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Reliability and risk function improvement of bulk cargo transportation system

Keywords

reliability, reliability improvement, multistate system, large system

Abstract

In the paper the basic notions of the ageing multistate systems reliability analysis are introduced. The multistate system reliability functions are defined and the mean values of the multistate system lifetimes in the reliability state subsets and in the particular reliability states are determined. The notion of the multistate system risk function and the moment of the system exceeding the critical reliability state are introduced. Further, in the developed reliability models, it is assumed that the system's components have the multistate Weibull reliability functions with various parameters in their different reliability state subsets. Under this assumption, the proposed multistate system reliability models are applied in maritime transport to the reliability analysis of a bulk cargo transportation system and its reliability function, moreover other main characteristics are determined.

1. Introduction

Taking into account the complexity of the failure processes of real technical systems, it seems reasonable to expand the two-state approach to multi-state approach [2] in the system reliability analysis. The assumption that the system is composed of multistate components with reliability states degrading in time [1] gives possibility for more precise analysis of its reliability. This assumption allows us to distinguish a system reliability critical state, which exceeded is either dangerous for the environment or does not assure the necessary level of the effectiveness of its operations process. Then, an important system reliability characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multistate reliability function, which is a basic characteristic of the multistate system. Next, the assumption that the system's components have single hot reserve gives the possibility to improve the system's reliability and extend the time to the moment of exceeding the system reliability critical state. This approach may be successfully applied to a wide class of real ageing technical systems, for instance to reliability analysis, identification and prediction of the bulk cargo terminal transportation system, which is demonstrated in the paper.

2. Multistate approach to system reliability analysis

In the multistate reliability analysis to define a system composed of n, $n \in N$ ageing components we assume that:

- E_i, $i = 1,2,...,n$, are components of a system,
- all components and a system under consideration have the set of reliability states $\{0,1,...,z\}$, $z \geq 1$,
- the reliability states are ordered, the state 0 is the worst and the state z is the best,
- the component and the system reliability states degrade with time t,
- $T_i(u)$, $i = 1,2,...,n$, $n \in N$, are independent random variables representing the lifetimes of components E_i in the reliability state subset $\{u,u + 1,...,z\}$, while they were in the reliability state z at the moment $t = 0$,
- $T(u)$ is a random variable representing the lifetime of a system in the reliability state subset $\{u,u + 1,...,z\}$, while it was in the reliability state z at the moment $t = 0$,

– $s_i(t)$ is a component E_i reliability state at the moment t, $t\in\langle 0,\infty)$, given that it was in the reliability state z at the moment $t = 0$,
– $s(t)$ is the system reliability state at the moment t, $t\in\langle 0,\infty)$, given that it was in the reliability state z at the moment $t = 0$.

The above assumptions mean that the reliability states of the ageing system and components may be changed over time only from better to worse.

Definition 1. A vector

$$R_i(t,\cdot) = [R_i(t,0), R_i(t,1), \ldots, R_i(t,z)], \qquad (1)$$

where

$$R_i(t,u) = P(s_i(t) \geq u \mid s_i(0) = z) = P(T_i(u) > t) \qquad (2)$$

for $t\in\langle 0,\infty)$, $i = 1,2,...,n$, $u = 0,1,\ldots,z$, is the probability that the component E_i is in the reliability state subset $\{u, u+1,..., z\}$ at the moment t, $t\in\langle 0,\infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multistate reliability function of a component E_i.

Definition 2. A vector

$$\boldsymbol{R}(t,\cdot) = [\boldsymbol{R}(t,0), \boldsymbol{R}(t,1), \ldots, \boldsymbol{R}(t,z)],\ t\in \langle 0,\infty), \qquad (3)$$

where

$$\boldsymbol{R}(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t), \qquad (4)$$

for $t\in\langle 0,\infty)$, $u = 0,1,...,z$, is the probability that the system is in the reliability state subset $\{u,u + 1,...,z\}$ at the moment t, $t\in\langle 0,\infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multistate reliability function of a system. The reliability functions $R_i(t,u)$ and $\boldsymbol{R}(t,u)$, $t\in\langle 0,\infty)$, $u = 0,1,...,z$, defined by (2) and (4) are called the coordinates of the components and the system multistate reliability functions $R_i(t,\cdot)$ and $\boldsymbol{R}(t,\cdot)$ are given by respectively (1) and (3). It is clear that from Definition 1 and Definition 2, for $u = 0$, we have $R_i(t,0) = 1$ and $\boldsymbol{R}(t,0) = 1$.

Under the above definitions, the mean value of the system lifetime $T(u)$ in the reliability state subset $\{u,u+1,...,z\}$ is given by

$$\mu(u) = \int_0^\infty \boldsymbol{R}(t,u)dt,\ u = 1,2,\ldots,z \qquad (5)$$

Moreover, the mean values of the system lifetimes in particular reliability states are given by

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),\ \bar{\mu}(z) = \mu(z), \qquad (6)$$
$$u = 0,1,...,z-1,$$

where $\mu(u)$ $u = 0,1,...,z$ are given by (5).

Further, if r is the system critical reliability state, then the system risk function is given by [1], [3]

$$\boldsymbol{r}(\mathrm{t}) = 1 - \boldsymbol{R}(\mathrm{t},\mathrm{r}),\ t\in\langle 0,\infty), \qquad (7)$$

and if $\boldsymbol{\tau}$ is the moment when the system risk function exceeds a permitted level δ, then if $\boldsymbol{r}^{-1}(\mathrm{t})$ exists we have

$$\boldsymbol{\tau} = \boldsymbol{r}^{-1}(\delta), \qquad (8)$$

where $\boldsymbol{r}^{-1}(\mathrm{t})$ is the inverse function of the risk function $\boldsymbol{r}(\mathrm{t})$.

Now, after introducing the notion of the multistate reliability analysis, we may define basic multistate reliability structures.

Definition 3. A multistate system is called a series if its lifetime $T(u)$ in the reliability state subset $\{u, u + 1,...,z\}$ is given by $T(u) = \min_{1\leq i\leq n}\{T_i(u)\}$, $u = 1,2,...,z$.

The reliability function of the multistate series system is given by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1),..., \boldsymbol{R}(t,z)] \qquad (9)$$

with the coordinates

$$\boldsymbol{R}(t,u) = \prod_{i=1}^{n} R_i(t,u),\ t\in <0,\infty),\ u = 1,2,...,z. \qquad (10)$$

Definition 4. A multistate system is called series-parallel if its lifetime $T(u)$ in the reliability state subset $\{u,u + 1,...,z\}$ is given by

$$T(u) = \max_{1\leq i\leq k_n}\{\min_{1\leq j\leq l_i}\{T_{ij}(u)\}\},\ u = 1,2,...,z.$$

The reliability function of the regular multistate series-parallel system is given by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1),..., \boldsymbol{R}(t,z)], \qquad (11)$$

with the coordinates

$$\boldsymbol{R}(t,u) = 1-\prod_{i=1}^{k_n}[1-\prod_{j=1}^{l_i} R_{ij}(t,u)],\ t\in(-\infty,\infty), \qquad (12)$$

$u = 1,2,...,z,$

where k_n is the number of series subsystems linked in parallel and l_i is the number of components in the series subsystem.

3. Multistate approach to system reliability improvement analysis

Definition 5. A multi-state series system is called a series system with a hot reserve of its components if its lifetime $T(u)$ in the state subset $\{u,u+1,...,z\}$ is given by [5] $T(u) = \min_{1\le i\le n}\{\max_{1\le j\le 2}\{T_{ij}(u)\}\}$, $u = 1,2,...,z$,

where $T_{i1}(u)$ are the lifetimes of the system's basic components and $T_{i2}(u)$ are the lifetimes of their reserve components.

The scheme of this kind of series system is shown in *Figure 1*.

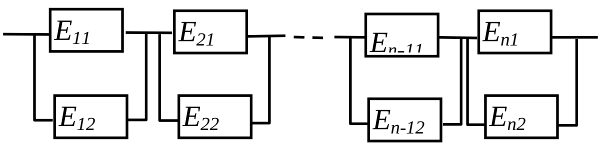

Figure 1. The scheme of a series system with a hot reserve of its components

The reliability function of the non-homogeneous multi-state series system with a hot reserve of its components is given by a vector

$$\boldsymbol{IR}(t,\cdot) = [1,\ \boldsymbol{IR}(t,1),...,\ \boldsymbol{IR}(t,z)], \qquad (13)$$

where

$$\boldsymbol{IR}(t,u) = \prod_{i=1}^{n}[1-[F_i(t,u)]^2],\ t\in<0,\infty), \qquad (14)$$

$u = 1,2,...,z.$

Definition 5. A multistate system is called series-parallel system with a hot reserve of its components if its lifetime $T(u)$ in the reliability state subset $\{u,u+1,...,z\}$ is given by $T(u) = \max_{1\le i\le k_n}\{\min_{1\le j\le l_i}\{\max_{1\le k\le 2}\{T_{ijk}(u)\}\}\}$, $u = 1,2,...,z$, where $T_{ij1}(u)$ are the lifetimes of the system's basic components and $T_{ij2}(u)$ are the lifetimes of the reserve components.

The reliability function of the regular multistate series-parallel system with a hot reserve of its components is given by the vector

$$\boldsymbol{IR}(t,\cdot) = [1, \boldsymbol{IR}(t,1),..., \boldsymbol{IR}(t,z)], \qquad (15)$$

with the coordinates

$$\boldsymbol{IR}(t,u) = 1-\prod_{i=1}^{k_n}[1-\prod_{j=1}^{l_i}[1-[F_{ij}(t,u)]^2], \qquad (16)$$

$t\in(-\infty,\infty)$, $u = 1,2,...,z,$

where k_n is the number of series subsystems linked in parallel and l_i is the number of components in the series subsystem.

4. Bulk cargo transportation system - technical description

The considered bulk cargo terminal placed at the Baltic seaside is designated for storage and reloading of bulk cargo such as different kinds of fertilizers e.g.: ammonium sulphate, but its main area of activity is to load bulk cargo on board the ships for export. There are two independent transportation systems: 1. The system of reloading rail wagons. 2. The system of loading vessels.

Cargo is brought to the terminal by trains consisting of self discharging wagons, which are discharged to a hopper and then by means of conveyors are transported into the one of four storage tanks (silos). Loading of fertilizers from storage tanks on board the ship is done by means of a special reloading system, which consists of several belt conveyors and one bucket conveyor, which allows for the transfer of bulk cargo in a vertical direction. Researched system is a system of belt conveyors, reffered to as the transport system. In the conveyor reloading system, we distinguish three bulk cargo transportation subsystems, the belt conveyors S_1, S_2 and S_3. The conveyor loading system is composed of six bulk cargo transportation subsystems, the dosage conveyor S_4, the horizontal conveyor S_5, the horizontal conveyor S_6, the sloping conveyor S_7, the dosage conveyor with buffer S_8, and the loading system S_9.

The bulk cargo transportation subsystems are built, respectively:

- the subsystem S_1 composed of 1 rubber belt, 2 drums, a set of 121 bow rollers, and a set of 23 belt supporting rollers,

- the subsystem S_2 composed of 1 rubber belt, 2 drums, a set of 44 bow rollers, and a set of 14 belt supporting rollers,
- the subsystem S_3 composed of 1 rubber belt, 2 drums, a set of 185 bow rollers, and a set of 60 belt supporting rollers,
- the subsystem S_4 composed of three identical belt conveyors, each composed of 1 rubber belt, 2 drums, a set of 12 bow rollers, and a set of 3 belt supporting rollers,
- the subsystem S_5 composed of 1 rubber belt, 2 drums, a set of 125 bow rollers, and a set of 45 belt supporting rollers,
- the subsystem S_6 composed of 1 rubber belt, 2 drums, a set of 65 bow rollers, and a set of 20 belt supporting rollers,
- the subsystem S_7 composed of 1 rubber belt, 2 drums, a set of 12 bow rollers, and a set of 3 belt supporting rollers,
- the subsystem S_8 composed of 1 rubber belt, 2 drums, a set of 162 bow rollers, and a set of 53 belt supporting rollers,
- the subsystem S_9 composed of 3 rubber belts, 6 drums, a set of 64 bow rollers, and a set of 20 belt supporting rollers.

The scheme of the bulk cargo transportation system is presented in *Figure 2*.

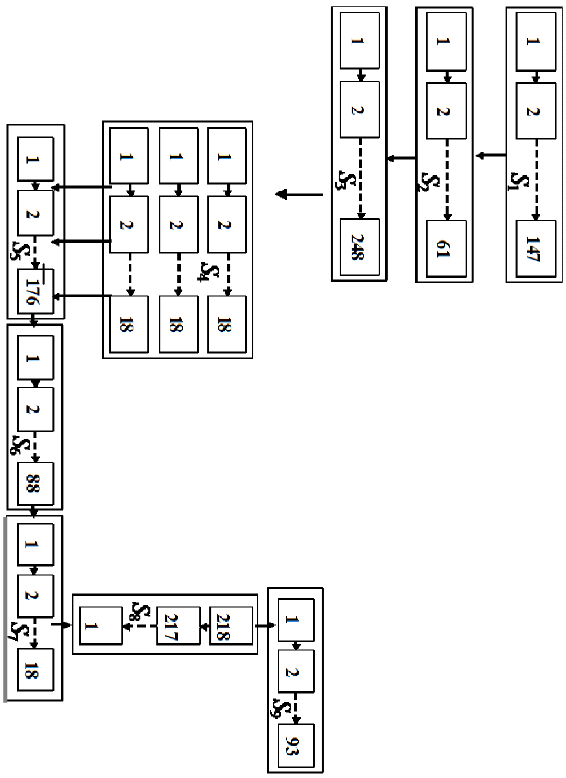

Figure 2. The scheme of port bulk cargo transportation system

Further, assuming that the system is in the reliability state subset {u,u+1,…,z}, if all of its subsystems are in this subset of reliability states, we conclude that the bulk cargo transportation system is a series system [4] of series subsystems S_1, S_2, S_3, S_5, S_6, S_7, S_8, S_9 and series-parallel subsystem S_4, with a scheme presented in *Figure 3*.

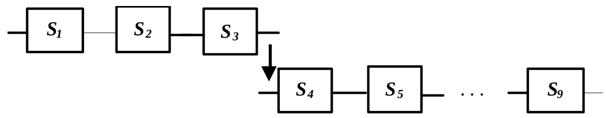

Figure 3. The scheme of port bulk cargo transportation system reliability structure

5. Reliability parameters of bulk cargo transportation system

After discussion with experts, in the reliability analysis of the bulk cargo transportation system we distinguish the following four reliability states (z = 3) of the considered system and its components:

- reliability state 3 – ensuring the highest efficiency of the conveyor,
- reliability state 2 – ensuring lower efficiency of the working conveyor, which is spilling cargo out of the belt caused by partial damage to some of the rollers or misalignment of the belt,
- reliability state 1 – ensuring lower efficiency of the working conveyor controlled directly by an operator,
- reliability state 0 – the conveyor is unable to work, which my be caused by e.g.: breakage of the belt or failure of the rollers.

We assume that the transitions between the components reliability states are possible only from a better to a worse state and we fix the system and its components critical reliability state to be r = 2.
Moreover, we assume that the system elements and their reserve elements $E_i^{(\upsilon)}$, $E_{ij}^{(\upsilon)}$ having the lifetimes $T_i^{(\upsilon)}(u) = T_{i1}^{(\upsilon)}(u) = T_{i2}^{(\upsilon)}(u)$, $T_{ij}^{(\upsilon)}(u) =$ $T_{ij1}^{(\upsilon)}(u) = T_{ij2}^{(\upsilon)}(u)$, $i = 1,2,\ldots,i^{(\upsilon)}$, $j = 1,2,\ldots, j_i^{(\upsilon)}$, $u = 1,2,3$, $\upsilon = 1,2,\ldots,9$, in the reliability states subsets {1,2,3}, {2,3}, {3} respectively, have the reliability functions respectively given by the vector:

$$R_i^{(\upsilon)}(t,\cdot) = [1, R_i^{(\upsilon)}(t,1), R_i^{(\upsilon)}(t,2), R_i^{(\upsilon)}(t,3)]$$

or

$$R_{ij}^{(\upsilon)}(t,\cdot) = [1, R_{ij}^{(\upsilon)}(t,1), R_{ij}^{(\upsilon)}(t,2)\ R_{ij}^{(\upsilon)}(t,3)]$$

with the Weibull probability functions:

$$R_i^{(\upsilon)}(t,u) = P(T_i^{(\upsilon)}(u) > t) = \exp[-\lambda_i^{(\upsilon)}(u)t^2],\quad t \in \langle 0,\infty),$$

or

$$R_{ij}^{(\upsilon)}(t,u) = P(T_{ij}^{(\upsilon)}(u) > t) = \exp[-\lambda_{ij}^{(\upsilon)}(u)t^2],\ t \in \langle 0,\infty),$$

with the parameters $\lambda_i^{(\upsilon)}(u)$, $\lambda_{ij}^{(\upsilon)}(u)$, $i = 1,2,...,i^{(\upsilon)}$, $j = 1,2,..., j_i^{(\upsilon)}$, $u = 1,2,3$, $\upsilon = 1,2…,9$, presented in *Tables 1-3*.

Table 1. Bulk cargo transportation subsystem S_1, S_2, S_3, component parameters $\lambda_i^{(k)}(u)$, $u = 1,2,3$

S_1	$\lambda_i^{(1)}(u)$	S_2	$\lambda_i^{(2)}(u)$	S_3	$\lambda_i^{(3)}(u)$
$i = 1$		$i = 1$		$i = 1$	
$u = 1$	0,01208	$u = 1$	0,01208	$u = 1$	0,01208
$u = 2$	0,02190	$u = 2$	0,02190	$u = 2$	0,02190
$u = 3$	0,04909	$u = 3$	0,04909	$u = 3$	0,04909
$i = 2,3$		$i = 2,3$		$i = 2,3$	
$u = 1$	0,00189	$u = 1$	0,00189	$u = 1$	0,00189
$u = 2$	0,00238	$u = 2$	0,00238	$u = 2$	0,00238
$u = 3$	0,00292	$u = 3$	0,00292	$u = 3$	0,00292
$i = 4,...,124$		$i = 4,...,47$		$i = 4,...,188$	
$u = 1$	0,00739	$u = 1$	0,00739	$u = 1$	0,00739
$u = 2$	0,01208	$u = 2$	0,01208	$u = 2$	0,01208
$u = 3$	0,02112	$u = 3$	0,02112	$u = 3$	0,02112
$i = 125,...,147$		$i = 48,...,61$		$i = 189,...,248$	
$u = 1$	0,00204	$u = 1$	0,00204	$u = 1$	0,00204
$u = 2$	0,00246	$u = 2$	0,00246	$u = 2$	0,00246
$u = 3$	0,00302	$u = 3$	0,00302	$u = 3$	0,00302

Table 2. Bulk cargo transportation subsystem S_4, S_5, S_6 component parameters $\lambda_i^{(k)}(u)$, $\lambda_{ij}^{(k)}(u)$, $u = 1,2,3$

S_4	$\lambda_{ij}^{(4)}(u)$ $j = 1,2,3$	S_5	$\lambda_i^{(5)}(u)$	S_6	$\lambda_i^{(6)}(u)$
$i = 1$		$i = 1$		$i = 1$	
$u = 1$	0,01208	$u = 1$	0,01208	$u = 1$	0,01208
$u = 2$	0,02190	$u = 2$	0,02190	$u = 2$	0,02190
$u = 3$	0,04909	$u = 3$	0,04909	$u = 3$	0,04909
$i = 2,3$		$i = 2,3$		$i = 2,3$	
$u = 1$	0,00189	$u = 1$	0,00189	$u = 1$	0,00189
$u = 2$	0,00238	$u = 2$	0,00238	$u = 2$	0,00238
$u = 3$	0,00292	$u = 3$	0,00292	$u = 3$	0,00292
$i = 4,...,15$		$i = 4,...,128$		$i = 4,...,67$	
$u = 1$	0,02806	$u = 1$	0,00739	$u = 1$	0,00739
$u = 2$	0,02986	$u = 2$	0,01208	$u = 2$	0,01208
$u = 3$	0,03205	$u = 3$	0,02112	$u = 3$	0,02112
$i = 16,...,18$		$i = 129,...,173$		$i = 68,...,88$	
$u = 1$	0,00594	$u = 1$	0,00204	$u = 1$	0,00204
$u = 2$	0,01003	$u = 2$	0,00246	$u = 2$	0,00246
$u = 3$	0,02011	$u = 3$	0,00302	$u = 3$	0,00302

Table 3. Bulk cargo transportation subsystem S_7, S_8, and S_9, component parameters $\lambda_i^{(k)}(u)$, $u = 1,2,3$

S_7	$\lambda_i^{(7)}(u)$	S_8	$\lambda_i^{(8)}(u)$	S_9	$\lambda_i^{(9)}(u)$
$i = 1$		$i = 1$		$i = 1,2,3$	
$u = 1$	0,01208	$u = 1$	0,01208	$u = 1$	0,01208
$u = 2$	0,02190	$u = 2$	0,02190	$u = 2$	0,02190
$u = 3$	0,04909	$u = 3$	0,04909	$u = 3$	0,04909
$i = 2,3$		$i = 1,2,3$		$i = 4,..,9$	
$u = 1$	0,00189	$u = 1$	0,00189	$u = 1$	0,00189
$u = 2$	0,00238	$u = 2$	0,00238	$u = 2$	0,00238
$u = 3$	0,00292	$u = 3$	0,00292	$u = 3$	0,00292
$i = 4,...,15$		$i = 4,...,165$		$i = 10,...,73$	
$u = 1$	0,02806	$u = 1$	0,00739	$u = 1$	0,00466
$u = 2$	0,02986	$u = 2$	0,01208	$u = 2$	0,00754
$u = 3$	0,03205	$u = 3$	0,02112	$u = 3$	0,01208
$i = 16,...,18$		$i = 167,...218$		$i = 74,...,93$	
$u = 1$	0,00594	$u = 1$	0,00204	$u = 1$	0,00119
$u = 2$	0,01003	$u = 2$	0,00246	$u = 2$	0,00181
$u = 3$	0,02011	$u = 3$	0,00302	$u = 3$	0,00238

3. Bulk cargo transportation system reliability prediction

Considering that the described system is a series system composed of subsystems S_υ, $\upsilon = 1,2,…,9$, after applying the formulae (9)-(12), (14)-(15), its reliability function is given by

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \boldsymbol{R}(t,2), \boldsymbol{R}(t,3)],\ t \in \langle 0,\infty), \quad (17)$$

where the coordinates have the following forms

$$\boldsymbol{R}(t,u) = \prod_{\upsilon=1}^{9} \boldsymbol{R}^{(\upsilon)}(t,u),\ t \in \langle 0,\infty),\ u = 1,2,3,$$

$\upsilon = 1,2,…,9$,

$$\boldsymbol{R}^{(1)}(t,u) = \prod_{i=1}^{147} R_i^{(1)}(t,u), \quad (18)$$

$$\boldsymbol{R}^{(2)}(t,u) = \prod_{i=1}^{61} R_i^{(2)}(t,u), \quad (19)$$

$$\boldsymbol{R}^{(3)}(t,u) = \prod_{i=1}^{248} R_i^{(3)}(t,u), \quad (20)$$

$$\boldsymbol{R}^{(4)}(t,u) = 1 - [1 - \prod_{i=1}^{18} R_{ij}^{(4)}(t,u)]^3 \quad (21)$$

$$\boldsymbol{R}^{(5)}(t,u) = \prod_{i=1}^{173} R_i^{(5)}(t,u), \quad (22)$$

$$\boldsymbol{R}^{(6)}(t,u) = \prod_{i=1}^{88} R_i^{(6)}(t,u), \quad (23)$$

$$\boldsymbol{R}^{(7)}(t,u) = \prod_{i=1}^{18} R_i^{(7)}(t,u), \quad (24)$$

$$\boldsymbol{R}^{(8)}(t,u) = \prod_{i=1}^{218} R_i^{(8)}(t,u), \quad (25)$$

$$\boldsymbol{R}^{(9)}(t,u) = \prod_{i=1}^{93} R_i^{(9)}(t,u) . \quad (26)$$

Applying the formulas (18)-(26) and considering the parameters given in Tables 1-3, the coordinates of the system reliability function given by (17) are as follows:

$$\boldsymbol{R}(t,1) = \exp[-0.95697t^2] \cdot \exp[-0.36958t^2]$$
$$\exp[-1.50541t^2] \cdot \ [3\exp[-0.37040t^2]$$
$$-3\exp[-0.740801t^2] + \exp[-1.11120t^2]]$$
$$\exp[-1.03141t^2] \cdot \exp[-0.53701t^2]$$
$$\exp[-0.37040t^2] \cdot \exp[-1.32116t^2]$$
$$\exp[-0.36962t^2] = 3\exp[-6.83196t^2]$$
$$-3\exp[-7.20236t^2] + \exp[-7.57276t^2] , \quad (27)$$

$$\boldsymbol{R}(t,2) = \exp[-1.54492t^2] \cdot \exp[-0.59262t^2]$$
$$\exp[-2.40906t^2] \cdot [3\exp[-0.41507t^2]$$
$$-3\exp[-0.830141t^2] + \exp[-1.24521t^2]]$$
$$\exp[-1.64736t^2] \cdot \exp[-0.86108t^2]$$
$$\exp[-0.41507t^2] \cdot \exp[-2.11400t^2]$$
$$\exp[-0.59874t^2] = 3\exp[-10.59790t^2]$$
$$-3\exp[-11.01297t^2] + \exp[-11.42804t^2] , \quad (28)$$

$$\boldsymbol{R}(t,3) = \exp[-2.67991t^2] \cdot \exp[-1.02649t^2]$$
$$\exp[-4.143331t^2] \cdot [3\exp[-0.49986t^2]$$
$$-3\exp[-0.999721t^2] + \exp[-1.49958t^2]]$$
$$\exp[-2.83083t^2] \cdot \exp[-1.48813t^2]$$
$$\exp[-0.49986t^2] \cdot \exp[-3.63643t^2]$$
$$\exp[-0.98551t^2] = 3\exp[-17.79035t^2]$$
$$-3\exp[-18.29021t^2] + \exp[-18.79007t^2] . \quad (29)$$

Their graphs are presented in *Figure 4*.

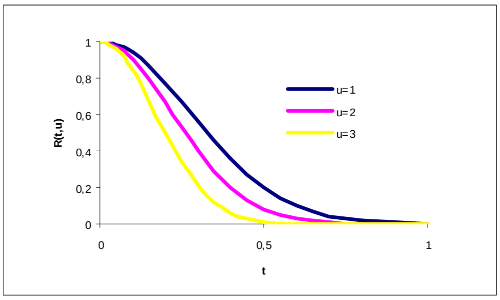

Figure 4. The graph of bulk cargo transportation system reliability function

Considering (27)-(29), the expected values of the bulk cargo transportation system lifetimes in the reliability states subsets {1,2,3}, {2,3}, {3}, according to (5), are respectively

$$\mu(1) \cong 0.3486, \ \ \mu(2) \cong 0.2777, \ \mu(3) \cong 0.2131. \quad (30)$$

And further, the standard deviations of this system lifetimes in the reliability state subsets {1,2,3}, {2,3}, {3}, according to (6) are

$$\sigma(1) \cong 0.0176, \ \sigma(2) \cong 0.0112, \ \sigma(3) \cong 0.0066. \quad (31)$$

Consequently, according to (6), the mean values of the maritime ferry technical system lifetimes in the particular reliability states 1, 2, 3, are respectively:

$$\bar{\mu}(1) \cong 0.0709, \ \bar{\mu}(2) \cong 0.0646, \ \bar{\mu}(3) \cong 0.2121. \quad (32)$$

Since the critical reliability state is $r = 2$, the system risk function of the bulk cargo transportation system, according to (7), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,2),$$

where $\boldsymbol{R}(t,2)$ is given by (28) and $\boldsymbol{r}(t)$ is illustrated in *Figure 6*.

Hence, by (8), the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is given as follows

$$\boldsymbol{\tau} = \boldsymbol{r}^{-1}(\delta) \cong 0.0710. \quad (33)$$

7. Bulk cargo transportation system reliability improvement prediction

To obtain better expected values of the bulk cargo transportation system lifetimes in the reliability states subsets {1,2,3}, {2,3}, {3}, we assume that the described bulk cargo transportation system is a series system with hot single redundancy of bow rollers and supporting rollers in all subsystems S_υ, $\upsilon = 1,2,\ldots,9$. After applying the formulae (9)-(10), (13)-(14) and (15)-(16), its reliability function is given by

$$\boldsymbol{IR}(t,\cdot) = [1, \boldsymbol{IR}(t,1), \ \boldsymbol{IR}(t,2), \boldsymbol{IR}(t,3)], \ t \in \langle 0,\infty), \quad (34)$$

where

$$\boldsymbol{IR}(t,u) = \prod_{\upsilon=1}^{9} \boldsymbol{IR}^{(\upsilon)}(t,u), \ \upsilon = 1,2,\ldots,9,$$

$$\boldsymbol{IR}^{(1)}(t,u) = R_1^{(1)}(t,u)[R_2^{(1)}(t,u)]^2 \prod_{i=4}^{147} [1-[F_{i1}^{(1)}(t,u)]^2],$$

$$\boldsymbol{IR}^{(2)}(t,u)=R_1^{(2)}(t,u)[R_2^{(2)}(t,u)]^2\prod_{i=4}^{61}[1-[F_{i1}^{(2)}(t,u)]^2], \quad (35)$$

$$\boldsymbol{IR}^{(3)}(t,u)=R_1^{(3)}(t,u)[R_2^{(3)}(t,u)]^2\prod_{i=4}^{248}[1-[F_{i1}^{(3)}(t,u)]^2], \quad (36)$$

$$\boldsymbol{IR}^{(4)}(t,u)=1-[1-R_1^{(4)}(t,u)[R_2^{(4)}(t,u)]^2\prod_{i=4}^{18}[1-[F_{i1}^{(4)}(t,u)]^2]]^3, \quad (37)$$

$$\boldsymbol{IR}^{(5)}(t,u)=R_1^{(5)}(t,u)[R_2^{(5)}(t,u)]^2\prod_{i=4}^{173}[1-[F_{i1}^{(5)}(t,u)]^2], \quad (38)$$

$$\boldsymbol{IR}^{(6)}(t,u)=R_1^{(6)}(t,u)[R_2^{(6)}(t,u)]^2\prod_{i=4}^{88}[1-[F_{i1}^{(6)}(t,u)]^2], \quad (39)$$

$$\boldsymbol{IR}^{(7)}(t,u)=R_1^{(7)}(t,u)[R_2^{(7)}(t,u)]^2\prod_{i=4}^{18}[1-[F_{i1}^{(7)}(t,u)]^2], \quad (40)$$

$$\boldsymbol{IR}^{(8)}(t,u)=R_1^{(8)}(t,u)[R_2^{(8)}(t,u)]^2\prod_{i=4}^{218}[1-[F_{i1}^{(8)}(t,u)]^2], \quad (41)$$

$$\boldsymbol{IR}^{(9)}(t,u)=[R_1^{(9)}(t,u)]^3[R_2^{(9)}(t,u)]^6\prod_{i=10}^{93}[1-[F_{i1}^{(9)}(t,u)]^2], \quad (42)\ (43)$$

for $t\in\langle 0,\infty)$, u = 1,2,3.

Applying the formulas (35)-(43) and considering the parameters given in Tables 1-3, the coordinates reliability function of the system with hot single reserve of its components given by (34) are as follows:

$$\begin{aligned}\boldsymbol{IR}(t,1) &= \exp[-6.46156t^2]\,(2-\exp[-0.00739t^2])^{702}\\ &(2-\exp[-0.00204t^2])^{215}\,(2-\exp[-0.02806t^2])^{12}\\ &(2-\exp[-0.00466t^2])^{64}\,(2-\exp[-0.00594t^2])^{3}\\ &(2-\exp[-0.00119t^2])^{20}\,[3\exp[-0.3704t^2]\\ &(2-\exp[-0.02806t^2])^{12}\,(2-\exp[-0.00594t^2])^{3}\\ &-3\exp[-0.7408t^2]\,(2-\exp[-0.02806t^2])^{24}\\ &(2-\exp[-0.00594t^2])^{6}+\exp[-1.1112t^2]\\ &(2-\exp[-0.02806t^2])^{36}\,(2-\exp[-0.00594t^2])^{9}],\end{aligned} \quad (44)$$

$$\begin{aligned}\boldsymbol{IR}(t,2) &= \exp[-10.18283t^2]\\ &(2-\exp[-0.01208t^2])^{702}\,(2-\exp[-0.00246t^2])^{215}\\ &(2-\exp[-0.01003t^2])^{3}\,(2-\exp[-0.00754t^2])^{64}\\ &(2-\exp[-0.02986t^2])^{12}\,(2-\exp[-0.00181t^2])^{20}\\ &[3\exp[-0.41507t^2]\,(2-\exp[-0.02986t^2])^{12}\\ &(2-\exp[-0.01003t^2])^{3}-3\exp[-0.83014t^2]\\ &(2-\exp[-0.02986t^2])^{24}\,(2-\exp[-0.01003t^2])^{6}\\ &+\exp[-1.24521t^2]\,(2-\exp[-0.02986t^2])^{36}\\ &(2-\exp[-0.01003t^2])^{9}]\end{aligned} \quad (45)$$

$$\begin{aligned}\boldsymbol{IR}(t,3) &= \exp[-17.29049t^2]\\ &(2-\exp[-0.02112t^2])^{702}\,(2-\exp[-0.00302t^2])^{215}\\ &(2-\exp[-0.02011t^2])^{3}\,(2-\exp[-0.01208t^2])^{64}\\ &(2-\exp[-0.03205t^2])^{12}\,(2-\exp[-0.00238t^2])^{20}\\ &[3\exp[-0.49986t^2]\,(2-\exp[-0.03205t^2])^{12}\\ &(2-\exp[-0.02011t^2])^{3}-3\exp[-0.9972t^2]\\ &(2-\exp[-0.03205t^2])^{24}\,(2-\exp[-0.02011t^2])^{6}\\ &+\exp[-1.49958t^2]\,(2-\exp[-0.03205t^2])^{36}\\ &(2-\exp[-0.02011t^2])^{9}].\end{aligned} \quad (46)$$

Their graphs are presented in *Figure 5*

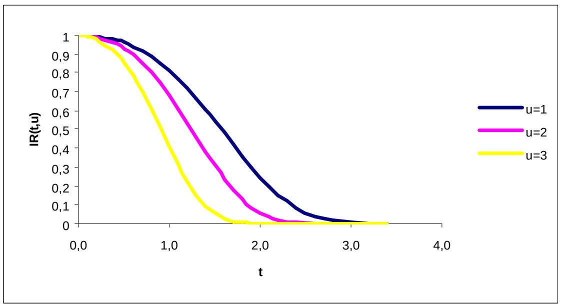

Figure 5. The graph of reliability function of the bulk cargo transportation system with hot single reserve of its components

Considering (44)-(46), the expected values of the bulk cargo transportation system lifetimes in the reliability states subsets {1,2,3}, {2,3}, {3}, according to (5), respectively are

$$\mu(1)\cong 1.5604,\ \ \mu(2)\cong 1.2375,\ \ \mu(3)\cong 0.9101. \quad (47)$$

And further, the standard deviations of this system lifetimes in the reliability state subsets {1,2,3}, {2,3}, {3}, are

$$\sigma(1)\cong 0.5973,\ \ \sigma(2)\cong 0.4839,\ \ \sigma(3)\cong 0.3698. \quad (48)$$

Consequently, according to (7), the mean values of the maritime ferry technical system lifetimes in the particular reliability states 1, 2, 3, are respectively:

$$\bar{\mu}(1)\cong 0.3229,\ \ \bar{\mu}(2)\cong 0.3274,\ \ \bar{\mu}(3)\cong 0.9101. \quad (49)$$

Since the critical reliability state is r = 2, then the system risk function of the bulk cargo transportation system with hot single redundancy, according to (7), is given by

$$\boldsymbol{r_h}(t) = 1 - \boldsymbol{IR}(t,2),$$

where $\boldsymbol{IR}(t,2)$ is given by (44) and $\boldsymbol{r_h}(t)$ is illustrated in *Figure 6*.

Hence, by (9), the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, is given as follows

$$\boldsymbol{\tau} = \boldsymbol{r_h}^{-1}(\delta) \cong 0.4224. \quad (50)$$

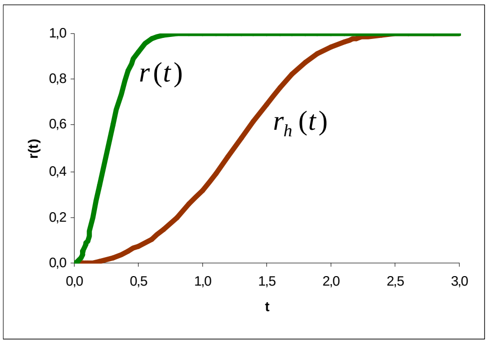

Figure 6. The graph of risk functions of a bulk cargo transportation system

8. Conclusion

The multistate approach to system reliability analysis and improvement, and the reliability models of typical multistate system structures, like that considered in the paper, can be applied in the reliability analysis of a wide class of complex technical systems. This possibility is illustrated through an example of a bulk cargo transportation system, for which reliability analysis and reliability improvement and reliability characteristics predictions were achieved. From the graph of the system risk functions of a system without reserve and a system with hot reserve, we can see how the quantitative redundancy prolongs the time to the moment when the system exceeds the critical level and the mean values of system lifetimes in particular states. Finally, we can search for a factor in the Weibull reliability functions, which will allow us to improve system's basic components and obtain the same risk function and mean values like for a system with hot reserve.

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