

Roman STRYCZEK
Bogusław PYTLAK

MULTI-OBJECTIVE OPTIMIZATION WITH ADJUSTED PSO METHOD ON EXAMPLE OF CUTTING PROCESS OF HARDENED 18CrMo4 STEEL

OPTYMALIZACJA WIELOKRYTERIALNA SKORYGOWANĄ METODĄ PSO NA PRZYKŁADZIE PROCESU SKRAWANIA STALI 18CrMo4 W STANIE ZAHARTOWANYM*

In this paper a Modified Particle Swarm Optimization (PSO) method for multi-objective (MO) problems with a discrete decision space is proposed. In the PSO method the procedure to determine inertia weight, learning factor and social factor is modified. In addition, both an elitism strategy and innovative deceleration mechanism preventing the particles from going beyond the limits of decision space are introduced. The proposed approach has been applied to a series of currently used test functions as well as to optimization problems connected with finish hard turning operation, where the obtained results have been compared with those obtained by means of Genetic Algorithms (GA). The results indicate that the proposed approach is relatively quick, and thus it is highly competitive with other optimization methods. The authors have obtained a very good diversity, convergence and a maximum range of the Pareto front in the criteria space. In order to assess the quality of the generated Pareto set for each of presented examples, a rating has been determined based on the entropy measurement and inverted generational distance (IGD).

Keywords: hard turning, particle swarm optimization (PSO) method, evolutionary computations, multi-objective optimization, entropy.

W pracy zaproponowano zmodyfikowaną metodę optymalizacji wielocząsteczkowej (PSO) dla problemów optymalizacji wielokryterialnej z dyskretną przestrzenią decyzyjną. W metodzie PSO zmieniono sposób określania momentu bezwładności, współczynnika uczenia oraz współczynnika społecznego. Dodatkowo wprowadzono elitaryzm oraz innowacyjny mechanizm hamowania cząstek chroniący je przed przekraczaniem dopuszczalnych granic przestrzeni decyzyjnej. Zaproponowane podejście zostało zweryfikowane na szeregu aktualnych funkcjach testowych oraz problemie optymalizacji procesu skrawania stali 18CrMo4 w stanie zahartowanym, gdzie porównano je z wynikami uzyskanymi za pomocą algorytmów genetycznych (GA). Uzyskane wyniki wskazują, że zaproponowane podejście jest względnie szybkie i wysoce konkurencyjne w stosunku do innych metod optymalizacji. Autorzy uzyskali bardzo różnorodne, zbieżne i w pełnym zakresie przebiegi frontu Pareto w przestrzeni kryteriów. W celu oceny jakości wygenerowanego zbioru Pareto dla każdego z prezentowanych przykładów wyznaczono ocenę opartą na pomiarze entropii oraz wskaźnika jakości IGD.

Słowa kluczowe: toczenie na twardo, metoda optymalizacji wielocząsteczkowej (PSO), obliczenia ewolucyjne, optymalizacja wielokryterialna, entropia.

1. Introduction

The search of optimal decision poses a problematic issue from the perspective of many, often conflicting criteria. Usually, the search results in a large set of solutions. Typical methods of single criterion optimization usually give one solution in a single run of the calculation process, and therefore such methods are useless in multi-objective optimization. In order to obtain many solutions in a single run of calculation process the unconventional methods must be employed. However, only a few of these make it possible to obtain an evenly distributed, coherent and complete set of solutions.

Nowadays, the most popular of these methods are based on evolutionary techniques; Genetic Algorithms (GA) in particular. Generally, these techniques are based on metaheuristics, improving the current situation of an individual in the population, increasing its chances of survival and/or enabling it to inherit the genetic code. The Particle Swarm Optimization (PSO) method is one of those techniques. It has become widely accepted, since its introduction in 1995 [11] and is used in many fields [19].

PSO has also become the major alternative for GA in the area of multi-objective optimization. The comparison to genetic algorithm and ant colony optimization algorithm indicates that PSO is more effective than the others because of its faster convergence rate [14]. The number of publications describing the use of PSO has grown exponentially for the last few years [20]. The success of this method results from its intuitive nature, the algorithm which is easy to use for programming, and the fact that it is liable to modification, which makes it an excellent tool for experimental research. Reyes-Sierra and Coello Coello [25] have provided a complete taxonomy of existing MOPSOs' algorithms. They studied the main features of MOPSOs such as: the existence of an external archive for non-dominated solutions, the election strategy of non-dominated solutions as the leaders guiding the swarm, the neighbourhood topology, and the existence or non-existence of a mutation operator. During the past few years, several efficient multi-objective variants of PSO have been proposed. Interesting proposals for improving the original PSO algorithm appear every year [21]. More than thirty different Multi-Objective Particle Swarm Opti-

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mizers (MOPSOs) have been described in the literature [5]. New approaches are still being put forward, some focusing on the successful and improved results achieved by the basic algorithm MOPSO [16]. Zhang et al. [28] proposed a new multi-swarm cooperative multi-objective particle swarm optimization algorithm. To better its performance, several improved techniques such as the Pareto dominance-based species technique, the escape strategy of mature species, and the local MOPSO algorithm have also been introduced. The proposed algorithm can produce solution sets that are highly competitive as far as the convergence, diversity, and distribution are concerned. Kaveh, and Laknejadi [10] proposed a hybrid method which is a combination of the particle swarm method and a recently developed algorithm charge system search (CSS). Combining the proposed method with a mutation operator and particle redistribution strategy strengthens the search ability of the proposed algorithm. Magnus and Pedersen [15] suggest a table of PSO parameters which may be used by a researcher in the first place when optimizing new problems. Chakraborty et al. [3] present an analysis of the general Pareto-based MOPSO and find conditions on its most important control parameters (the inertia factor and acceleration coefficients) that govern the convergence behaviour of the algorithm to the optimal Pareto front in the objective function space. Many multi-objective optimization problems in real world engineering applications involve discrete and/or discontinuous parameters [7].

We think that the three following features have a significant impact on the efficiency of MOPSO: the control method of approach and the attempts to exceed the limits of decision space by the particles, consideration of continuous or discrete decision space, and control of the movement speed of particles.

A quality assessment of the Pareto front generated during the multi-objective optimization is not easy [1]. There are many different approaches here and their wider description can be found in the work [29]. In order to compare effectiveness of various approaches, special tests have been developed. Usually, this assessment includes three aspects: convergence meant as the minimum distance between adjacent solutions on the Pareto front, diversification understood as a uniform distribution of the solutions in criteria space, and a maximum range of Pareto front in the criteria space. A suitably chosen metric of decision space has a crucial meaning for the correct assessment in reference to the first and the second aspect. De Carvalho and Pozo [4] performed an empirical analysis of measure by means of three quality indicators (generational distance, inverted generational distance and spacing) to examine how the many-objective technique named control of dominance area of solutions (CDAS) affects the convergence and diversity of MOPSO algorithms. Also Pradhan and Panda [22] used some performance metrics, such as: set coverage metric, generational distance, maximum Pareto-optimal front error, spacing and spread.

One of the main issues dealt with by the researchers seeking effective, intelligent methods for multi-objective optimization is how to obtain a complete and coherent set of Pareto solutions. A current quality assessment of the Pareto front may serve as a determinant of its proper shape. The Entropy-based Multi-Objective Genetic Algorithm (E-MOGA) method, which strongly improves the convergence and uniformity of the Pareto front in comparison to the Multi-Objective Genetic Algorithm (MOGA), can serve as an example here. To assess the quality of solutions, the approach described in this work also uses two forms of entropy: external and internal, as well as inverted generational distance (IGD). This approach does not modify the method of entropy calculation, but it makes the comparison of different tests possible. Our research confirms the following conclusion: "a solution set with a higher entropy is spread more evenly throughout the feasible region and provides a better coverage of the space" [8].

In practical problems the decision space is limited by technical capabilities. The limit values are often optimal. Therefore, the proposed PSO algorithm approach has a built-in mechanism of the particle de-

celeration, in order to prevent them from exceeding the limits of decision space by its better penetration of the values near the boundary. In industry the decision maker consider the most often a discrete decision space. In design, he is usually able to analyse only a very limited number of solutions [6]. A continuous decision space is only relevant for theoretical consideration. Therefore, the proposed approach takes into account the position change of a particle in one direction only as a multiple of some fixed discrete value.

The speed control of particles movement is one of the main, but little explored parameter influencing the efficiency of the PSO algorithm. Nebro et al. [17] have proposed a new MOPSO algorithm which includes a velocity constriction mechanism. In our approach, as described below, heuristics strategy has also been used for the particles' speed control.

2. Methodology

Let us imagine N -dimensional, discrete space of decision-making, in which each point of the space is represented by a vector \mathbf{x} . Each component x_i of the vector \mathbf{x} has the specified range of variation ΔL_i , corresponding to the interval $[L_i^-, L_i^+]$, and a constant step of discretization d_i in the range of the variation. The discrete area of decision-making is dictated with the practical reasons in mind, because in reality, the designer determines the precision of the settings. Realization of the PSO method in discrete space somewhat complicates the algorithm. On the other hand, it eliminates oversized concentrations of solutions in a certain areas of the Pareto front, which tend to elongate the calculation process. Due to the fact that individual ranges of the variation can differ from one another considerably, which results in differences d_i , it was assumed that d_i creates a new unit for the given dimension. This assumption establishes an appropriate metric to calculate the distance between the points in the decision space and at the same time enables the quality assessment of the generated Pareto set, by measuring the entropy.

Formally, the multi criteria optimization problem can be expressed in the following way: we require a vector $\mathbf{x}^*=[x_1^*, x_2^*, \dots, x_N^*]^T$, which satisfies:

K inequality constraints

$$g_k(\mathbf{x}) \geq 0 \quad \text{for } k=1, 2, \dots, K \quad (1)$$

and M equality constraints

$$h_m(\mathbf{x}) = 0 \quad \text{for } m=1, 2, \dots, M \quad \text{and } M < N \quad (2)$$

and optimizes the vector of the objective function $\mathbf{f}(\mathbf{x})=[f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_l(\mathbf{x})]^T$, where $\mathbf{x}=[x_1, x_2, \dots, x_N]^T$ is the vector of decision variables.

The PSO method is often subjected to various modifications. One of the questions that researchers ask themselves is: how should the social factor in the generation of successive positions of the particle be taken into account? In case of a single criterion optimization, one can choose the movement in the direction of the best located neighbour in a specified surrounding of the analyzed particle, or the movement towards the best individual from the whole population. In case of multiobjective optimization, one can additionally select a movement toward the Pareto front. In this paper a few variant approaches were tested. The best results were achieved when the movement toward the nearest located solution on the Pareto front had been chosen.

In the canonical version of PSO, a particle is associated with the position attribute, the velocity attribute and the individual experience attribute. The position of a particle is always updated in every step using the equation (3)

$$x_i = x_i^0 + v_i \quad (3)$$

and the velocity is updated in the following way:

$$v_i = w \cdot v_i^0 + c_1 \cdot r_1 \cdot (x_i^{bp} - x_i) + c_2 \cdot r_2 \cdot (x_i^{gb} - x_i) \quad (4)$$

where: x_i^0 – i -th component of the position vector in the previous step, v_i^0 – i -th component of the velocity vector in the previous step, w – inertia weight, c_1, c_2 – acceleration coefficients, $r_1, r_2 \in [0,1]$ are random values, x_i^{bp} – best particle position, x_i^{gb} – best global position.

In multi-objective algorithms, a set containing a representation of all non-dominated solutions (leaders) is maintained.

The general structure of code of the proposed algorithm is shown as follows:

```

BEGIN
  Initialize Swarm
  Initialize Particles_Best
  Initialize Leaders_archive
  FOR t=1 to Number_of_Iteration
    FOR p=1 to Population_Size
      Find_Leader_p
      Move_Particle_p
      Evaluate_new_position_of_Particle_p
      Update_Particle_Best_p
      IF new_Leader=TRUE
        Update_Leaders_Archive
      ENDIF
    NEXT p
  NEXT t
  Output_Leaders_Archive
END
    
```

where, t denotes the generation index, p denotes particle index.

The proposed approach to determination of the Pareto front with the PSO method consists of the following steps:

Step 1. Generating of the initial population of particles. For each particle $p=1, 2, \dots, N_{pop}$ The components of the decision variables vector of \mathbf{x}_p and the initial speed vector \mathbf{v}_p for i -the dimension of the decision space are generated randomly:

$$\left. \begin{aligned} x_{ip} &= L_i^- + d_i \cdot \text{round}(\text{rand}()) \\ v_{ip} &= 2 \cdot \text{rand}() - 1 \end{aligned} \right\} \text{ for } i=1, 2, \dots, N \quad (5)$$

Meanwhile, the position of the particle in the criteria space (fitness space) is calculated $best_p = [f_1(\mathbf{x}_p), f_2(\mathbf{x}_p), \dots, f_l(\mathbf{x}_p)]^T$, and analysis regarding the location of the solution against the Pareto front is performed. The location generated in this step is stored as the $best_p$, which means the best position of the particle p achieved up to now. The Initial Pareto front is determined during this step.

Step 2. Accomplishment of successive iterations during which the particles are moving in the decision space. During the next iterations, the course of the Pareto front is being constantly modified if necessary.

What follows for each particle p is:

Step 2.1. Calculation of the distance d_p^f (Fig. 1) of the particle p from the current Pareto Set (from the current leader for the particle p):

$$d_p^f = \min(d_1^f, d_2^f, \dots, d_{L_p}^f) \quad (6)$$

where:
$$d_l^f = \sqrt{\sum_{i=1}^N \left(\frac{x_i - x_i^f}{d_i} \right)^2} \text{ for } l=1, 2, \dots, L_p \quad (7)$$

where: x_i – current location of the particle in the i -dimension, x_i^f – the location of the nearest point on the forehead of Pareto (leader) in the i -dimension.

Step 2.2. Determining coefficients w, c_1, c_2 , (Fig. 2) that are used for calculation of velocity components of particle p motion. The coefficient w is actually the weight, and it is taken into account when considering the current direction of particle motion. In case of the coefficient w , its value is set as a reference value and is determined at constant level equal to 0.5. The coefficient c_1 decides how closely the particle will try to return to its the best position. It was assumed that in the first phase of iterations this coefficient will play a decisive role, guaranteeing penetration of the decision space by the particle near its current location. In the final phase of iterations this coefficient reaches the value of 0, because the main task in this phase is to direct all particles near to the Pareto front. The coefficient c_2 takes into account the social impact of the particle, enabling it to choose the direction of the particle's movement toward better located particles, especially those on the Pareto front. In the proposed approach, the partial components of particle movement associated with the coefficient c_2 point at the shortest way toward the Pareto set. In the first iteration cycles the social impact is ignored, allowing the particle to move in random directions and thus better penetrate its environment. In the second phase, the social coefficient becomes decisive. In the presented approach the oscillation of the coefficient c_2 value, was used so as to enable a particle to temporarily abandon the close surrounding of the Pareto set. Therefore the particle can leave the Pareto set and penetrate the area near to the Pareto set better, and consequently its further movement in the right direction is possible. To determine current values of the coefficients, the following formulas were adopted:

$$w = 0.5 = const \quad (8)$$

$$c_1 = 1 - \frac{1}{e^{-20q/(N_q - 0.5)}} \quad (9)$$

$$c_2 = \frac{0.5 + 0.5 \cdot \cos(34.5q/N_q)}{1 + e^{-10q/(N_q - 0.5)}} \quad (10)$$

where: q – number of successive iteration, N_q – number of all iterations.

Step 2.3. Determining the particle deviation d_p^b from its the best position. It is calculated from the formula:

$$d_p^b = \sqrt{\sum_{i=1}^N \left(\frac{x_i - x_i^b}{d_i} \right)^2} \quad (11)$$

where: x_i^b – i -th component of the best position of particle so far.

Step 2.4. Determining directional vector components for the best particle position so far:

$$p_i^b = r_1 \cdot \frac{x_i^b - x_i}{d_p^b} \text{ for } i=1, 2, \dots, N \quad (12)$$

where: $r_1 = \text{rand}()$ – random number, the same for all components.

Step 2.5. Determining directional vector components of the particle p for the Pareto set (leader):

$$p_i^f = r_2 \cdot \frac{x_i^f - x_i}{d_i^f} \quad \text{for } i=1, 2, \dots, N \quad (13)$$

where: $r_2 = \text{rand}()$ – random number, the same for all components.

Step 2.6. Calculating the elements of movement speed vector:

$$v_i = w \cdot v_i^0 + c_1 \cdot p_i^b + c_2 \cdot p_i^f \quad \text{for } i=1, 2, \dots, N \quad (14)$$

where: v_i^0 – i -th component of the speed vector in the previous step.

Step 2.7. The normalization of the movement speed vector components:

$$v_i^n = \frac{v_i}{\sum_{i=1}^N v_i^2} \quad \text{for } i=1, 2, \dots, N \quad (15)$$

Step 2.8. Correction of the particle speed resulting from the possibility of exceeding of the permitted movement area. The speed correction is carried out by the inhibition mechanism of the particle according to the following formulas:

$$dv_i = a_i \cdot v_i^n \cdot \text{abs} \left(x_i - \frac{L_i^+ + L_i^-}{2} - \text{sgn} \left(v_i^n \right) \cdot \frac{L_i^+ - L_i^-}{2} \right) \quad (16)$$

where: a_i – coefficient of movement speed of the particle. This coefficient should be chosen in such a way to guarantee similar mobility of the particles in all directions. The coefficient value is chosen from the range of (0,1].

Step 2.9. Calculation of new components of particle position:

$$x_i' = d_i \cdot \text{round} \left(\frac{x_i - dv_i}{d_i} \right) \quad \text{for } i=1, 2, \dots, N \quad (17)$$

Step 2.10. Generation of a new location of the particle in the criteria space. In case when the criteria are not within the assumed constraints, the next part of this step is omitted. Checking whether the new position of the particle in the modified metric is the best position so far. If the current particle position is located on the Pareto front, such position is added to the Pareto set and all the solutions which are predominated by the new solution are removed.

Step 2.11. In case of failure to achieve desired number of iterations, return to step 2.1.

The term “entropy” appears in many areas of science and is associated with the assessment of disorder or arrangement. Individual entropy treated as the amount of information (Hartley 1928) can be determined by the formula: $h_i = -\ln(p_i)$, where p_i – the probability of an event. Absolute entropy of n events is the weighted arithmetic average of the amount of information received with the occurrence of individual events, where the probability of these events constitute the weights $H = -\sum(p_i \cdot \ln(p_i))$ (Shannon 1948). In turn, relative entropy H_r is expressed by the formula $H_r = H / \ln(n)$.

In the paper the two different definitions of entropy are used: external and internal. The external entropy is measured by means of assessing how close a given set is to the reference set of solutions. In this case, as the reference set we chose the ideal Pareto set, i.e. the complete set of solutions possible to be achieved in a given discrete space of the decision. To meet this requirement the authors calculated

the reference Pareto set. The generated Pareto set was used as the set comparable with the calculated set. However, by the internal entropy we mean the measure of entropy defined on the generated Pareto set, taking into account mutual distances of particular solutions from their nearest neighbours in this set. In practice, we have a possibility of calculating only the internal entropy. The objective of this work is to show, however, that when the internal entropy reaches a sufficiently high level, the external entropy reaches a satisfactory level, too.

Assess the external entropy one should define the elements’ interaction function s from the set S with the element f from the set F . It was assumed, that this function (Fig. 3) is expressed by the formula:

$$I_f = \max_s \left(e^{-b \cdot (r_{f \rightarrow s})^2} \right) \quad (18)$$

where: $r_{f \rightarrow s}$ is the distance between the element f and the element s measured in the assumed metric decision space, b – coefficient controlling the range of impact, in conducted tests was established as 1.

In practice, it is sufficient to find the distance $r_{f \rightarrow s}$ from the current Pareto set in the decision space for each solution f belonging to ideal Pareto front F .

$$r_{f \rightarrow s} = \min \left(\sqrt{\sum_{i=1}^N \left(\frac{x_i^s - x_i^f}{d_i} \right)^2} \right) \quad (19)$$

where: $f \in F, s \in S$.

To be able to compare different experiments’ results, it is preferable to normalize values of the influence function, to have their sum equal 1. As a result, the maximum entropy can amount to 1:

$$I_f^n = \frac{I_f}{\sum_f I_f} \quad (20)$$

Hence, the quality assessment of current Pareto front in the form of external entropy is:

$$H^e = \frac{-\sum_f I_f^n \cdot \ln(I_f^n)}{\ln(n_f)} \quad (21)$$

where: n_f – the power of the set F .

As mentioned above, an ideal Pareto front remains unknown in the course of practical calculations. Hence, actual evaluation of the quality of the Pareto front should be determined by the measure of the internal entropy H^i . The Agglomeration method was used for the calculation of the H^i . In the first step, a randomly chosen solution from the set S is moved to initially empty set S' . In the following steps, the next solutions $s \in S$ (which are located in the nearest distance to the set S' in the decision space) are moved to the set S' . Figure 4 illustrates the method of calculating mutual distances in the decision space. At the same time, the distance D_s of a transferred solution to the set S' is written on the stack during each step.

The influence function of the solution s onto neighbouring solutions takes the form:

$$I_s = e^{-b(D_s - 1)^2} \quad (22)$$

where: it was assumed that $b=1$.

After normalization:

$$I_s^n = \frac{I_s}{\sum_s I_s} \quad (23)$$

Internal entropy of the set S is calculated by the formula:

$$H^i = \frac{-\sum I_s^n \cdot \ln(I_s^n)}{\ln(n_s - 1)} \quad (24)$$

where: n_s – the power of the set S .

For example, the value of internal entropy for the sequence of 0-1-2-4-5 (Fig. 4).

where: $I_{0 \rightarrow 1} = I_{0 \rightarrow 2} = 1$, $I_{2 \rightarrow 4} = 0.135$, $I_{4 \rightarrow 5} = 0.035$, 0-1-2-4-5 chain length is 2.17, $I_{0 \rightarrow 1}^n = I_{0 \rightarrow 2}^n = 0.46$, $I_{2 \rightarrow 4}^n = 0.062$, $I_{4 \rightarrow 5}^n = 0.016$, is:

$$H^i = -(0.46 \ln(0.46) + 0.46 \ln(0.46) + 0.062 \ln(0.062) + 0.016 \ln(0.016)) / \ln(4) = 0.694$$

Additionally, the inverted generational distance (IGD) is used in assessing the performance of the algorithms in our experimental studies.

3. Experiments and results

3.1. Test functions

The modified particle swarm optimization method proposed here for the multi-objective problems has been applied to the solve several currently used test functions.

The first of the test problems was presented in the paper [13]. The objective functions for the particular criteria of the optimization were described by the formulas:

$$f_1 = x_1 + \frac{2}{|J_1|} \sum_{j \in J_1} \left(x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right) \right)^2 \rightarrow \min$$

$$f_2 = 1 - \sqrt{x_1} + \frac{2}{|J_2|} \sum_{j \in J_2} \left(x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right) \right)^2 \rightarrow \min \quad (25)$$

where: $J_1 = \{j \mid j \text{ is odd and } 2 \leq j \leq n\}$ and $J_2 = \{j \mid j \text{ is even and } 2 \leq j \leq n\}$, and the decision space $\Omega = [0, 1] \times [-1, 1]^{n-1}$ and $n=3$.

After 500 iterations 251 solutions were found and they are presented in the Figures 5 and 6. Additionally, the Figure 7 shows a graph of entropy values, power of Pareto set and IGD (Inverted Generational Distance) in the function of iteration.

The second test problem was also presented in the paper [13]. The objective functions for the particular optimization criteria were expressed by the formulas:

$$f_1 = x_1 + \frac{2}{|J_1|} \sum_{j \in J_1} \left(x_j - \left(0.3x_1^2 \cos\left(24\pi x_1 + \frac{4j\pi}{n}\right) + 0.6x_1 \right) \cdot \cos\left(6\pi x_1 + \frac{j\pi}{n}\right) \right)^2 \rightarrow \min$$

$$f_2 = 1 - \sqrt{x_1} + \frac{2}{|J_2|} \sum_{j \in J_2} \left(x_j - \left(0.3x_1^2 \cos\left(24\pi x_1 + \frac{4j\pi}{n}\right) + 0.6x_1 \right) \cdot \sin\left(6\pi x_1 + \frac{j\pi}{n}\right) \right)^2 \rightarrow \min \quad (26)$$

where: $J_1 = \{j \mid j \text{ is odd and } 2 \leq j \leq n\}$ and $J_2 = \{j \mid j \text{ is even and } 2 \leq j \leq n\}$, and the decision space $\Omega = [0, 1] \times [-1, 1]^{n-1}$ and $n=3$.

500 iterations found 204 solutions, which are presented in the Figures 8 and 9 for $d_1 = d_2 = d_3 = 0.01$. There were 491 solutions for

$d_1 = d_2 = d_3 = 0.002$ and they are presented in the Figures 10 and 11. Additionally, the Figures 12 and 13 show a graph of entropy values, power of the Pareto set and IGD in the function of iteration for values of $d_1 = d_2 = d_3 = 0.01$ and 0.002 accordingly.

The third test problem was presented in the work [27]. The objective functions for the particular optimization criteria were described by the formulas:

$$f_1 = x_1 \rightarrow \min$$

$$f_2 = g(x) \left[1 - \sqrt{\frac{f_1(x)}{g(x)}} - \frac{f_1(x)}{g(x)} \sin(10\pi x_1) \right] \rightarrow \min \quad (27)$$

$$g(x) = 1 + \frac{9 \left(\sum_{i=2}^n x_i \right)}{(n-1)}$$

where: $x = (x_1, \dots, x_n)^T \in [0, 1]^n$ and $n=3$.

After 500 iterations we found 134 solutions, which are presented in Figures 14 and 15. Additionally, the Figure 16 shows a graph of the entropy values, power of the Pareto set and IGD in the function of iteration.

As shown in the above figures, the modified particle swarm optimization algorithm has generated better sets of Pareto solutions (PSO solutions) than those presented in the publications cited above and in the works [13, 27]. For comparison, in the above figures, an ideal Pareto set is presented (all solutions). The analysis of Pareto solution sets shows that the PSO method can find the most solutions from the ideal Pareto set in a very short time period (after 500 iterations).

3.2. Multi-criteria optimization of hard turning operation of hardened 18CrMo4 steel

In the next step, a modified particle swarm optimization algorithm for multi-objective optimization problems has been used for solving the problems of multi-objective optimization in finish hard turning of hardened steel. The obtained experimental results were compared with the results from the work by [24], where GA with Modified Distance Method (MDM) [18] were used to solve the problem of multi-objective optimization of hard turning operation.

Technological progress in the area of cutting materials has made it possible to machine hardened materials with use of cutting tools with specified contour and angles of the edge. However, this operation is relatively rare in industry, due to the very high cost of tools made of cubical boron nitride (CBN) and the necessity to use the machine tools with appropriately high rigidity. Therefore, this operation should be performed with the optimal values of cutting parameters and many optimization criteria should be taken into account [9]. This will increase profitability and the number of industrial applications.

Hard finish turning operation of hardened (58HRC) 18CrMo4 steel machined with the use of CBN tools with Wiper geometry was subjected to optimization. 18CrMo4 steel (C – 0.18%, Mn – 0.32%, Si – 0.31%, P – 0.012%, S – 0.003%, Cr – 1.02%, Ni – 0.14%, Cu – 0.28%, Ti – 0.071%) is used for toothed elements. The research included: the effect of cutting speed $v_c = 100-200$ m/min, feed $f = 0.1-0.3$ mm/rev, depth of cut $a_p = 0.1-0.2$ mm, and length of cutting distance L on: unit production cost K_j , time per unit t_j , resultant cutting force F and selected parameters of the surface roughness: R_a , R_z and R_{max} [24]. The research was carried out with respect to the machined surface mating with sealing rings (Radial shaft seal), where the following parameters are recommended: $R_a = 0.2-0.8$ μm , $R_z = 1-4$ μm i $R_{max} \leq 6.3$ μm [12]. On the basis of experimental research results formulas [24] were developed for:

- unit production cost K_j ,

$$K_j = \frac{1.25\pi}{v_c f} \left(0.35 + \frac{v_c^{1.672184} f^{0.036654} a_p^{0.072133}}{423.326} \right) + 0.56 \quad (28)$$

– time per unit t_j ,

$$t_j = \frac{1.25\pi}{v_c f} \left(1 + \frac{v_c^{1.672184} f^{0.036654} a_p^{0.072133}}{85219.833} \right) + 1.6 \quad (29)$$

– resultant cutting force F ,

$$F = \sqrt{\left(993.402604 \cdot v_c^{-0.200600} \cdot f^{0.623620} \cdot a_p^{0.660314} \cdot L^{0.158012} \right)^2 + \left(320.402695 \cdot v_c^{-0.213734} \cdot f^{0.293166} \cdot a_p^{0.373255} \cdot L^{0.240112} \right)^2 + \left(814.912603 \cdot v_c^{-0.388726} \cdot f^{0.386365} \cdot a_p^{1.502916} \cdot L^{0.285235} \right)^2} \quad (30)$$

– arithmetical mean of roughness profile ordinates R_a ,

$$R_a = 0.634065 - 0.004599v_c - 1.221571f - 1.125925a_p + 0.000010v_c^2 + 2.265811f^2 + 0.007205v_c \cdot f + 0.007658v_c \cdot a_p + 0.0000001v_c \cdot L + 6.020526f \cdot a_p - 0.039339v_c \cdot f \cdot a_p \quad (31)$$

– average, maximal height of roughness from 5 elementary sectors R_z ,

$$R_z = 2.678195 - 0.015906v_c - 3.961551f - 4.844458a_p + 0.000027v_c^2 + 3.816645f^2 + 0.046635v_c \cdot f + 0.033541v_c \cdot a_p + 28.458806f \cdot a_p + 0.001495a_p \cdot L - 0.243340v_c \cdot f \cdot a_p - 0.008445f \cdot a_p \cdot L + 0.000049v_c \cdot f \cdot a_p \cdot L \quad (32)$$

– maximum height of roughness R_{max} ,

$$R_{max} = 2.775193 - 0.016948v_c - 2.123522f + 0.000042v_c^2 + 5.430604f^2 + 0.028112v_c \cdot f - 0.058322v_c \cdot f \cdot a_p + 0.000008v_c \cdot a_p \cdot L \quad (33)$$

In the equations (30)-(33), cutting distance L is present apart from cutting parameters. The value L was derived from the dependency $VB_C=g(v_c, f, a_p, L)=0.2$ mm, and was inserted into the above specified equations as the constant [23].

As a result of the optimization performed with the use of the modified PSO method for multi-objective optimization problems, after 50 iterations a set of non-dominated solutions was obtained as having 197 solutions and it is presented in Figure 17. For comparison, the set of non-dominated solutions obtained with the help of GA with MDM is presented as well (Fig. 18). Here the result of performed optimization was a Pareto set consisting of 106 non-dominated solutions.

As can be seen, the PSO method has detected much more solutions, and such solutions are better than the ones generated with the help of GA with MDM (Fig. 19). This confirms supremacy of the PSO over the AG and the opinion expressed in the work [2, 26], where the authors compared the PSO method with the ACO, TS, SA, MA, GA methods. In each case of turning and milling operations, the PSO has proved to be the best for selecting the optimal cutting method.

Figure 19 shows the entropy graph and the power of the Pareto set in the function of iteration.

4. Conclusions

In this article, only a discrete space of particle movement is considered. In practice, we always define the location in a particular dimension with a specific, rational accuracy. As has been confirmed by experimental studies, the decision space discretization is essential. The method proved effective only after discretization. The conducted in the work research show that such an approach is correct.

A great number of experiments has been conducted for both constant and variable in time coefficients w , c_1 and c_2 . For the particle inertia coefficient determined as the base coefficient (hence its constant value), experiments have been conducted in order to adjust the remaining two coefficients correctly. Obviously, these formulas are heuristic. They cannot be treated as universal. Nevertheless, we think that introduction of oscillation and the reduction of chaos (randomness) in successive iterations has a positive effect on efficiency of the method. This has been confirmed by the conducted experiments.

The evaluation of determined Pareto front quality is executed by the measurement of internal entropy. As pointed out by the performed tests, internal entropy at levels greater than 0.9 corresponds to the external entropy on a very similar level. Because the external entropy for multidimensional multi-objective optimization problems is not known, we can refer to the level of the internal entropy, not only as an assessment degree of the generated Pareto front, but also as a criterion for interrupting the calculation when the internal entropy exceeds a certain, satisfactory level. The external and internal entropy show significant changes during the calculations, in contrast to the parameter of IGD, which stabilizes quickly. The only negative phenomenon here is a longer time needed for computation, when the number of solutions on the Pareto front increases. Hence, we should choose appropriate steps of discretization of the d_i in order to avoid this problem.

Further research connected with the proposed method should tend towards the determination of the parameter a_i at the appropriate level. It has a very large impact on the mobility of particles, which should take the referred average interval. It seems that suitable values are very closely linked to the specific character of the problems being solved. Therefore, it is difficult to determine these values a priori. Instead, one should develop a mechanism of its adaptive selection during calculations. Another important factor for the effectiveness of the PSO method is a suitable choice of ranges in the decision space. In the work this issue has not been studied deeply. Nevertheless, on the base of the tests performed one can conclude that the appropriate narrowing of the decision space to the range corresponding to the position of particles on the Pareto front is very beneficial. When evaluating the obtained tests' results, we can draw the conclusion that the developed modified PSO method is highly competitive when compared to the previous proposals of the PSO, and should find numerous practical applications.

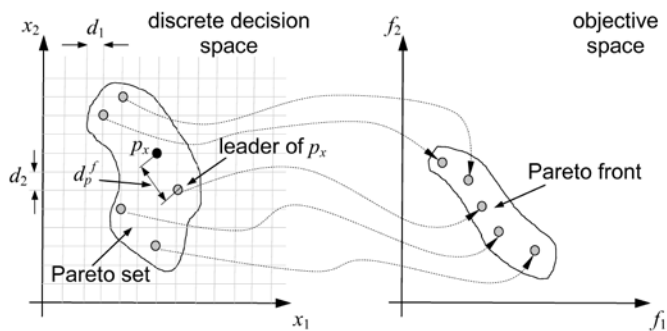


Fig. 1. Leader of particle in discrete decision space

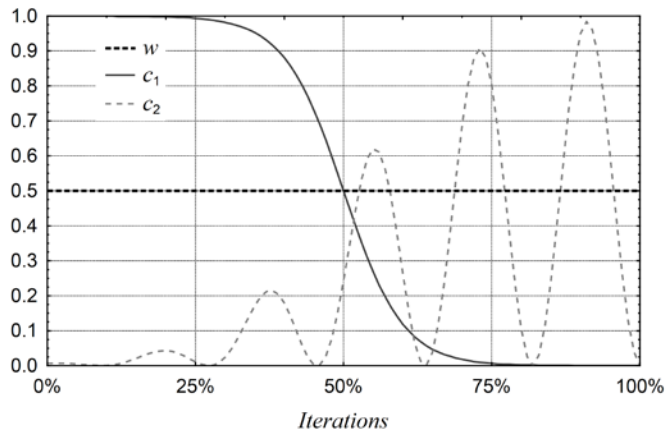


Fig. 2. Changes of w, c₁, c₂ coefficients value during calculations

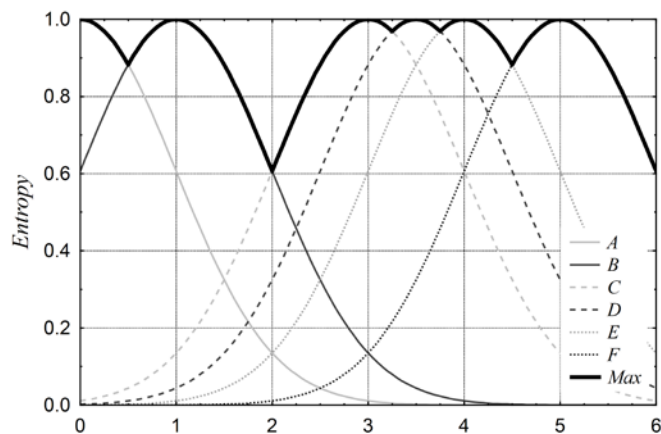


Fig. 3. Determination of the influence function value

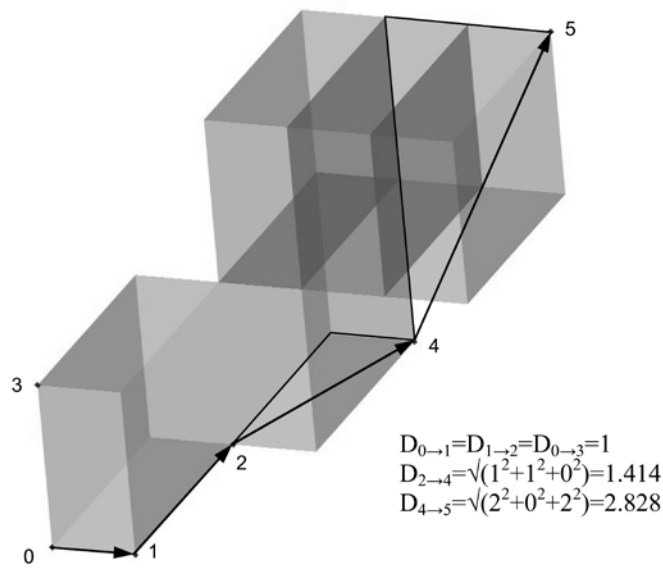


Fig. 4. Determination of the distance in 3-dimensional decision space

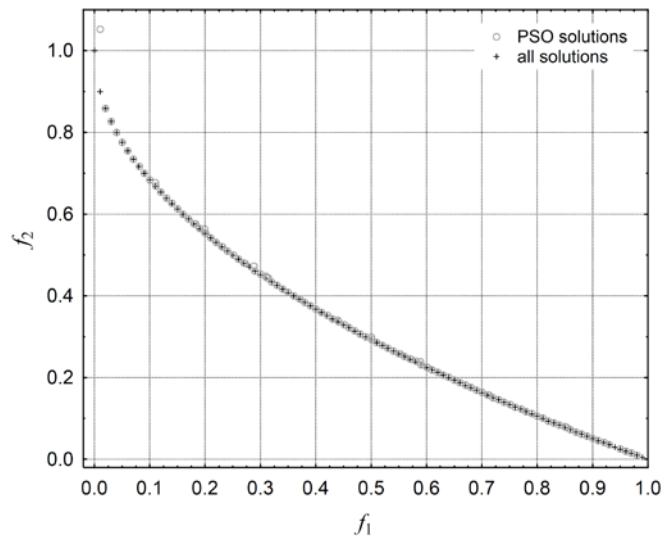


Fig. 5. Set of Pareto-optimal solutions for the first test problem generated for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.01$

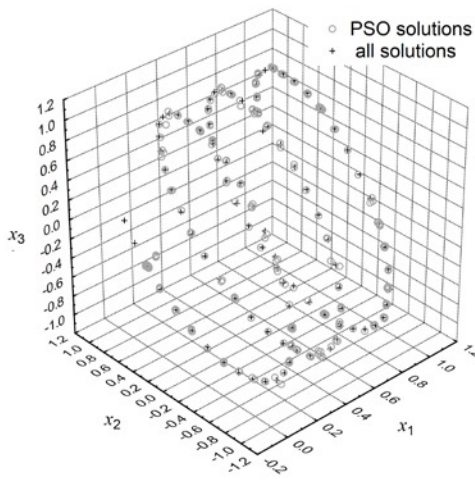


Fig. 6. Set of Pareto-optimal solutions for the first test problem in the decision space for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.01$

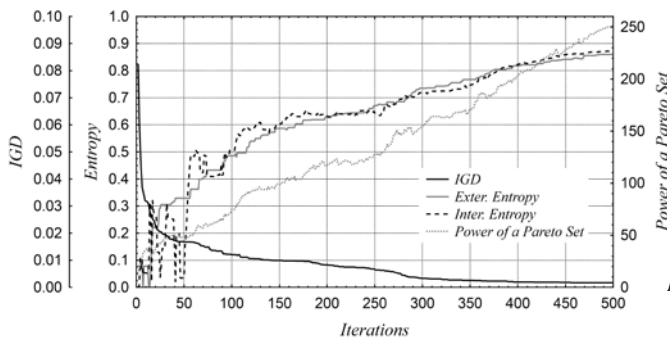


Fig. 7. Values of external and internal entropy, power of Pareto set and IGD in the function of iteration for the first test problem for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.01$

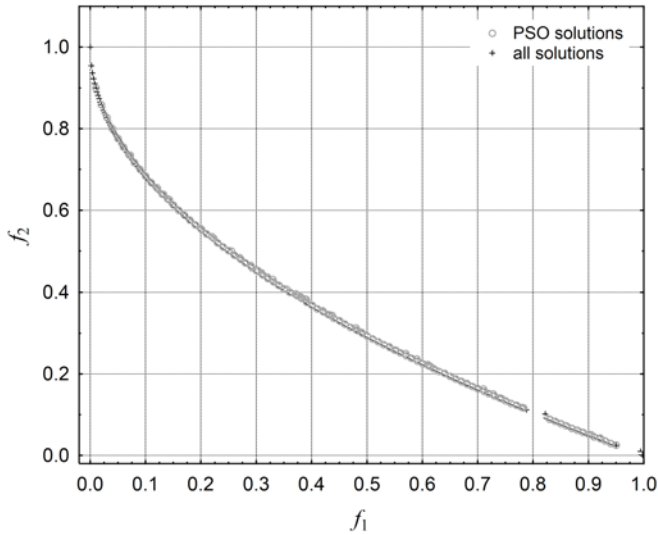


Fig. 8. Set of Pareto-optimal solutions for the second test problem generated for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.01$

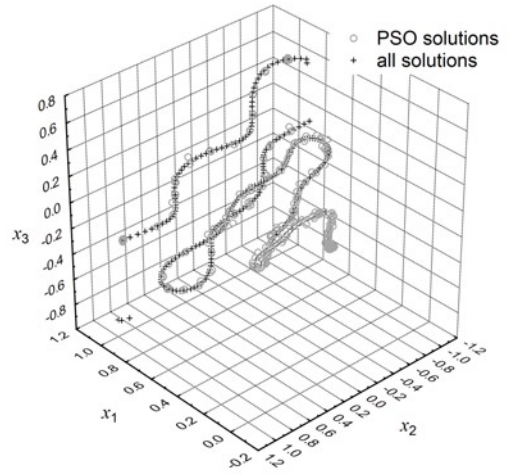


Fig. 9. Set of Pareto-optimal solutions for the second test problem in the decision space for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.01$

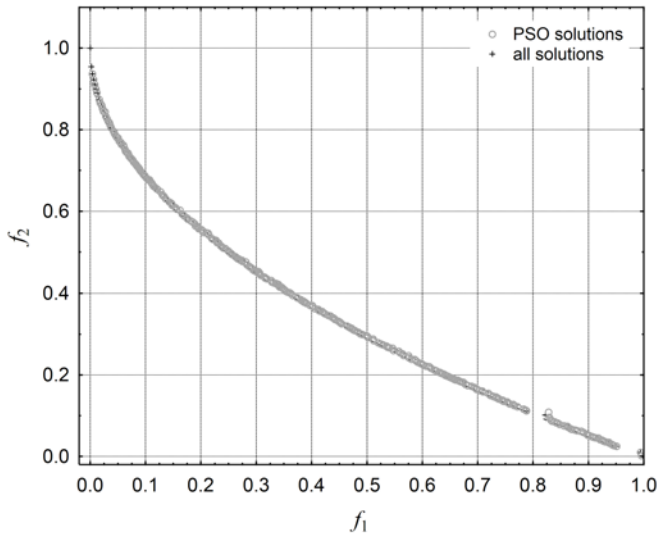


Fig. 10. Set of Pareto-optimal solutions for the second test problem generated for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.002$

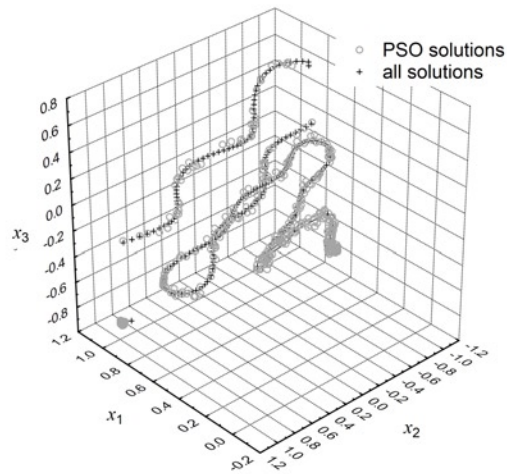


Fig. 11. Set of Pareto-optimal solutions for the second test problem in the decision space for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.00$

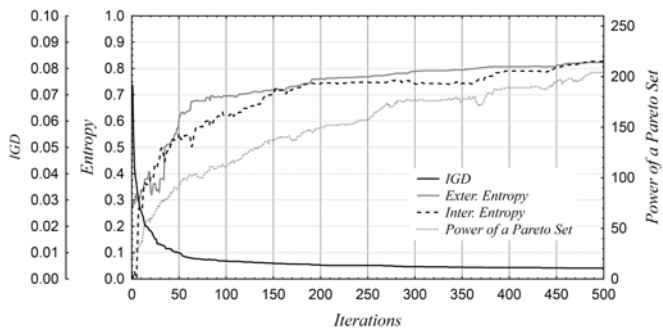


Fig. 12. Values of external and internal entropy, power of Pareto set and IGD in the function of iteration for the second test problem for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.01$

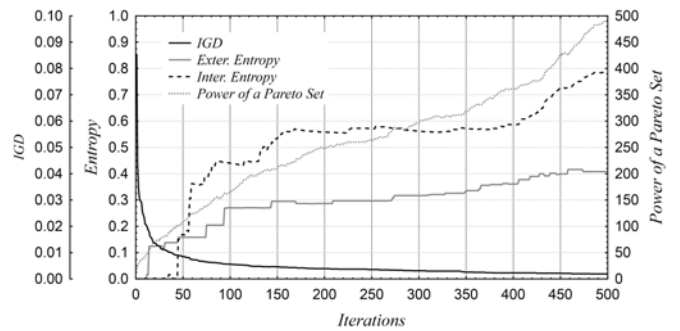


Fig. 13. Values of external and internal entropy, power of Pareto set and IGD in the function of iteration for the second test problem for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.002$

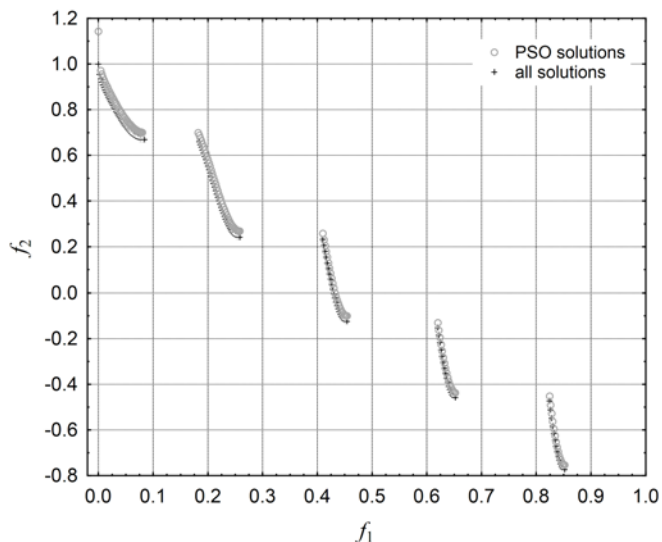


Fig. 14. Set of Pareto-optimal solutions for the third test problem generated for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.002$

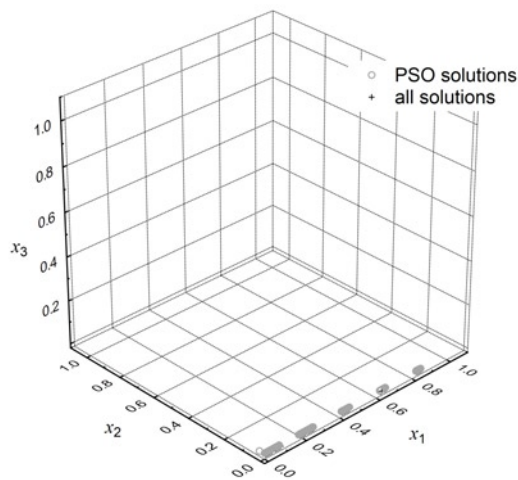


Fig. 15. Set of Pareto-optimal solutions for the third test problem in the decision space for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.002$

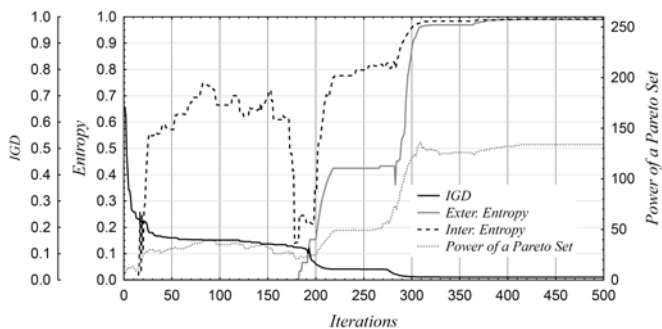


Fig. 16. Values of external and internal entropy values, power of Pareto set and IGD in the function of iteration for the third test problem for the parameters: $N_{pop}=200$, $N_q=500$, $a=const=0.2$, $d_1=d_2=d_3=0.002$

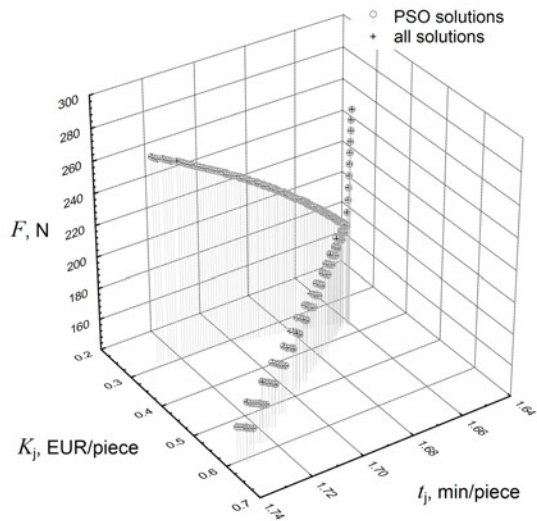


Fig. 17. Set of Pareto-optimal solutions generated with help of the PSO method for the parameters: $N_{pop}=200$, $N_q=50$, $a=const=0.8$, $d_1=1$, $d_2=d_3=0.01$

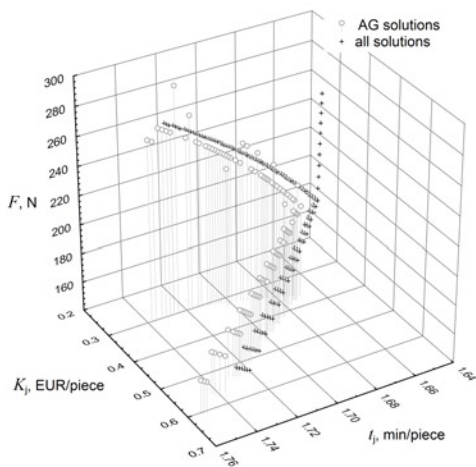


Fig. 18. Set of Pareto-optimal solutions generated with help of the genetic algorithms [24]

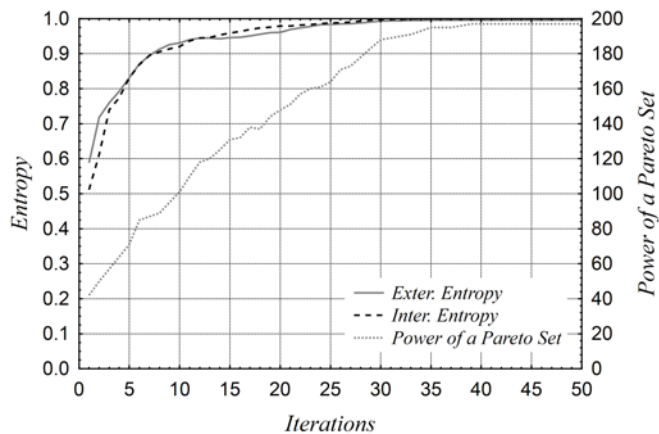


Fig. 19. Values of external and internal entropy, and power of the Pareto set in the function of iteration

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Roman STRYCZEK

Bogusław PYTLAK

Department of Manufacturing Technology and Automation
 University of Bielsko-Biala,
 43–309 Bielsko-Biała, ul. Willowa 2, Poland
 E-mails: rstryczek@ath.bielsko.pl, bpytlak@ath.bielsko.pl
