METHOD FOR ENHANCED ACCURACY IN MACHINING FREE-FORM SURFACES ON CNC MILLING MACHINES

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Abstract: The present article describes a method for enhanced accuracy in machining free-form surfaces produced on CNC milling machines. In this method, surface patch machining programs are generated based on their nominal CAD model. After the pretreatment, coordinate control measurements are carried out. The obtained results of the measurements contain information on the values and distribution of observed machining deviations. These data, after appropriate processing, are used to build a corrected CAD model of the surface produced. This model, made using reverse engineering techniques, compensates for the observed machining deviations. After regeneration of machining programs, the object processing and control measurements are repeated. As a result of the conducted procedure, the accuracy of the manufacture of the surface object is increased. This article also proposes the introduction of a simple procedure for the filtration of measurement data. Its purpose is to minimise the effect of random phenomena on the final machining error correction. The final part of the article presents the effects of the proposed method of increasing the accuracy of manufacturing on 'raw' and filtered measurement data. In both cases, a significant improvement in the accuracy of the machining process was achieved, with better final results obtained from the filtered measurement data. The method proposed in the article has been verified for three-axis machining with a ball-end cutter.

Key words: free-form surface, milling, coordinate measurements, CAD model, data filtration, accuracy improvement

1. INTRODUCTION

The machining of objects containing curved geometries is currently used in the industry for the production of various types of cams, blanking dies and electrodes for electrical discharge machining (EDM). When producing these elements, it is necessary to maintain high accuracy. This fact often requires a machining error compensation procedure. Various approaches are currently in use to improve manufacturing accuracy [1, 2]. One of these approaches is to determine the geometrical errors of the CNC (computer numerical control) machine tool and use them for the correction of machining programs [3, 4]. This approach requires a series of machining tests and control measurements aimed at the determination of models describing the machine tool errors [5, 6, 7]. These models are used to correct machining programs before machining [8, 9].

A similar approach is to identify and eliminate geometric errors in a CNC machine tool. For example, the recent researches focus on development of a method to accurately identify geometric errors of five-axis CNC machines. A theoretical model for identification of geometric errors is created. In this model, both position-independent errors and position-dependent errors are considered as the error sources. Experiments on a five-axis CNC machine tool also demonstrate significant reduction in the volumetric error after error compensation [10].

There is another interesting approach in which the cutter/workpiece engagement is taken into account. It varies with the tool orientation continuously including lead and tilt angles during machining, which results in the obvious time-varying characteristic

for consecutive cutting forces. Considering tool orientation, actual cutter runout and cutter motion process, an accurate calculation model for instantaneous cutter/workpiece engaging process in five-axis ball-end milling is calculated based on an improved analytical method. Then, based on the cutting force model, the tool orientation optimisation strategies with a flexible cutter and rigid workpiece for roughing and finishing milling operation are elaborated [11].

An interesting and complicated approach is to take into account the geometrical aspects of the tool and the workpiece. An example is a precise approach to the generation of optimised collision-free and gouging-free tool paths for five-axis CNC machining of freeform NURBS (Non-Uniform Rational B-Spline) surfaces using flat-end and rounded-end (bull nose) tools having cylindrical shank. A global optimisation is performed to find the tool path that maximises the approximation quality of the machining [12]. Another example of this approach is five-axis machining of freeform surfaces, where the degrees of freedom in selecting and moving the cutting tool allow one to adapt the tool motion optimally to the surface to be produced. A careful geometric analysis of curvature-adapted machining via so-called second-order line contact between tool and target surface is performed. As a result, better toolpath generation results are obtained, which results in improved machining accuracy [13].

Another approach is to analyse the errors, which are the source of the machining process itself and the accompanying phenomena [14]. These are mainly deformations of the tool and workpiece, originating from cutting forces and inertial forces [15]. Literature analysis related to this approach indicates that many methods have been developed to increase manufacturing accura-

cy. One of these is the design of the machining process, in which the cutting forces are controlled by adjusting parameters such as the feed or the width of the machined layer [16]. As a result, the tool does not bend beyond a certain limit. Adaptive control systems are being developed to correct the position of the tool in real time [17]. They require the machine tool to be set up with sensors to control the parameter values affecting the accuracy of machining (e.g. tool deflection, displacement of the machine's executive unit, etc.). Another way is the method of modifying the tool path based on the calculated tool deflection [18]. Deflection is determined by processing the process simulation models that have been developed. These models take into account deformation of the tool, the tool holder and the spindle when compensating the tool path due to the cutting forces.

Another universal method of increasing the accuracy of manufactured elements is the method using coordinate measurements. The correction of the machining process is based on the results of measurements made on a CNC machine tool [19] or a coordinate measuring machine (CMM) [20]. Measurements performed on the machine tool use machine tool probes. The advantage of this approach is that machining and measurements are made without changing the position of the workpiece in the same coordinate system. In the case of measurements made on a CMM, the workpiece must be removed from the processing station and transferred to the measuring station. A very important element of the measurement process in this case is reconstruction of the (coordinate system) setting of the workpiece on a CMM [21]. Measurement data obtained in two ways are usually compared to nominal CAD models of workpieces [22]. On this basis, machining deviations are determined and are then used in the process of correction of manufacturing errors.

The method of increasing the accuracy of manufacturing proposed in the article has been applied to the treatment of a free-form surface on a CNC milling machine. In this method, the creation of functions describing the distribution of machining deviations is omitted. Correction of machining programs is carried out indirectly by converting the nominal CAD model of the object into a corrected geometrical model, taking into account the deviations observed after the initial machining of the workpiece. Reconstruction of the CAD model takes place through the direct use of the results of coordinate measurements (observed deviations) carried out after the initial treatment. The expected effects of the proposed method are an increase in the accuracy of free surface preparation and simplification of the entire process of increasing the machining accuracy. An additional aspect proposed in the article is the introduction of a simple procedure for the filtration of measurement data. Its purpose is to minimise the effect of random phenomena on the final machining error correction. The method proposed in the article has been verified for three-axis machining with a ball-end cutter.

2. METHOD FOR INCREASING THE ACCURACY OF MANUFACTURING

2.1. Procedure

For increasing the accuracy of free-form surfaces produced on CNC milling machines, a method that is based on coordinate measurements performed outside the machine tool is presented in this article (Fig. 1). The results of measurements carried out after

machining are data on the basis of which the values and distribution of observed machining deviations are determined. In this method, the nominal geometric model of the surface object is created first. This model accurately reproduces the shape and dimensions of the surface area being produced. On this basis, programs controlling milling centre machining are created in the CAD/CAM system. In the next step, after the machine is properly equipped, the parts are machined. After the processing, coordinate control measurements of the created object are carried out.
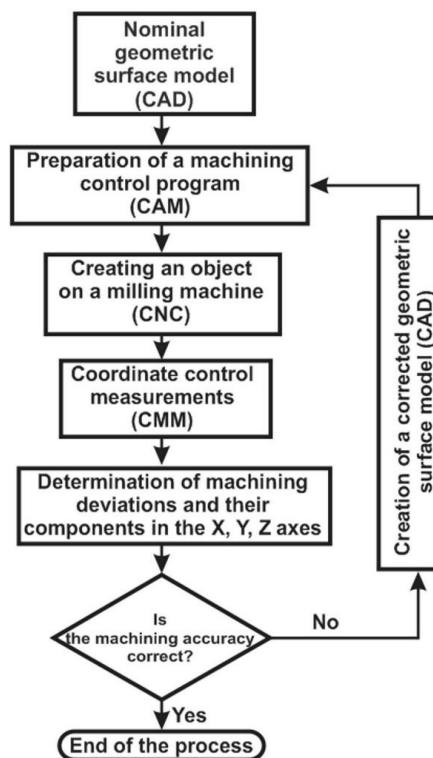

Fig. 1. Method for increasing the accuracy of the manufacturing of shaped surfaces

In the next stage of the process, based on the results of coordinate measurements, observed deviations and their components in individual axes are determined. The achieved machining accuracy can be estimated. The observed machining errors are compared with the required accuracy. If the machining accuracy meets the expectations, the manufacturing process is finished. In the case when the obtained accuracy is not satisfactory, machining errors are compensated. This compensation requires the reconstruction of the nominal geometric model of the manufactured object into a model that corrects the machining errors that occur, the re-creation of machining control programs and repetition of machining. The reconstruction of the geometrical model uses the results of coordinate measurements. These data contain information such as coordinates of nominal and observed points, deviations observed and directional cosines describing the direction of deviations observed. On the basis of this information, the components of deviations observed in the X, Y and Z axes are determined. These components are used in creating the corrected geometric model of the object. The method of determining the components of machining deviations and the reconstruction of the nominal surface model are described later in this article. The corrected surface geometric model is used for the re-creation of machining programs. After they are obtained, parts processing

and coordinate control measurements are repeated. If, after the repetition of machining, the accuracy obtained is still unsatisfactory, the correction process can be repeated again. It should be noted that in control coordinate measurements, machining deviations are always determined in relation to the initial, nominal geometric surface model.

2.2. Determining machining deviations and reconstructing the geometrical model

Control coordinate measurements of a free-form surface with a rectangular contour can be performed with a bidirectional uniform distribution of measurement points. In this way, the *nxm* grid of observed points is obtained. For this purpose, automatic surface scanning procedures can be used, for example, *UVScan* or *Grid* (PC-DMIS system). The number of measuring points is adjusted to the degree of complexity of the shape of the object.

In order to estimate the accuracy of making the free-form surface describing the object to be manufactured, machining deviations should be determined at the measuring points. The measure of the determined deviations are the distances between the points on the surface of the CAD model (nominal surface) and the corresponding points observed as a result of control measurements on the coordinate machine. Deviations are determined in the normal direction to the work surface (Fig. 2).
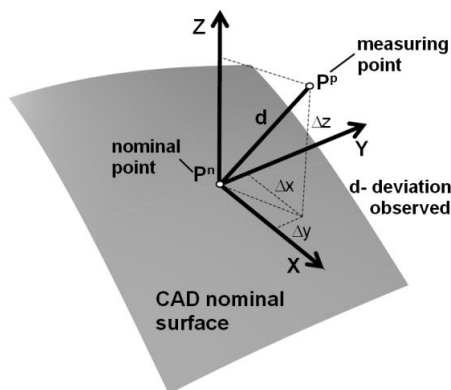

Fig. 2. Graphical representation of machining deviation

The data necessary for the determination of deviations is included in the measurement program controlling the CMM. After the measurements, information on the coordinates of the nominal points and the points observed during the measurement, observed machining deviations and directional cosines at the measuring points is available.

On the basis of this information, the correction of the surface describing the object being manufactured is made. First, the components of the observed machining deviations in the individual axes of the coordinate system are determined. The following relationships are used for calculations:

$$\Delta x_{ij} = d_{ij} * \cos \alpha_{ij}$$
$$\Delta y_{ij} = d_{ij} * \cos \beta_{ij} \qquad (1)$$
$$\Delta z_{ij} = d_{ij} * \cos \gamma_{ij}$$

where:
- Δx_{ij}, Δy_{ij}, Δz_{ij} are the components of observed machining deviations;

- d_{ij} is deviation observed at the measuring point;
- $\cos \alpha_{ij}$, $\cos \beta_{ij}$, $\cos \gamma_{ij}$ are directional cosines at the measuring points;
- i, j are coefficients describing the location of the observed point.

Determining the components of machining deviations makes it possible to calculate the corrected coordinates of points.

If the correction is carried out on 'raw' measurement data, the corrected coordinates will be determined from the following relationships:

$$x_{ij}^{cor} = x_{ij}^{nom} - \Delta x_{ij}$$
$$y_{ij}^{cor} = y_{ij}^{nom} - \Delta y_{ij} \qquad (2)$$
$$z_{ij}^{cor} = z_{ij}^{nom} - \Delta z_{ij}$$

where: $x_{ij}^{cor}, y_{ij}^{cor}, z_{ij}^{cor}$ are the coordinates of the corrected surface patch and $x_{ij}^{nom}, y_{ij}^{nom}, z_{ij}^{nom}$ are the coordinates of points on the nominal surface (CAD model).

The presented approach is the simplest, but it does not guarantee achieving the best final result. Due to the complexity of the machining and measurement process, deviations observed may contain significant effects of random phenomena. These deviations have two components: determined and random. The introduction of the measurement data filtration makes it possible to minimise the influence of random deviations on the final effect of machining deviations' correction. Equation (1) changes its form, and the components of the adjusted deviations are determined according to the following relationship:

$$\Delta x_{ij} = d_{ij}^{f} * \cos \alpha_{ij}$$
$$\Delta y_{ij} = d_{ij}^{f} * \cos \beta_{ij} \qquad (3)$$
$$\Delta z_{ij} = d_{ij}^{f} * \cos \gamma_{ij}$$

where: d_{ij}^{f} are the filtered components of observed machining deviations (determined components).

The methods of data filtration are many. The key to choosing the right method should be the simplicity of its use. The next part of the article will present an approach taken from the techniques used in image filtering.

The determined corrected coordinates are used to create the corrected surface patch. It contains information about manufacturing errors. In the construction of the patch of the corrected surface, reverse engineering techniques are used [23]. Firstly, a net of *nxm* corrected points is created (Fig. 3a). Next, a series of curves is interpolated onto the grid (Fig. 3b), on which, subsequently, a surface patch is applied (Fig. 3c). It is important to maintain the same *uv* parameterisation directions as in the nominal model when creating a corrected surface patch. The surface thus constructed compensates for the machining deviations that occur. It is necessary to create modified machining programs.
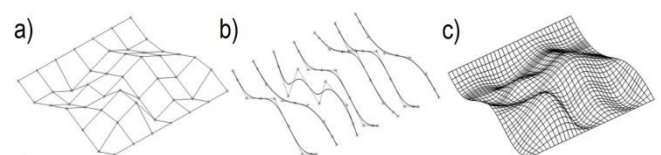

Fig. 3. Creation of the surface: (a) grid of points, (b) series of curves, (c) surface patch

3. EXPERIMENTAL VERIFICATION OF THE PROPOSED METHOD OF INCREASING THE ACCURACY OF MANUFACTURING

The method of correction of machining errors described in the previous part of the article was verified using the example of an object described using the NURBS surface patch (Fig. 4). The surface patch was built on a control grid built on 49 control points. The degree of B-spline functions in two directions of the *uv* surface patch parameterisation equalled 3. As can be seen in the attached drawing, the surface smoothly reproduced the shape of the control grid. The surface model was the basis for the preparation of programs controlling the machining of parts and measurement programs used to control the accuracy of manufacturing.
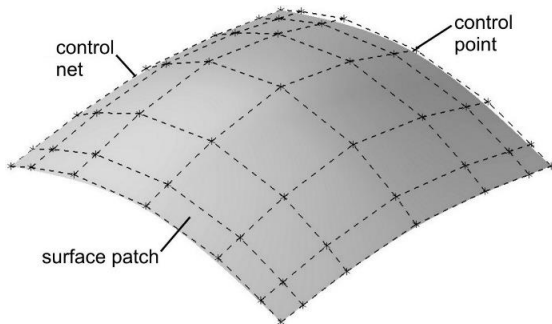

Fig. 4. A surface model describing the object being manufactured

The object was made of aluminium 2017A (Fig. 5). The milling centre control program included roughing, shaping and finishing. After the shaping treatment, a 0.3 mm allowance was left on the machined surface. To remove this allowance, a spherical cutter with a diameter of 6 mm was used to process the aluminium. Parallel tool passes with 0.2 mm spacing were programmed. Finishing was carried out at a spindle speed of 7.500 rpm and a feed of 300 mm/min. The surface produced was contained within a square of 45 mm sides.
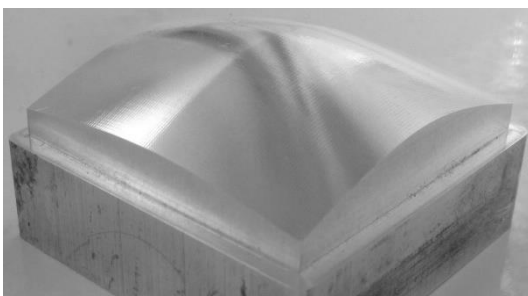

Fig. 5. The created object

After completion of the manufacturing stage, the test object was subjected to control measurements. The control measurements were carried out on a Hexagon Metrology Global Performance measuring machine (PC-DMIS software, MPEE = 1.5 + L/333 [μm], Renishaw SP25M measuring head, 20-mm-long stylus with a spherical 2-mm-diameter tip). Detailed information on coordinate measurements of different examples of machined surfaces can be found in references [20] and [21].

Due to the fact that the shape of the manufactured object was described by the patch of the NURBS surface of the third degree, the surface produced did not show any sudden changes in shape. This allowed the use of one of the automatic surface scanning procedures available in the PC-DMIS system. This procedure, called *UVScan*, allows for an even distribution of measuring points. Finally, the control measurements were programmed for a grid of 45×45 measurement points (distance between points = 1 mm). The distribution of measuring points on the measured surface is shown in Fig. 6.
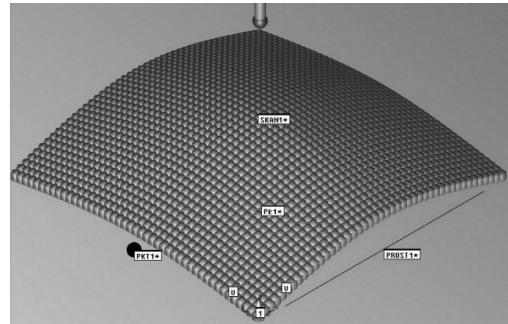

Fig. 6. Distribution of measurement points

As a result of the measurements carried out, information on 2025 deviations that were observed was obtained. The map and the distribution of spatial deviations are presented in Fig. 7. All determined deviations were within the range (−0.03, +0.045) mm.
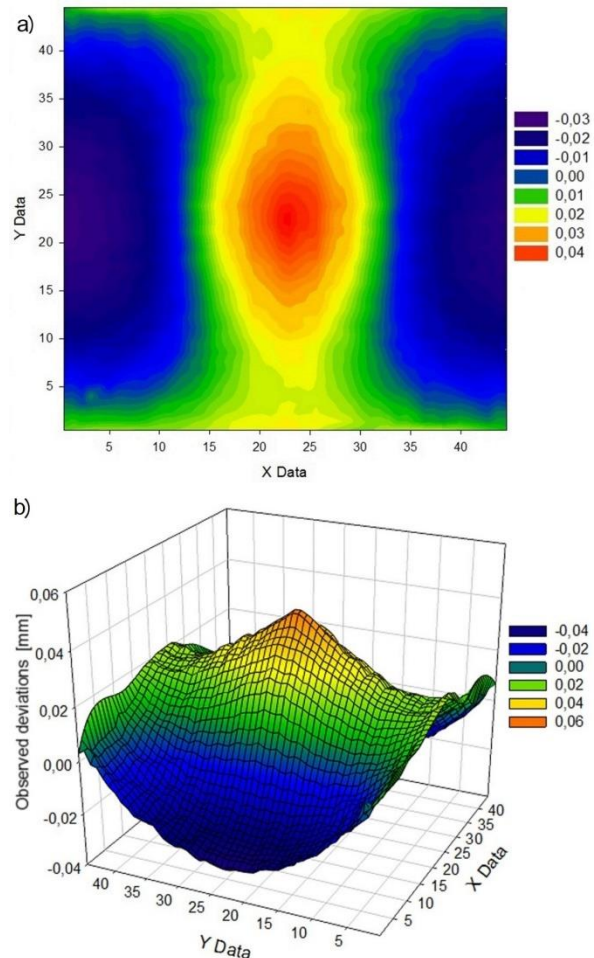

Fig. 7. Observed deviation: (a) deviation map, (b) spatial diagram

Table 1 presents a summary of the coordinate measurements of the treated area. The obtained accuracy was at a relatively good level. At the same time, it leaves a margin to apply the method proposed in the article and to obtain better final effects of the treatment.

Tab. 1. Results of preliminary coordinate measurements

Deviations observed	Value (mm)
Max. negative	−0.030
Max. positive	+0.045
Average value	0.0021
Standard deviation	0.0173

3.1. Correction based on 'raw' measurement data

In accordance with the procedure described in the previous section of the article, the construction of the corrected part of the geometric model was started. Firstly, the nominal coordinates and coordinates of the 2025 measurement points were separated from the measurement program. Using equations (1) and (2), the corrected coordinates were determined for each point. Based on these, in the MASTERCAM system, a corrected geometric model of the produced surface was created. In the beginning, a series of 45 interpolated curves was created on the grid of points (Fig. 8a). In the next step, the surface patch was spread on the series of curves obtained (Fig. 8b).
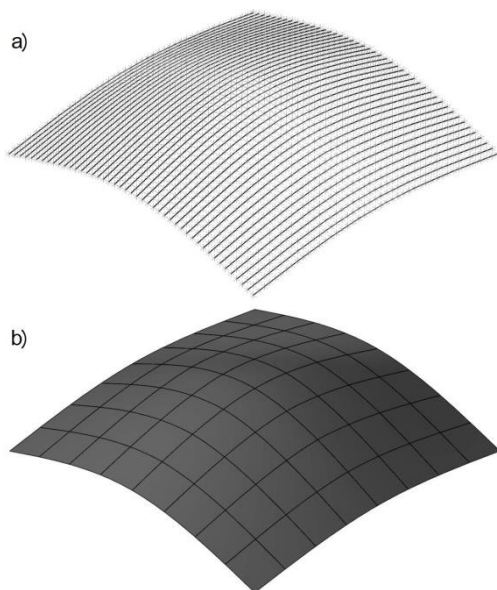

Fig. 8. Creation of the corrected geometric model of the object:
(a) corrected points and series of curves,
(b) patch of corrected surface

Based on the corrected geometric model of the parts, machining programs were re-created. The same tools and parameters used for the previous treatment of the object were used. The re-machined surface patch was subjected to coordinate measurements. The maps of obtained machining deviations (Fig. 9a) and their spatial distribution (Fig. 9b) indicate a significant improvement in the accuracy of manufacturing.
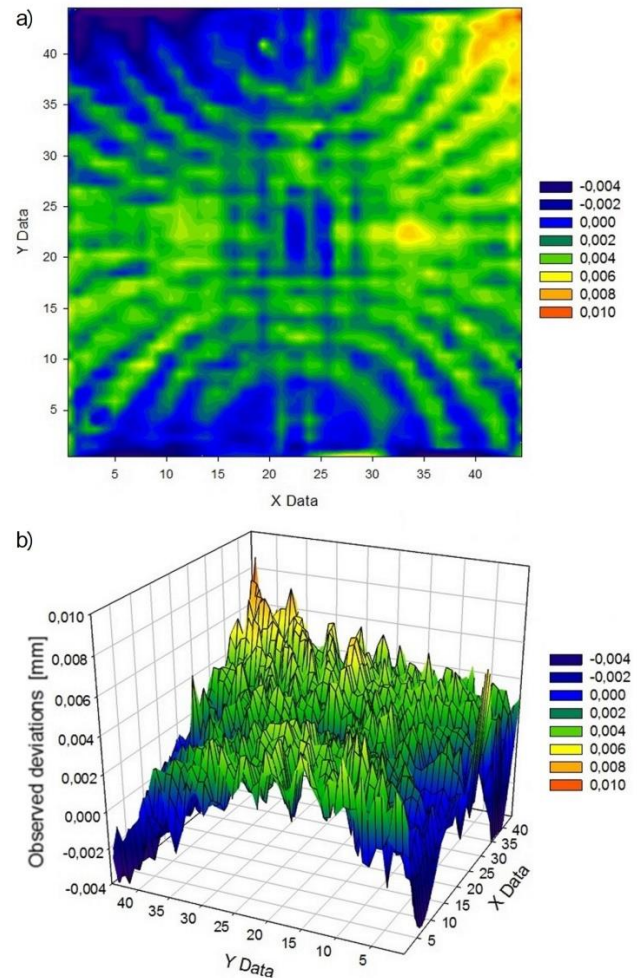

Fig. 9. Deviations observed after correction on 'raw' measurement data:
(a) deviation map, (b) spatial diagram

Table 2 contains the most important information on the results obtained using the method of increasing accuracy proposed in the article. A significant decrease in the maximum positive and negative deviations is observed. Also, the mean value and standard deviation indicate a small spread of the results obtained with respect to nominal data.

Tab. 2. Results of machining deviations correction

Deviations observed	Value (mm
Max. negative	−0.005
Max. positive	+0.009
Average value	0.0023
Standard deviation	0.0018

3.2. Correction based on filtered measurement data

Another approach in the construction of a revised surface model of the manufactured object is the introduction of a filtration procedure for measurement data. Its purpose is to minimise the influence of random components of observed machining deviations on the final effect of the procedure of increasing the accuracy of manufacturing.

This article proposes the use of procedures used in image filtration [24, 25]. The use of filters in the processing of measurement data means that the values of points from its surroundings are taken into account in calculating the new point value. Each measuring point from the environment contributes a weight during the calculation.

These weights are saved in the form of a mask. Typical mask sizes are 3×3, 5×5 and 7×7. The dimensions of the masks are usually odd because the measuring point in the middle represents the point at which the filter conversion operation is performed. The following is an example of data filtration based on a 3×3 filter.

$f_{-1,-1}$	$f_{0,-1}$	$f_{1,-1}$
$f_{-1,0}$	$f_{0,0}$	$f_{1,0}$
$f_{-1,1}$	$f_{0,1}$	$f_{1,1}$

The deviations observed in the measurement points have the form of a grid consisting of n columns and m rows. The new value of the $d_{i,j}$ elements is calculated with the coordinates (i, j) according to the following procedure. First, the weighted sum of the point component and all neighbours is calculated according to the weights indicated by the filter mask.

$$d'_{ij} = f_{-1,-1} * d_{i-1,j-1} + f_{0,-1} * d_{i,j-1} + f_{1,-1} * d_{i+1,j-1} +$$
$$f_{-1,0} * d_{i-1,j} + f_{0,0} * d_{i,j} + f_{1,0} * d_{i+1,j} + f_{-1,1} * d_{i-1,j+1} +$$
$$f_{0,1} * d_{i,j+1} + f_{1,1} * d_{i+1,j+1} \tag{4}$$

The sum obtained in this way is divided by the sum of all mask weights, if it is different from 0.

$$d_{ij}^f = \frac{d'_{ij}}{f_{-1,-1}+f_{0,-1}+f_{1,-1}+f_{-1,0}+f_{0,0}+f_{1,0}+f_{-1,1}+f_{0,1}+f_{1,1}} \tag{5}$$

The process of normalising the component value of the observed machining deviation results in a smoother deviation distribution being obtained and minimises the influence of random components on the final result of the machining error correction. In order to test the procedure proposed in the article, a 5×5 mask was used. Due to the even distribution of the measurement points, it was assumed that the impact of all points surrounding the deviation being processed is the same (all weights equal to 1). The format of the mask used is presented below.

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

The effect of data filtration is presented in Fig. 10. A modified map of machining deviations is presented in Fig. 10a. Comparing it with the map of 'raw' deviations (Fig. 7), a significant smoothing of contours, representing individual levels of machining deviations, can be seen. This is due to the separation of random components (Fig. 10b) generated during the machining and measurement process.
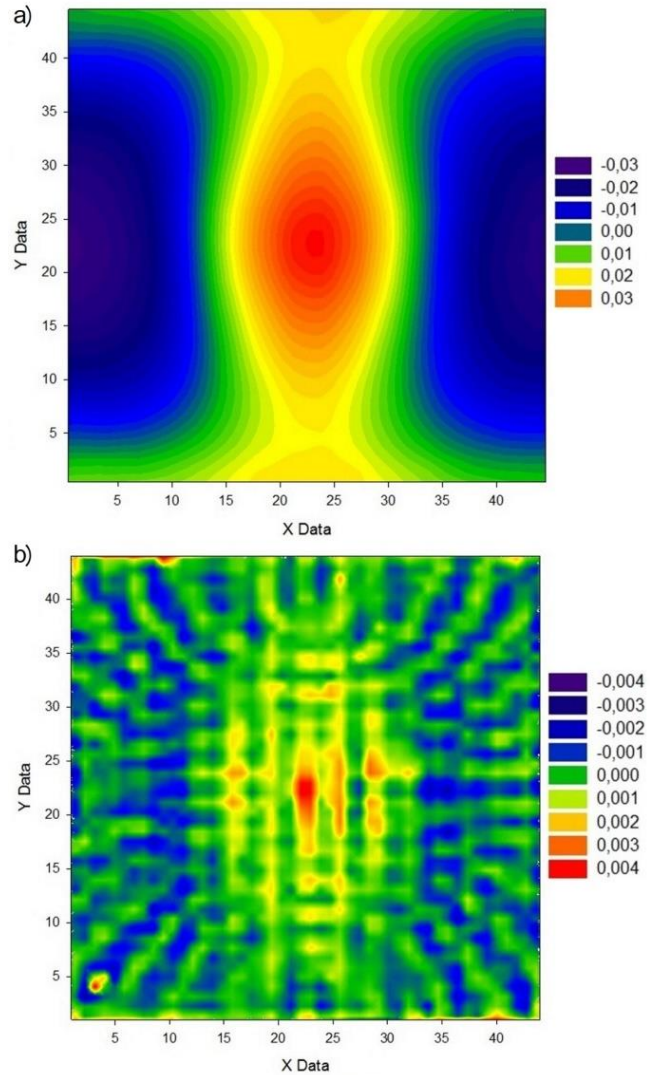

Fig. 10. Filtering effect of observed machining deviations:
(a) map of components after filtration,
(b) map of filtered components

Table 3 contains numerical values illustrating the change of 'raw' deviations after applying data filtration. A change in the maximum values of deviations after filtration can be observed. The filtered components are in the interval (−0.0027; 0.005), and their dispersion relative to the mean value (standard deviation) is insignificant.

Tab. 3. Filtration results of observed machining deviations

Deviations observed (mm)	'Raw' deviations	Corrected deviations	Filtered components
Max. negative	−0.030	−0.028	−0.00271
Max. positive	+0.045	+0.040	+0.005
Average value	0.0021	0.0021	−2.1E−05
Standard deviation	0.0173	0.0168	0.0009

On comparing the map of machining deviations obtained after the correction of machining based on 'raw' deviations (Fig. 9a) and the map of filtered components (Fig. 10b), one can notice some similarity. This indicates the effect of random components of

machining deviations on the final effect of increasing machining accuracy. Therefore, it can be assumed that the filtration of the coordinate measurement results carried out, and the correction of manufacturing errors based on the corrected deviations, will positively influence the final result of the procedure.

After the data filtration procedure, the adjusted machining deviations were determined using equations (1) and (3). The further course of action was analogous to that of 'raw' measurement data. On the basis of 2025 corrected points, a series of 45 curves was created, on which the surface patch was spread. On the basis of the corrected surface patch, the machining programs and the surface patch were regenerated. The process ended with coordinate control measurements. The final results are presented in Fig. 11. The map view of the obtained machining deviations (Fig. 11a), in combination with the post-correction deviation map on the raw data base (Fig. 9a) indicates an improvement in surface accuracy. The results obtained after the correction of the filtered data are better than in the case of correction for 'raw' measurement data. The map of machining deviations does not show such a similarity to the map of filtered components (Fig. 10b). This indicates minimisation of the influence of random components of observed machining deviations on the final effect of increasing the accuracy of manufacturing.
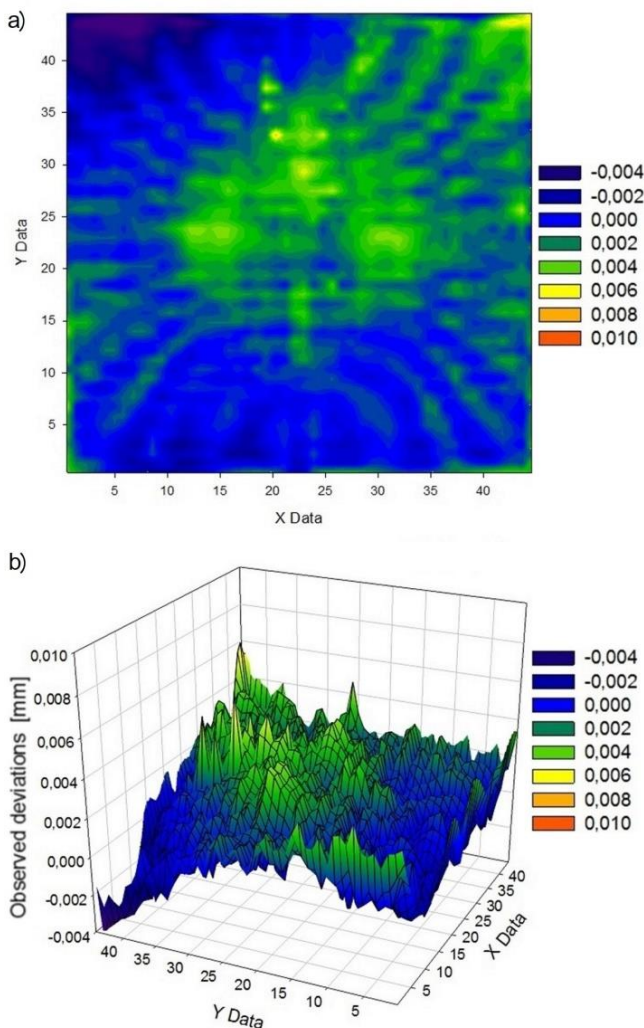

Fig. 11. Deviations observed after correction of filtered measurement data: (a) deviation map, (b) spatial diagram.

Table 4 presents the results of repeated corrections of machining deviations. The obtained machining deviations are in this range in the interval (−0.004; 0.007) and are smaller than those obtained in the correction for 'raw' measurement data. Other values, that is, the mean value and standard deviation, also decreased. This indicates a greater convergence of the surface area produced with its nominal CAD model. They also show a smaller spread over the nominal data, as illustrated in Fig. 11b.

Tab. 4. Results of correction of machining deviations based on filtered data

Deviations observed	Value (mm)
Max. negative	−0.004
Max. positive	+0.007
Average value	0.0013
Standard deviation	0.0014

4. CONCLUSIONS

The implementation of the method of increasing the accuracy of manufacturing of shaped surfaces presented in this article allowed the accuracy of manufacturing to be significantly increased. Table 5 presents the results observed before and after the correction process using two methods. In both cases, they indicate the effectiveness of the proposed procedure. The presented results show a clear decrease in the maximum observed machining deviations. The introduction of filtration of measurement data made it possible to improve the final result. The observed maximum machining deviations are the smallest in this case. The deviation of standard deviations and their mean indicates an additional positive effect. The surface produced shows the greatest similarity to the nominal CAD model. The measurement data filtration makes it possible to reduce the influence of random components of observed machining deviations on the process of increasing the accuracy of manufacturing.

Tab. 5. List of obtained results

	'Raw' deviations (mm)	Correction 1 – 'raw' deviations (mm)	Correction 2 – filtered deviations (mm)
Max. negative	−0.030	−0.005	−0.004
Max. positive	0.045	0.009	0.007
Average value	0.0022	0.0023	0.0013
Standard deviation	0.0173	0.0018	0.0014

The implementation of the presented method of correction of manufacturing errors is relatively simple. It is based on typical hardware and software used in enterprises (CAD/CAM systems, CNC machine tools, CMMs). The implementation of machining error correction is additionally facilitated by the parametric linking of technological and geometric data in modern CAD/CAM systems. This means that once developed, technological data do not need to be re-entered into the system. As a consequence, after rebuilding the geometric model of the object, the tool path is automatically rebuilt.

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