Transients in Transformers with Non-Uniform Inductance and Capacitance Distributions

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Summary: **The electromagnetic transients in transformer windings exhibiting location–dependent inductances and capacitances are investigated in the time domain. Analytical functions describing this dependence are assumed and incorporated in the two integro–differential equations governing the transient voltage and current distributions. The boundary conditions are available from the source initiating the transients and the winding's end termination. A numerical procedure is applied in order to get frequency domain solutions for the voltage and current in the form of Interpolating and Parametric Functions. The numerical Laplace inversion is then applied to these *s*–domain expressions. Results pertinent to transients initiated by step- and double-exponential impulse sources are presented and discussed. All possible transformers' neutral connections are considered. The possible error introduced by neglecting either or both of the inductance and capacitance non-uniformities is addressed. Results indicate that the main error is attributed to neglecting the inductance non-uniformity, whereas the impact of the capacitance non-uniformity is relatively small. In most cases, the winding's copper and insulation losses have a small effect on the transient response.**

Key words:
distributed parameter circuits, electromagnetic transients, integro-differential equations, modeling, mutual parameters, non-uniform, power transformers, transient response, winding

I. INTRODUCTION

The time- and frequency-domain analyses of transformer windings were the topic of numerous studies such as those documented in [1–14] and [17]. In particular, the frequency, and the complex s-domain approach, has been successfully used in order to derive analytical expressions for the voltages current distributions along the windings and for identifying their resonance frequencies [4–6].

In reference [7], the winding is represented by cascading a number of generally non-identical ladder circuits. This is followed by solving the corresponding simultaneous differential and algebraic equations. The model could be refined by applying an alternative concentrated-parameter recursive *s*-domain analytical solution technique [8].

References [10], [11] suggest a more accurate and efficient approach based on the distributed parameter analysis utilizing the frequency- and location-dependent *A, B, C, D* circuit constants, adopted from the transmission line theory.

In [12], a direct analytical procedure for analyzing windings with non-uniform series inductance is presented based on the assumption of a quadratic inductance distribution. The analysis of windings exhibiting non-uniform inter-turn insulation capacitance is given in [13]. It is based on a time-domain numerical solution of partial differential equations with location-dependent coefficients. An analytical solution is suggested in [14] based on the cascade connection of equivalent transmission lines. Relations addressing transformers with abruptly changing insulation characteristics at junctions between uniform winding sections are presented in [17]. This method could be successfully applied to the more realistic situations involving windings with gradually changing capacitance which can be described by simple mathematical functions.

It should be noted that the calculation of the non-uniform inductance addressed in [12] was based on the very simplifying assumption that the current is uniform along the winding. The mutual flux linkage between any two turns or sections depends, however, on the geometrical details as well as the instantaneous current distribution, which is a part of the still unknown solution. Similar statements can be also made regarding the inter-turn capacitive coupling. This paper tries to provide a rigorous approach to this problem. The procedure will start with formulating the two governing integro-differential equations of voltage and current, in terms of the winding data, the voltage or current sources, the neutral point treatment and the complex frequency *s*.

Analytical relations expressing the non-uniformity of the winding's mutual inductive and capacitive couplings will be suggested. They will be helpful in deriving direct solutions of the two equations utilizing the powerful built-in features of modern computer packages such as *Mathematica* [15], [16]. The candidate equations for the inter-turn mutual couplings will be derived from measured winding data published elsewhere [2]. In principle, numerical solutions of the integro-differential equations may be possible using one of the lengthy available standard procedures such as those in [18-20]. Paper [21] suggests, however, an alternative straight forward and easily programmable solution technique in the frequency domain. Hosono algorithm, [22], will then be used in order to perform the numerical Laplace inversion of the derived s-domain expressions for the current and voltage distributions. These expressions are available as the solution output of the *Mathematica* program in the form of two Interpolating and Parametric Functions. Some modifications will be needed in order to reduce the current and voltage integro-differential equations, which deal originally with two independent variables to make them in terms of just one single independent variable. In addition to results of case studies addressing different wave forms for the sources initiating the transients and the possible transformer's neutral treatments, the impact of each of parameters' non-uniformities (inductance or capacitance) on the accuracy of computation will be addressed.

II. METHOD OF ANALYSIS

According to Figure 1, adopted from [21], the considered winding of a total length 1 per unit connects the supply point at its sending end with the winding's neutral point. Their co-ordinates are $x = 0$ and $x = 1$, respectively. The voltage and current distributions along the winding are denoted $v(x)$ and $i(x)$, respectively. The source voltage is $v(0) = e$ pu. The condition at the neutral point is described by: $v(1) = Z_N.i\ (1)$, where Z_N is the impedance connecting the neutral point to ground.

The winding circuit parameters and their assumed numerical values are as follows:

r_{series} series resistance = 2.198 Ω
c_o shunt (earth) capacitance = 20.7 nF

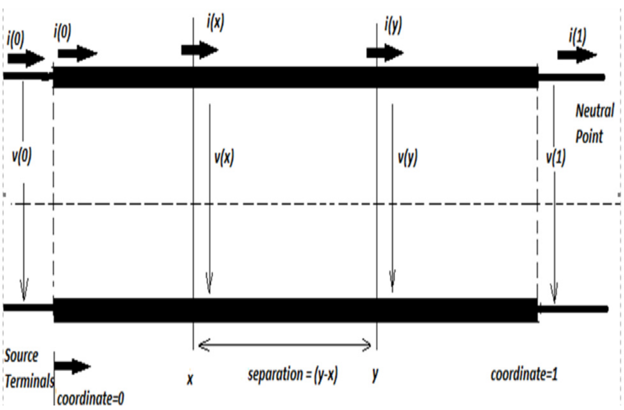


Fig. 1. The considered transformer winding.

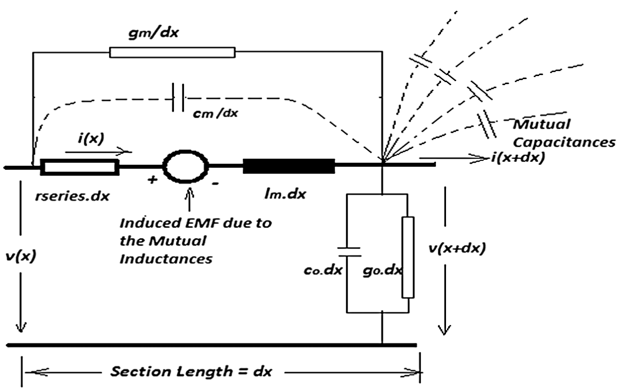


Fig. 2. The equivalent circuit of an infinitesimal winding section of length *dx* per unit.

g_o shunt conductance = 0.15 nS
c_m series capacitance (mean value) = 2.07 pF
Its maximum value is $c_s/0.6934$ as explained later.
g_m winding insulation conductance = 2.07 nS
l_m the winding's self inductance (mean value) = 0.021 H
The maximum value is $l_m/0.6934$.
These parameters are adopted from [8], [13] and [21].

The detailed equivalent circuit of an infinitesimal winding section of per unit length *dx* is depicted in Figure 2. In addition to the resistances, conductances, the self and mutual capacitances, it includes a voltage source. It represents the sum of all the electromotive forces (EMF) induced due to the distributed mutual inductances.

Figure 3 depicts the dependence of the measured values, adopted from [2], of the pu mutual inductance between any two sections of coordinates x and y, on the magnitude of their separation $(y-x)$ pu. The base value of the mutual inductance is 29 mH. The value decreases steadily from 29 mH for zero separation to about 0.292 mH between the first and last sections. In order to simplify the analysis, this relation is replaced by the approximate separation function:

$$f(x,y)=[2.8396-1.8396\,[1+k(y-x)^2/2!]] \qquad (1)$$

This function has to be an even one with respect to $(y-x)$. The coefficient k can assume the values $k=0$ or $k=1$ to describe the cases in which the non-uniformity of the mutual coupling is either neglected or taken into account, respectively. The average value of $f(x,y)$ is 0.6934 per unit. As will be discussed later, other possible mathematical expressions for this function can be similarly adopted.

The accuracy of the analysis can be further improved if more terms are such as $k(y-x)^4/4!$, $k(y-x)^6/6!$, etc. are added to equation (1). This will also increase the complexity of the derivations. Another possible simpler approximation for the separation function is described by:

$$f(x,y)=\cos\left[\frac{\pi(y-x)}{2}\right] \qquad (2)$$

Both approximations (1) and (2) yielded almost identical numerical results.

The rate of voltage drop at the location x includes the resistive and self-inductive parts due to the local current $i(x)$ and the integration of all the inductively induced components due to the currents at the other points along the entire winding. After some modifications, the voltage integro-differential equation will be:

$$v'(x)=-2.198\,i(x)-s.l_m[(1-0.9198\,k\,x^2)\times \\ \times\int_0^1 i(x)dx-0.9198\,k\int_0^1 x^2 i(x)dx+1.8396\,k\,x\int_0^1 x\,i(x)dx \qquad (3)$$

The integro-differential equation of the current $i(x)$ can also be derived as follows:

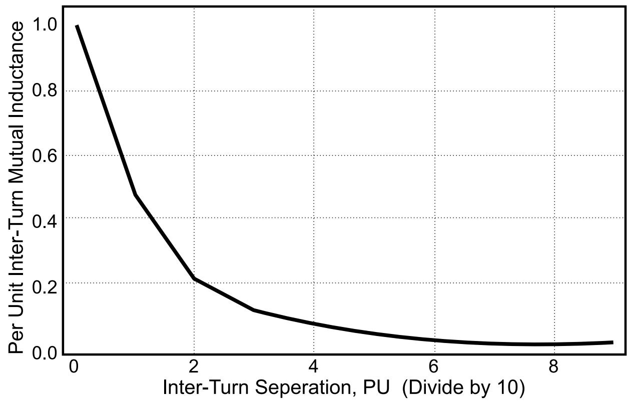


Fig. 3. Relation between the per unit mutual inductance between two winding section and the magnitude of their per unit separation (*y*-*x*).

$$i'(x) = -v(x).(s.co + g_o) + \int_0^1 \left[v(y) - v(x) \right] . \\ .\left(s.c_{m.} f(x,y) + g_m \right).dy \tag{4}$$

After substituting $f(x,y)$ from equation (2) and with some simplifications,

$$i'(x) = -v(x).(s.\mathrm{co} + g_o) + (s.\mathrm{c}_m + g_m)((0.9198\,k\,x^2 - 1).v(x) + \\ + \left(1 - 0.9198\,k\,x^2\right)\int_0^1 v(x)\mathrm{d}x - 0.9198\,k\int_0^1 x^2 v(x)\mathrm{d}x + \\ + 0.3066\,kv(x) - 0.9198\,k\,x\,v(x) + 1.8396kx\int_0^1 x\,v(x)\mathrm{d}x) \tag{5}$$

in which s denotes the complex frequency, equal to $j\omega$ for steady state sinusoidal conditions.

If the alternative expression of $f(x,y)$ given by equation (2) is used, the resulting voltage integro-differential equation will be

$$v'(x) = -r_{series}.i(x) + s.l_m \int_{y=0}^{1} i(y).\cos\left[(x-y)m\right]dy = \\ = -r_{series}.i(x) + s.l_m \cos(mx)\int_0^1 i(y).\cos(my)dy + \\ + s.l_m \sin(mx)\int_0^1 i(y).\sin(my)dy = \\ = -r_{series}.i(x) + s.l_m \cos(mx)\int_0^1 i(x).\cos(mx)dx + \\ + s.l_m \sin(mx)\int_0^1 i(x).\sin(mx)dx \tag{6}$$

with $m = \dfrac{\pi}{2}$ and for the current:

$$i'(x) = -v(x)[g_o + s.c_g] + \\ + s.c_m \int_0^1 v(y)[\cos(mx)cos(my) + \sin(mx)\sin(my)]dy - \\ - s.c_m.v(x)\int_0^1 [\cos(mx)cos(my) + \sin(mx)\sin(my)]dy = \\ = -v(x)[g_o + s.c_g] + s.c_m[\cos(mx)\int_0^1 v(x)\cos(mx)dx + \\ + \sin(mx)\int_0^1 v(x)\sin(mx)dx] - \\ - s.c_m v(x)(\frac{2}{\pi})[\cos(mx) + \sin(mx)] \tag{7}$$

Either one of the two equation pairs (3) & (4) or (6) & (7) could be solved by using the *Mathematica* command (ParametricNDSolve). The output will give the results in the form of two Interpolating and Parametric Functions for the voltages and current at any winding point in terms of its co-ordinate x and the angular frequency $j\omega$. More details are available in [15], [16] and [21]. The Laplace or s-expressions $v[s,x]$ and $i[s,x]$ can be easily obtained by substituting $\omega = -js$.

The numerical inversion can be obtained according to Hosono:

$$f(t) \approx (\frac{e^a}{t})[\sum_{n=1}^{l-1} F_n + \left(\frac{1}{2^{m+1}}\right)\sum_{n=0}^{m} A_{m\,n} F_{l+n}]$$

where

$$F_n = (-1)^n\, Im[F\left(a + \frac{j(n-0.5)\mathrm{p}}{t}\right)]$$

where

$$a >> 1, A_{m\,m} = 1, A_{m\,n-1} = A_{m\,n} + \binom{m+1}{n}$$

III. SAMPLE RESULTS

The following are sample results describing the transient response of the transformer winding to both a unit-step and an impulse voltage source of the waveform $e(t) = 1.034\,[\exp(-t.10^6/16.8) - \exp(-t.10^6/0.085)$ where t is the time in seconds. This double-exponential function has a peak value of 1 per unit occurring at $t = 1.2$ μs.

Four possible connections of the neutral point will be considered: solid-earthing, isolated neutral, resistive earthing and inductive earthing via a Petersen coil.

The results depict the solutions with and without taking the parameter non-uniformities into account.

A. Step- Response

1. Solidly-Earthed Neutral

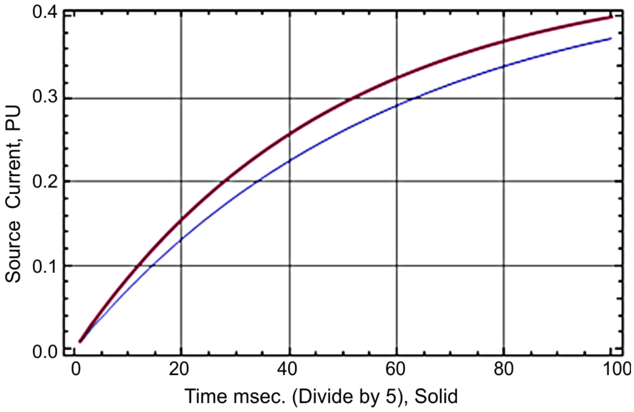


Fig. 4. The unit-step response of solidly-earthed transformer:
Thin curve: both non-uniformities considered,
Thick curve: both non-uniformities neglected.

Figure 4 shows the source current over the time range $0 \leq t \leq 20$ ms. The effect of the different winding capacitances cannot be easily recognized over this time scale. In fact, it exhibits some higher frequency oscillations that diminish during the first milliseconds. The transient is dominated by the resistive and inductive elements. Both curves are close to the response of simple series *RL*-circuits. Neglecting the non-uniformity in both the capacitances and inductances would lead to faster response (the thick curve). In both cases, the current final value was found to be 0.455 per unit (or approximately 1 V/2.198 Ω), as expected.

2. Isolated Neutral

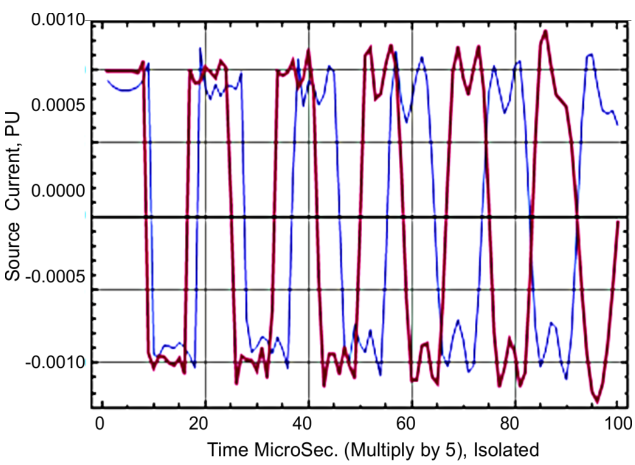


Fig. 5. The unit-step response of the transformer with isolated neutral:
Thin curve: both non-uniformities considered,
Thick curve: both non-uniformities neglected.

The wave form of the source current given in Figure 5 indicates that the magnitude of the input impedance immediately after switching is about 1 kΩ. From the wave reflections, the winding travel time is slightly shorter than 22 μs. The natural frequency is accordingly slightly higher than 11 kHz. This is in good agreement with the results of reference [21]. Again here, the response is faster if the parameters' non-uniformities are not considered. Due to the energy dissipation in the winding's copper and the dielectric losses, the current curves will asymptotically approach the expected final value g_o = 0.15 nA/1 V.

3. Resistively-Earthed Neutral

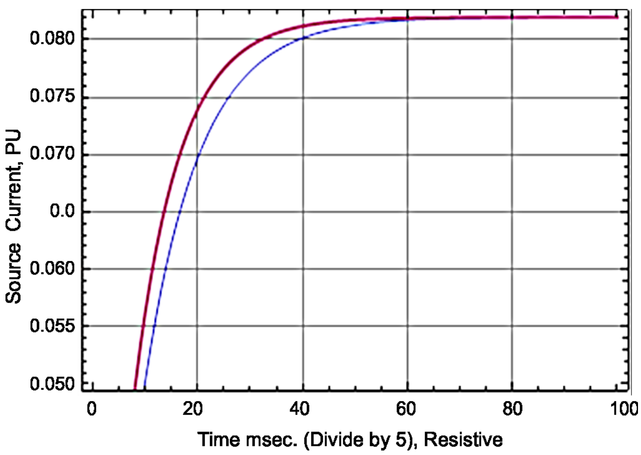


Fig. 6. The unit-step response of the transformer with a 10 Ω neutral earthing resistance:
Thin curve: both non-uniformities considered,
Thick curve: both non-uniformities neglected.

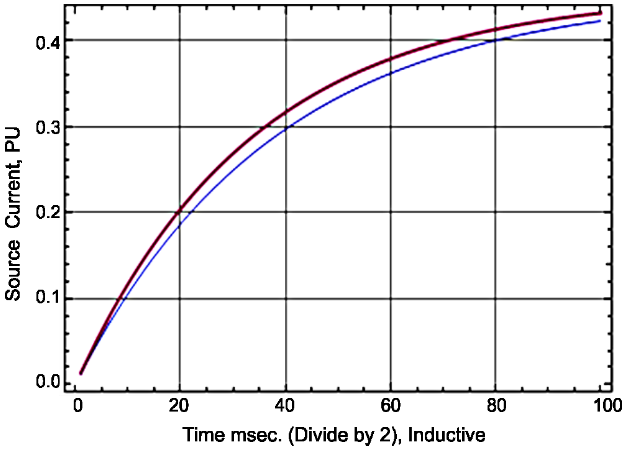
According to Figure 6, both curves of the current source exhibit the expected final value of 1 V/(10+ 2.198) Ω = 82 mA. The circuit has an approximate time constant of 21 mH/(10 + 2.198) Ω = 1.72131 ms.

An approximate estimate for the neutral voltage can be obtained by multiplying the current values by the earthing resistance 10 Ω, with a time delay of about 25 μs.

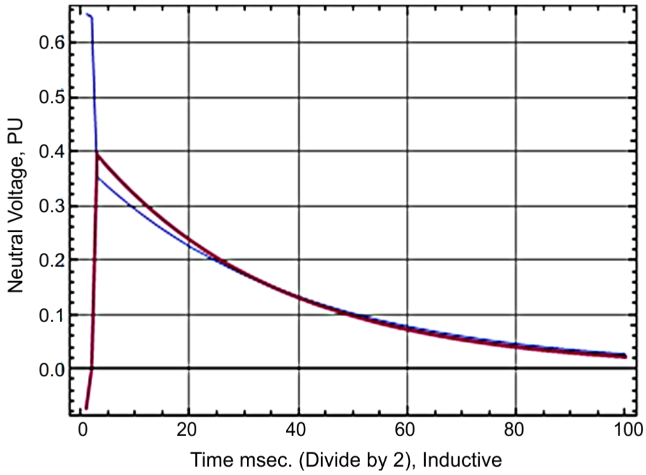
4. Inductively-Earthed Neutral

The plots in Figure 7 depict the transients following the application of a unit-step source voltage, if the transformer's neutral point is earthed via a pure inductive Petersen coil of a reactance of 5 Ω (at 50 Hz). Its inductance is accordingly 16 mH.

The source current plots in Fig, 7-(a) indicate a slower increase in comparison with those in Fig. 4. This is due to the increase of the circuit inductance from 21 mH to 37 mH. The final values of both current curves are 0.455 per unit, as expected from DC analysis. The neutral point voltage computed with- and without- considering the parameters' non-uniformities are given in plots 7-(b). The initial point after the travel time of about 22 μs is around 0.40 per unit, in good agreement with the potential division among the series connected winding's and the Petersen coil's inductances. The neutral voltage will then decay almost exponentially to zero.



(a) The source current



(b) The voltage at the neutral point

Fig. 7. The unit-step response of the transformer with Inductive neutral earthing:
Top: The source current,
Bottom: The voltage at the neutral point.
Thin curves: both non-uniformities considered, Thick curves: both non-uniformities neglected,

B. Response to an Impulse Voltage

The following results illustrate the winding's response to an impulse voltage source of the waveform

$e(t) = 1.034\,[\exp(-t.10^6/16.8) - \exp(-t.10^6/0.085)$

where t is the time in seconds. This double-exponential function has a peak value of 1 per unit occurring at $t = 1.2$ μs.

1. Solidly-Earthed Neutral

As mentioned earlier, the input impedance immediately after switching is about 1 kΩ. This explains the initial value of the source current of about 0.001 per unit. After some exponential decay over a time span equal to double the winding's travel time of 22 μs, the current then jumps to more than 0.002 per unit. This is due to the wave reflection at the neutral with a reflection coefficient of –1. The response is faster if the non-uniformities are neglected. The current peaks decrease with time due to the winding's copper and dielectric losses. The final value of the current is zero.

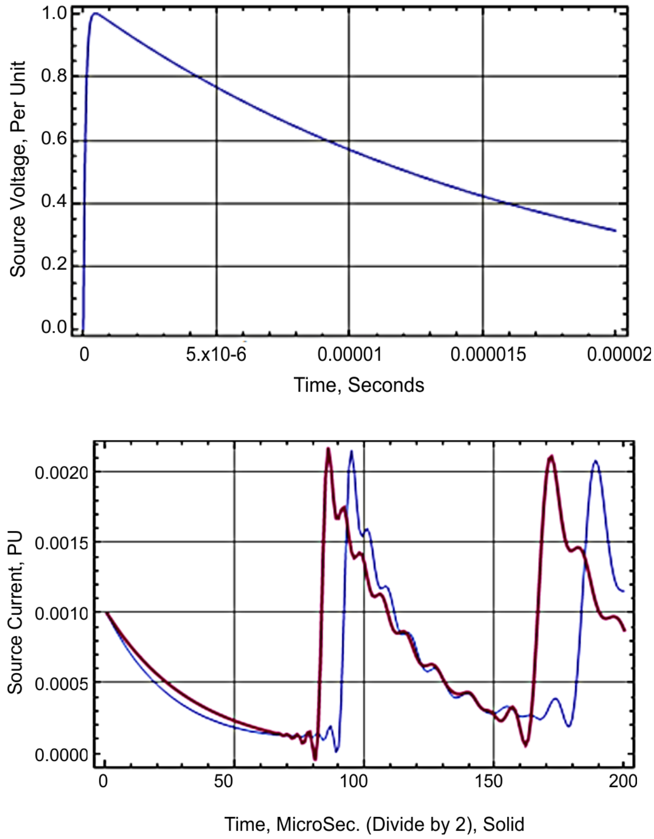


Fig. 8. The response of the solidly-earthed transformer to an impulse voltage source of the waveform
$e(t) = 1.034\,[\exp(-t.10^6/16.8) - \exp(-t.10^6/0.085)$, t is the time in seconds (shown on the top)
Thin curve: both non-uniformities considered,
Thick curve: both non-uniformities neglected.

2. Isolated Neutral

Figure 9 illustrates the source current in the case of isolated neutral. Over the time range $0 \le t \le 44$ μs, i.e.

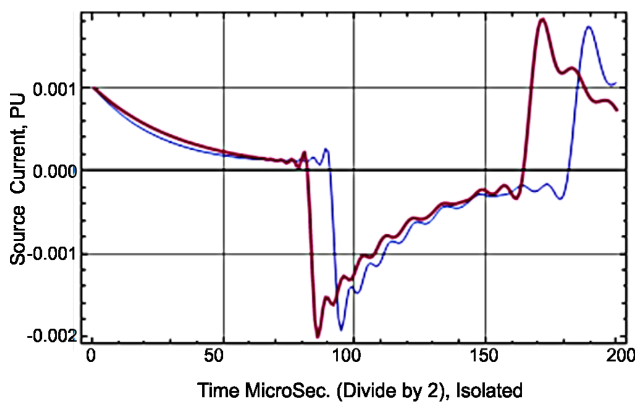
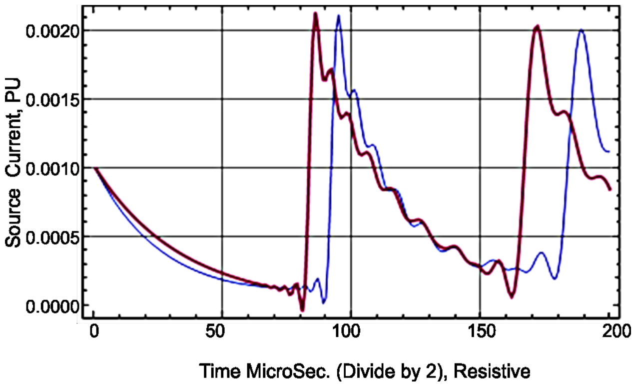
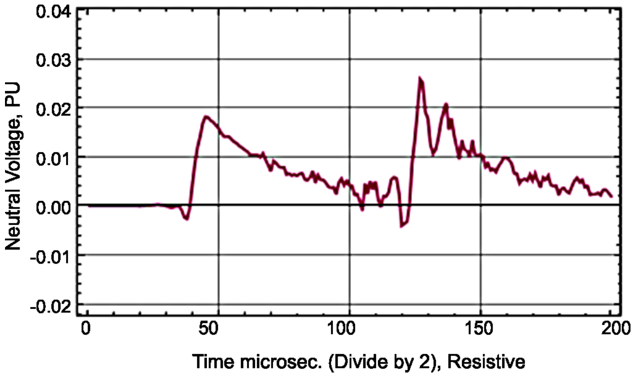


Fig. 9. The response of the transformer with isolated neutral to the impulse voltage source
Thin curve: both non-uniformities considered,
Thick curve: both non-uniformities neglected.



(a) Source Current



(b) The voltage at the neutral point

Fig. 10. Response of the transformer source current (top) and neutral voltage (bottom)with a resistively-earthed neutral to the impulse voltage source
Thin curve: both non-uniformities considered,
Thick curve: both non-uniformities neglected.

before twice the winding's travel or delay time, there is no difference between Figures 8 and 9. The reflection at the neutral point is manifested by the negative current peak of around –0.0018 per unit close to t = 44 µs. The magnitude of the positive and negative peaks decrease due to the damping introduced by the losses. The final value of the source's current is zero.

3. Resistively-Earthed Neutral

Plots (a) and (b) of Figure 10 show the source current and neutral voltage if the transformer's neutral is earthed through a 10 Ω ohmic resistance. Due to this relatively small earthing resistance, with respect to the winding's surge impedance of about 1 kΩ, there is no difference between these current curves and those of the solid-earthing case, Fig. 8. Additional results indicate that the parameters' non-uniformities have a negligible effect on the neutral voltage (right plot). It shows that the voltage starts to change only after the travel time of 22 µs elapses. After double this duration, i.e. after wave return from a round excursion to the source and back, a second voltage rise occurs. The neural current can be obtained by dividing the voltage by the 10 Ω earthing resistance.

4. Inductively-Earthed Neutral

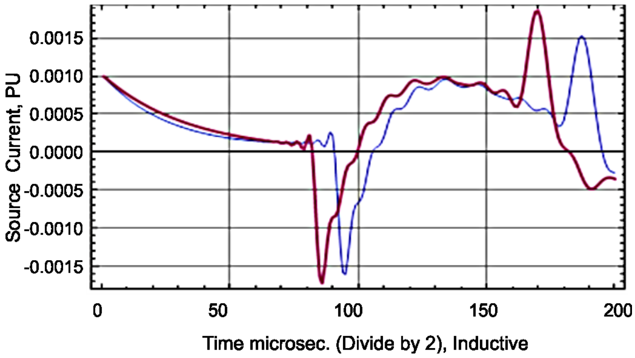


Fig. 11. The response of the transformer with an inductively-earthed neutral to the impulse voltage source
Thin curve: both non-uniformities considered,
Thick curve: both non-uniformities neglected.

The transformer's neutral point is earthed via a 16 mH pure inductive Petersen coil. The transient source current is shown in Figure 11 and is identical to the previous cases for $0 \leq t \leq 44$ µs. The importance of taking the non-uniformities into account can be clearly recognized. Neglecting them would lead to an erroneous time lead of about 5 µs in addition to a considerable increase in the positive and negative current peaks.

IV. ACCURACY CONSIDERATIONS

The Relative Impact of Each Non-Uniformity

This section tries to assess the error resulting from ignoring one or both of the parameters' non-uniformities. For each of the above-mentioned cases, an additional run was made neglecting only the non-uniformity of the winding's capacitance distribution. Typical results are given in Figure 12 addressing the response of the transformer (with isolated neutral point) to a double-exponential impulse source voltage. The plot includes three curves for the source current. The thick one is pertinent to the results when neglecting both inductance and capacitance non-uniformities. The second (thin) trace corresponds to the computations with both non-uniformities included. The third (dotted) curve gives the results when

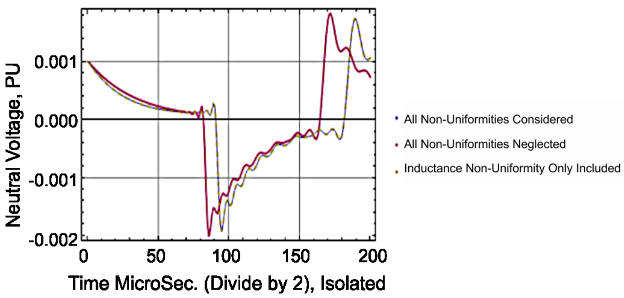
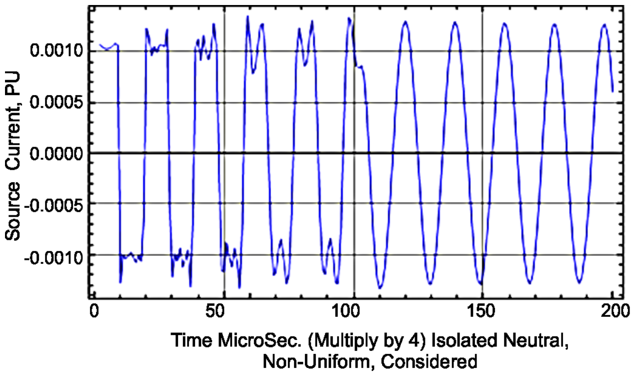
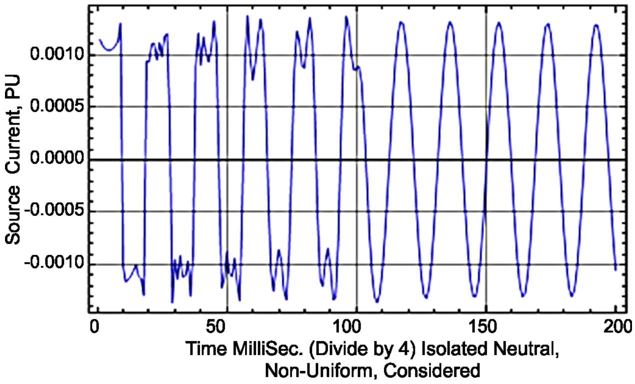


Fig. 12. The response of the transformer with an inductively-earthed neutral to the impulse voltage source, $0 \leq t \leq 400$ µs.



(a) Using equations (3) & (4)



(b) Using equations (5) & (6)

Fig. 13. The response of the transformer with an inductively-earthed neutral to the unit-step voltage source.

only the inductance non-uniformity is considered. It can be seen that there is practically no difference between the thin and dotted curves. This implies that the error resulting from neglecting the capacitance non-uniformity is much less than that of not taking the inductance non-uniformity into account.

The Choice of The Separation Function $f(x,y)$

In this section, the results of using the two suggested expressions for the separation function $f(x,y)$ are compared. The first one are the quadratic equations for the inductance and capacitance distributions given by Eqn. (1) leading to the two integro-differential equations (3) and (5). The second alternative expression uses the cosine function, yielding the two other alternative equations (6) and (7). The corresponding results for the source current (with isolated neutral) are depicted in the plots (a) and (b) of Figure 13, respectively. The source voltage has a unit-step waveform. The two plots are almost identical.

V. SUMMARY AND CONCLUSIONS

1. The electromagnetic transients in transformer windings exhibiting location-dependent inductance and capacitance distributions are investigated in the time domain. Functions describing this dependence are assumed and incorporated in the two integro-differential equations governing the transient voltage and current. They have two independent variables and include the winding data, the voltage sources, the neutral point treatment and the complex frequency s. Some modifications were needed in order to make them in terms of just one single independent variable.
2. A numerical procedure is applied in order to get frequency domain solutions in the form of interpolating functions. Numerical Laplace inversion is then applied to the frequency–domain expressions. Results pertinent to transients initiated by step- and double-exponential impulse sources are presented. All possible transformers' neutral connections are considered.
3. The error introduced by neglecting either or both of the inductance and capacitance non-uniformities is addressed. Results indicate that the main error component is attributed to neglecting the inductance non-uniformity, whereas the impact of the capacitance non-uniformity is relatively small. In most cases, the winding's copper and insulation losses have a small effect on the transient response.
4. Neglecting the non-uniformity in both the capacitances and inductances would lead to an erroneous faster response.
5. The magnitude of the winding's input impedance immediately after switching is about 1 kΩ and the winding travel time is slightly shorter than 22 µs. This explains the initial value of the source current of about 0.001 per unit and a natural frequency which is slightly higher than 11 kHz.
6. The results of using two suggested expressions for the separation function are compared. The first one is a quadratic equation for the inductance and capacitance distributions, whereas the second alternative expression uses the cosine function. The two approaches yield almost identical solutions.

REFERENCES

[1] A. Greenwood, Electrical Transients in Power Systems, 2nd ed. Wiley-Interscience, 1991, ch. 11.

[2] R.C. Degeneff, "A General Method for Determining Resonances in Transformer Windings," *IEEE Trans. Power App. Syst.*, vol. 96, no. 2, pp. 423–430, Mar. 1977.

[3] S. Hosseini, M. Vakilian and G. Gharehpetian, "Comparison of Transformer Detailed Models for Fast and Very Fast Transient Studies," *IEEE Trans. Power Del.*, vol. 23, no. 2, pp. 733–741, Apr. 2008.

[4] M.M. Saied and A.S. AlFuhaid, "Electromagnetic Transients in Line-Transformer Cascade by a Numerical Laplace Transform Technique," *IEEE Trans. Power App. Syst.*, vol. 104, no. 10, pp. 2901–2909, Oct. 1985.

[5] M.M. Saied and A.S. AlFuhaid, "Frequency Response of Two-Winding Transformers Obtained by a Distributed-Parameter s-Domain Method," *J. Electric Power Components and Systems*, vol. 32, no. 8, pp. 755–766, Aug. 2004.

[6] R. Jayaratchagan and B. Shriram, "Impulse Voltage Distribution", *International Journal of Engineering Research & Technology (IJERT)*, vol. 2, no. 4, pp. 1203–1207, Apr. 2013.

[7] M.M. Saied, "New Solution Technique for the Frequency and Transient Response of Transformer Windings with All Inter-

Turn Mutual Inductances and Capacitances Included," *Trends In Electrical Engineering*, vol. 3, no. 1, 2013.
[8] M. Saied, "A Contribution to the Frequency Analysis and the Transient Response of Power Transformers' Windings," *J. Electric Power Components and Systems*, vol. 42, no. 11, pp. 1143–1151, Jul. 2014.
[9] M. Popov, L. Sluis and R. Smeets, "Complete analysis of Very Fast Transients in Layer-type Transformer Windings," presented at the International Conference on Power Systems Transients (IPST'07), Lyon, France, 2007.
[10] M.M. Saied, "Transformer Modeling for The Frequency and Transient Analyses, with Non-Uniform Inductance Emulating The Inter-Turn Magnetic Coupling," *Electrical Power and Utilisation J.*, vol. 17, no. 1, pp. 1–11, Jun. 2014.
[11] A. Predota, Z. Benesova and L. Koudela, "Analysis of Transients in Transformer Winding Respecting Space-Varying Inductance", *Przegląd Elektrotechniczny*, vol. 88, no. 7b, pp. 220–222, Jul. 2012.
[12] M.M. Saied, "Modeling of Transformer Windings having Nonuniform Inductance Distribution," *J. of Power Electronics & Power Systems*, vol. 3, no. 3, pp. 12–25, 2014.
[13] A. Predota and Z. Benesova, "Fast Transient Overvoltage in Transformer Winding," *Przegląd Elektrotechniczny*, vol. 87, no. 5, pp. 142–145, May 2011.
[14] M.M. Saied, "The Transient Response and Frequency Characteristics of Power Transformers Having Non-Uniform Winding Insulation," *J. of Power Electronics & Power Systems*, vol. 4, no. 1, pp. 37–52, 2014.
[15] Wolfram Research Inc. (2014), *Documentation on the software Mathematica 10*. [On-line]. Available: http://www.wolfram.com/mathematica/?source=nav [2014].
[16] Wolfram Research Inc. (2008), *Wolfram Mathematica Tutorial Collection: Advanced Numerical Differential Equation Solving in Mathematica*. [On-line]. Available: http://www.wolfram.com/learningcenter/tutorialcollection/AdvancedNumericalDifferentialEquationSolvingInMathematica/ [2008].
[17] M.M. Saied, "An Analytical Method for the Analysis of Transformer Windings Having Non-Uniform Capacitance Distribution," *J. of Power Electronics & Power Systems*, vol. 5, no. 2, pp. 1–14, 2015.
[18] J. Manafian, "Solving the integro-differential equations using the modified Laplace Adomian decomposition method", *J. of Mathematical Extension*, vol. 6, no. 1, pp. 1–15, 2012.
[19] E. Hesameddini and E. Asadolhifard,"Solving Systems of Linear Volterra Integro-Differential Equations by Using Sinc-Collocation Method," *Int. J. of Mathematical Engineering and Science*, vol. 2, no. 7, pp. 1–9, Jul. 2013.
[20] M. Turkyilmazoglu, "High-order nonlinear Volterra–Fredholm-Hammerstein integro-differential equations and their effective computation," *Applied Mathematics and Computation*, vol. 247, pp. 410–416, Nov. 2014.
[21] M. Saied, "Frequency Characteristics of Transformer Windings with Separation-Dependent Inter-Turn Mutual Parameters," *J. of Power Electronics & Power Systems*, vol. 6, no. 3, pp. 72–81, 2016.
[22] T. Hosono, "Numerical Inversion of Laplace Transform with some Applications to Wave Optics," *Radio Science*, vol. 16, no. 6, pp. 1015–1019, Nov.-Dec. 1981.

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