RECONFIGURABLE DOUBLE INVERTED PENDULUM APPLIED TO THE MODELLING OF HUMANOID ROBOT MOTION

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Abstract:
A simplified model of a human body is proposed. Model is built according to anthropomorphic data. Basic movements (gait and run) are analysed, center of mass trajectories for upper and lower part of the body were calculated. Analysis of the center of mass trajectories in terms of correlation coefficient is done. This proves the concept of the influence of arm movements over the whole body movement. Non-dimensional analysis was done, in terms of relation between appropriate pendulum's lean and the normalised position of pendulum's mass.

Keywords: *human body simplified model, double inverted pendulum, anthropomorphic data, trajectory of center of mass, Pearson's correlation coefficient*

1. Introduction

1.1. State of the Art

First humanoid robots were not able to react to unexpected perturbations. The reason for that was, that the gait was a simple playback – angular trajectories captured from human by motion tracking system were used [1]. With such method, on-line modifications were not possible.

The robot body is highly nonlinear, multidimensional system, the coordination of the body parts' motion is not a trivial task – especially when the postural balance and human like configuration must be kept.

The humanoid robot's motion generation is a very important and interdysciplinary topic. There is a variety of methods used, for example: using captured movement of human subject and motion generation based on trajectories produced by Central Patterns generators: [18], [6]. In those methods trajectories of every joint are generated, taking into account the structure of the whole robot. There is also different approach to motion generation: motion generation using simplified robot's models (inverted pendulum based models), [14]. In this concept, complex models and regularities are presented in a condensed manner and only basic characteristics are imitated. The whole system is treated as a one entity.

Nowadays, researchers are searching for such control methods that will directly consider the postural stability when modifying motion trajectories with disturbances rejection (on-line methods). Active control of postural stability considering the whole robot structure is an undoubted advantage, but unfortunately complicated model bears high computation cost, what is a significant drawback in real-time applications. Therefore simplified descriptions of the robots are needed. The structure of humanoid robot is similar to the structure of a human body. Both a human and a robot have a pair of arms, two lower extremities, a trunk and a head. Due to that, simplified models of a human body are considered as proper models of robots. Those models allow imitating basic characteristics of human gait and, among others, generation of body point mass trajectories. The first simplified model of human gait was Linear Inverted Pendulum (LIP) [2], [13]. LIP (Fig. 1) is used to model the human body dynamics during slow gaits where the overall centre of mass takes the highest position in the middle of support phase. LIP is also used for describing the human body sway in standing posture [7]. It was also applied for modelling the dancer's body dynamics [17].

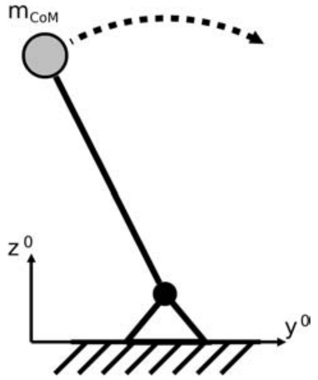


Fig. 1. Simplified model – Linear Inverted Pendulum (LIP)

The Spring-Loaded Inverted Pendulum (SLIP) (Fig. 2) model describes the centre of mass (CoM) dynamics for high-speed locomotion in a variety of insects and animals [14] and it is often used in robotics [4].

Despite of the fact that the SLIP model is able to imitate the characteristics for both – gait and run, LIP is still used [3]. LIP with constant pendulum length models "stiff" character of the gait, while SLIP models compliant character of the run. Unlike LIP, SLIP takes into account the change of pendulum length because linear spring is a part of the pendulum and point mass is located on the top of this spring. In [21] energetic considerations are presented. Authors claim that inverted pendulum models are enough to dynamic walking synthesizing. It is shown that vertical motion of the CoM is not a drawback, but a undoubted advan-

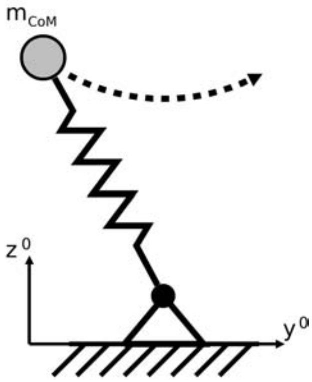


Fig. 2. Simplified model – Spring Loaded Inverted Pendulum (SLIP)

tage from the efficient walking point of view. Vertical motion of CoM (considering LIP) reduces the energy consumption.

Both simplified models are widely used for humanoid robots motion generation. In [15] walking pattern generator using inverted pendulum model was presented. In [5], [16] modified version of 3D LIP is discussed. Such model offers the possibility to simulate (and to investigate) pathological gait. Balancing strategy during running based on the model of inverted pendulum on a wheeled cart was discussed in [6]. Biologically inspired balancing algorithm based on double inverted pendulum was described in [7], in [8] movement synergies are emphasized. Simplified models are widely used even when synthesising more complex motions using the Capture Point [9] or the Zero Moment Point [10] approaches.

In [20] comparison of two pendulum based models is presented: linear inverted pendulum (LIP) model and double inverted pendulum model. Linear inverted pendulum model is a standard model, in which the whole body is concentrated at one point mass, located at the top of the pendulum. Double inverted pendulum is also a standard LIP with an additional body – ”Head and Trunk” segment. Authors claimed, that both models describe well the human motion, e.g. the ground reaction force profile is well reconstructed by those two models. However, double inverted pendulum model mentioned in this article lacks consideration of upper extremities – the upper part of a human body is simplified to HaT (”Head and Trunk”) segment.

Neglecting segmented structure of a robot and replacing it by one pendulum with point mass on the top results in many limitations. One of such limitations is ignoring the contribution of the arms movement to the postural balance [11]. Standard models referred above can only describe simple motions such as slow walk or run. More complex motions – for example situations when pushed person is losing balance are impossible to consider. During such motion a human makes a correcting step to come back to stable position. Introducing corrective movements of the arms and trunk is rather not a case in robots. Such problem cannot be analysed using simplified models – they cannot imitate the movements of upper part of the body because the whole body is considered as only one point mass.

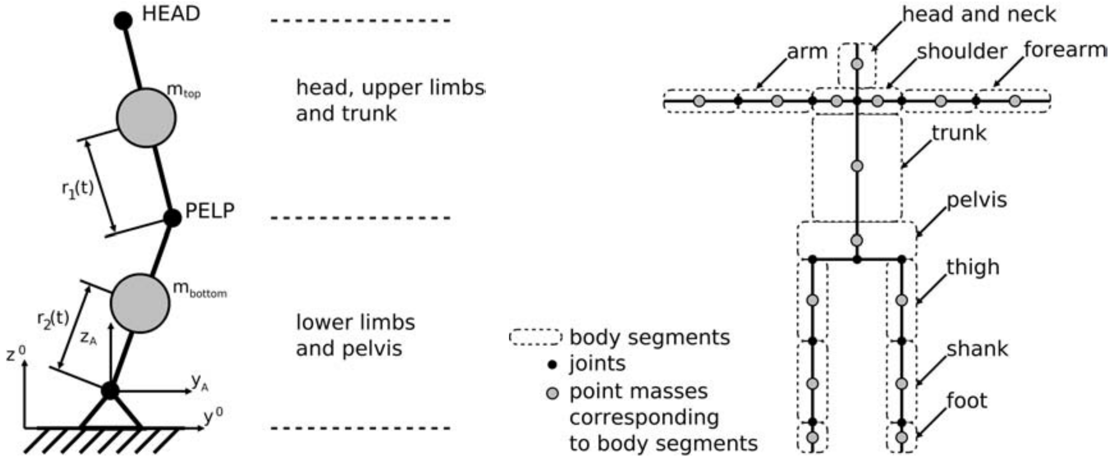
The development of simplified models is becoming visible in the field of humanoid robot motion synthesis. The basic example of simple models' development is the extension of planar inverted pendulum model to the three dimensional one. Such extension offers the possibility of simulating the pathological gaits [16].

There is necessity to create the models of humanoid structure which will allow to consider upper body movements. Such models should allow to investigate different movement situations – not only gait and run but also non-rhythmic behaviours.

The most important drawback of simplified models is neglecting the arm's movement, what is a far-reaching simplification. Moreover such models bring the whole complex system of a humanoid robot to the one point mass. The influence of arms movements on the position of the total center of mass is therefore missed. Arms movement is especially important in situations when object is losing its stability (for example, when human is pushed backwards) and is taking corrective movements to keep the balance. Even in cases of very simple, periodic human motions, there is clearly seen contribution of the arms movement. What is more, there are problems with motion generation for "transient" situations, what means for situations of the transition from walking to running. Model proposed by us overcomes limitiations mentioned above due to the introduction of moving masses concept. Different configurations of proposed model are suitable for representing both standard movements (gait, run) and for nonstandard situations.

Based on that conclusion we proposed model that imitates the basic features of human movements including the arms motion. Acquiescing to the fact, that inverted pendulums based descriptions are well representing the synergies in human body motion, that concept was adopted. Double inverted pendulum model with masses moving along the pendulum's rod was proposed in this paper. The lower part of the model represents legs and pelvis and the upper one – trunk, head and arms. For reference, it should be mentioned that in [9] inverted pendulum with a reaction wheel was used. Authors claimed, that such model can imitate well different body configurations, not only that typical for rhythmic gait or run. Unfortunately, this model is still reducing the whole body to one point mass. In our concept, typical inverted pendulum is enhanced by the second segment that allows to considers the upper part of the body. Point masses located on those pendulums can change their location by moving along the pendulum's rod.

The novelty of our work concerns the reconfigurability of proposed model. In standard models, the masses location is strictly determined, in our model there is the possibility to intentionally change the masses location. This results in the ability to incorporate arms movements in whole body motion.



(a)
a

(b)
b

Fig. 3. (a) Simplified model of humanoid robot's structure, (b) human body segmented model

1.2. Double Inverted Pendulum as a Model of a Human Body

With double inverted pendulum model (Fig. 3(a)) the structure of humanoid is divided into two segments with point masses selected in accordance to anthropometric data [12]. Positions of those masses are actively controlled. It is a new concept comparing to standard LIP and SLIP models, where mass position is determined by the pendulum's length (in LIP) and the spring stiffness (in SLIP). Actively changing the masses location allows us to simulate human motion features and to influence accordingly the postural stability. This is especially important in situations where arms movement is significant and its contribution to postural stability is significant.

Our model is depicted in the Fig. 3(a). It allows imitating different movements like gait, run or push recovery. We proposed a model that might be used to capture nature of human movements and to show the motion synergies of moving segments.

2. Considered Models and Movements

2.1. Human Body Model

Considered model of human body is shown in the Fig 3(b). This model describes tested person: a woman of 1.65 m height and 55.3 kG weight. Model was created using experimental data that were recorded using professional motion capture system. Human body model was divided to 15 segments: head and neck (1 segment), shoulders (2 segments), arms (2 segments), forearms with hands (2 segments), abdomen with chest (1 segment), hips (1 segment), thighs (2 segments), shanks (2 segments), feet (2 segments). For each of those segments a point mass located according to anthropometric data (12) was assigned.

2.2. Simplified Model

15-segments-model of a human body (presented in the Fig. 3(b)) was simplified to 2-segment-model (an upper and lower part of the body) connected by revolute planar joint located in the pelvis ("PELP" point). Mass of lower part of the body is a sum of feet, shanks, thighs and pelvis masses. Mass of the upper part is the sum of remaining segments. Length of the lower pendulum was defined as equal to distance between ankle and hip joints. Length of the upper part was considered as equal to distance between pelvis "PELP" point and point in the middle of the head. Simplification to a double inverted pendulum instead to a single inverted pendulum allows us to consider the influence of the upper limbs to the human postural stability.

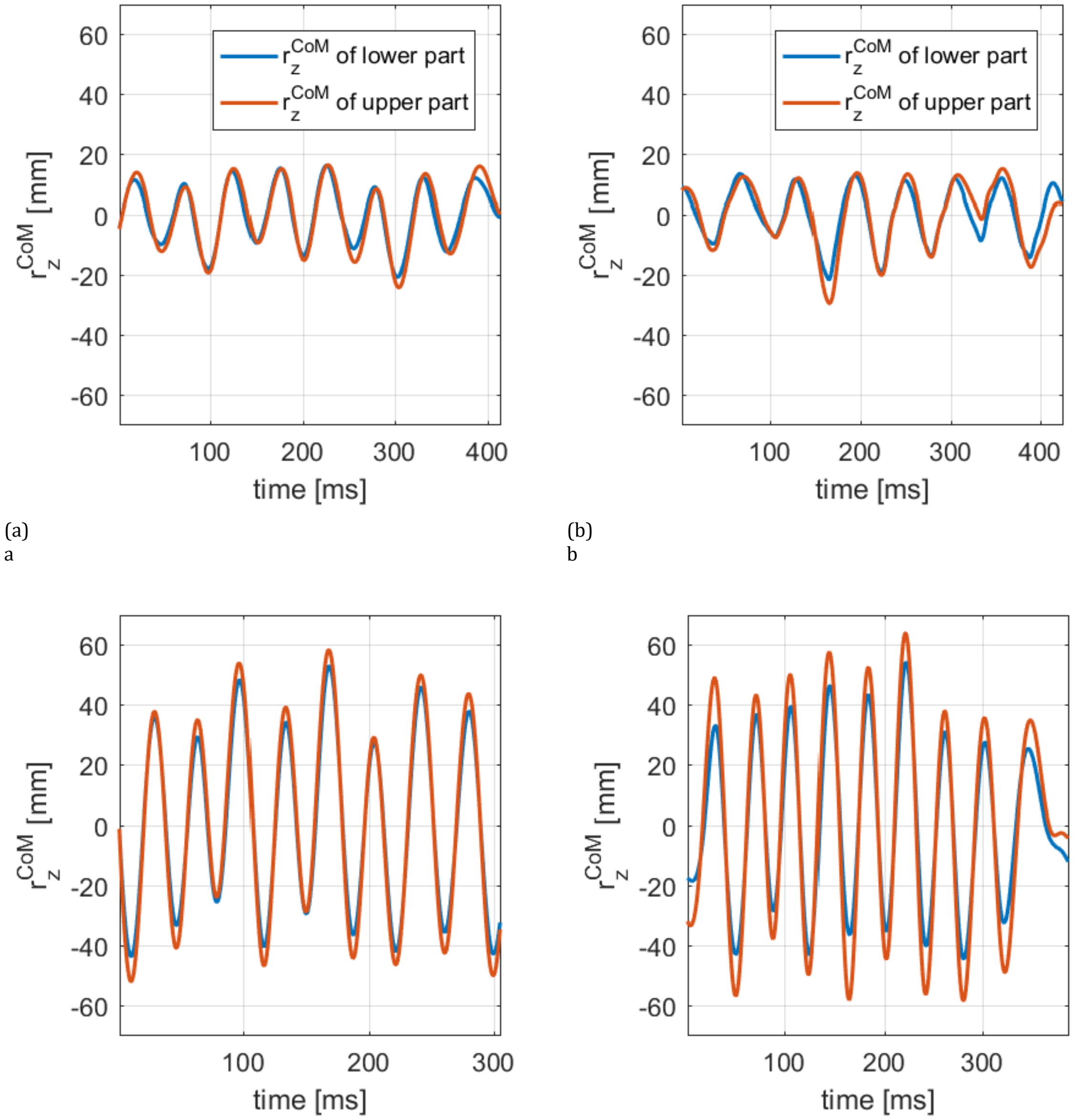
2.3. Analysed Movements

The proposed model is a planar model which means that the movement can be presented in the sagittal plane. To validate this model, we analysed situations that can be simplified to planar ones (e.g. walk and run). Each movement was analysed in two conditions: when upper limbs move freely and when they are affixed to the trunk. Such situations were recorded using Vicon motion capture system.

Presented studies were conduced using double inverted pendulum model with masses moving along the rods (Fig. 3(a)). Segmented model (Fig. 3(b)) is presented only to indicate the way of determining the point masses and characteristic points of the model.

3. Analysis of Center of Mass Trajectories

Using motion data gathered by Vicon motion capture system, the trajectories of human body point masses were calculated. That allowed to investigate the trends in human movement. Simplified human body model – double inverted pendulum was then used to imitating such trends. Obtained good coincidence between human and pendulum's trajectories confirmed that the double inverted pendulum with moving masses allows to incorporate the effect of arms movements sufficiently for reflecting the dynamics of more complex motion.



(a)
a

(b)
b

(c)
c

(d)
d

Fig. 4. Point mass trajectories for: (a) gait with arms tied to the body, (b) natural gait (with arms moving freely), (c) run with arms tied to the body, (d) natural run (with arms moving freely). Blue solid line is CoM trajectory for lower part of the body and red solid line is CoM trajectory for upper part of the body

Location of total center of mass (CoM) was calculated using:

$$r_{CoM}^{0} = \frac{\Sigma_{i=1}^{n} m_i r_m^i}{\Sigma_{i=1}^{n} m_i}, \quad (1)$$

where m_i is mass of i-th segment and r_i is coordinate of that mass in global reference frame. Division of the body into segments is presented in the Fig. 3 and parameters describing centres of mass location for each segment are in accordance to anthropometric data taken from [12]. Obtained point mass trajectories of individual segments in 2D reference frame are located at different heights (levels). For the purpose of comparison, we brought them to the same reference level.

Vicon system allows precise 3D movement measure using the real markers placed on subject body. This system provides also information about the total body centre of mass trajectory. For our purpose, we needed also trajectories of point masses for lower and upper part of the body. After obtaining necessery data we built segmented model of the body (Fig. 3(b)), to which appropriate point masses (corresponded to the anthropometric data, [12]) were assigned. Only with such model and taking into account the motions of all segments, the resultant trajectories of point masses for lower and upper part of the body were obtained. Those point masses correspond to the center of masses of double inverted pendulum.

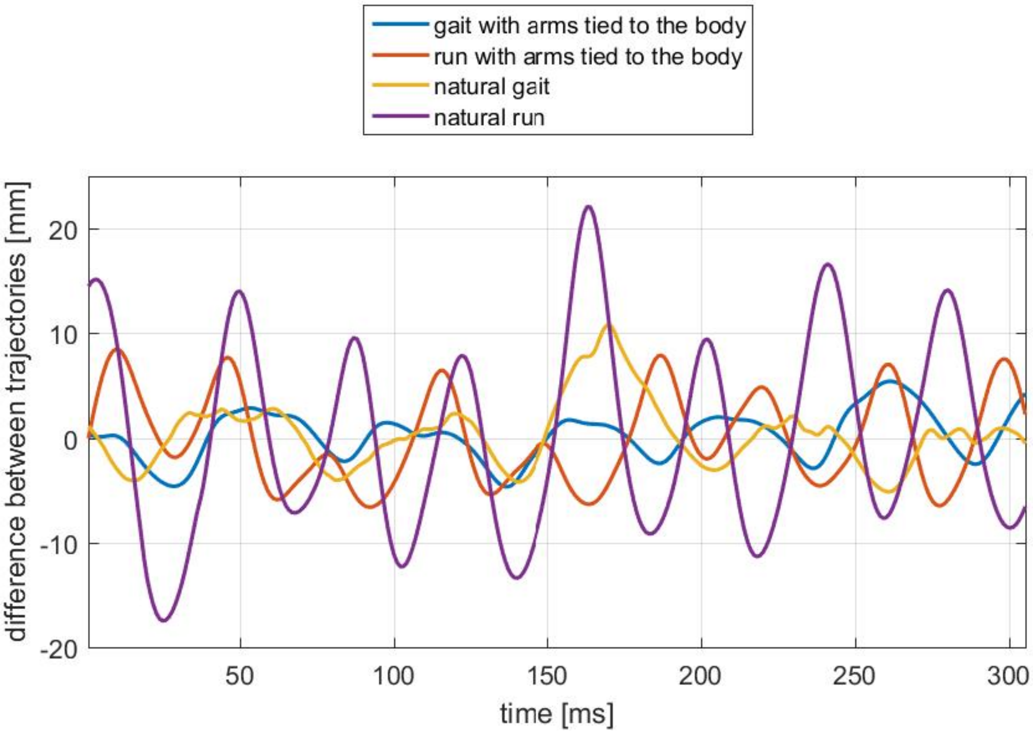


Fig. 5. The differences between CoM trajectories for upper part of the body and for lower part of the body for: gait with arms tied to the body (blue line), run with arms tied to the body (red line), natural gait with arms moving freely (yellow line) and natural run with arms moving freely (violet line)

3.1. Trajectories Comparison in the Time Domain

Point mass trajectories calculated for four basic movements are shown in Fig. 4: walk with arms tied to the body (Fig. 4(a)), walk with arms moving freely (Fig. 4(b)), run with arms tied to the body (Fig. 4(c)) and run with arms moving freely (Fig. 4(d)). Horizontal axis denotes time expressed in milliseconds and vertical axis shows the CoM trajectories brought to the zero (reference) level. Two trajectories are placed in each plot: CoM trajectory of the lower part of the body (blue solid line) and CoM trajectory of the upper part of the body (red solid line).

It is clearly seen that there is a strong coincidence between trajectories. Both CoM trajectories of the lower part and the upper part of the body are showing sinusoidal trend. This applies to all movements situations shown in the Fig. 4. Trajectories of the upper part of the body and lower part of the body are in phase and there is no time shift. During movements with arms tied to the body trajectories of the upper part of the body and of the lower part of the body are almost identical. In the normal gait with arms moving freely that trajectories differs a bit, moreover if we deal with the normal run we can see even bigger difference between trajectories. Such result was expected – when arms are tied to the body they not influence separately the CoM trajectory of the upper body part. In normal gait, arms are moving freely, so there is arms influence to CoM localization. In case of natural run, we have very fast and sudden arms movements, so they strongly affects the CoM localization.

The differences between CoM trajectories for upper and for lower part of the body are shown in the Fig. 6. Basing on those observation, we can conclude that the biggest difference in CoM positions is for natural run, while the gait with arms tied to the body is characterized by the smallest discrepancies.

3.2. Relation Between the Trajectories in Terms of Correlation

Summarized above observations were quantified by correlation coefficient. Assuming that the CoM trajectory of the lower part of the body is our first variable (x) and the CoM trajectory of the upper part of the body is our second variable (y), we investigated the linear dependence of those variables using Pearson's correlation coefficient. The estimator of the correlation coefficient ρ is expressed by:

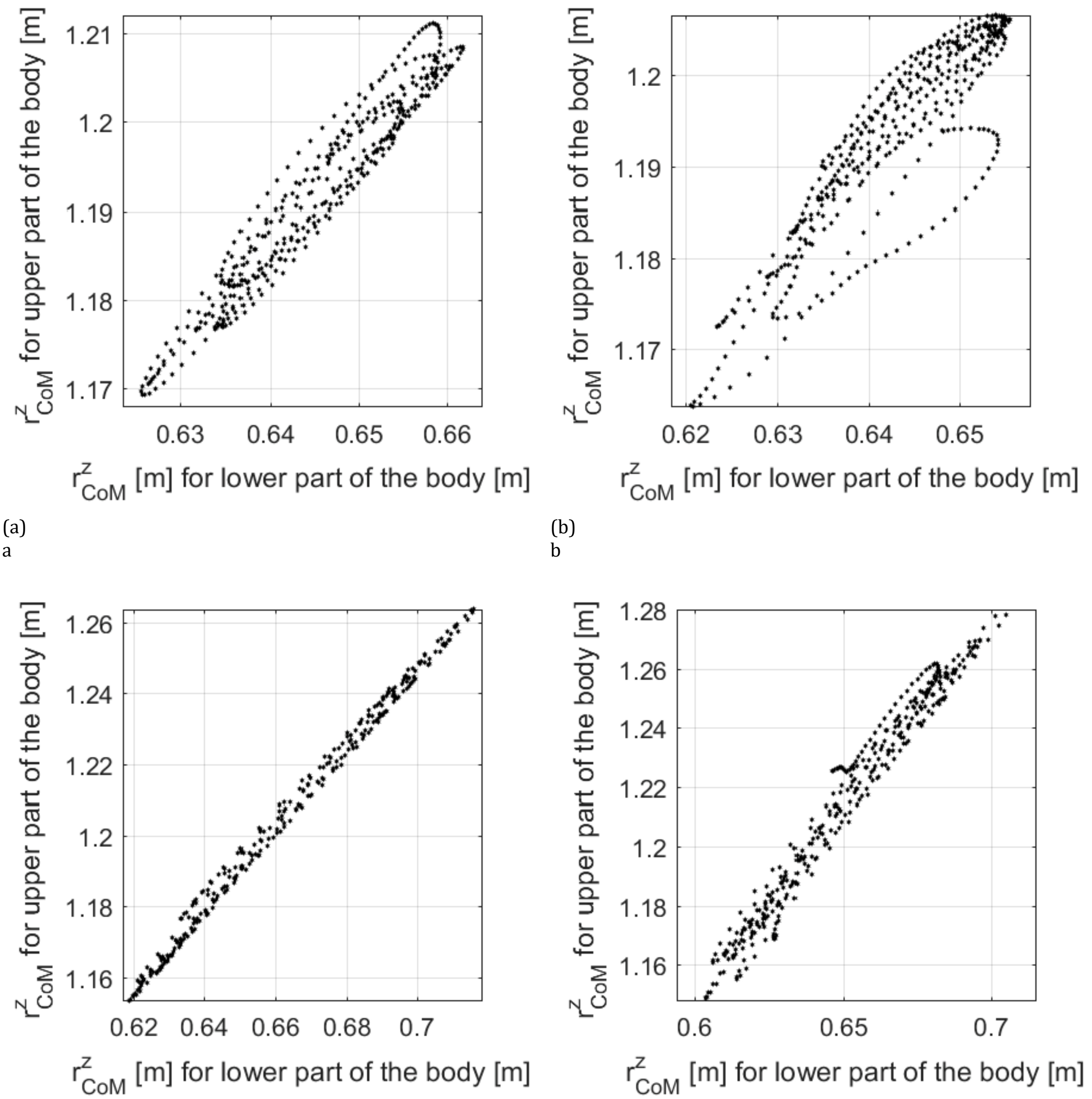
$$\rho = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\Sigma_{i=1}^{n}(y_i - \bar{y})^2}}, \qquad (2)$$

where x_i and y_i are the values of the CoM trajectories in i-th point and $\bar{x}$, $\bar{y}$ are mean values:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad (3)$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i. \qquad (4)$$

Correlation coefficient allows to indicate differences between motion of upper part of the body and lower part of the body – it is mathematical expression of the "strength" of movement patterns dependence in time domain shown in the Fig. 4 and in the Fig. 5.



(a)
a

(b)
b

(c)
c

(d)
d

Fig. 6. Scatterplots for following movement situations: (a) gait with arms tied to the body, (b) natural gait (with arms moving freely), (c) run with arms tied to the body, (d) natural run (with arms moving freely)

Tab. 1. Pearson's correlation coefficients for four basic movements

Movement	Correlation coefficient
Gait with arms tied to the body	$\rho_1 = 0.9701$
Gait with arms moving naturally	$\rho_2 = 0.9255$
Run with arms tied to the body	$\rho_3 = 0.9972$
Run with arms moving naturally	$\rho_4 = 0.9878$

The biggest correlation coefficient was obtained for movement with arms tied to the body, what was expected. The stronger independence of the arms movements of the rest of the body, the lower is the correlation coefficient – it means that arms movement influences the CoM trajectories.

Because both body parts are strictly affixed to each other and positions of considered by us two point masses are mainly influenced by masses of trunk parts, only small fluctuations in obtained correlation coefficients can be seen. The correlation coefficient is in general high in value. The masses of lower limbs and pelvis are included to the mass of lower part of our double pendulum, the rest is included to the mass to the upper part. With such division, for more considerable movement of upper extremities', the correlation coefficient is relatively smaller and vice versa. Therefore, for running with arms tied to the body, the correlation coefficient was the biggest – the arms were not moving differently than the torso. The smallest correlation coefficient was obtained for natural gait, because arms were moving naturally in that case.

Difference in correlations obtained for different movements confirms our hypothesis that arms movement should be considered in our model.

3.3. Relation Between Trajectories Presented on Scatterplots

The plots given in Fig. 6 illustrate quantified differences (using our indicators) in considered movements.

Scatterplots (Fig. 6) present straight-forward relation between trajectories of lower and upper part of the body. On horizontal axis position of lower body point mass is presented, while on vertical axis position of centre of mass for upper part of body is indicated. For such coordiante frame plots present localisation of upper body mass (y_i) depends on position of lower body mass (x_i). Each point (x_i, y_i) concerns the data for i-th time frame. Plots presented in Fig. 6 confirm important dependence mentioned while discussing the plots from Fig. 4. When centre of mass for lower part of the body moves downward the centre of mass for upper part also moves down. It was depicted in Fig. 4 by overlapping of waves (with no phase shift). Therefore we can conclude that for gaits and runs there is synergy in position for both point masses.

3.4. Analysis in Terms of Non-dimensional Quantities

In this section, analysis of natural gait with arms moving freely in terms of non-dimensional quantities is presented. Position of mass center is set together with the lean angle of appropriate part of the body, everything is parametrized by time.

Non-dimensional factor representing pendulums' point masses positions was evaluated using:

$$k_i = \frac{d_i^0}{l} \cdot 100\%, \tag{5}$$

where d_i^0 is the position of centre of mass in i-th time instant (expressed in global reference frame), l is the length of the appropriate pendulum. This gives the information where centre of mass is located with respect to the reference points (ankle or pelvis point), and how big is the displacement taking into account the pendulum's length. Lean of the appropriate part of pendulum is expressed with respect to the vertical axis, as it is shown in the Fig. 3(a). The results are presented in 3D space (Fig. 7) with the axes indicating the time, the lean angle and the normalized position of the point mass (according to formula (5)). Analysing the plot in the Fig. 7(a) we can notice, that the lean of the first pendulum (representing the lower part of the body) stays in range from $-6°$ to $+8°$ (backward – forward sway). In the same time the ratio k (showing the change of location) stays between 69% and 73%. It means, that for considered movement, centre of mass of the lower part of the body remains in position which is near to 70% from the origin (from the ankle joint). The maximal displacement of the mass is about 4%, what means that for the pendulum length equal to 1m it is 0.04 m. Such result corresponds to the anthropomorphic data (Fig. 4(b)). Analysing the plot shown in the Fig. 7(b), we can notice, that the lean of the pendulum corresponding to the upper part of the body stays in range from $-6°$ to $+6°$. The ratio k (showing the change of mass location) is between 20% and 26%. It means that for this type of movement centre of mass of the upper part of the body is located in the position which is about 24% from the pendulum's joint (from the pelvis). The maximal displacement of mass centre is 6%, that means for the pendulum length 0.8 m it will be about 0.05 m. Such result corresponds to the anthropomorphic data (Fig. 4(b)) as well.

4. Results

Analysis presented in section 3 confirmed significant relation between point mass trajectories for upper and lower parts of the body in four basic movements: gait with arms tied to the body, natural gait with arms moving freely, run with arms tied to the body and natural run with arms moving freely. The CoM trajectories are in phase with each other and they differ only in amplitude. The strong relation between trajectories is well illustrated on scatterplots – increase of the value in one trajectory is accompanied by increase of the value in the other. This observation brings the idea of synergy.

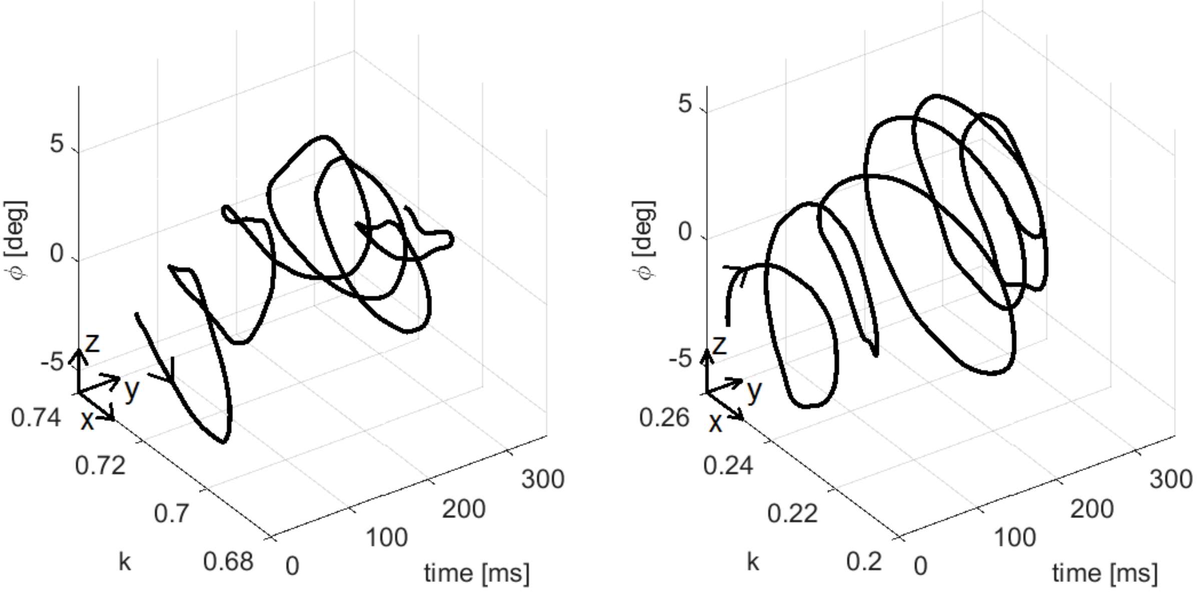
The greater the arms movements, the bigger is the difference – the correlation coefficient between CoM trajectories is smaller for movements with arms moving freely. This supports the fact that arms motion influences the localization of CoM. However, the correlation coefficient is high (more than 0.9) for all four situation. Analysis presented in the section 4 proved the idea of using double inverted pendulum. Non-dimensional analysis gave the comprehensive picture of pendulums behaviour.

5. Conclusions

Correlation coefficient expresses significance of the arms movement. When the arms are moving freely the trajectory of the upper mass becomes more different than the lower mass trajectory – what is reflected by smaller value of correlation coefficient.

Introduced by us normalized factor displayed in 3D space allows to illustrate double inverted pendulum reconfigurations during different movements. Additionally, the comparison of the movement of both body parts was made using Pearson's coefficient.

Obtained results made the first step for the description of motion synergy. Identification of synergetic strength fluctuations taking into account the movements of upper and lower parts of the body is important for robot motion synthesis. Knowing that hu-



(a)
a

(b)
b

Fig. 7. Relation between centres of masses positions and leans: (a) for upper part of the body, (b) for lower part of the body

man motion exhibits regularities, we decided to quantify them.

Our work confirmed that expanding simplified model of human gait (LIP model) into a bit more complex one is useful. Using anthropometric data, we showed that it is possible to reconstruct the human movement taking into account proposed by us model. The important drawback when doing humanoid robots' motion synthesis using simplified models is the lack of arm's movement incorporation, what is a far-reaching simplification. Simplified models reduce the whole complex humanoid robot system to one point mass. That neglects the arms movement contribution to the postural stabilization and to the overall motion dynamics. Importance of arm motion can be especially observed in situations, when the object is losing its stability (e.g., when the human is pushed backwards) and produces corrective movements regaining balance. Even for a very simple, periodic fast human gait, we can clearly notice the influence of the arms on the whole motion dynamics.

Double inverted pendulum model requires comprehensive description of its behaviour for different movements. In our investigation the scatterplots and the 3D plots gave such information.

Next steps will be devoted to the analysis of the wider variety of body movements and postures. In the further research we will study more movements delivering more complete conclusions on the double pendulums' configurations. It is expected that it will result in more complete description of synergies using not only 3D plots but dedicated indicators.

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