the conditions of lubrication and speed.

- Setting with a direct measurement of force: Because the aim behind the setting of the bearing is establishing a specific preload in it, it seems most appropriate to use one of the methods in which the force can be directly produced and measured (with the use of dislocating, up to achieving preload or the moment of friction), because they are simple and can be carried out with a smaller effort.
- Individual setting has this advantage that particular elements can be produced with normal tolerance, and the required preload can be achieved with a properly high level of accuracy.

With the method of system setting, which can be also called "random statistical setting," the bearings, shafts and fittings, rings and spacer sleeves, etc. are produced in normal quantities and assembled randomly, with full exchangeability of parts. In case of tapered roller bearings, this replaceability also includes outer rings and the systems of inner rings. Tolerance limit values are assumed, which statistically rarely appear together, so that there appears to be no necessity for uneconomical production of very accurate bearings and cooperating elements. Yet if the force of preload with the minimum spread has to be achieved, the production tolerances have to become tighter. It is an advantage of system setting that, during assembling of bearings, no inspection or additional equipment is required **[L. 8]**.

CALCULATING METHOD

As mentioned in **[L. 4]**, in order to solve the problem, a number of issues with mutual interrelations have to be connected. The most important of these issues is to determine elastic displacements in a bearing. Dislocations in bearings cause reactions in them.

A method of determining dislocations in a bearing has been presented in the studies **[L. 2, 5, 6, 7]**.

Reactions of a bearing (radial, axial as well as reaction moments) emerge from the internal forces acting between the elements of the bearing. Internal forces depend on the mutual dislocations of the elements of a bearing.

Knowing the forces Q operating between the balls of a bearing and the rings, it is possible to determine all reactions of a bearing understood generally, i.e. reaction forces in three directions of a coordinate system and reaction moments in relation to the axes perpendicular in relation to the axis of rotation of a bearing. The proceeding of reactions is illustrated in **Fig. 1**.

The upper part of the illustration presents, in perspective, the curve of the centres of the courses curvature of the external ring in the bearing. Forces Q, operating between the ball and the race of the ring, are applied to the points located on the curve. They are deflected from the plane y-z with the angle of operation α. The following formulas result from projecting the force Q upon the directions x–y–z:

$$Q_x = Q \cdot \sin\alpha \tag{1}$$

$$Q_r = Q \cdot \cos\alpha \tag{2}$$

$$Q_y = Q_r \cdot \cos\alpha \cdot \cos\psi \tag{3}$$

$$Q_z = Q_r \cdot \sin\psi \cdot \cos\psi \tag{4}$$

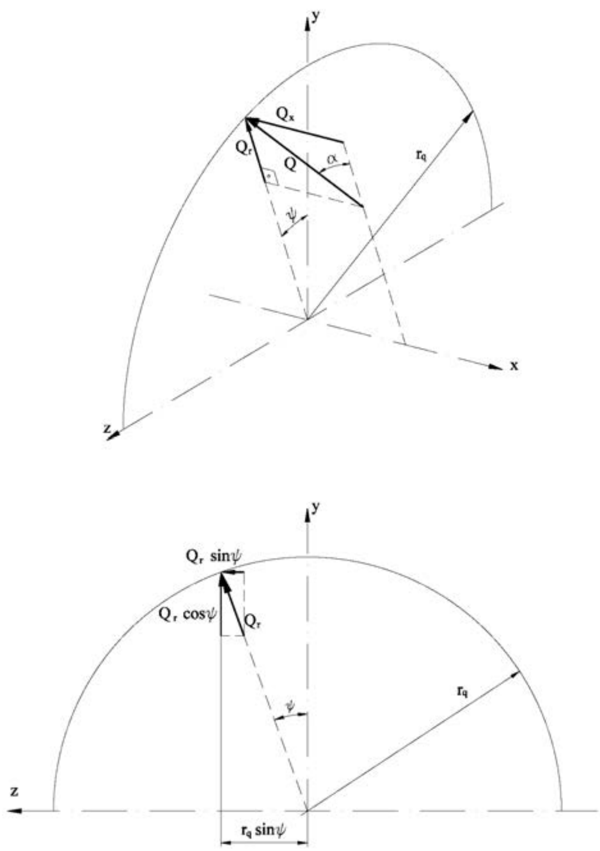


Fig. 1. Illustration of determining the reaction of a bearing

Rys. 1. Ilustracja wyznaczania reakcji łożyska

A bending moment resulting from the force Q has to be calculated in relation to the nodal point of the nominal reaction of bearing, because, in accordance with the assumptions, this point is assumed to be the support point of the shaft by the bearing. The above calculation is illustrated in **Fig. 2**. The nodal point is marked as W. The force Q is a reaction of the external ring to the ball; thus, it is applied in the middle of the course curvature of the external ring, marked q. The moment of force Q coming from a freely chosen ball (here a ball located on the plane of the illustration was selected) equals the following:

$$M = Q \cdot r \tag{5}$$

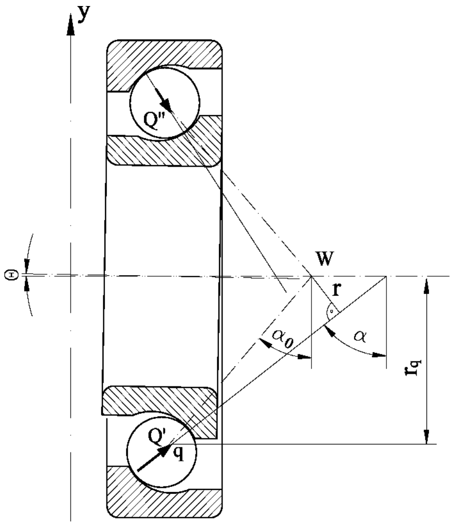


Fig. 2. Illustration of determining reaction moment of a bearing
Rys. 2. Ilustracja wyznaczania momentu reakcyjnego łożyska

A minor cathetus r in a marked right triangle is an arm of force Q in relation to point W.

$$r = qW \cdot \sin(\alpha - \alpha_0) \tag{6}$$

Hypotenuse qW is

$$qW = \frac{r_q}{\cos\alpha_0} \tag{7}$$

whereas

$$r_q = 0.5\, d_{bz} - r_{bz} \tag{8}$$

$$\cos\alpha_0 = \frac{w}{r_{bw} + r_{bz} - D_k} \tag{9}$$

Thus the minor cathetus is

$$r = \frac{r_q \times \left(r_{bw} + r_{bz} - D_k\right)}{w} \cdot \sin(\alpha - \alpha_0) \tag{10}$$

Moments of forces Q are determined on two planes: x-y and x-z. Taking into account all possible locations of the considered ball, determined by the angle ψ, these moments are specified according to the following relations:

$$\begin{gathered} M_{xz} = Q \cdot r \cdot \sin\psi \\ M_{xy} = Q \cdot r \cdot \cos\psi \end{gathered} \tag{11}$$

out of which the following final expressions result:

$$M_{xz} = Q \cdot \frac{r_q \cdot \left(r_{bw} + r_{bz} - D_k\right)}{w} \cdot \sin(\alpha - \alpha_0) \cdot \sin\psi \tag{12}$$

$$M_{xz} = Q \cdot \frac{r_q \cdot \left(r_{bw} + r_{bz} - D_k\right)}{w} \cdot \sin(\alpha - \alpha_0) \cdot \cos\psi \tag{13}$$

The expressions (1)–(6) specify the forces and moments coming from one (any) ball.

Total reactions of a bearing and total reaction moments of the bearing are the effect of the influence of all bearings. These values are calculated by summing the forces and moments applied by all these balls (and only these), which are a subject to normal deformations, i.e. are loaded by non-zero forces.

$$R_x = \Sigma(Q \cdot \sin\alpha) \tag{14}$$

$$R_r = \Sigma(Q \cdot \cos\alpha) \tag{15}$$

$$R_y = \Sigma(Q \cdot \cos\alpha \cdot \cos\psi) \tag{16}$$

$$R_z = \Sigma(Q \cdot \cos\alpha \cdot \sin\psi) \tag{17}$$

$$M_{xy} = \Sigma\left[Q \cdot \frac{r_q \cdot \left(r_{bw} + r_{bz} - D_k\right)}{w} \cdot \sin(\alpha - \alpha_0) \cdot \sin\psi\right] \tag{18}$$

$$M_{xy} = \Sigma\left[Q \cdot \frac{r_q \cdot \left(r_{bw} + r_{bz} - D_k\right)}{w} \cdot \sin(\alpha - \alpha_0) \cdot \cos\psi\right] \tag{19}$$

The presented formulas were used to determine the reaction of both the bearings of the shaft separately on the basis of their separate inner deformations.

ANALYTICAL CALCULATIONS

For the analysis, a system of two angular contact ball bearings has been adopted in accordance with the model presented in the first part of the study **[L. 4]**, with the bearings of the size 7212B and of the parameters as given in **Table 2**. Dynamic load rating C was determined from **[L. 10]**.

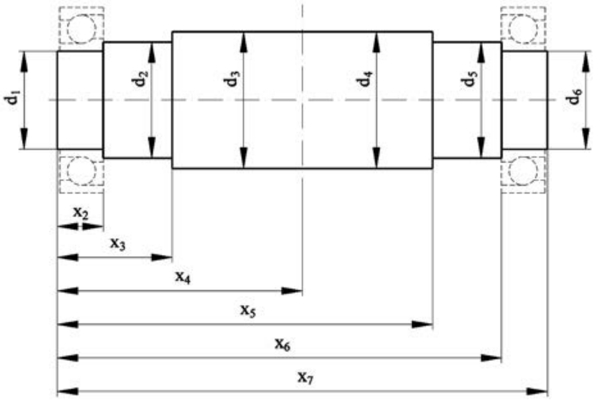


Fig. 3. Draft of a model shaft [L. 4]
Rys. 3. Szkic modelowego wału **[L. 4]**

A coordinate of the beginning of a shaft x_1 equal to zero has been adopted for all model shafts.

Table 1. Dimensional parameters of model shafts [m] [L. 4]

Tabela 1. Parametry wymiarowe modelowych wałów [mm] **[L. 4]**

Parameter	The size of the parameter
x_2	22
x_3	100
x_4	200
x_5	300
x_6	378
x_7	400
d_1	60
d_2	67
d_3	75
d_4	75
d_5	67
d_6	60

Moreover, the loading of the shaft associated with circumferential force F_{c1}, equalling respectively: 0.075 C; 0.1 C; 0.125 C, was assumed as in the first part of the paper **[L. 4]**. The values of the remaining forces are: $F_{c1} = F_{c2}$, $F_{p1} = F_{p2}$, $F_{x1} = F_{x2}$, radial force F_p equals 0.364 of the circumferential force for the assumption that the angle of contact in the mesh for the gear is approx. 20°, and axial force F_x has the following values: 0, $0.049 \cdot F_c$, $0.098 \cdot F_c$, $0.196 \cdot F_c$, $0.392 \cdot F_c$. These forces are established in relations to the circumferential force.

Table 2. Dimensions of working areas of the bearings adopted for calculations

Tabela 2. Wymiary powierzchni roboczych łożysk przyjętych do obliczeń

Bearing →	7212B
D_k [mm]	15.875
d_{bw} [mm]	68.976
d_{bz} [mm]	101.059
r_{bw} [mm]	8.180
r_{bz} [mm]	8.330
The number of rolling elements Z	15
Dynamic load C [N]	57200

Locations of planes of loads were assumed in relations to the length of L_w shaft equal the dimension x_7 = 400 mm, and they are respectively (**Fig. 4**) the following:

– For variation I of the load: $x_L = 0.4 \cdot L_w$, $x_L = 0.5 \cdot L_w$ $x_L = 0.6 \cdot L_w$,
– For variation II of the load: $x_{L1} = 0.4 \cdot L_w$, $x_{L2} = 0.6 \cdot L_w$.

The moment of friction in a roller bearing consists of mechanical friction and hydraulic resistances in lubricant. In the system of angular contact ball bearings, the moment of mechanical friction is proportional to the following expression:

$$M = M_{1A} + M_{1B} \approx \left(\Sigma Q_A\right)^{\frac{4}{3}} + \left(\Sigma Q_B\right)^{\frac{4}{3}} \tag{20}$$

where

ΣQ_A – sum of internal forces as a result of deformations in A bearing,
ΣQ_B – sum of internal forces as a result of deformations in B bearing.

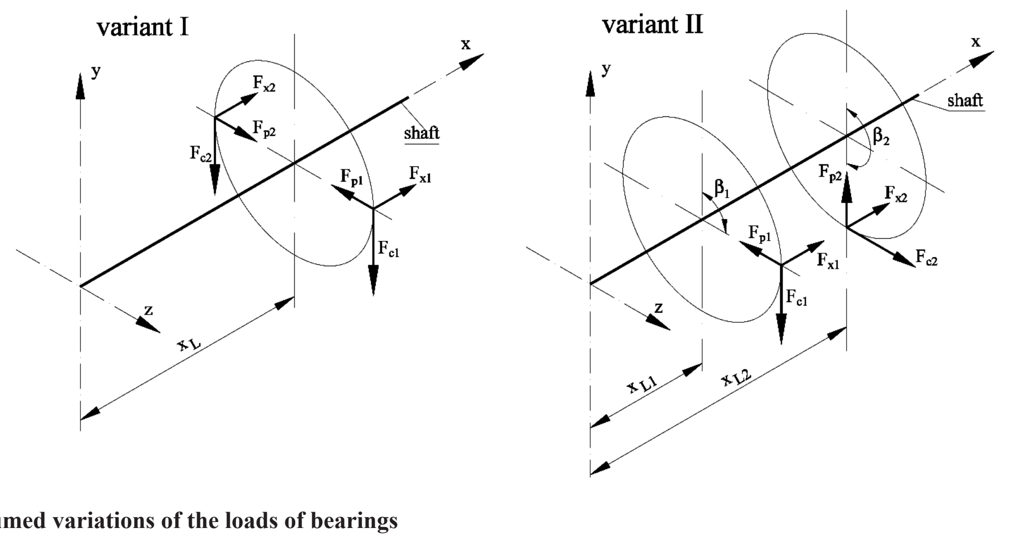


Fig. 4. Assumed variations of the loads of bearings

Rys. 4. Przyjęte warianty obciążeń łożyskowania

It has been assumed as the starting point that, in accordance with the formula (21) **[L. 9],** the moment of the friction of a specific angular contact ball-bearing in series B (with a given diameter d_m) is proportional to the product of the ratio of friction μ_1 and the equivalent load P_0.

$$M_1 = \mu_1 \cdot f_1 \cdot P_0 \cdot \frac{d_m}{2} \tag{21}$$

where

P_0 – equivalent resting load of a bearing;
f_1 – ratio depending on the type of bearing, e.g., for angular contact ball bearings in series B, it equals 1;
d_m – pitch diameter of the bearing;
μ_1 – ratio of friction, e.g., for angular contact ball bearings in series B, it is defined by the following formula **[L. 9]**:

$$\mu_1 = 0{,}0015\left(\frac{P_0}{C_0}\right)^{\frac{1}{3}} \tag{22}$$

According to the formula (22), the ratio of the friction of a specific bearing (with a given static load C_0) is proportional to the square root of the third degree from equivalent load: $\mu_1 \approx P_0^{\frac{1}{3}}$. Thus, the moment of the friction of a specific bearing can be expressed with the following proportionality:

$$M_1 \approx P_0 \cdot P_0^{\frac{1}{3}} \tag{23}$$

$$M_1 \approx P_0^{\frac{4}{3}} \tag{24}$$

The equivalent load of a bearing is proportional to the sum of load of all balls in the bearing. Consequently, it can be stated that the moment of mechanical friction M_1 is proportional to the sum of the loading of all balls in the bearing at the power of 4/3.

$$M_1 \approx (\Sigma Q)^{\frac{4}{3}} \tag{25}$$

The moment of mechanical friction of the whole bearing system consisting of two bearings (Bearing A and Bearing B) is a sum of the frictional moments of both the bearings. Determining the right side of the expression (20) is totally sufficient to achieve the objectives presented in the study where it has to be stated how the preload influences work parameters of a bearing system. Thanks to calculating the value of expression $(\Sigma Q_A)^{\frac{4}{3}} + (\Sigma Q_B)^{\frac{4}{3}}$ a relevant (percentage) change in the frictional moment depending on the applied preload can be found.

The dimension has been called the frictional moment W_M. **Fig. 5** presents exemplary characteristics of the frictional moment W_M (dotted line) together with the indicator W_T (continuous line, the method of determining indicator W_T was presented in **[L. 4]**) for the system accepted for the analysis.

It is easy to notice by the exemplary characteristics that all characteristics of W_M indicator are presented by lines ascending together with the increase of preload. They prove great similarity of shape. It has been decided to analyse, in detail, the course of the characteristics in case of one typo number of bearings and then to determine the degree of proportionality of characteristics W_M in relation to the size of bearings.

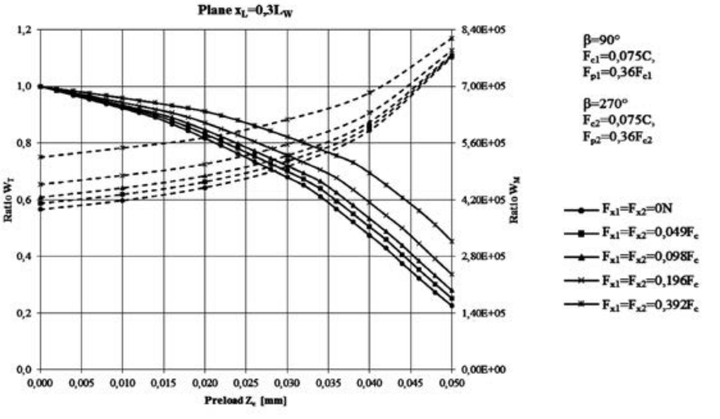
The increase of the resistance causes increased energy use, heat emitting, and worse fluidity of movement in delicate equipment (e.g., the measuring one). The ascending character of curves W_M indicates that the value of preload must be restricted due to the increase of the movement resistances in the bearing system. Therefore, the following question emerges: How high of an increase of movement resistances is acceptable? There exists no clear answer to this question, because a lot depends on a specific application. For the needs of this study, it has been arbitrarily decided that the accepted increase of the mechanical friction moment is 10% in relation to the state without preload ($Z_c = 0$).

In order to develop graphs of acceptable values of preload for considered characteristics of the W_M indicator, such values of preload Z_c were read for which there was an increase of the characteristics by 10%, and these points were transferred to the next graphs. Thus, the developed graphs are presented in **Fig. 6**.

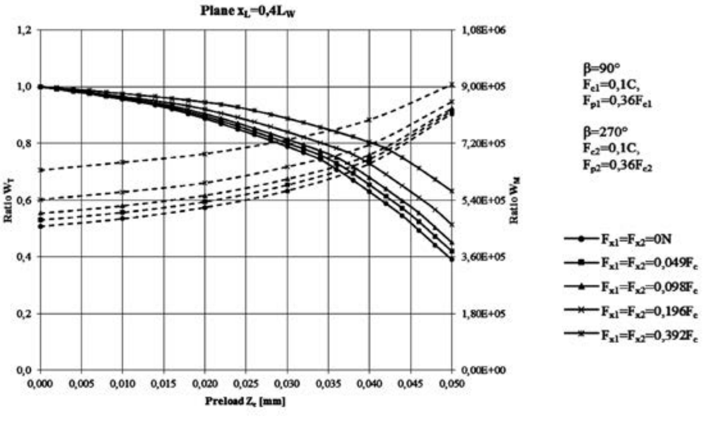
The following observations result from the graphs:

1. If the resultant of axial forces loads the bearing of the system that is less loaded transversely (as described by characteristics with the coordinates: $x_L = 0.3\ L_W$ and $0.4\ L_W$), the higher are the values of acceptable preload, the higher is the relative transversal load of F_c/C system, and the higher is the share of axial forces in relation to circumferential F_x/F_c. The characteristics can be considered linear. The line responding to $F_c = 0.1\ C$ begins from the value $Z_c \approx 0.018$ mm.
2. If both the identical bearings of the system are identically transversally loaded (in case of central location of the area of load, i.e. $x_L = 0.5\ L_W$), characteristics Z_c deviate a little from the linear ones. In their initial range, they run almost horizontally, and no sooner than after exceeding a relative axial load $F_x/F_c \approx 0.4$ they deflect towards the ascending direction. Inclination in the ascending part is almost identical as for $x_L < 0.5\ L_W$. Moreover, the proportionality of the value Z_c to relative transversal load is similar to the one in Conclusion 1 above. The line responding to $F_c = 0.1\ C$ begins with the value $Z_c \approx 0.017$ mm.

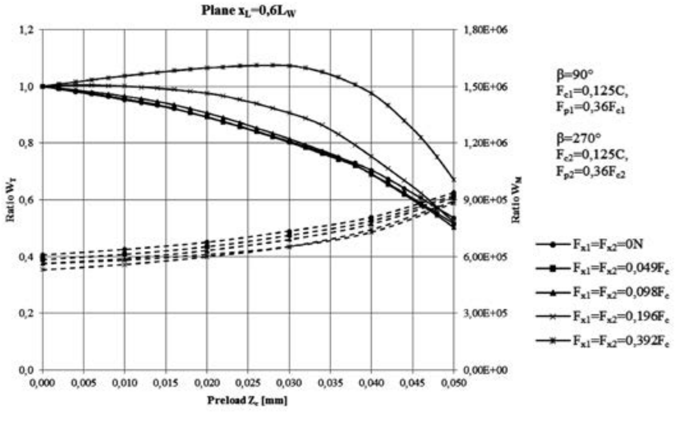
a)


Plane x_L=0,3L_W
Ratio W_T
Ratio W_M
Preload Z_v [mm]
β=90°
F_{c1}=0,075C,
F_{p1}=0,36F_{c1}
β=270°
F_{c2}=0,075C,
F_{p2}=0,36F_{c2}
F_{x1}=F_{x2}=0N
F_{x1}=F_{x2}=0,049F_c
F_{x1}=F_{x2}=0,098F_c
F_{x1}=F_{x2}=0,196F_c
F_{x1}=F_{x2}=0,392F_c

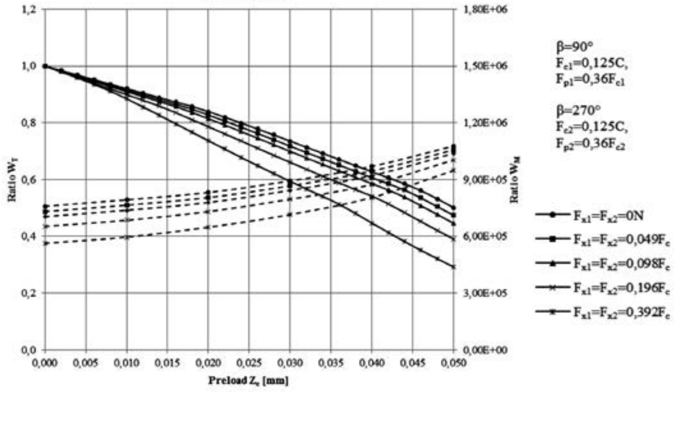
b)


Plane x_L=0,4L_W
Ratio W_T
Ratio W_M
Preload Z_v [mm]
β=90°
F_{c1}=0,1C,
F_{p1}=0,36F_{c1}
β=270°
F_{c2}=0,1C,
F_{p2}=0,36F_{c2}
F_{x1}=F_{x2}=0N
F_{x1}=F_{x2}=0,049F_c
F_{x1}=F_{x2}=0,098F_c
F_{x1}=F_{x2}=0,196F_c
F_{x1}=F_{x2}=0,392F_c

c)


Plane x_L=0,6L_W
Ratio W_T
Ratio W_M
Preload Z_v [mm]
β=90°
F_{c1}=0,125C,
F_{p1}=0,36F_{c1}
β=270°
F_{c2}=0,125C,
F_{p2}=0,36F_{c2}
F_{x1}=F_{x2}=0N
F_{x1}=F_{x2}=0,049F_c
F_{x1}=F_{x2}=0,098F_c
F_{x1}=F_{x2}=0,196F_c
F_{x1}=F_{x2}=0,392F_c

d)


Plane x_L=0,7L_W
Ratio W_T
Ratio W_M
Preload Z_v [mm]
β=90°
F_{c1}=0,125C,
F_{p1}=0,36F_{c1}
β=270°
F_{c2}=0,125C,
F_{p2}=0,36F_{c2}
F_{x1}=F_{x2}=0N
F_{x1}=F_{x2}=0,049F_c
F_{x1}=F_{x2}=0,098F_c
F_{x1}=F_{x2}=0,196F_c
F_{x1}=F_{x2}=0,392F_c

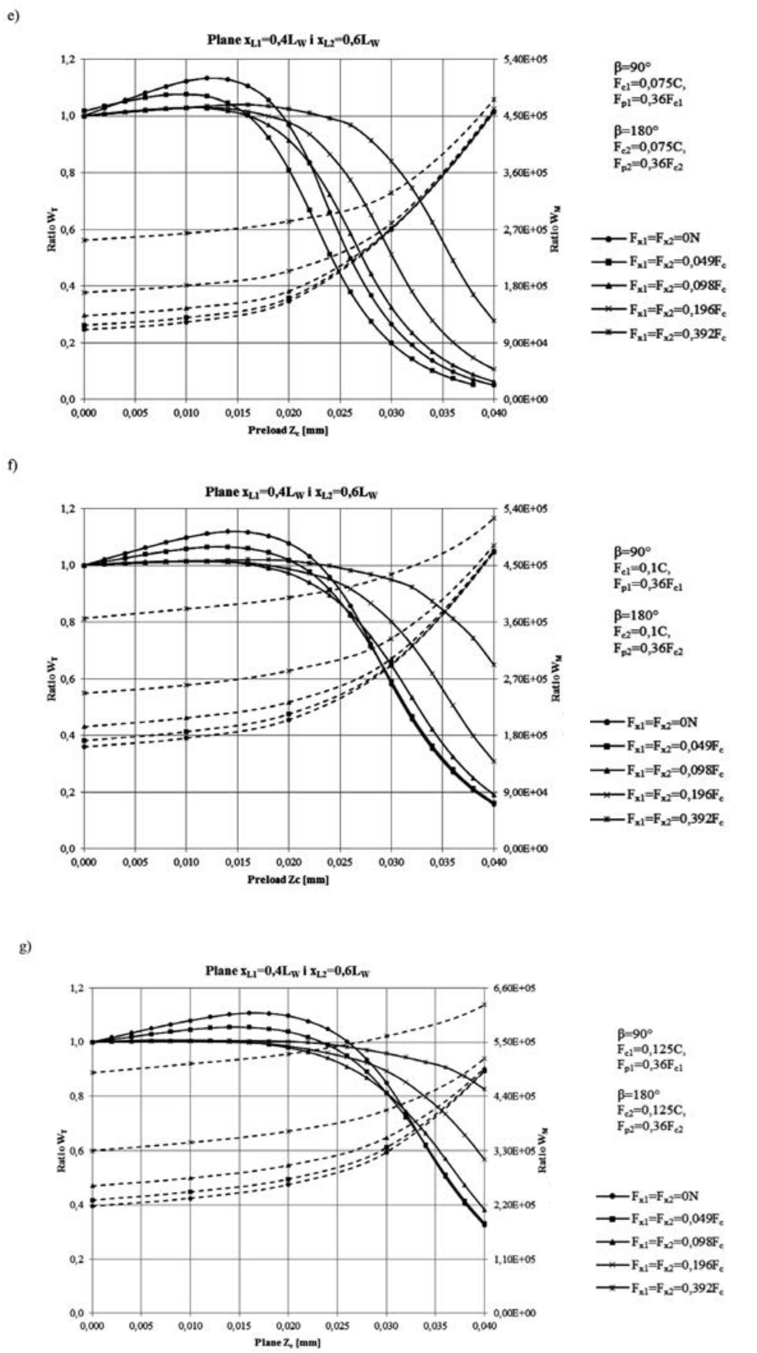
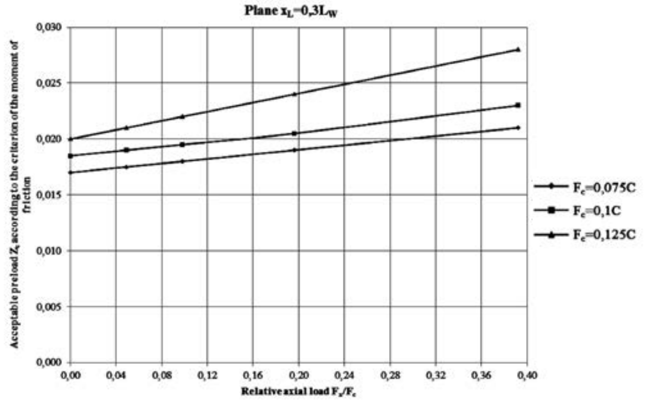


Fig. 5. Exemplary characteristics of W_M indicator for the assumed variation of load (continuous line – indicator of fatigue life W_T, dotted line – indicator of the frictional moment W_M)

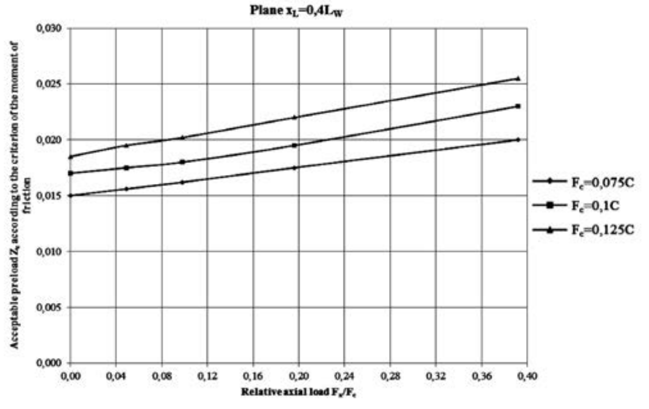
Rys. 5. Przykładowe charakterystyki wskaźnika W_M dla przyjętego wariantu obciążenia (linia ciągła – wskaźnik W_T, linia przerywana – wskaźnik W_M)

3. If the resultant of axial forces loads the bearing of the system that is a little more loaded transversally (which results from the location of the plane of load defined by the coordinate $x_L = 0.6\ L_W$), characteristics of the acceptable Z_c fall in the left part, and after that, from a certain point, they begin to rise. For a certain value of relative axial load, a minimum for curves Z_c is observed. Proportionality Z_c to the relative transversal load F_c/C is similar to the one in previous cases. The line representing $F_c = 0.1$ C begins from the value $Z_c \approx 0.017$ mm.
4. Further moving of the plane of loads away from the centre of the spacing of bearings towards the bearing loaded by axial forces (determined by the coordinate of the plane of load $x_L = 0.7\ L_W$) causes the opposite course of characteristics than in the previous case.

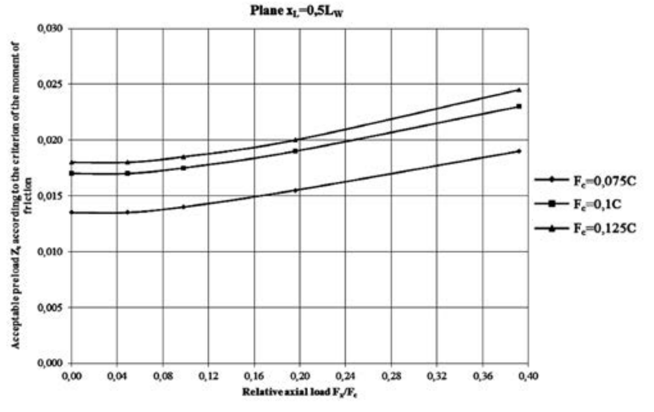
a)


Plane x_L=0,3L_W
Acceptable preload Z_a according to the criterion of the moment of friction
Relative axial load F_a/F_c
F_c=0,075C
F_c=0,1C
F_c=0,125C

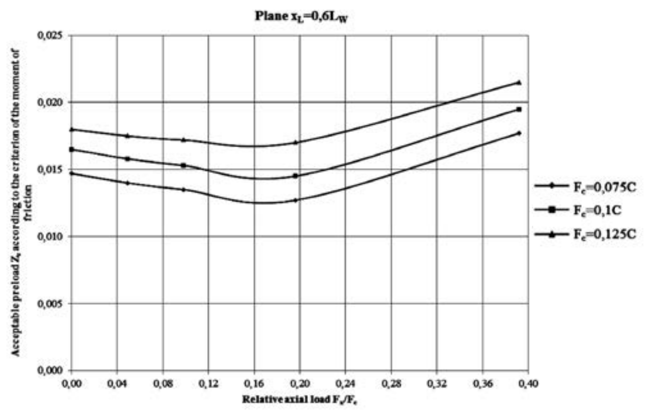
b)


Plane x_L=0,4L_W
Acceptable preload Z_a according to the criterion of the moment of friction
Relative axial load F_a/F_c
F_c=0,075C
F_c=0,1C
F_c=0,125C

c)


Plane x_L=0,5L_W
Acceptable preload Z_a according to the criterion of the moment of friction
Relative axial load F_a/F_c
F_c=0,075C
F_c=0,1C
F_c=0,125C

d)


Plane x_L=0,6L_W
Acceptable preload Z_a according to the criterion of the moment of friction
Relative axial load F_a/F_c
F_c=0,075C
F_c=0,1C
F_c=0,125C

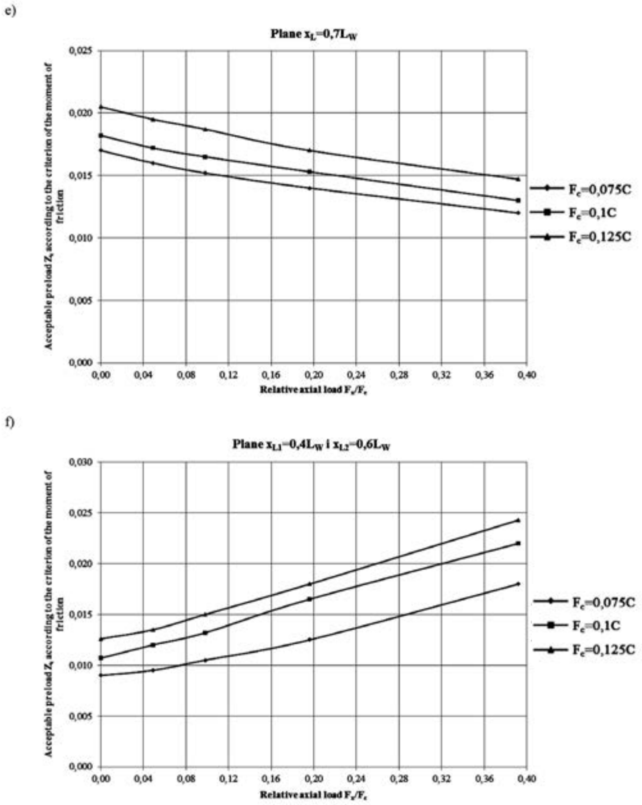


Fig. 6. Acceptable preload Z_c according to the criterion of the moment of friction in the function of relative axial load F_x/F_c

Rys. 6. Dopuszczalny zacisk wstępny Z_c wg kryterium momentu tarcia w funkcji względnego obciążenia osiowego F_x/F_c

The characteristics represent a descending course of a shape similar to the linear one. Similar to previous cases, the proportionality of the value Z_c to the relative transversal load F_c/C has been maintained.

5. In case of the appearance of two planes of loads arranged symmetrically (It is Variation II of the load, in accordance with **Fig. 6b**), characteristics Z_c have indirect courses between the cases described in Points 1 and 2, i.e. the ascending ones, but on a lower level of values, because the line responding to F_c = 0.1 C begins from the value $Z_c \approx 0.01$ mm. Proportionality Z_c to the relative transversal load in the system of bearings F_c/C is similar to all the previous cases.

SUMMARY

Four types of characteristics result from the collected graphs: two rising on different levels, falling–rising and falling. The first rising characteristics can be attributed to the first three locations of the plane of load according to Variation I of load, i.e. x_L = (0.3–0.5) L_W. The second rising characteristic responds to Variation II of the loads' location. Rising-falling characteristics respond to Variation I of the load with the coordinate x_L = 0.6 L_W; whereas, the falling ones respond to x_L = 0.7 L_W. Thus, selected characteristics responding to the relative transversal load of the bearing F_c/C = 0.1 are presented together in **Fig. 7**.

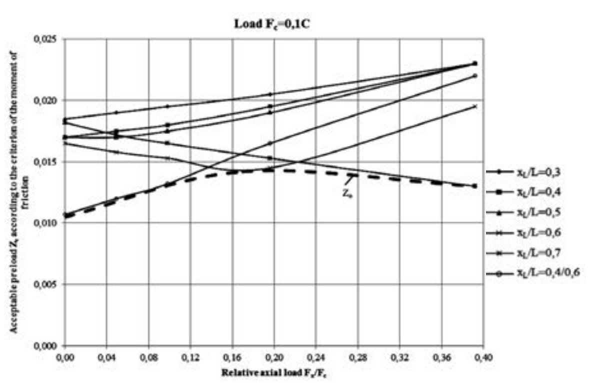


Fig. 7. Dependence of the preload Z_c on the axial load, for different locations of loads

Rys. 7. Zależność granicznego zacisku wstępnego Z_c od obciążenia osiowego, przy różnych położeniach obciążeń

The dotted line represents the values of limit preload Z_C, which is appropriate for the possibly widest range of loads locations.

Connecting together the conclusions from this study with the conclusions from study **[L. 4]** will be a subject for further analyses used for developing the rules of selection of the value of preload of angular contact ball bearings.

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