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Stress–Dilatancy For Crushed Latite Basalt

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Abstract: In this article, the stress–dilatancy relationship for crushed latite basalt is analysed by using *Frictional State Theory*. The relationship is bilinear, and the parameters α and β determine these two straight lines. At the initial stage of shearing, the mean normal stress increment mainly influences breakage, but at the advanced stage, it is shear deformation that influences breakage. At the advanced stage of shearing, the parameter α_{pt} represents energy consumption because of breakage and β_{pt} mainly represents changes in volume caused by breakage during shear. It is also shown that breakage effect is significant at small stress levels and the η-D^p plane is important to fully understand the stress–strain behaviour of crushed latite basalt in triaxial compression tests.

Keywords: ballast; dilatancy; frictional state; breakage.

1 Introduction

Crushed latite basalt is commonly used as railway ballast in Australia [5, 12]. It is well established that breakage of grains influences the strength and deformation behaviour of soils [5, 7, 8, 9, 11, 12]. The intensity of breakage is a function of granulometry, stress level and deformation process [6, 9]. Cyclic loading significantly intensifies particle breakage [6]. Marsal's breakage index [8] and ballast breakage index [4] are the widely used parameters of ballast breakage.

The breakage of particles reduces angularity and thus reduces friction as well as the critical state friction angle of ballast [2, 5, 15]. The critical state fiction angle of sands is independent of breakage of grains [1, 3, 10]. The peak friction angle is also reduced by the growth of confining pressure without particle breakage. The separation of stress level influence and particle breakage influence on the behaviour of crushed latite basalt is very difficult and has not been proposed in literature until now. The large-scale triaxial testing of latite basalt at large axial strains exceeding 20–25% shows the continual breakage of particles. There were little or no volume changes at almost constant stress at large strains, so the reduction of critical state parameters is natural in the modelling of latite basalt behaviour.

During monotonic shearing, the total plastic work's components are purely shearing, some part of volume changes (not natural dilatancy) during shear and particle breakage. The natural dilatancy, characteristic for each soil, caused by shear has no influence on energy dissipation [13]. Salim and Indraratna [12] stated that the increment in energy consumption because of particle breakage per volume unit is proportional to the breakage index increment.

Szypcio [13] developed *Frictional State Theory* to properly describe stress–dilatancy relationships for soils at different deformation modes at drained and undrained conditions. This theory is used in this article to describe the stress–dilatancy relationship for crushed latite basalt based on the experimental data published by Indraratna et al. [5] and Salim and Indraratna [12]. In analysing the experimental data of tests conducted using large-size triaxial apparatus [5, 12], it is shown that the stress–dilatancy relationship for crushed latite basalt is bilinear. The points representing maximum curvature of the stress–dilatancy relationship are named *transformation points*. These points are situated on almost a straight line. For many granular soils, the critical frictional state angle (Φ^o) is equal to critical state angle (Φ'_{cv}) [14]. It is shown that critical state friction angle is independent of particle breakage (stress level). It is also shown that breakage of particles of crushed latite basalt appears at low stress level during shear.

2 Stress–Dilatancy Relationship For Drained Triaxial Compression

The stress–dilatancy relationship (1.a) for drained triaxial compression developed from *Frictional State Theory* has the simple form [14]:

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$$\eta = Q - AD^p \tag{1.a}$$

where

$$\eta = q/p' \tag{1.b}$$

$$Q = M^o - \alpha\, A^o \tag{1.c}$$

$$A = \beta\, A^o \tag{1.d}$$

$$D^p = \frac{\delta\varepsilon_\upsilon^p}{\delta\varepsilon_q^p} \tag{1.e}$$

$$M^o = M_c^o = \frac{6\sin\Phi^o}{3 - \sin\Phi^o} \tag{1.f}$$

$$A^o = A_c^o = 1 - \frac{1}{3} M_c^o \tag{1.g}$$

Φ^o is the critical state friction angle and α and β are the experimental parameters [13]. For latite basalt, as a non-cohesive material, we can write [14]

$$\frac{\sigma_1'}{\sigma_3'} = \frac{3 + 2M_c^o - 2A_c^o\left(\alpha + \beta\, D^p\right)}{3 - M_c^o + A_c^o\left(\alpha + \beta\, D^p\right)} \tag{2}$$

and the value of the mobilised angle of shear resistance

$$\Phi' = 2\tan^{-1}\sqrt{\frac{\sigma_1'}{\sigma_3'}} - \frac{\pi}{2} \tag{3}$$

The value Φ^o=53.7° best fulfils *Frictional State Theory* requirements.

The plastic part of volume ($\delta\varepsilon_\upsilon^p$) and shear ($\delta\varepsilon_q^p$) deformation increments are calculated using equations (4) and (5) as the difference between total values and elastic parts of increments, respectively:

$$\delta\varepsilon_\upsilon^p = \delta\varepsilon_\upsilon - \delta\varepsilon_\upsilon^e \tag{4}$$

$$\delta\varepsilon_q^p = \delta\varepsilon_q - \delta\varepsilon_q^e \tag{5}$$

The elastic parts of deformation increments are calculated from equations (6) and (7):

$$\delta\varepsilon_\upsilon^e = \frac{\kappa}{\vartheta}\frac{\delta p'}{p'} \tag{6}$$

$$\delta\varepsilon_q^e = \frac{2}{9}\,\frac{1+\nu}{1-2\nu}\,\frac{\kappa}{\vartheta}\,\frac{\delta q}{p'} \tag{7}$$

where

$$\vartheta = 1 + e \tag{8}$$

and for conventional triaxial tests ($\delta\sigma_3' = 0$),

$$\delta p' = \frac{1}{3}\delta\sigma_1' \tag{9}$$

$$\delta q = \delta\sigma_1' \tag{10}$$

e is the void ratio, ν is the Poisson's ratio and κ is the Cam clay model parameter that represents slope of unloading–reloading line in ϑ-lnp' plane. In this article, it is assumed, as most appropriate values, κ = 0.002 and ν = 0.15 for crushed latite basalt.

3 Experiments And Results

The crushed latite basalt was tested by Salim and Indraratna [12] and Indraratna et al. [5] at drained conditions using a large-scale cylindrical triaxial apparatus. The initial void ratio was e = 0.76. The relationships η-D^p were developed [5, 12] for different confining pressure constant during tests. The published experimental relationships $(\sigma_1'/\sigma_3') - \varepsilon_q$ and $\varepsilon_\upsilon - \varepsilon_q$ were approximated by a high degree of polynomials. The polynomials were later used for analysis. The values of major stress ratio (σ_1'/σ_3'), volume deformations (ε_υ), stress ratio (η) and plastic dilatancy (D^p) were calculated with the use of these polynomials.

In Figure 1, the relationships $(\sigma_1'/\sigma_3') - \varepsilon_q$, $\varepsilon_\upsilon - \varepsilon_q$ are shown for crushed latite basalt tests conducted by Indraratna et al. [5]. In Figure 2, the relationships η-D^p for these tests are shown.

Similar relationships for crushed latite basalt tested by Salim and Indraratna [12] are shown in Figures 3 and 4.

The points representing maximum curvature of η-D^p lines were chosen and shown in Figures 2 and 4. These points are named *transformation points* (T). The appropriate points are also shown in Figures 1 and 3. The (T) points were collected and shown in Figure 5. It may be accepted that (T) points lay on a straight line (*transformation line*) defined by equation (1) with

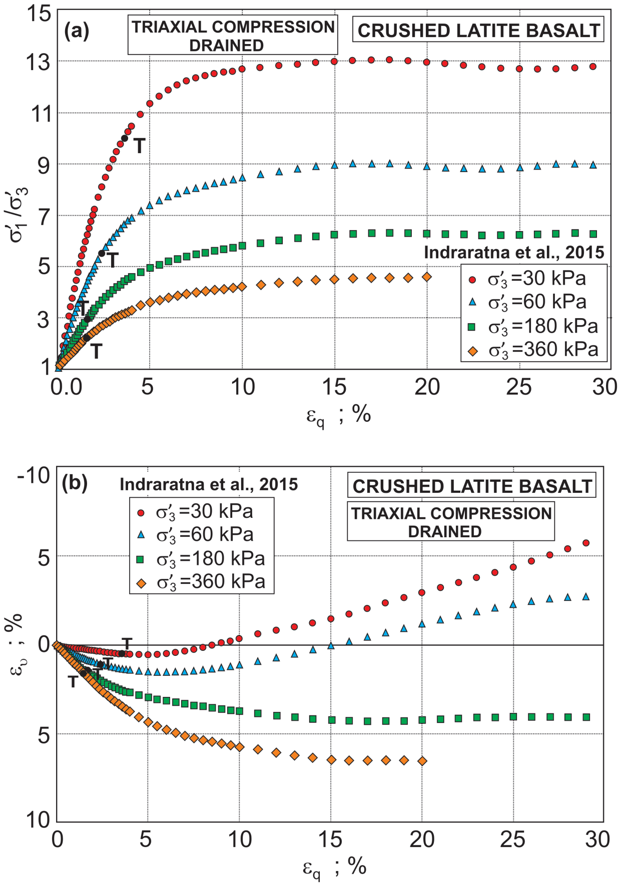

Figure 1: The relationships between stress ratio, volume deformations and shear strain: (a) $(\sigma_1'/\sigma_3') - \varepsilon_q$; (b) $\varepsilon_\upsilon - \varepsilon_q$ (adopted from Indraratna et al., 2015).

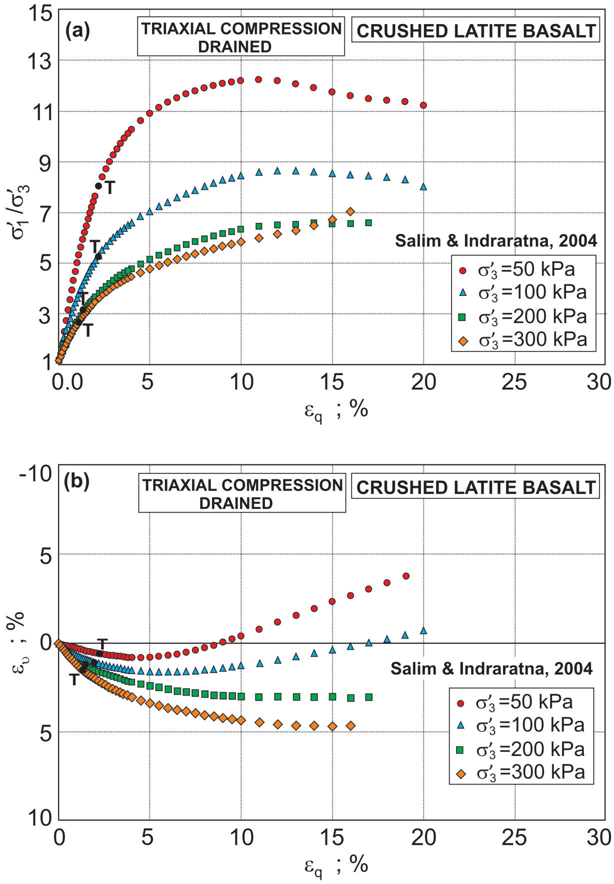

Figure 3: The relationships between stress ratio, volume deformations and shear strain: (a) $(\sigma_1'/\sigma_3') - \varepsilon_q$; (b) $\varepsilon_\upsilon - \varepsilon_q$ (adopted from Salim and Indraratna, 2004).

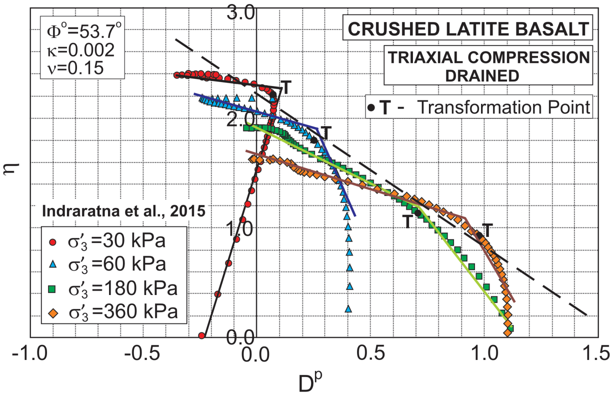

Figure 2: The relationships η-D^p for crushed latite basalt.

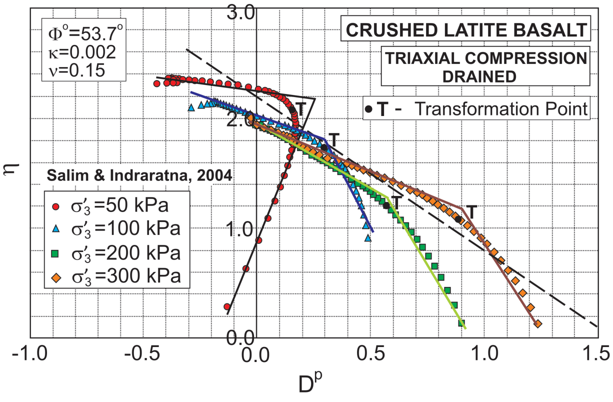

Figure 4: The relationships η-D^p for crushed latite basalt.

$\Phi^o = 53.7^\circ$, $\alpha = 0$ and $\beta = \beta^* = 5.29$. In Figure 5, the *frictional state line* ($\Phi^o = 53.7^\circ$, $\alpha = 0$, $\beta = 1.0$) is also shown.

In the author's opinion, if there is no particle breakage, the *transformation line* must be a *frictional state line*. So, the area marked in grey in Figure 5 represents the influence of particle breakage on η-D^p relationship. This influence is significant for crushed latite basalt even at low confining pressure and small values of plastic dilatancy. This is probably due to a low number of contact points between particles. In these contact points, there appeared very high stress and intense crushing.

Table 1: The values of α and β for triaxial compression of crushed latite basalt.

Parameters	**Indraratna et al. (2015)**				**Salim and Indraratna (2004)**			
α and β	**Confining pressure σ_c (kPa)**							
	30	**60**	**180**	**360**	**50**	**100**	**200**	**300**
α_{bt}	2.50	−3.60	−3.30	−7.70	5.00	−3.00	−4.00	−7.00
β_{bt}	−25.0	18.00	10.00	13.00	−19.00	15.00	13.00	12.00
α_{pt}	−0.35	0.55	1.10	2.00	−0.20	0.65	0.90	0.95
β_{pt}	1.00	2.20	3.80	2.40	1.00	2.70	4.50	3.20

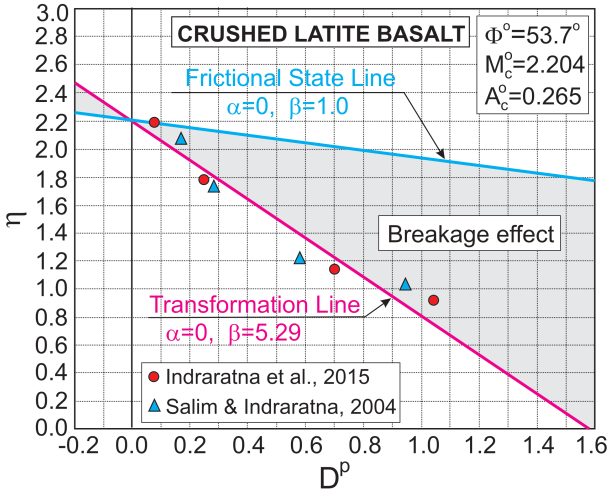

Figure 5: Transformation line for triaxial compression of crushed latite basalt.

At the initial phase of shearing, the mean normal stress increments are high and play a dominant role in particle breakage. At the advanced phase of shearing, the mean normal stress increments are relatively small and shear deformation plays a dominant role in particle breakage. The *transformation line* represents the border between these two phases.

It may be accepted that η-D^p relationships for crushed latite basalt are bilinear. One straight line represents points of experimental data below the *transformation line* and the second straight line represents points higher up this line. The parameters α and β of *Frictional State Theory* [13] defining position and slopes of these lines are marked as α_{bt}, β_{bt} and α_{pt}, β_{pt}, respectively. The values of α and β are summarised in Table 1.

At the initial stage of deformation, parameters α_{bt} and β_{bt} represent the summary effect of structure degradation and particle breakage on stress–dilatancy relationships. On the basis of only *Frictional State Theory*, it is not possible to separate these two effects. At the advanced stage of shearing, the structure of granular soil is erased and natural dilatancy [13] as well as particle breakage has influence on the stress–dilatancy (η-D^p) relationships. In the author's opinion, α_{pt} represents the influence of energy consumption because of breakage and (β_{pt}-1) represents the influence of volume changes (volume decrease) caused by particle breakage. If $\alpha_{pt} > 0$ ($\sigma_c > 55$ kPa), the influence of energy consumed for breakage is more significant than the influence of volume changes on the stress–dilatancy relationships.

On the contrary, if $\alpha_{pt} < 0$ ($\sigma_c < 55$ kPa) then the influence of energy consumed on breakage is smaller than the volume changes due to breakage. If $\alpha = 0$ then the two effects are balanced. However, these remarks need more research in the future.

In *Frictional State Theory*, the values

$$\chi_1 = \frac{p'}{p^o} - 1 \qquad (11)$$

$$\chi_2 = \frac{q}{q^o} - 1 \qquad (12)$$

represent the difference between current and reference (frictional) state. The relationships $\chi_1 - \varepsilon_q$, $\chi_2 - \varepsilon_q$ for tests conducted by Indraratna et al. [5] are shown in Figure 6a and b, respectively.

At *transformation point*, the values χ_1 and χ_2 jump for σ_c=30 kPa or increments significantly rise for $\sigma_c > 30$ kPa. It was noticed by the author that ratio χ_2/χ_1 is constant during shear and $\chi_2/\chi_1 = \tan\Phi^o$ for crushed latite basalt for drained triaxial compression.

The value of Φ^o angle was assumed based on this finding.

4 Conclusions

Stress–strain behaviour of crushed latite basalt may be successfully analysed by using *Frictional State Theory*. The critical state friction angle $\Phi^o = 53.7^o$ is independent of confining pressure or breakage of particles. The

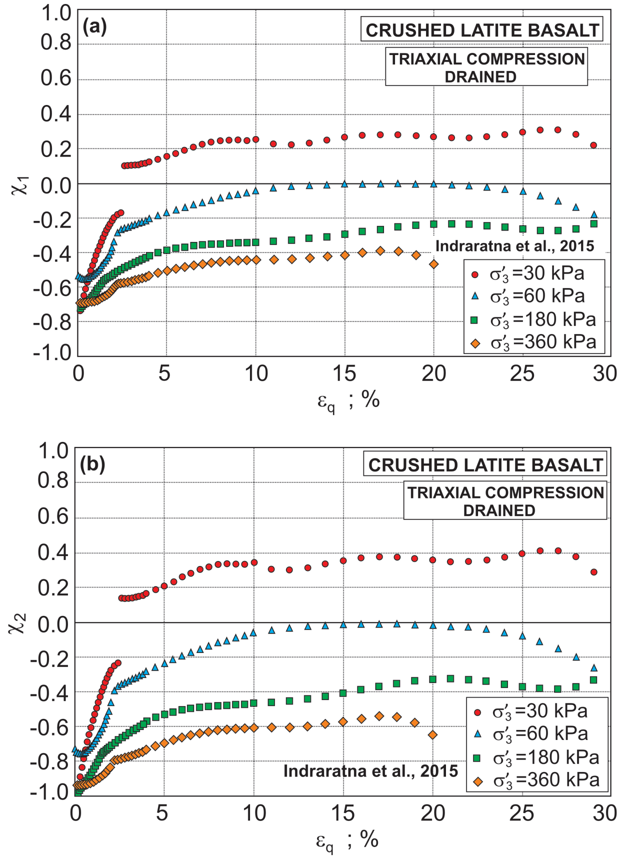

Figure 6: Relationships $\chi - \varepsilon_q$: (a) $\chi_1 - \varepsilon_q$; (b) $\chi_2 - \varepsilon_q$.

parameters α_{pt} and β_{pt} represent the influence of particle breakage on stress–strain behaviour at the advanced stage of shearing. The parameter α_{pt} represents the influence of energy consumed on crushing, and the parameter β_{pt} represents the influence of volume changes due to breakage on stress–dilatancy relationships.

The stress–dilatancy relationship is bilinear. At the initial stage of shearing, the breakage of particles is predominantly caused by the mean normal stress increments. At the advanced stage, the breakage of particles is mainly caused by shear deformation.

Particle breakage significantly influences the stress–dilatancy relationship for crushed latite basalt, even at a low stress level.

The results of this article show that *Frictional State Theory* may be used for describing the stress–strain behaviour of latite basalt and building a new model with small number of model parameters in the future.

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