Kołowrocki Krzysztof, 0000-0002-4836-4976
Gdynia Maritime University, Gdynia, Poland, k.kolowrocki{at}wn.umg.edu.pl
Magryta-Mut Beata, 0000-0002-4189-1853
Gdynia Maritime University, Gdynia, Poland, b.magryta{at}wn.umg.edu.pl

Operation cost and safety optimization of maritime transportation system

Keywords

operation cost, safety, optimization, maritime ferry technical system

Abstract

The model of system operation total cost during the fixed operation time is introduced and the procedure of its minimization is presented. The model of system safety impacted by operation process is introduced and the procedure of its safety maximization is proposed. To analyse jointly the system operation cost and safety optimization, we propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find the minimal operation total cost during the fixed operation time, through applying the created system general operation cost model in the fixed operation time and linear programming. Next, to find the system conditional safety indicators, corresponding to this system minimal total operation cost during the fixed operation time, we replace the limit transient probabilities in particular operation states, existing in the formula for the system safety function coordinates, by their optimal values existing in the formula for the system minimal total cost during the fixed operation time. Further, applying this formula for the system conditional safety function coordinates, related to the system minimal operation total cost during the fixed operation time, we find the remaining system conditional safety indicators. The created models are applied to the maritime ferry technical system to minimize the mean value of the system operation total cost during the fixed operation time of one month and to maximize its safety indicators. After that, the ferry technical system safety indicators corresponding to its minimal mean value of operation total cost are found. The evaluation of results achieved is performed and the perspective for future research in the field of the complex systems including critical infrastructures operation costs and safety joint analysis and optimization is given.

1. Introduction

To tie the investigation of the complex technical system operation cost together with the investigation of its safety, the semi-Markov process model (Ferreira & Pacheco, 2007; Glynn et al., 2006; Grabski, 2002, 2014; Limnios & Oprisan, 2005; Mercier, 2008; Tang et al., 2007) can be used to describe this system operation process (Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020).

Having the system operation process characteristics and the system conditional instantaneous operation costs at the operation states, it is possible to create the system general operation total cost model during the fixed operation time (Kołowrocki & Magryta, 2020a, 2021). Using this system operation cost model, it is possible to change the system operation process through applying the linear programming (Klabjan & Adelman, 2006) for minimizing the system operation cost (Kołowrocki & Magryta, 2020a, 2021)

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and finding optimal values of the system limit transient probabilities in the particular operation states.

The system operation process model, under the assumption on the system safety structure multi-state model (Xue, 1985; Xue & Yang, 1995), can be used to construct the general safety model of the complex multistate system changing its safety structure and its components safety parameters during variable operation process (Kołowrocki, 2014; Kołowrocki & Magryta, 2020a; Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020). Further, using this general model, it is possible to define the complex system main safety characteristics such as the system safety function, the mean values and standard deviations of the system lifetimes in the system safety state subsets and the system mean lifetimes in system safety subsets (Dąbrowska, 2020a-b; Kołowrocki, 2014, 2020; Kołowrocki & Soszyńska-Budny, 2010a-b, 2011/2015). Other system safety indicators, like the system risk function, the system fragility curve, the moment when the system risk function exceeds a permitted level, the system intensity of ageing, the coefficient of system operation impact on system intensity of ageing and the system resilience indicator to operation process impact, can be introduced as well (Gouldby et al., 2010; Kołowrocki, 2014; Kołowrocki & Soszyńska-Budny, 2018a-b, 2019a-b; Lauge et al., 2015; Szymkowiak, 2018a-b, 2019).

To analyse jointly the system operation cost and safety optimization, we firstly apply the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that minimize the system operation total cost during the fixed operation time. Next, to find the system conditional safety indicators, corresponding to this system minimal total operation cost during the fixed operation time, we replace the limit transient probabilities at the system particular operation states, existing in the formula for the system safety function, by their optimal values existing in the formula for the system minimal operation total cost during the fixed operation time in order to get thee formula for the system conditional safety function related to this system minimal operation total cost. Further, applying this formula for the system conditional safety function, we find the remaining system conditional safety indicators.

The created model for minimizing of the system operation total cost during the fixed operation time is applied to the maritime ferry technical system to find the minimal mean value of the system operation total cost during the fixed operation time of one month. Next, the ferry technical system safety indicators corresponding to this minimal operation total cost are found.

The chapter is organized into 7 parts, this Introduction as Section 1, Sections 2–6 and Conclusion as Section 7. In Section 2, the model of system operation total cost during the fixed operation time is introduced and the procedure of its minimization is presented. In Section 3, the model of system safety impacted by operation process is introduced and the procedure of the system safety maximization is proposed. In Section 4, the maritime ferry technical system operation process and operation total cost during one month are analyzed and the minimal value of this system operation total during the examined operation time is determined. In Section 5, the maritime ferry technical system operation process influence on its safety indicators is examined and the maximal values of this system safety indicators impacted by its operation process are determined. In Section 6, joint analysis of the ferry technical system operation total cost during one month minimizing and its conditional safety indicators corresponding to this system minimal operation total cost is performed. The system minimal operation total cost is fixed and the system safety indicators corresponding to this minimal operation total cost are determined. In Conclusion, the evaluation of results achieved is done and the perspective for future research in the field of the complex systems including critical infrastructures and their operation costs joint analysis and optimization is proposed.

2. System operation cost

2.1. System operation cost model

We assume that the system is operating at ν, $\nu > 1$, operation states z_b, $b = 1,2,\ldots,\nu$, that have influence on the system functional structure and on the system operation cost. Assuming semi-Markov model of the system operation process $Z(t)$, $t \geq 0$, it is possible to find this process two basic characteristics (Grabski, 2014; Kołowrocki & Magryta, 2020a; Kołowrocki & Soszyńska-

Budny, 2011/2015):

- the vector of limit values

$$p_b = \lim_{t\to\infty} p_b(t),\ b = 1,2,\dots,v, \qquad (1)$$

of transient probabilities

$$p_b(t) = P(Z(t) = z_b),\ t \ge 0, b = 1,2,\dots,v, \qquad (2)$$

of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\dots,v$,

- the vector $[\hat{M}_b]_{1xn}$ of the mean values

$$\hat{M}_b = E[\hat{q}_b] \cong p_b q,\ b = 1,2,\dots,v, \qquad (3)$$

of the total sojourn times $\hat{q}_b$, $b = 1,2,\dots,v$, of the system operation process $Z(t)$, $t \ge 0$, at the particular operation states z_b, $b = 1,2,\dots,v$, during the fixed the system operation time q, $q > 0$, where p_b, $b = 1,2,\dots,v$, are defined by (1)–(2).

Further, we may define the system instantaneous operation cost. Namely, we define the instantaneous system operation cost in the form of the vector

$$C(t) = [[C(t)]^{(1)}, C(t)]^{(2)}, \dots, [C(t)]^{(n)}],\ t \ge 0, \qquad (4)$$

with the coordinates

$$[C(t)]^{(b)},\ t \ge 0,\ b = 1,2,\dots,v, \qquad (5)$$

that are the system conditional instantaneous operation costs at the system operation states z_b, $b = 1,2,\dots,v$.

It is natural to assume that the system operation total cost during the fixed operation time depends significantly on the system operation total costs at the particular operation states. This dependency is clearly expressed in mean value of the system total operation cost during the system operation time q, given by

$$\boldsymbol{C}(q) = \sum_{b=1}^{n} p_b [C(q)]^{(b)},\ q > 0, \qquad (6)$$

where p_b, $b = 1,2,\dots,v$, are limit transient probabilities at operation states defined by (1)–(2), and $[C(q)]^{(b)}$, $b = 1,2,\dots,v$, are the mean values of the system conditional operation total costs at the particular system operation states z_b, $b = 1,2,\dots,v$, given by

$$[C(q)]^{(b)} = \int_0^{\hat{M}_b} C[(t)]^{(b)} dt,\ q > 0, b = 1,2,\dots,v, \qquad (7)$$

where $\hat{M}_b$, $b = 1,2,\dots,v$, are given by (3), and $[C(t)]^{(b)}$, $t \ge 0$, $b = 1,2,\dots,v$, are defined by (5).

2.2. System operation cost optimization model

In Section 2.1 we described system operation cost model. From the linear equations (6), we can see that the mean value of the system total unconditional operation cost $\boldsymbol{C}(q)$, $q > 0$, is determined by the limit values of transient probabilities p_b, $b = 1,2,\dots,v$, of the system operation process at the operation states z_b, $b = 1,2,\dots,v$, defined by (1)–(2) and by the mean values $[C(q)]^{(b)}$, $q > 0$, $b = 1,2,\dots,v$, of the system conditional total operation costs at the particular system operation states z_b, $b = 1,2,\dots,v$, determined by (7). Therefore, the system operation cost optimization based on the linear programming (Klabjan & Adelman, 2006), can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\dots,v$, of the limit transient probabilities p_b, $b = 1,2,\dots,v$, of the system operation process at the operation states to minimize the mean value $\boldsymbol{C}(\theta)$ of the system unconditional operation total cost under the assumption that the mean values $[C(q)]^{(b)}$, $b = 1,2,\dots,v$, of the system conditional total operation costs at the particular operation states z_b, $b = 1, 2,\dots,v$, are fixed.

Thus, we may formulate the optimization problem as a linear programming model with the objective function of the form given by (6) with the bound constraints

$$\breve{p}_b \le p_b \le \hat{p}_b,\ b = 1,2,\dots,v,\ \sum_{b=1}^{n} p_b = 1, \qquad (8)$$

where

$$[C(q)]^{(b)},\ [C(q)]^{(b)} \ge 0,\ b = 1,2,\dots,v, \qquad (9)$$

are fixed mean values of the system conditional operation costs at the operation states z_b, $b = 1,2,\ldots,v$, determined according to (7) and

$$\breve{p}_b,\ 0 \le \breve{p}_b \le 1 \text{ and } \hat{p}_b,\ 0 \le \hat{p}_b \le 1,\ \breve{p}_b \le \hat{p}_b,$$

$$b = 1,2,\ldots,v, \tag{10}$$

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,\ldots,v$, respectively.

Now, we can find the optimal solution of the formulated by (6), (8)–(10) the linear programming problem, i.e. we can determine the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,\ldots,v$, that minimize the objective function given by (6). The minimizing procedure is described in (Magryta, 2021).

Finally, after applying this procedure, we can get the minimum value of the system total unconditional operation cost, defined by the linear form (6), in the following form

$$\dot{C}(q) = \sum_{i=1}^{n} \dot{p}_b [C(q)]^{(b)}. \tag{11}$$

3. System safety

3.1. System safety model

Considering the safety function of the system impacted by operation process

$$S(t,\cdot) = [S(t,1), S(t,2),\ldots, S(t,z)],\ t \ge 0, \tag{12}$$

coordinate given by (Kołowrocki & Soszyńska-Budny, 2011/2015)

$$S(t,u) \cong \sum_{b=1}^{n} p_b [S(t,u)]^{(b)},\ t \ge 0,$$

$$u = 1,2,\ldots,z, \tag{13}$$

where p_b, $b = 1,2,\ldots,v$, are the limit transient probabilities of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and

$$[S(t,u)]^{(b)} = P([T(u)]^{(b)} > t),\ t \ge 0,\ u = 1,2,\ldots,z,$$

$$b = 1,2,\ldots,v,$$

at these operation states are the conditional safety functions of the system and $[T(u)]^{(b)}$, are the system conditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the operation states z_b, $b = 1,2,\ldots,v$, it is natural to assume that the system operation process has a significant influence on the system safety.

From the expression (12), the mean values of the system unconditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, are of the form

$$\mu(u) = \sum_{b=1}^{n} p_b [\mu(u)]^{(b)} \text{ for } u = 1,2,\ldots,z. \tag{14}$$

The values of the variances of the system unconditional lifetimes in the system safety state subsets are

$$[\sigma(u)]^2 = 2\int_0^{\infty} t S(t,u)dt - [\mu(u)]^{(2)},\ u = 1,2,\ldots,z \tag{15}$$

where $\mu(u)$ is given by (14) and $S(t,u)$ is given by (13).

The expressions for the mean values of the system unconditional lifetimes in the particular safety states are

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),\ u = 1,2,\ldots,z-1,$$

$$\bar{\mu}(z) = \mu(z). \tag{16}$$

The system risk function and the moment when the risk exceeds a permitted level d, respectively are given by (Kołowrocki & Soszyńska-Budny, 2011/2015):

$$r(t) = 1 - S(t,r),\ t \ge 0, \tag{17}$$

and

$$t = r^{-1}(d), \tag{18}$$

where $S(t,r)$ is given by (13) for $u = r$ and $r^{-1}(t)$, if it exists, is the inverse function of the risk function $r(t)$.

The mean values of the system intensities of ageing (departure from the safety state subset $\{u,u+1,\ldots,z\}$), are defined by

$$\lambda(u) = \frac{1}{\mu(u)},\ u = 1,2,\ldots,z. \tag{19}$$

Considering the values of the system without operation impact intensities of ageing $\lambda^0(u)$, defined in (Kołowrocki & Soszyńska-Budny, 2018b; Kołowrocki & Magryta-Mut, 2020c), the coefficients of the operation process impact on the system intensities of ageing are given by

$$\rho(u) = \frac{\lambda(u)}{\lambda^{\circ}(u)}, \ \ u = 1,2,...,z. \tag{20}$$

Finally, the system resilience indicators, i.e. the coefficients of the system resilience to operation process impact, are

$$\boldsymbol{RI}(u) = \frac{1}{\rho(u)}, \ \ u = 1,2,...,z. \tag{21}$$

3.2. System safety optimization

Considering the safety function of the system impacted by operation process $\boldsymbol{S}(t,\cdot)$, $t \geq 0$, coordinate given by (13), it is natural to assume that the system operation process has a significant influence on the system safety. This influence is also clearly expressed in the equation (14) for the mean values of the system unconditional lifetimes in the safety state subsets. From the linear equation (14), we can see that the mean value of the system unconditional lifetime $\boldsymbol{\mu}(u)$, $u = 1,2,\ldots,z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and the mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at these operation states. Therefore, the system lifetime optimization based on the linear programming can be proposed (Klabjan & Adelman, 2006). Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states to maximize the mean value $\boldsymbol{\mu}(u)$ of the unconditional system lifetime in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, under the assumption that the conditional mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in the safety state subsets at the particular operation states are fixed. As a special case of the above formulated system lifetime optimization, if r, $r = 1,2,\ldots,z$, is a system critical safety state, we want to find the optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the system operation states to maximize the mean value $\boldsymbol{\mu}(r)$ of the unconditional system lifetime in the safety state subset $\{r,r+1,\ldots,z\}$, $r = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{\mu}(r)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in this safety state subset at the particular operation states are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\boldsymbol{\mu}(r) = \sum_{b=1}^{n} p_b [\mu(r)]^{(b)}, \tag{22}$$

for a fixed $r \in \{1,2,\ldots,z\}$ and with the following bound constraints

$$\breve{p}_b \leq p_b \leq \hat{p}_b, \ b = 1,2,\ldots,v, \tag{23}$$

$$\sum_{b=1}^{n} p_b = 1, \tag{24}$$

where

$$[m(r)]^{(b)}, \ [m(r)]^{(b)} \geq 0, \ b = 1,2,\ldots,v, \tag{25}$$

are fixed mean values of the system conditional lifetimes in the safety state subset $\{r,r+1,\ldots,z\}$ and

$$\breve{p}_b, \ 0 \leq \breve{p}_b \leq 1 \text{ and } \hat{p}_b, \ 0 \leq \hat{p}_b \leq 1, \ \breve{p}_b \leq \hat{p}_b,$$

$$b = 1,2,\ldots,v, \tag{26}$$

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,\ldots,v$, respectively.

Now, we can obtain the optimal solution of the formulated by (22)–(26) the optimization problem, i.e. we can find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,\ldots,v$, that maximize the objective function given by (22). The maximizing procedure is described in (Kołowrocki & Magryta, 2020b; Magryta-Mut, 2020).

Finally, after applying this procedure, we can get the maximum value of the system total mean lifetime in the safety state subset $\{r,r+1,\ldots,z\}$ defined by the linear form (22), in the following form

$$\dot{\mu}(r) = \sum_{b=1}^{n} \dot{p}_b [m(r)]^{(b)} \tag{27}$$

for a fixed $r \in \{1,2,\ldots,z\}$.

Further, by replacing the limit transient probabilities p_b, $b = 1,2,\ldots,v$, existing in the formulae (12)–(14) by their optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, we get the optimal form of the system safety and the expressions for all remaining safety indicators considered in Section 3.1.

4. Maritime ferry technical system operation cost

4.1. Operation process

We will examine the operation cost of a selected maritime ferry technical system that is a member of the shipping critical infrastructure. The considered maritime ferry is described in (Kołowrocki et al., 2016; Kołowrocki & Magryta-Mut, 2020c; Magryta-Mut, 2020).

The maritime ferry operation process $Z(t)$, $t \geq 0$, was identified and specified in (Kołowrocki et al., 2016). Having regards to the opinions of experts on the varying in time operation process of the pondered maritime ferry system, we identify the eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to “GD” buoy,
- an operation state z_4 – navigation at restricted waters from “GD” buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to “Angoering” buoy,
- an operation state z_6 – navigation at restricted waters from “Angoering” buoy to “Verko” berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ferry turning at Karlskrona Port,
- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to “Angoering” buoy,
- an operation state z_{13} – navigation at open waters from “Angoering” buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to “GD” buoy,
- an operation state z_{15} – navigation from “GD” buoy to turning area,
- an operation state z_{16} – ferry turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

To identify the unknown parameters of the ferry technical system operation process the suitable statistical data coming from its real realizations should be collected. It is possible to collect these data because of the high frequency of the ferry voyages that result in a large number of its technical system operation process realizations. The ferry technical system operation process is very regular in the sense that the operation state changes are from the particular state z_b, $b = 1,2,\ldots,17$, to the neighboring state z_{b+1}, $b = 1,2,\ldots,17$, and from z_{18} to z_1 only.

The ferry technical system operation process $Z(t)$ characteristics are:

- the limit values of transients probabilities p_b, of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,18$, (Kołowrocki & Magryta, 2020a, 2021; Kołowrocki & Soszyńska-Budny, 2011/2015):

$$p_1 = 0.038,\ p_2 = 0.002,\ p_3 = 0.026,$$
$$p_4 = 0.036,\ p_5 = 0.363,\ p_6 = 0.026,$$
$$p_7 = 0.005,\ p_8 = 0.016,\ p_9 = 0.037,$$
$$p_{10} = 0.002,\ p_{11} = 0.003,\ p_{12} = 0.016,$$
$$p_{13} = 0.351,\ p_{14} = 0.034,\ p_{15} = 0.024,$$
$$p_{16} = 0.003,\ p_{17} = 0.005,\ p_{18} = 0.013. \tag{28}$$

4.2. Operation cost

According to the information coming from experts, the approximate values of the instantaneous operation cost per hour of the subsystems arbitrarily assumed in comparison to the unknown amount c are:

- for the navigational subsystem S_1 used at the operation state z_b, b = 1, 2,…, 18, is constant and amounts 20c, whereas, the cost of this subsystem when is not used equal to 10c;
- for the propulsion and controlling subsystem S_2 used at the operation state z_b, b = 2, 3, 6, 7, 10, 11, 15, 16, 17, is constant and amounts 75c, at the operation state z_b, b = 4, 5, 12, 13, 14, is constant and amounts 55c, whereas, the cost of this subsystem when not used is equal to 25c;
- for the loading and unloading subsystem S_3 used at the operation state z_b, b = 1, 18, is constant and amounts 30c, at the operation state z_b, b = 8, 9, is constant and amounts 20c, whereas, the cost of this subsystem when not used is equal to 10c;
- for the stability control subsystem S_4 used at the operation state z_b, b = 1, 4, 5, 6, 8, 9, 12, 13, 14, 18, is constant and amounts 13c, whereas, the cost of this subsystem when not used is equal to 10c;
- for the anchoring and mooring subsystem S_5 used at the operation state z_b, b = 2, 7, 10, 17, is constant and amounts 30c, whereas, the cost of this subsystem when is not used equal to 5c.

Through (3) and (28), the approximate mean values $\hat{M}_b$, of total sojourn times of the ferry technical system at the particular operation states during the operation time θ = 1 month = 30 days = 720 hours:

$$\hat{M}_1 = 27.36,\ \hat{M}_2 = 1.44,\ \hat{M}_3 = 18.72,$$
$$\hat{M}_4 = 25.92,\ \hat{M}_5 = 261.36,\ \hat{M}_6 = 18.72,$$
$$\hat{M}_7 = 3.6,\ \hat{M}_8 = 11.52,\ \hat{M}_9 = 26.64,$$
$$\hat{M}_{10} = 1.44,\ \hat{M}_{11} = 2.16,\ \hat{M}_{12} = 11.52,$$
$$\hat{M}_{13} = 252.72,\ \hat{M}_{14} = 24.48,\ \hat{M}_{15} = 17.28,$$
$$\hat{M}_{16} = 2.16,\ \hat{M}_{17} = 3.6,\ \hat{M}_{18} = 9.36. \quad (29)$$

Hence, the subsystems S_1, S_2, S_3, S_4, S_5 use at particular operation states implies that the system at the particular operation states conditional instantaneous operation costs per hour $[C(t)]^{(b)}$, $t \in \langle 0,1 \rangle$, b = 1,2,…,18, during the operation time interval of $q = 1$ month, introduced by (5), are:

$$[C(t)]^{(1)} = 93c,\ [C(t)]^{(2)} = 145c,$$
$$[C(t)]^{(3)} = 120c,\ [C(t)]^{(4)} = 103c,$$
$$[C(t)]^{(5)} = 103c,\ [C(t)]^{(6)} = 123c,$$
$$[C(t)]^{(7)} = 145c,\ [C(t)]^{(8)} = 83c,$$
$$[C(t)]^{(9)} = 83c,\ [C(t)]^{(10)} = 145c,$$
$$[C(t)]^{(11)} = 120c,\ [C(t)]^{(12)} = 103c,$$
$$[C(t)]^{(13)} = 103c,\ [C(t)]^{(14)} = 103c,$$
$$[C(t)]^{(15)} = 120c,\ [C(t)]^{(16)} = 120c,$$
$$[C(t)]^{(17)} = 145c,\ [C(t)]^{(18)} = 93c. \quad (30)$$

Applying the formula (7) to (30) and (29), we get the approximate mean values $[C(\theta)]^{(b)}$, b = 1,2,…,18, of the total operation costs at the operation state z_b, b = 1,2,…,18, during the operation time $q = 1$ month = 720 hours:

$$[C(t)]^{(1)} = 93c \cdot 27.36 = 2554.48c,$$
$$[C(t)]^{(2)} = 145c \cdot 1.44 = 208.8c,$$
$$[C(t)]^{(3)} = 120c \cdot 18.72 = 2246.4c,$$
$$[C(t)]^{(4)} = 103c \cdot 25.92 = 2669.76c,$$
$$[C(t)]^{(5)} = 103c \cdot 261.36 = 26920.08c,$$
$$[C(t)]^{(6)} = 123c \cdot 18.72 = 2302.56c,$$
$$[C(t)]^{(7)} = 145c \cdot 3.6 = 522c,$$
$$[C(t)]^{(8)} = 83c \cdot 11.52 = 965.16c,$$
$$[C(t)]^{(9)} = 83c \cdot 26.64 = 2211.12c,$$
$$[C(t)]^{(10)} = 145c \cdot 1.44 = 208.8c,$$
$$[C(t)]^{(11)} = 120c \cdot 2.16 = 259.2c,$$
$$[C(t)]^{(12)} = 103c \cdot 11.52 = 1186.56c,$$
$$[C(t)]^{(13)} = 103c \cdot 252.72 = 26030.16c,$$
$$[C(t)]^{(14)} = 103c \cdot 24.48 = 2521.44c,$$
$$[C(t)]^{(15)} = 120c \cdot 17.28 = 2073.6c,$$
$$[C(t)]^{(16)} = 120c \cdot 2.16 = 259.2c,$$
$$[C(t)]^{(17)} = 145c \cdot 3.6 = 522c,$$
$$[C(t)]^{(18)} = 93c \cdot 9.36 = 870.48c. \quad (31)$$

Considering the values of the total costs $[C(\theta)]^{(b)}$, b = 1,2,…,18, from (31) and the values of transient probabilities p_b, b = 1,2,…,18, given by (28), the ferry technical system total operation mean cost during the operation time

θ = 1 month = 720 hours, according to (6), is given by

$$\begin{aligned}\boldsymbol{C}(\theta) &\cong p_1[C(\theta)]^{(1)} + p_2[C(\theta)]^{(2)} + p_3[C(\theta)]^{(3)} \\ &+ p_4[C(\theta)]^{(4)} + p_5[C(\theta)]^{(5)} + p_6[C(\theta)]^{(6)} \\ &+ p_7[C(\theta)]^{(7)} + p_8[C(\theta)]^{(8)} + p_9[C(\theta)]^{(9)} \\ &+ p_{10}[C(\theta)]^{(10)} + p_{11}[C(\theta)]^{(11)} + p_{12}[C(\theta)]^{(12)} \\ &+ p_{13}[C(\theta)]^{(13)} + p_{14}[C(\theta)]^{(14)} + p_{15}[C(\theta)]^{(15)} \\ &+ p_{16}[C(\theta)]^{(16)} + p_{17}[C(\theta)]^{(17)} + p_{18}[C(\theta)]^{(18)} \\ &\cong 0.038\cdot 2554.48c + 0.002\cdot 208.8c \\ &+ 0.026\cdot 2246.4c + 0.036\cdot 2669.76c \\ &+ 0.363\cdot 26920.08c + 0.026\cdot 2302.56c \\ &+ 0.005\cdot 522c + 0.016\cdot 965.16c \\ &+ 0.037\cdot 2211.12c + 0.002\cdot 208.8c \\ &+ 0.003\cdot 259.2c + 0.016\cdot 1186.56c \\ &+ 0.351\cdot 26030.16c + 0.034\cdot 2521.44c \\ &+ 0.024\cdot 2073.6c + 0.003\cdot 259.2c \\ &+ 0.005\cdot 522c + 0.013\cdot 870.48c \\ &\cong 19490.17c. \qquad (32)\end{aligned}$$

4.3. Cost optimization

Applying (32), to find the minimum value of the ferry technical system mean cost, we define the objective function given by (6), in the following form

$$\begin{aligned}\boldsymbol{C}(\theta) &\cong p_1\cdot 2554.48c + p_2\cdot 208.8c \\ &+ p_3\cdot 2246.4c + p_4\cdot 2669.76c \\ &+ p_5\cdot 26920.08c + p_6\cdot 2302.56c \\ &+ p_7\cdot 522c + p_8\cdot 965.16c + p_9\cdot 2211.12c \\ &+ p_{10}\cdot 208.8c + p_{11}\cdot 259.2c + p_{12}\cdot 1186.56c \\ &+ p_{13}\cdot 26030.16c + p_{14}\cdot 2521.44c \\ &+ p_{15}\cdot 2073.6c + p_{16}\cdot 259.2c \\ &+ p_{17}\cdot 522c + p_{18}\cdot 870.48c. \qquad (33)\end{aligned}$$

The lower $\check{p}_b$, and upper $\hat{p}_b$ bounds of the unknown optimal values of transient probabilities p_b, b = 1,2,…,18, respectively are (Kołowrocki & Magryta, 2020a, 2021; Kołowrocki & Soszyńska-Budny, 2011/2015):

$$\begin{aligned}&\check{p}_1 = 0.0006,\ \check{p}_2 = 0.001,\ \check{p}_3 = 0.018, \\ &\check{p}_4 = 0.027,\ \check{p}_5 = 0.286,\ \check{p}_6 = 0.018, \\ &\check{p}_7 = 0.002,\ \check{p}_8 = 0.001,\ \check{p}_9 = 0.001, \\ &\check{p}_{10} = 0.001,\ \check{p}_{11} = 0.002,\ \check{p}_{12} = 0.013, \\ &\check{p}_{13} = 0.286,\ \check{p}_{14} = 0.025,\ \check{p}_{15} = 0.018, \\ &\check{p}_{16} = 0.002,\ \check{p}_{17} = 0.002,\ \check{p}_{18} = 0.001,\end{aligned}$$

$$\begin{aligned}&\hat{p}_1 = 0.056,\ \hat{p}_2 = 0.002,\ \hat{p}_3 = 0.027, \\ &\hat{p}_4 = 0.056,\ \hat{p}_5 = 0.780,\ \hat{p}_6 = 0.024, \\ &\hat{p}_7 = 0.018,\ \hat{p}_8 = 0.018,\ \hat{p}_9 = 0.056, \\ &\hat{p}_{10} = 0.003,\ \hat{p}_{11} = 0.004,\ \hat{p}_{12} = 0.024, \\ &\hat{p}_{13} = 0.780,\ \hat{p}_{14} = 0.043,\ \hat{p}_{15} = 0.024, \\ &\hat{p}_{16} = 0.004,\ \hat{p}_{17} = 0.007,\ \hat{p}_{18} = 0.018. \qquad (34)\end{aligned}$$

Therefore, according to (9)–(10), we assume the following bound constraints

$$\begin{aligned}&0.0006 \le p_1 \le 0.056,\ 0.001 \le p_2 \le 0.002, \\ &0.018 \le p_3 \le 0.027,\ 0.027 \le p_4 \le 0.056, \\ &0.286 \le p_5 \le 0.780,\ 0.018 \le p_6 \le 0.024, \\ &0.002 \le p_7 \le 0.018,\ 0.001 \le p_8 \le 0.018, \\ &0.001 \le p_9 \le 0.056,\ 0.001 \le p_{10} \le 0.003, \\ &0.002 \le p_{11} \le 0.004,\ 0.013 \le p_{12} \le 0.024, \\ &0.286 \le p_{13} \le 0.780,\ 0.025 \le p_{14} \le 0.043, \\ &0.018 \le p_{15} \le 0.024,\ 0.002 \le p_{16} \le 0.004, \\ &0.002 \le p_{17} \le 0.007,\ 0.001 \le p_{18} \le 0.018, \\ &\sum_{b=1}^{18} p_b = 1. \qquad (35)\end{aligned}$$

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities p_b, b = 1,2,…,18, that minimize the objective function (33), we arrange the mean values of the ferry technical system conditional operation costs $[C(\theta)]^{(b)}$, b = 1,2,…,18, determined by (31), in non-decreasing order

$$\begin{aligned}&208.8c \le 208.8c \le 259.2c \le 259.2c \le 522c \\ &\le 522c \le 870.48c \le 965.16c \le 1186.56c \\ &\le 2073.6c \le 2211.12c \le 2246c \le 2302.56c \\ &\le 2521.44c \le 2544.48c \le 2669.76c \le 26030.16c \\ &\le 26920.08c,\end{aligned}$$

i.e.

$$\begin{aligned}&[C(q)]^{(2)} \le [C(q)]^{(10)} \le [C(q)]^{(11)} \le [C(q)]^{(16)} \\ &\le [C(q)]^{(7)} \le [C(q)]^{(17)} \le [C(q)]^{(18)} \le [C(q)]^{(8)} \\ &\le [C(q)]^{(12)} \le [C(q)]^{(15)} \le [C(q)]^{(9)} \le [C(q)]^{(3)} \\ &\le [C(q)]^{(6)} \le [C(q)]^{(14)} \le [C(q)]^{(1)} \le [C(q)]^{(4)} \\ &\le [C(q)]^{(13)} \le [C(q)]^{(5)}. \qquad (36)\end{aligned}$$

Further, we substitute

$$x_1 = p_2,\ x_2 = p_{10},\ x_3 = p_{11},\ x_4 = p_{16},\ x_5 = p_7,$$
$$x_6 = p_{17},\ x_7 = p_{18},\ x_8 = p_8,\ x_9 = p_{12},\ x_{10} = p_{15},$$
$$x_{11} = p_9,\ x_{12} = p_3,\ x_{13} = p_6,\ x_{14} = p_{14},\ x_{15} = p_1,$$
$$x_{16} = p_4,\ x_{17} = p_{13},\ x_{18} = p_5, \tag{37}$$

and

$$\breve{x}_1 = \breve{p}_2 = 0.001,\ \breve{x}_2 = \breve{p}_{10} = 0.001,$$
$$\breve{x}_3 = \breve{p}_{11} = 0.002,\ \breve{x}_4 = \breve{p}_{16} = 0.002,$$
$$\breve{x}_5 = \breve{p}_7 = 0.002,\ \breve{x}_6 = \breve{p}_{17} = 0.002,$$
$$\breve{x}_7 = \breve{p}_{18} = 0.001,\ \breve{x}_8 = \breve{p}_8 = 0.001,$$
$$\breve{x}_9 = \breve{p}_{12} = 0.013,\ \breve{x}_{10} = \breve{p}_{15} = 0.018,$$
$$\breve{x}_{11} = \breve{p}_9 = 0.001,\ \breve{x}_{12} = \breve{p}_3 = 0.018,$$
$$\breve{x}_{13} = \breve{p}_6 = 0.018,\ \breve{x}_{14} = \breve{p}_{14} = 0.025,$$
$$\breve{x}_{15} = \breve{p}_1 = 0.0006,\ \breve{x}_{16} = \breve{p}_4 = 0.027,$$
$$\breve{x}_{17} = \breve{p}_{13} = 0.286,\ \breve{x}_{18} = \breve{p}_5 = 0.286,$$

$$\hat{x}_1 = \hat{p}_2 = 0.002,\ \hat{x}_2 = \hat{p}_{10} = 0.003,$$
$$\hat{x}_3 = \hat{p}_{11} = 0.004,\ \hat{x}_4 = \hat{p}_{16} = 0.004,$$
$$\hat{x}_5 = \hat{p}_7 = 0.018,\ \hat{x}_6 = \hat{p}_{17} = 0.007,$$
$$\hat{x}_7 = \hat{p}_{18} = 0.018,\ \hat{x}_8 = \hat{p}_8 = 0.018,$$
$$\hat{x}_9 = \hat{p}_{12} = 0.024,\ \hat{x}_{10} = \hat{p}_{15} = 0.024,$$
$$\hat{x}_{11} = \hat{p}_9 = 0.056,\ \hat{x}_{12} = \hat{p}_3 = 0.027,$$
$$\hat{x}_{13} = \hat{p}_6 = 0.024,\ \hat{x}_{14} = \hat{p}_{14} = 0.043,$$
$$\hat{x}_{15} = \hat{p}_1 = 0.056,\ \hat{x}_{16} = \hat{p}_4 = 0.056,$$
$$\hat{x}_{17} = \hat{p}_{13} = 0.78,\ \hat{x}_{18} = \hat{p}_5 = 0.78, \tag{38}$$

and we minimize with respect to x_i, i = 1,2,…,18, the linear form (33) takes the form

$$\begin{aligned} \boldsymbol{C}(\theta) = {} & x_1 \cdot 208.8c + x_2 \cdot 208.8c + x_3 \cdot 259.2c \\ & + x_4 \cdot 259.2c + x_5 \cdot 522c + x_6 \cdot 522c \\ & + x_7 \cdot 870.48c + x_8 \cdot 956.16c + x_9 \cdot 1186.56c \\ & + x_{10} \cdot 2073.6c + x_{11} \cdot 2211.12c + x_{12} \cdot 2246.4c \\ & + x_{13} \cdot 2302.56c + x_{14} \cdot 2521.44c \\ & + x_{15} \cdot 2544.48c + x_{16} \cdot 2669.76c + \\ & x_{17} \cdot 26030.16c + x_{18} \cdot 26920.08c, \end{aligned} \tag{39}$$

with the following bound constraints

$$0.001 \le x_1 \le 0.002,\ 0.001 \le x_2 \le 0.003,$$
$$0.002 \le x_3 \le 0.004,\ 0.002 \le x_4 \le 0.004,$$
$$0.002 \le x_5 \le 0.018,\ 0.002 \le x_6 \le 0.007,$$
$$0.001 \le x_7 \le 0.018,\ 0.001 \le x_8 \le 0.018,$$
$$0.013 \le x_9 \le 0.024,\ 0.018 \le x_{10} \le 0.024,$$
$$0.001 \le x_{11} \le 0.056,\ 0.018 \le x_{12} \le 0.027,$$
$$0.018 \le x_{13} \le 0.024,\ 0.025 \le x_{14} \le 0.043,$$
$$0.0006 \le x_{15} \le 0.056,\ 0.027 \le x_{16} \le 0.056,$$
$$0.286 \le x_{17} \le 0.78,\ 0.286 \le x_{18} \le 0.78,$$
$$\sum_{i=1}^{18} x_i = 1. \tag{40}$$

We calculate

$$\breve{x} = \sum_{i=1}^{18} \breve{x}_i = 0.7046,$$

$$\hat{y} = 1 - \breve{x} = 1 - 0.7046 = 0.2954 \tag{41}$$

and we find

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0,\ \hat{x}^0 - \breve{x}^0 = 0,$$
$$\breve{x}^1 = 0.001,\ \hat{x}^1 = 0.002,\ \hat{x}^1 - \breve{x}^1 = 0.001,$$
$$\breve{x}^2 = 0.002,\ \hat{x}^2 = 0.005,\ \hat{x}^2 - \breve{x}^2 = 0.003,$$
$$\breve{x}^3 = 0.004,\ \hat{x}^3 = 0.009,\ \hat{x}^3 - \breve{x}^3 = 0.005,$$
$$\breve{x}^4 = 0.006,\ \hat{x}^4 = 0.013,\ \hat{x}^4 - \breve{x}^4 = 0.007,$$
$$\breve{x}^5 = 0.008,\ \hat{x}^5 = 0.031,\ \hat{x}^5 - \breve{x}^5 = 0.023,$$
$$\breve{x}^6 = 0.01,\ \hat{x}^6 = 0.038,\ \hat{x}^6 - \breve{x}^6 = 0.028,$$
$$\breve{x}^7 = 0.011,\ \hat{x}^7 = 0.056,\ \hat{x}^7 - \breve{x}^7 = 0.045,$$
$$\breve{x}^8 = 0.012,\ \hat{x}^8 = 0.074,\ \hat{x}^8 - \breve{x}^8 = 0.062,$$
$$\breve{x}^9 = 0.025,\ \hat{x}^9 = 0.098,\ \hat{x}^9 - \breve{x}^9 = 0.073,$$
$$\breve{x}^{10} = 0.043,\ \hat{x}^{10} = 0.122,\ \hat{x}^{10} - \breve{x}^{10} = 0.079,$$
$$\breve{x}^{11} = 0.044,\ \hat{x}^{11} = 0.178,\ \hat{x}^{11} - \breve{x}^{11} = 0.134,$$
$$\breve{x}^{12} = 0.062,\ \hat{x}^{12} = 0.205,\ \hat{x}^{12} - \breve{x}^{12} = 0.143,$$
$$\breve{x}^{13} = 0.08,\ \hat{x}^{13} = 0.229,\ \hat{x}^{13} - \breve{x}^{13} = 0.149,$$
$$\breve{x}^{14} = 0.105,\ \hat{x}^{14} = 0.272,\ \hat{x}^{14} - \breve{x}^{14} = 0.167,$$
$$\breve{x}^{15} = 0.1056,\ \hat{x}^{15} = 0.328,\ \hat{x}^{15} - \breve{x}^{15} = 0.2224,$$
$$\breve{x}^{16} = 0.1326,\ \hat{x}^{16} = 0.384,\ \hat{x}^{16} - \breve{x}^{16} = 0.2514,$$
$$\breve{x}^{17} = 0.4186,\ \hat{x}^{17} = 1.164,\ \hat{x}^{17} - \breve{x}^{17} = 0.7454,$$
$$\breve{x}^{18} = 0.7046,\ \hat{x}^{18} = 1.944,\ \hat{x}^{18} - \breve{x}^{18} = 1.2394, \tag{42}$$

From the above, the expression takes the form

$$\hat{x}^I - \breve{x}^I < 0.2954, \tag{43}$$

then it follows that the largest value $I \in \{0,1,\ldots,18\}$ such that this inequality holds is

$I = 16$. Therefore, we fix the optimal solution that minimize linear function (33). Namely, we get

$$\dot{x}_1 = \breve{x}_1 = 0.002,\ \dot{x}_2 = \breve{x}_2 = 0.003,$$
$$\dot{x}_3 = \breve{x}_3 = 0.004,\ \dot{x}_4 = \breve{x}_4 = 0.004,$$
$$\dot{x}_5 = \breve{x}_5 = 0.018,\ \dot{x}_6 = \breve{x}_6 = 0.007,$$
$$\dot{x}_7 = \breve{x}_7 = 0.018,\ \dot{x}_8 = \breve{x}_8 = 0.018,$$
$$\dot{x}_9 = \breve{x}_9 = 0.024,\ \dot{x}_{10} = \breve{x}_{10} = 0.024,$$
$$\dot{x}_{11} = \breve{x}_{11} = 0.056,\ \dot{x}_{12} = \breve{x}_{12} = 0.027,$$
$$\dot{x}_{13} = \breve{x}_{13} = 0.024,\ \dot{x}_{14} = \breve{x}_{14} = 0.043,$$
$$\dot{x}_{15} = \breve{x}_{15} = 0.056,\ \dot{x}_{16} = \breve{x}_{16} = 0.056,$$
$$\dot{x}_{17} = \hat{y} - \breve{x}^{16} + \breve{x}^{16} + \breve{x}_{17} = 0.2954 - 0.384 + 0.1326$$
$$+ 0.286 = 0.33,$$
$$\dot{x}_{18} = \breve{x}_{18} = 0.286. \qquad (44)$$

Finally, after making the substitution inverse to (37), we get the optimal transient probabilities

$$\dot{p}_1 = \dot{x}_{15} = 0.056,\ \dot{p}_2 = \dot{x}_1 = 0.002,$$
$$\dot{p}_3 = \dot{x}_{12} = 0.027,\ \dot{p}_4 = \dot{x}_{16} = 0.056,$$
$$\dot{p}_5 = \dot{x}_{18} = 0.286,\ \dot{p}_6 = \dot{x}_{13} = 0.024,$$
$$\dot{p}_7 = \dot{x}_5 = 0.018,\ \dot{p}_8 = \dot{x}_8 = 0.018,$$
$$\dot{p}_9 = \dot{x}_{11} = 0.056,\ \dot{p}_{10} = \dot{x}_2 = 0.003,$$
$$\dot{p}_{11} = \dot{x}_3 = 0.004,\ \dot{p}_{12} = \dot{x}_9 = 0.024,$$
$$\dot{p}_{13} = \dot{x}_{17} = 0.33,\ \dot{p}_{14} = \dot{x}_{14} = 0.043,$$
$$\dot{p}_{15} = \dot{x}_{10} = 0.024,\ \dot{p}_{16} = \dot{x}_4 = 0.004,$$
$$\dot{p}_{17} = \dot{x}_6 = 0.007,\ \dot{p}_{18} = \dot{x}_7 = 0.018, \qquad (45)$$

that minimize the mean value of the ferry technical system total operation cost $\boldsymbol{C}(\theta)$ during the operation time θ = 1 month = 720 hours, expressed by the linear form (32) and (45), its optimal value is

$$\dot{C}(\theta) \cong 0.056 \cdot 2554.48c + 0.002 \cdot 208.8c$$
$$+ 0.027 \cdot 2246.4c + 0.056 \cdot 2669.76c$$
$$+ 0.286 \cdot 26920.08c + 0.024 \cdot 2302.56c$$
$$+ 0.018 \cdot 522c + 0.018 \cdot 965.16c + 0.056 \cdot 2211.12c$$
$$+ 0.003 \cdot 208.8c + 0.004 \cdot 259.2c + 0.024 \cdot 1186.56c$$
$$+ 0.33 \cdot 26030.16c + 0.043 \cdot 2521.44c$$
$$+ 0.024 \cdot 2073.6c + 0.004 \cdot 259.2c + 0.007 \cdot 522c$$
$$+ 0.018 \cdot 870.48c \cong 17056.54c. \qquad (46)$$

5. Maritime ferry technical system safety

5.1. Safety characteristics

We assume, that the maritime ferry incorporates a number of main technical subsystems having an crucial impact on its safety, further termed the ferry technical system:

- S_1 – a navigational subsystem,
- S_2 – a propulsion and controlling subsystem,
- S_3 – a loading and unloading subsystem,
- S_4 – a stability control subsystem,
- S_5 – an anchoring and mooring subsystem.

The subsystems S_1, S_2, S_3, S_4, S_5, are forming a general series safety structure of the ferry technical system shown in Figure 1.

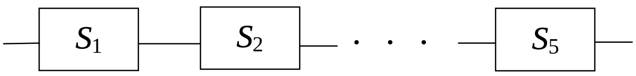

Figure 1. The general structure of the ferry technical system safety.

After analyzing the matter with help of experts and taking into consideration the safety of the operation of the ferry, we identify the five safety states of the ferry technical system and its components:

- a safety state 4 – the ferry operation is fully safe,
- a safety state 3 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution,
- a safety state 2 called a critical safety state – the ferry operation is less safe and more dangerous because of the possibility of environment pollution and causing small accidents,
- a safety state 1 – the ferry operation is much less safe and much more dangerous because of the possibility of serious environment pollution and causing extensive accidents,
- a safety state 0 – the ferry technical system is destroyed (Kołowrocki & Soszyńska-Budny, 2011/2015).

Furthermore, we presume that only possible transitions between the components' safety states are those from better to worse and that that critical safety state for the system and its components is $r = 2$.

Applying (12)–(13) and using (28) the safety function of maritime ferry technical system is given by

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \geq 0, \quad (47)$$

and

$$\boldsymbol{S}(t,u) = 0.038\cdot[\boldsymbol{S}(t,u)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,u)]^{(2)} + 0.026\cdot[\boldsymbol{S}(t,u)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,u)]^{(4)} + 0.363\cdot[\boldsymbol{S}(t,u)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,u)]^{(6)} + 0.005\cdot[\boldsymbol{S}(t,u)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,u)]^{(8)} + 0.037\cdot[\boldsymbol{S}(t,u)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,u)]^{(10)} + 0.003\cdot[\boldsymbol{S}(t,u)]^{(11)} + 0.0016\cdot[\boldsymbol{S}(t,u)]^{(12)} + 0.351\cdot[\boldsymbol{S}(t,u)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,u)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,u)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,u)]^{(16)} + 0.005\cdot[\boldsymbol{S}(t,u)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,u)]^{(18)},\ t \geq 0, \text{ for } u = 1,2,\ldots,4. \quad (48)$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $b = 1,2,\ldots,18$, are the system conditional safety functions at the operation state z_b, $b = 1,2,\ldots,18$, determined in (Kołowrocki & Magryta-Mut, 2020c, 2021; Magryta-Mut, 2020, 2021).
Hence, in particular for $u = 2$, we have

$$\boldsymbol{S}(t,2) = 0.038\cdot[\boldsymbol{S}(t,2)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(2)} + 0.026\cdot[\boldsymbol{S}(t,2)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,2)]^{(4)} + 0.363\cdot[\boldsymbol{S}(t,2)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,2)]^{(6)} + 0.005\cdot[\boldsymbol{S}(t,2)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,2)]^{(8)} + 0.037\cdot[\boldsymbol{S}(t,2)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(10)} + 0.003\cdot[\boldsymbol{S}(t,2)]^{(11)} + 0.0016\cdot[\boldsymbol{S}(t,2)]^{(12)} + 0.351\cdot[\boldsymbol{S}(t,2)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,2)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,2)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,2)]^{(16)} + 0.005\cdot[\boldsymbol{S}(t,2)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,2)]^{(18)},\ t \geq 0, \quad (49)$$

where $[\boldsymbol{S}(t,2)]^{(b)}$, $t \geq 0$, $b = 1,2,\ldots,18$, are the system conditional safety functions at the operation state z_b, $b = 1,2,\ldots,18$, determined in (Kołowrocki & Magryta-Mut, 2020c, 2021; Magryta-Mut, 2020, 2021).
Further, the expected values of the analyzed system conditional lifetimes in the safety state subset not worse than the critical safety state {2,3,4} at the operation states b, $b = 1,2,\ldots,18$, respectively are (Kołowrocki & Magryta-Mut, 2020c, 2021; Kołowrocki & Soszyńska-Budny, 2018a; Magryta-Mut, 2020):

$$[\boldsymbol{\mu}(2)]^{(1)} \cong 1.47,\ [\boldsymbol{\mu}(2)]^{(2)} \cong 1.33,\ [\boldsymbol{\mu}(2)]^{(3)} \cong 1.40,\ [\boldsymbol{\mu}(2)]^{(4)} \cong 1.39,\ [\boldsymbol{\mu}(2)]^{(5)} \cong 1.39,\ [\boldsymbol{\mu}(2)]^{(6)} \cong 1.38,\ [\boldsymbol{\mu}(2)]^{(7)} \cong 1.28,\ [\boldsymbol{\mu}(2)]^{(8)} \cong 1.44,\ [\boldsymbol{\mu}(2)]^{(9)} \cong 1.44,\ [\boldsymbol{\mu}(2)]^{(10)} \cong 1.33,\ [\boldsymbol{\mu}(2)]^{(11)} \cong 1.34,\ [\boldsymbol{\mu}(2)]^{(12)} \cong 1.40,\ [\boldsymbol{\mu}(2)]^{(13)} \cong 1.39,\ [\boldsymbol{\mu}(2)]^{(14)} \cong 1.39,\ [\boldsymbol{\mu}(2)]^{(15)} \cong 1.40,\ [\boldsymbol{\mu}(2)]^{(16)} \cong 1.34,\ [\boldsymbol{\mu}(2)]^{(17)} \cong 1.28,\ [\boldsymbol{\mu}(2)]^{(18)} \cong 1.46 \text{ years.} \quad (50)$$

The standard deviation of the considered system conditional lifetimes in the safety state subset {2,3,4} at the operation state z_b, $b = 1,2,\ldots,18$, respectively are (Kołowrocki & Magryta, 2020a; Kołowrocki & Magryta-Mut, 2020c; Kołowrocki & Soszyńska-Budny, 2018a; Magryta-Mut, 2020):

$$[\boldsymbol{\sigma}(2)]^{(1)} \cong 1.45,\ [\boldsymbol{\sigma}(2)]^{(2)} \cong 1.31,\ [\boldsymbol{\sigma}(2)]^{(3)} \cong 1.38,\ [\boldsymbol{\sigma}(2)]^{(4)} \cong 1.38,\ [\boldsymbol{\sigma}(2)]^{(5)} \cong 1.37,\ [\boldsymbol{\sigma}(2)]^{(6)} \cong 1.37,\ [\boldsymbol{\sigma}(2)]^{(7)} \cong 1.26,\ [\boldsymbol{\sigma}(2)]^{(8)} \cong 1.42,\ [\boldsymbol{\sigma}(2)]^{(9)} \cong 1.42,\ [\boldsymbol{\sigma}(2)]^{(10)} \cong 1.31,\ [\boldsymbol{\sigma}(2)]^{(11)} \cong 1.31,\ [\boldsymbol{\sigma}(2)]^{(12)} \cong 1.38,\ [\boldsymbol{\sigma}(2)]^{(13)} \cong 1.39,\ [\boldsymbol{\sigma}(2)]^{(14)} \cong 1.38,\ [\boldsymbol{\sigma}(2)]^{(15)} \cong 1.37,\ [\boldsymbol{\sigma}(2)]^{(16)} \cong 1.31,\ [\boldsymbol{\sigma}(2)]^{(17)} \cong 1.26,\ [\boldsymbol{\sigma}(2)]^{(18)} \cong 1.45 \text{ years.} \quad (51)$$

Thus, applying (14) and considering (28) and (49), the value of the ferry technical system unconditional lifetime in the safety state subset not worse than this critical safety state {2,3,4} is

$$\boldsymbol{\mu}(2) = p_1\cdot 1.47 + p_2\cdot 1.33 + p_3\cdot 1.40 + p_4\cdot 1.39 + p_5\cdot 1.39 + p_6\cdot 1.38 + p_7\cdot 1.28 + p_8\cdot 1.44 + p_9\cdot 1.44 + p_{10}\cdot 1.33 + p_{11}\cdot 1.34 + p_{12}\cdot 1.40 + p_{13}\cdot 1.39 + p_{14}\cdot 1.39 + p_{15}\cdot 1.40 + p_{16}\cdot 1.34 + p_{17}\cdot 1.28 + p_{18}\cdot 1.46 \cong 1.395. \quad (52)$$

Further, considering (51)–(52), the corresponding standard deviation of the analyzed system unconditional lifetime in the state subset {2,3,4} is (Magryta-Mut, 2021)

$$\boldsymbol{\sigma}(2) \cong 1.383 \text{ years.} \quad (53)$$

As the ferry technical system's critical safety state is $r = 2$, then its system risk function is given by (Kołowrocki & Magryta, 2020a; Kołowrocki & Magryta-Mut, 2020c, 2021; Magryta-Mut, 2021):

$$\boldsymbol{r}(t) \cong 1 - \boldsymbol{S}(t,2),\ t \geq 0, \quad (54)$$

where $\boldsymbol{S}(t,2)$, $t \geq 0$, is given by (49).
Hence, and considering (52), the moment when the system risk function exceeds a permitted level, for instance $d = 0.05$, is

$$\tau = \boldsymbol{r}^{-1}(d) \cong 0.0727 \text{ year}. \qquad (55)$$

By (19), considering (51), the analyzed system's mean value of the intensity of ageing is

$$\lambda(2) = \frac{1}{\mu(2)} = \frac{1}{1.395} \cong 0.717. \qquad (56)$$

By (20), considering (56) and the values of the ferry technical system without operation impact intensity of ageing $\lambda^0(2) = 0.678$, determined in (Kołowrocki & Magryta-Mut, 2020c; Kołowrocki & Soszyńska-Budny, 2018b; Magryta-Mut, 2021), the coefficient of the operation process impact on the ferry technical system intensity of ageing is

$$r(2) = \frac{\lambda(2)}{\lambda^0(2)} = \frac{0.717}{0.678} \cong 1.058. \qquad (57)$$

Hence, applying (21), the ferry technical system resilience indicator, i.e. the coefficient of the ferry technical system resilience to operation process impact, is

$$\boldsymbol{RI}(2) = \frac{1}{r(2)} = \frac{1}{1.058} \cong 0.945 = 94.5\%. \qquad (58)$$

5.2. Optimal safety characteristics

Applying the optimization procedure from Section 3.2, we obtain the optimal mean value of the ferry technical system lifetime is (Magryta-Mut, 2020)

$$\dot{\mu}(2) = \dot{p}_1 \cdot 1.47 + \dot{p}_2 \cdot 1.33 + \dot{p}_3 \cdot 1.40 + \dot{p}_4 \cdot 1.39$$
$$+ \dot{p}_5 \cdot 1.39 + \dot{p}_6 \cdot 1.38 + \dot{p}_7 \cdot 1.28 + \dot{p}_8 \cdot 1.44$$
$$+ \dot{p}_9 \cdot 1.44 + \dot{p}_{10} \cdot 1.33 + \dot{p}_{11} \cdot 1.34 + \dot{p}_{12} \cdot 1.40$$
$$+ \dot{p}_{13} \cdot 1.39 + \dot{p}_{14} \cdot 1.39 + \dot{p}_{15} \cdot 1.40 + \dot{p}_{16} \cdot 1.34$$
$$+ \dot{p}_{17} \cdot 1.28 + \dot{p}_{18} \cdot 1.46 \cong 1.399 \text{ years}, \qquad (59)$$

where

$\dot{p}_1 = 0.056, \dot{p}_2 = 0.001, \dot{p}_3 = 0.027,$
$\dot{p}_4 = 0.056, \dot{p}_5 = 0.382, \dot{p}_6 = 0.018,$
$\dot{p}_7 = 0.002, \dot{p}_8 = 0.018, \dot{p}_9 = 0.056,$
$\dot{p}_{10} = 0.001, \dot{p}_{11} = 0.002, \dot{p}_{12} = 0.024,$
$\dot{p}_{13} = 0.286, \dot{p}_{14} = 0.025, \dot{p}_{15} = 0.024,$
$\dot{p}_{16} = 0.002, \dot{p}_{17} = 0.002, \dot{p}_{18} = 0.018.$

Moreover, the corresponding optimal unconditional safety function of the ferry technical system takes the form

$$\dot{\boldsymbol{S}}(t,2) = 0.056 \cdot [\boldsymbol{S}(t,2)]^{(1)} + 0.001 \cdot [\boldsymbol{S}(t,2)]^{(2)}$$
$$+ 0.027 \cdot [\boldsymbol{S}(t,2)]^{(3)} + 0.056 \cdot [\boldsymbol{S}(t,2)]^{(4)}$$
$$+ 0.382 \cdot [\boldsymbol{S}(t,2)]^{(5)} + 0.018 \cdot [\boldsymbol{S}(t,2)]^{(6)}$$
$$+ 0.002 \cdot [\boldsymbol{S}(t,2)]^{(7)} + 0.018 \cdot [\boldsymbol{S}(t,2)]^{(8)}$$
$$+ 0.056 \cdot [\boldsymbol{S}(t,2)]^{(9)} + 0.001 \cdot [\boldsymbol{S}(t,2)]^{(10)}$$
$$+ 0.002 \cdot [\boldsymbol{S}(t,2)]^{(11)} + 0.024 \cdot [\boldsymbol{S}(t,2)]^{(12)}$$
$$+ 0.286 \cdot [\boldsymbol{S}(t,2)]^{(13)} + 0.025 \cdot [\boldsymbol{S}(t,2)]^{(14)}$$
$$+ 0.024 \cdot [\boldsymbol{S}(t,2)]^{(15)} + 0.002 \cdot [\boldsymbol{S}(t,2)]^{(16)}$$
$$+ 0.002 \cdot [\boldsymbol{S}(t,2)]^{(17)} + 0.018 \cdot [\boldsymbol{S}(t,2)]^{(18)},$$
$$t \geq 0, \qquad (60)$$

where $[\boldsymbol{S}(t,2)]^{(b)}$, $t \geq 0$, $b = 1,2,\ldots,18$, are determined in (Kołowrocki & Magryta-Mut, 2020c; Magryta-Mut, 2020, 2021).
Moreover, considering (59) and (60), the corresponding optimal standard deviations of the ferry technical system unconditional lifetime in the state subset not worse than the critical safety state is (Kołowrocki & Magryta-Mut, 2020c; Magryta-Mut, 2020, 2021)

$$\dot{\sigma}(2) \cong 1.386 \text{ years}. \qquad (61)$$

As the ferry technical system critical safety state is $r = 2$, then considering (17) and (60), its optimal system risk function, is given by

$$\dot{\boldsymbol{r}}(t) \cong 1 - \dot{\boldsymbol{S}}(t,2), \; t \geq 0. \qquad (62)$$

Considering (18) and (60) the moment when the optimal system risk function exceeds a permitted level, for instance $d = 0.05$, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(d) \cong 0.0729 \text{ year}. \qquad (63)$$

By (19) and (59) the ferry technical system mean value of the optimal intensity of ageing is

$$\dot{\lambda}(2) = \frac{1}{\dot{\mu}(2)} = \frac{1}{1.399} \cong 0.715. \qquad (64)$$

Considering (20) and (64) and the values of the analyzed system without operation impact intensity of ageing $\lambda^0(t)$ = 0.678, determined in (Kołowrocki & Magryta, 2020a; Kołowrocki & Magryta-Mut, 2020c; Magryta-Mut, 2021), the optimal coefficient of the operation process impact on the ferry technical system intensity of ageing is

$$\dot{\rho}(2) = \frac{\dot{\lambda}(2)}{\lambda^0(2)} = \frac{0.715}{0.678} \cong 1.055. \quad (65)$$

Hence, applying (21), the ferry technical system optimal resilience indicator, i.e. the optimal coefficient of the ferry technical system resilience to operation process impact is

$$\dot{\boldsymbol{RI}}(2) = \frac{1}{\dot{\rho}(2)} = \frac{1}{1.055} \cong 0.948 = 94.8\%. \quad (66)$$

6. Joint system operation cost and safety optimization

6.1. System safety corresponding to its minimal operation cost

To analyze jointly the system operation cost and safety it is possible to propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find the minimal operation total cost during the fixed operation time, through applying the proposed system operation cost model. Next, to find the system conditional safety indicators, corresponding to this system minimal total operation cost during the fixed operation time, we replace the limit transient probabilities in particular operation states, existing in the formula for the system safety function, by their optimal values existing in the formula for the system minimal operation total cost during the fixed operation time.

Thus, in Section 4, there is presented the procedure of determining the optimal values $\dot{p}_b$, b = 1, 2,…, ν, the limit transient probabilities of the system operation process $Z(t)$ at the particular operation states z_b, b = 1, 2,…, v, that allows to find the minimal operation total cost during the fixed operation time θ, through applying the system operation cost model and determining its value. To find the system conditional safety indicators, corresponding to this system minimal total operation cost during the fixed operation time θ, we replace p_b, b = 1, 2,…,v, existing in the formula (13) for the system safety function, by $\dot{p}_b$, b = 1, 2,…, v, existing in the formula (11) for its minimal operation cost during this fixed operation time.

6.2. Maritime ferry technical system safety corresponding to its minimal operation cost

In Section 4, we get the optimal limit transient probabilities of the ferry operation process $Z(t)$ at the particular operation states z_b, b = 1, 2,…, 18, determined by (45):

$$\dot{p}_1 = 0.056,\ \dot{p}_2 = 0.002,\ \dot{p}_3 = 0.027,$$
$$\dot{p}_4 = 0.056,\ \dot{p}_5 = 0.286,\ \dot{p}_6 = 0.024,$$
$$\dot{p}_7 = 0.018,\ \dot{p}_8 = 0.018,\ \dot{p}_9 = 0.056,$$
$$\dot{p}_{10} = 0.003,\ \dot{p}_{11} = 0.004,\ \dot{p}_{12} = 0.024,$$
$$\dot{p}_{13} = 0.33,\ \dot{p}_{14} = 0.043,\ \dot{p}_{15} = 0.024,$$
$$\dot{p}_{16} = 0.004,\ \dot{p}_{17} = 0.007,\ \dot{p}_{18} = 0.018, \quad (67)$$

that minimize the ferry technical system operation total cost during the fixed operation time of one month and determining its minimal value given by (46).

To find the ferry technical system conditional safety indicators, corresponding to this system optimal operation total cost, we replace p_b, b = 1, 2,…, 18, existing in the formula (48) for the coordinate $\boldsymbol{S}(t,2)$, $t \geq 0$, and in the formulae of the remaining coordinates (Magryta-Mut, 2021) of the system safety function, by $\dot{p}_b$, b = 1, 2,…, 18, defined by (67). This way, we get the conditional ferry technical system safety function, corresponding to this system optimal total operation cost during the fixed operation time, given by the vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \geq 0, \quad (68)$$

where according to (12)–(13) and considering the ferry technical system operation process optimal limit transient probabilities at the operation states determined by (67), the vector coordinates are given respectively by

$$\boldsymbol{S}(t,1) = 0.056\cdot[\boldsymbol{S}(t,1)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,1)]^{(2)}$$

$$+ 0.027\cdot[\boldsymbol{S}(t,1)]^{(3)} + 0.056\cdot[\boldsymbol{S}(t,1)]^{(4)} + 0.286\cdot[\boldsymbol{S}(t,1)]^{(5)} + 0.024\cdot[\boldsymbol{S}(t,1)]^{(6)} + 0.018\cdot[\boldsymbol{S}(t,1)]^{(7)} + 0.018\cdot[\boldsymbol{S}(t,1)]^{(8)} + 0.056\cdot[\boldsymbol{S}(t,1)]^{(9)} + 0.003\cdot[\boldsymbol{S}(t,1)]^{(10)} + 0.004\cdot[\boldsymbol{S}(t,1)]^{(11)} + 0.024\cdot[\boldsymbol{S}(t,1)]^{(12)} + 0.33\cdot[\boldsymbol{S}(t,1)]^{(13)} + 0.043\cdot[\boldsymbol{S}(t,1)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,1)]^{(15)} + 0.004\cdot[\boldsymbol{S}(t,1)]^{(16)} + 0.007\cdot[\boldsymbol{S}(t,1)]^{(17)} + 0.018\cdot[\boldsymbol{S}(t,1)]^{(18)}, \quad (69)$$

$$\boldsymbol{S}(t,2) = 0.056\cdot[\boldsymbol{S}(t,2)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(2)} + 0.027\cdot[\boldsymbol{S}(t,2)]^{(3)} + 0.056\cdot[\boldsymbol{S}(t,2)]^{(4)} + 0.286\cdot[\boldsymbol{S}(t,2)]^{(5)} + 0.024\cdot[\boldsymbol{S}(t,2)]^{(6)} + 0.018\cdot[\boldsymbol{S}(t,2)]^{(7)} + 0.018\cdot[\boldsymbol{S}(t,2)]^{(8)} + 0.056\cdot[\boldsymbol{S}(t,2)]^{(9)} + 0.003\cdot[\boldsymbol{S}(t,2)]^{(10)} + 0.004\cdot[\boldsymbol{S}(t,2)]^{(11)} + 0.024\cdot[\boldsymbol{S}(t,2)]^{(12)} + 0.33\cdot[\boldsymbol{S}(t,2)]^{(13)} + 0.043\cdot[\boldsymbol{S}(t,2)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,2)]^{(15)} + 0.004\cdot[\boldsymbol{S}(t,2)]^{(16)} + 0.007\cdot[\boldsymbol{S}(t,2)]^{(17)} + 0.018\cdot[\boldsymbol{S}(t,2)]^{(18)}, \quad (70)$$

$$\boldsymbol{S}(t,3) = 0.056\cdot[\boldsymbol{S}(t,3)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,3)]^{(2)} + 0.027\cdot[\boldsymbol{S}(t,3)]^{(3)} + 0.056\cdot[\boldsymbol{S}(t,3)]^{(4)} + 0.286\cdot[\boldsymbol{S}(t,3)]^{(5)} + 0.024\cdot[\boldsymbol{S}(t,3)]^{(6)} + 0.018\cdot[\boldsymbol{S}(t,3)]^{(7)} + 0.018\cdot[\boldsymbol{S}(t,3)]^{(8)} + 0.056\cdot[\boldsymbol{S}(t,3)]^{(9)} + 0.003\cdot[\boldsymbol{S}(t,3)]^{(10)} + 0.004\cdot[\boldsymbol{S}(t,3)]^{(11)} + 0.024\cdot[\boldsymbol{S}(t,3)]^{(12)} + 0.33\cdot[\boldsymbol{S}(t,3)]^{(13)} + 0.043\cdot[\boldsymbol{S}(t,3)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,3)]^{(15)} + 0.004\cdot[\boldsymbol{S}(t,3)]^{(16)} + 0.007\cdot[\boldsymbol{S}(t,3)]^{(17)} + 0.018\cdot[\boldsymbol{S}(t,3)]^{(18)}, \quad (71)$$

$$\boldsymbol{S}(t,4) = 0.056\cdot[\boldsymbol{S}(t,4)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,4)]^{(2)} + 0.027\cdot[\boldsymbol{S}(t,4)]^{(3)} + 0.056\cdot[\boldsymbol{S}(t,4)]^{(4)} + 0.286\cdot[\boldsymbol{S}(t,4)]^{(5)} + 0.024\cdot[\boldsymbol{S}(t,4)]^{(6)} + 0.018\cdot[\boldsymbol{S}(t,4)]^{(7)} + 0.018\cdot[\boldsymbol{S}(t,4)]^{(8)} + 0.056\cdot[\boldsymbol{S}(t,4)]^{(9)} + 0.003\cdot[\boldsymbol{S}(t,4)]^{(10)} + 0.004\cdot[\boldsymbol{S}(t,4)]^{(11)} + 0.024\cdot[\boldsymbol{S}(t,4)]^{(12)} + 0.33\cdot[\boldsymbol{S}(t,4)]^{(13)} + 0.043\cdot[\boldsymbol{S}(t,4)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,4)]^{(15)} + 0.004\cdot[\boldsymbol{S}(t,4)]^{(16)} + 0.007\cdot[\boldsymbol{S}(t,4)]^{(17)} + 0.018\cdot[\boldsymbol{S}(t,4)]^{(18)}, \quad (72)$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $t \geq 0$, $u = 1, 2, 3, 4$, $b = 1, 2, \ldots, 18$, are given respectively by in (Magryta-Mut, 2021).

The conditional expected values and standard deviations of the ferry technical system lifetimes in the safety state subsets {1, 2, 3, 4}, {2, 3, 4}, {3, 4}, {4} calculated from the results given by (69)–(72), considering (Magryta-Mut, 2021), corresponding to this system minimal total operation cost during the fixed operation time, respectively are:

$$\boldsymbol{\mu}(1) \cong 0.056\cdot1.70476 + 0.002\cdot1.60772 + 0.027\cdot1.68087 + 0.056\cdot1.6956 + 0.286\cdot1.69547 + 0.024\cdot1.67434 + 0.018\cdot1.54736 + 0.018\cdot1.72871 + 0.056\cdot1.72871 + 0.003\cdot1.60772 + 0.004\cdot1.6102 + 0.024\cdot1.70148 + 0.33\cdot1.69547 + 0.043\cdot1.6863 + 0.024\cdot1.68087 + 0.004\cdot1.61025 + 0.007\cdot1.54736 + 0.018\cdot1.70476 \cong 1.692 \text{ years}, \quad (73)$$

$$\boldsymbol{\sigma}(1) \cong 1.666 \text{ years}, \quad (74)$$

$$\boldsymbol{\mu}(2) \cong 0.056\cdot1.41708 + 0.002\cdot1.32879 + 0.027\cdot1.3912 + 0.056\cdot1.39303 + 0.286\cdot1.39292 + 0.024\cdot1.37699 + 0.018\cdot1.27865 + 0.018\cdot1.43719 + 0.056\cdot1.43719 + 0.003\cdot1.32879 + 0.004\cdot1.3336 + 0.024\cdot1.39692 + 0.33\cdot1.39292 + 0.043\cdot1.3854 + 0.024\cdot1.3912 + 0.004\cdot1.3336 + 0.007\cdot1.27865 + 0.018\cdot1.41708 \cong 1.394 \text{ years}, \quad (75)$$

$$\boldsymbol{\sigma}(2) \cong 1.376 \text{ years}, \quad (76)$$

$$\boldsymbol{\mu}(3) \cong 0.056\cdot1.22861 + 0.002\cdot1.18936 + 0.027\cdot1.24553 + 0.056\cdot1.24632 + 0.286\cdot1.24619 + 0.024\cdot1.23228 + 0.018\cdot1.15851 + 0.018\cdot1.26722 + 0.056\cdot1.26722 + 0.003\cdot1.18936 + 0.004\cdot1.19593 + 0.024\cdot1.24985 + 0.33\cdot1.24619 + 0.043\cdot1.23945 + 0.024\cdot1.24553 + 0.004\cdot1.19593 + 0.007\cdot1.15851 + 0.018\cdot1.22861 \cong 1.243 \text{ years}, \quad (77)$$

$$\boldsymbol{\sigma}(3) \cong 1.229 \text{ years}, \quad (78)$$

$$\boldsymbol{\mu}(4) \cong 0.056\cdot1.11601 + 0.002\cdot1.06574 + 0.027\cdot1.11512 + 0.056\cdot1.11522 + 0.286\cdot1.1151 + 0.024\cdot1.10301 + 0.018\cdot1.02847 + 0.018\cdot1.13163 + 0.056\cdot1.13163 + 0.003\cdot1.06574 + 0.004\cdot1.07262 + 0.024\cdot1.11836 + 0.33\cdot1.1151 + 0.043\cdot1.1091 + 0.024\cdot1.11512 + 0.004\cdot1.07262$$

$+ 0.007 \cdot 1.02847 + 0.018 \cdot 1.11601$
$\cong 1.1132$ years, (79)

$$\boldsymbol{\sigma}(4) \cong 1.101 \text{ years.} \quad (80)$$

And further, according to (16), considering (73), (75), (77) and (79), the conditional mean values of the system lifetimes in the particular safety states 1, 2, 3, 4, corresponding to this system minimal total operation cost during the fixed operation time, respectively are:

$$\bar{\boldsymbol{\mu}}(1) = \boldsymbol{\mu}(1) - \boldsymbol{\mu}(2) = 0.299,$$

$$\bar{\boldsymbol{\mu}}(2) = \boldsymbol{\mu}(2) - \boldsymbol{\mu}(3) = 0.151 \text{ year,} \quad (81)$$

$$\bar{\boldsymbol{\mu}}(3) = \boldsymbol{\mu}(3) - \boldsymbol{\mu}(4) = 0.130,$$

$$\bar{\boldsymbol{\mu}}(4) = \boldsymbol{\mu}(4) = 1.113 \text{ years.} \quad (82)$$

Since the critical safety state is $r = 2$ then the conditional system risk function, corresponding to this system optimal operation total cost during the fixed operation time, according to (17) is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t, 2),\ t \geq 0, \quad (83)$$

where $\boldsymbol{S}(t,2)$ is given by (70).
Hence, according to (18). the moment when the system conditional risk function exceeds a permitted level, for instance $d = 0.05$, corresponding to this system optimal operation total cost during the fixed operation time, is

$$t = \boldsymbol{r}^{-1}(d) \cong 0.0463 \text{ year.} \quad (84)$$

The conditional ferry technical system conditional intensities of ageing, corresponding to this system optimal operation total cost during the fixed operation time in the system safety subsets, are:

$$\lambda(1) \cong 0.591,\ \lambda(2) \cong 0.718,$$

$$\lambda(3) \cong 0.805,\ \lambda(4) \cong 0.898. \quad (85)$$

The conditional coefficients of the operation process impact on the ferry technical system intensities of ageing, corresponding to this system optimal operation total cost, are:

$$\boldsymbol{\rho}(1) \cong 1.046,\ \boldsymbol{\rho}(2) \cong 1.059,$$

$$\boldsymbol{\rho}(3) \cong 1.046,\ \boldsymbol{\rho}(4) \cong 1.046. \quad (86)$$

Finally, by (86), the ferry technical system conditional resilience indicator, i.e. the conditional coefficient of the ferry technical system resilience to the operation process impact, corresponding to this system optimal operation total cost during the fixed operation time, is

$$\boldsymbol{RI}(2) = 1/\boldsymbol{\rho}(2) \cong 0.944 = 94{,}4\%. \quad (87)$$

6.3. Discussion of results

The obtained in Section 4.3 minimal total operation cost of the ferry technical system given by (46) is significantly less than its operation total cost before optimization given by (32) in Section 4.2. Whereas, the conditional safety indicators of the ferry technical system corresponding to its minimal operation total cost, determined in Section 6.2, are slightly worse than those before the safety optimization given in Section 5.1 and those determined after the safety direct unconditional optimization determined in Section 5.2.
Thus, if reducing (minimizing) the ferry technical system's operation total cost has higher priority than maximizing its safety, we can modify this system operation process through replacing approximately the limit transient probabilities p_b, $b = 1,2,\ldots,18$, at the ferry particular operation states before the operation total cost minimization given by (28) by the values convergent to their optimal values $\dot{p}_b$, $b = 1, 2,\ldots, 18$, after the operation total cost minimization determined by (67).
In practice, it is easier to modify the considered system operation process through replacing approximately the system operation total time mean values in the particular operation states during the fixed operation time of $\theta = 1$ month $= 30$ days $= 720$ hours, defined by the approximate formula (Kołowrocki & Magryta-Mut, 2020c)

$$\hat{M}_b = p_b \cdot q,\ \ b = 1,2,\ldots,18, \quad (88)$$

and after considering (28) given in hours by (29), by the system total operation time mean values in the particular operation states during the fixed operation time of $\theta = 1$ month $= 30$ days $= 720$ hours, after the system operation total cost optimization, determined according to the approximate formula (Kołowrocki & Magryta-Mut, 2020c)

$$\breve{M}_b = \breve{p}_b \cdot q, \ b = 1,2,...,18, \quad (89)$$

and, considering (67), given in hours by:

$$\breve{M}_1 = 40.32, \ \breve{M}_2 = 1.44, \ \breve{M}_3 = 19.44,$$
$$\breve{M}_4 = 40.32, \ \breve{M}_5 = 205.92, \ \breve{M}_6 = 17.28,$$
$$\breve{M}_7 = 12.96, \ \breve{M}_8 = 12.96, \ \breve{M}_9 = 40.32,$$
$$\breve{M}_{10} = 2.16, \ \breve{M}_{11} = 2.88, \ \breve{M}_{12} = 17.28,$$
$$\breve{M}_{13} = 237.60, \ \breve{M}_{14} = 30.96, \ \breve{M}_{15} = 17.28,$$
$$\breve{M}_{16} = 2.88, \ \breve{M}_{17} = 5.04, \ \breve{M}_{18} = 12.96. \quad (90)$$

Equivalently, we can to modify the considered system operation process through replacing approximately the system total operation time mean values in the particular operation states during the fixed operation time of θ = 1 year = 365 days, determined by the approximate formula (88) and, after considering (28), given in days by:

$$\hat{M}_1 = 13.87, \ \hat{M}_2 = 0.73, \ \hat{M}_3 = 9.49,$$
$$\hat{M}_4 = 13.14, \ \hat{M}_5 = 132{,}495, \ \hat{M}_6 = 9.49,$$
$$\hat{M}_7 = 1.825, \ \hat{M}_8 = 5.84, \ \hat{M}_9 = 13.505,$$
$$\hat{M}_{10} = 0.73, \ \hat{M}_{11} = 1.095, \ \hat{M}_{12} = 5.84,$$
$$\hat{M}_{13} = 128.115, \ \hat{M}_{14} = 12.41, \ \hat{M}_{15} = 8.76,$$
$$\hat{M}_{16} = 1.095, \ \hat{M}_{17} = 1.825, \ \hat{M}_{18} = 4.745, \quad (91)$$

by the system total operation time mean values in the particular operation states during the fixed operation time of θ = 1 year = 365 days, after the system operation total cost minimization, determined according to the approximate formula (89) and after considering (67) given in days by:

$$\breve{M}_1 = 20.44, \ \breve{M}_2 = 0.73, \ \breve{M}_3 = 9.855,$$
$$\breve{M}_4 = 20.44, \ \breve{M}_5 = 104.39, \ \breve{M}_6 = 8.76,$$
$$\breve{M}_7 = 6.57, \ \breve{M}_8 = 6.57, \ \breve{M}_9 = 20.44,$$
$$\breve{M}_{10} = 1.095, \ \breve{M}_{11} = 1.46, \ \breve{M}_{12} = 8.76,$$
$$\breve{M}_{13} = 120.45, \ \breve{M}_{14} = 15.695, \ \breve{M}_{15} = 8.76,$$
$$\breve{M}_{16} = 1.46, \ \breve{M}_{17} = 2.555, \ \breve{M}_{18} = 6.57. \quad (92)$$

The procedure of the ferry technical system operation process can be performed for other than the above fixed operation times of 1 month and 1 year, dependently to the system operator comfort in the achievement of the best results of the system operation total times in the particular operation states convergence to their optimal values resulting from the performed system operation total cost minimization.

7. Conclusion

The procedure of using the operation total cost model and the general safety analytical model of complex multistate technical system related to its operation process (Kołowrocki, 2014) and the linear programming (Klabjan & Adelman, 2006) is presented and proposed to joint analysis of the system operation total cost minimization and the system safety corresponding to this cost evaluation. The mean value of the complex multistate system total operation cost is minimize through the system operation process modification. This operation process modification allows to find the complex system conditional safety indicators corresponding to the system minimal operation total cost during the fixed operation time. The proposed cost optimization procedure and finding corresponding system safety indicators gives practically important possibility of the system total operation cost minimizing and keeping the fixed corresponding conditional safety indicators during the operation through the system new operation strategy. The proposed system operation cost and system safety optimization separated and joint models and procedures are applied to the ferry technical system operation cost and safety examination. These procedures can be used in operation cost and safety optimization of members of various real complex systems and critical infrastructures (Gouldby et al., 2010; Habibullah et al., 2009; Kołowrocki et al., 2016; Kołowrocki & Magryta, 2020b; Kołowrocki & Magryta-Mut, 2020c; Lauge et al., 2015; Magryta-Mut, 2020). Further research can be related to considering other impacts on the system operation cost and safety, for instance related to climate-weather factors (Kołowrocki, 2021;

Kołowrocki & Kuligowska, 2018; Torbicki, 2019a-b; Torbicki & Drabiński; 2020) and resolving the issues of critical infrastructure (Lauge et al., 2015) operation cost and safety optimization and discovering optimal values of operation cost and safety and resilience indicators of system impacted jointly by the operation and climate-weather conditions (Kołowrocki, 2021). These developments can also benefit the mitigation of critical infrastructure accident consequences (Blokus & Kołowrocki, 2019, 2020; Bogalecka, 2020; Dąbrowska & Kołowrocki, 2019a-b; 2020a-c, Kołowrocki, 2021) and minimizing the system operation cost and improving critical infrastructure resilience to operation and climate-weather conditions (Kołowrocki, 2021; Kołowrocki & Kuligowska, 2018; Torbicki, 2019a-b; Torbicki & Drabiński; 2020).

The proposed optimization procedures and perspective of future research applied to system operation cost and to safety and resilience optimization of the complex systems and critical infrastructures can give practically important possibility of these systems effectiveness improvement through the proposing their new operation strategy.

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