Active Vibration Suppression of Axially Moving String via Distributed Force

Waldemar ŁATAS
Institute of Applied Mechanics, Faculty of Mechanical Engineering,
Cracow University of Technology, 31-864 Kraków, Al. Jana Pawła II 37, Poland,
waldemar.latas@pk.edu.pl

Abstract

In the paper the problem of suppression of the waves – traveling along the linear, axially moving string – by the active distributed force is presented. The control law is based upon the idea of wave cancellation. The distributed force density is assumed to be proportional to the string transverse velocity resulting from the original running wave, assumed in the form of packet wave with amplitude modulation. As an objective function of the optimization problem considered the energy dissipated by the damping force segment is taken. Simulation results included demonstrate the effectiveness of the control law assumed and superiority of the distributed damping force over the concentrated force.

Keywords: axially moving string, traveling waves, vibration suppression, active vibration control

1. Introduction

Axially moving string-like structures are important mass and energy transfer systems. The traveling string represents a typical model that has been utilized to describe the dynamics of many engineering devices such as power transmission chains, machinery belts, textile fibers, paper sheets, band saws, aerial cable tramways, lift cables, fluid pipes; primarily when the object is long and narrow enough and its flexural rigidity could be neglected.

The longitudinal and transverse oscillations associated with the devices – induced due to the external excitations and oscillations of supports – limit their utility in applications, so appropriate control methods should be introduced to suppress the vibration of the moving element.

Papers [1–3] deal with the mathematical models and numerical methods established for study of the transient response and stability of the translating strings under various excitations.

References [2,4] contain a survey of contributions related to the vibration control of axially moving strings.

The means of control of distributed parameter systems (DPS) can be systemized with regard to the criteria applied:

- active control or passive control [also combination (hybrid) or semi-active];
- different control techniques and control laws: modal control; parametric control; wave cancellation method; the Laplace'a transform domain approach; the Lyapunov energy method; the sliding-mode technique; adaptive methods;
- concentrated force control or distributed force control;

– boundary control or spatial domain control;
– feedback control or feedforward control;
– linear or nonlinear string model.

Generally speaking vibration control can be achieved in active [4–10] or passive way (or semi-active). As the first vibration suppression technique applied to the distributed parameter systems the so-called modal control was employed which approximates the string by a number of modes and utilizes the well-known control methods to design controllers.

Parametric control [8][11] refers to the adjustment of system parameters, mainly the string tension, being changed on-line to suppress vibration.

The idea of the wave cancellation (feedforward type) method [5–7,12] is to apply a control force to superimpose a secondary wave source to cancel the waves traveling in the structure. The technique of the Laplace'a transform was used in [5,7].

The Lyapunov function analysis implemented to boundary control can be used to derive control laws reducing the total mechanical energy of a moving system [8,13–15]. The variable structure control utilized to a traveling string was investigated in references [7,16–17].

In the distributed parameter systems the controllers are distributed or point forces applied to the domain or at the boundary. It occured that only one collocated point sensor and actuator suffices for control of the axially moving string. It was found that the DPS became incontrollable and unobservable when sensors or actuators are located at nodal points.

Distributed control enables to achieve better control performance than boundary control but is more difficult in implementations due to the distributed forces and feedback signals required. Boundary control seems to be more economic and easier applicable, the equations of the dynamic model don't require modifications after adding actuators but implementations may be limited due to the size of the support rollers.

There are few papers based on the nonlinear models of axially moving string for vibration control [13–15] comparing to the linear ones. It was shown in [18] that the non-linear string can be stabilized by a linear velocity boundary feedback control.

The present paper investigates active control of the linear axially moving string via a distributed force segment, the idea of wave cancellation is used. The original wave used in numerical simulations was assumed in the form of packet wave with amplitude modulation.

An analytical solution of the linear differential equation of axially moving string (at a constant velocity) was used and the derived formula for the string transverse velocity was utilized.

The energy balance of waves occurring in the system was employed to estimate the effectiveness of the proposed method of wave suppression.

The presented results of numerical calculations prove the effectiveness of the proposed method of vibrations suppression in the axially moving string, they are also evidence of the advantage of using the distributed force compared to the concentrated force applied.

2. Mathematical Model of Axially Moving String

The model used is linear, the transverse motion of axially moving string is governed by the equation:

$$\mu \frac{\partial^2 u}{\partial t^2} + 2\mu V \frac{\partial^2 u}{\partial x \partial t} + \left(\mu V^2 - T\right) \frac{\partial^2 u}{\partial x^2} = q(x,t) \tag{1}$$

where the following notations are introduced: μ – linear mass density, T – tension force, c – velocity of the travelling wave equal to $\sqrt{T/\mu}$, V – axial velocity of the moving string, $q(x,t)$ – distributed loading.

For the given initial conditions:

$$u_0(x) = u(x,t)\big|_{t=0}, \quad u_1(x) = \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} \tag{2}$$

the solution of Eq. (1) takes the form (for $V < c$):

$$\begin{aligned} u(x,t) = {} & \frac{1}{2}\left[u_0(x-(c+V)t) + u_0(x+(c-V)t)\right] \\ & + \frac{V}{2c}\left[u_0(x+(c-V)t) - u_0(x-(c+V)t)\right] \\ & + \frac{1}{2c}\int_{x-(c+V)t}^{x+(c-V)t} u_1(\xi)\,d\xi + \frac{1}{2c\mu}\int_0^t \int_{x-(c+V)(t-\tau)}^{x+(c-V)(t-\tau)} q(\xi,\tau)\,d\xi\,d\tau \end{aligned} \tag{3}$$

In the presented analysis it was assumed that the string is formally infinite, practically it means that the borders of the string are so far away from the loading segment that it is not necessary to take into account the effect of wave reflection. The solution given by the expression (3) can also be used for the problem of motion of the string of a finite length, this requires the determination of reflected waves resulting from the adopted initial conditions and the appropriate extension of the function describing the distributed loading.

Denoting the last component of the expression (3) by $u^*(x,t)$, it can be proved that:

$$\begin{aligned} \frac{\partial u^*(x,t)}{\partial t} = {} & \frac{1}{2\mu}\left(\frac{c+V}{c}\right)\int_0^t \left[q(x-(c+V)(t-\tau),\tau)\right] d\tau \\ & + \frac{1}{2\mu}\left(\frac{c-V}{c}\right)\int_0^t \left[q(x+(c-V)(t-\tau),\tau)\right] d\tau \end{aligned} \tag{4}$$

The above expression will be used to determine the energy of travelling waves in a moving string.

3. Optimization Problem

The problem generally is to find the efficient way to suppress the waves travelling along the string. The idea of the wave cancellation method is to apply a control force to superimpose a secondary wave source to cancel the waves traveling in the structure.

In the paper [19] it was shown for a stationary string, that the concentrated active damping force dissipates maximum the half of the incident original wave energy (with the reflected wave neglected). The optimal force magnitude is then proportional to the magnitude of the string velocity component associated with the original wave (the vectors directions are opposite), the constant of proportionality is equal to the ratio of the string tension to the travelling wave velocity.

The similar strategy was proposed in [20] for suppressing the travelling waves by the active damping segment, proving its effectiveness also for the case taking into account reflection of the waves from the border [21]. Based on the results obtained for the stationary string, an analogous method of suppressing vibrations for the moving string was adopted.

The secondary wave comes from the distributed load, the active distributed damping force is assumed to be proportional to the component of the string transverse velocity resulting from the motion of the original wave $u_0^*(x,t)$:

$$q(x,t) = -\alpha(x,t)\frac{\partial u_0^*(x,t)}{\partial t}H(x)H(L-x) \tag{5}$$

where $\alpha(\cdot)$ denotes the damping coefficient function, $H(\cdot)$ is the Heaviside function.

The original wave used in numerical simulations was assumed in the form of packet wave with amplitude modulation, convenient in modeling of disturbances observed in cables:

$$u_0^*(x,t) = A_0\left[\sin(k_0(x-(c+V)t+4\sigma))\exp\left(-\frac{(x-(c+V)t+4\sigma)^2}{4\sigma^2}\right)\right] \tag{6}$$

where: k_0 – wave number, σ – packet width parameter.

The traveling wave (6) corresponds to the sum of the first three components of expression (3), dependant on the appropriately adopted initial conditions (2), it satisfies the homogenous equation of motion (1) for $q(x,t) \equiv 0$.

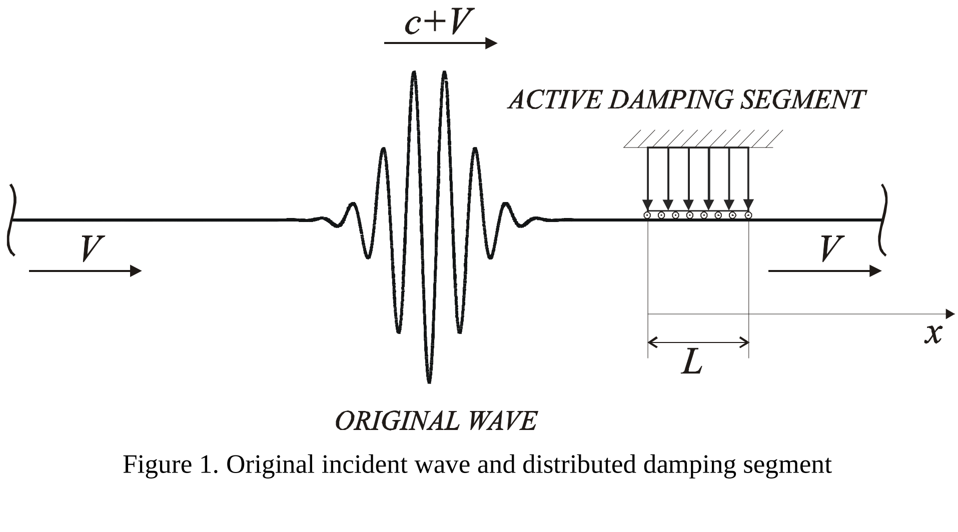

Figure 1. Original incident wave and distributed damping segment

Using the introduced notations the solution of Eq. (1) can be written in the form:

$$u(x,t) = u_0^*(x,t) + u^*(x,t) \tag{7}$$

Due to a shape of the packet wave function taken (Fig. 1), it is assumed that a width of the packet wave with non-zero amplitude of motion necessary to consider is 8σ, so the expression (6) displays the wave running to the right and reaching the damping segment at the moment $t = 0$.

The active distributed force applied to the string by the damping segment has the effect of injecting the two secondary waves into the structure, running toward the left and right, which are transmitted outside the segment. The first (reflected) wave travels in opposite direction to the incident wave, the second wave superposing with the incident wave finally forms the total passed wave.

The energy balance will be used in the calculations, containing the original wave energy E_O, the reflected wave energy E_R and the passed wave energy E_P:

$$E_O = \int_0^{\frac{8\sigma}{(c+V)}} \sqrt{T\mu} \left(\frac{\partial u_0^*(0,t)}{\partial t} \right)^2 dt \tag{8}$$

$$E_R = \int_0^{\frac{8\sigma+2L}{(c+V)}+\frac{L}{(c-V)}} \sqrt{T\mu} \left(\frac{\partial u^*(0,t)}{\partial t} \right)^2 dt \tag{9}$$

$$E_P = \int_{\frac{L}{(c+V)}}^{\frac{8\sigma+L}{(c+V)}} \sqrt{T\mu} \left(\frac{\partial u_0^*(L,t)}{\partial t} + \frac{\partial u^*(L,t)}{\partial t} \right)^2 dt \tag{10}$$

In the above integrals, instead of the infinite limits, the limits used in the numerical calculations are substituted.

From the energy balance the energy dissipated E_D by the active force exerted by the damping segment can be obtained:

$$E_D = E_O - \left(E_R + E_P\right) \tag{11}$$

The dissipated energy can be used as an objective function. The aim is to determine the optimal control function $\alpha(x,t)$ (Eq. 5), which maximizes the energy dissipated by the active damping force segment.

4. Numerical Calculations Results

Numerical simulations were performed to demonstrate the effectiveness of the proposed vibration control method via wave cancellation.

The dissipation efficiency η is defined as a ratio of the energy dissipated by the damping segment to the energy of the original wave:

$$\eta = \frac{E_D}{E_O} \tag{12}$$

The calculations were carried out assuming that the function describing the damping coefficient is constant over time and along the entire length of the damping segment: $\alpha(x,t) \equiv \alpha_0 = const$. The aim was to find the optimal value of the relative damping coefficient $\tilde{\alpha}_0 = \alpha_0/\mu$ [1/s] maximizing the dissipation efficiency η .

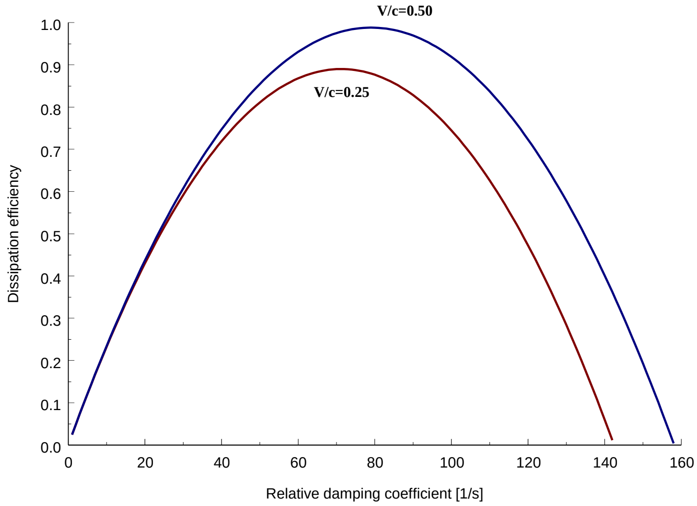

Figure 2. Dissipation efficiency η vs. relative damping coefficient $\tilde{\alpha}_0$: $\lambda/L = 3.70$, $\sigma/L = 3.0$

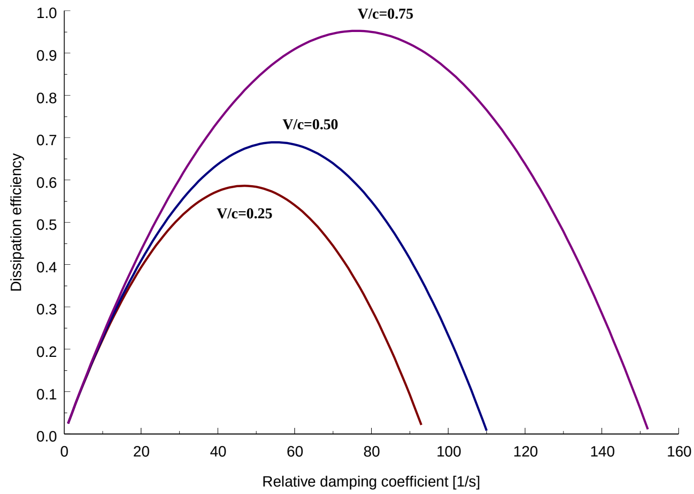

Figure 3. Dissipation efficiency η vs. relative damping coefficient $\tilde{\alpha}_0$: $\lambda/L = 9.22$, $\sigma/L = 3.0$

The graphs in Fig. 2 show the energy dissipation efficiency as a function of the relative damping coefficient, for the assumed wavelength $\lambda = 2\pi/k_0$ and for different velocities of the moving string.

From the plots it is evident that too low damping coefficient value means that the energy of the passed wave is too high, similarly too high damping coefficient value causes that the energy of the reflected wave is too high – both cases resulting in a small amount of the energy dissipated in the damping segment. Similar graphs in the Fig. 3 were made for a larger incident wavelength value.

Finding the maxima of the curves shown in Fig. 2 and Fig. 3, the optimal values of the damping coefficient can be determined. It is visible that the maximum achievable damping efficiency decreases as the wavelength increases, and increases as the velocity of the moving string increases.

For the given values of wave velocity c, velocity of axially moving string V, estimated wavelength λ and assumed damping segment width L, the optimal value of the damping coefficient α_0 can be determined. Given the measured transverse velocity of the string cross-section for the incident wave, this allows the distributed force acting in the segment to be determined as a function of position and time. The transverse string velocity must be measured at such a distance from the damping segment that the motion of the string is not disturbed by the resulting secondary waves.

5. Conclusions

The problem of optimal active distributed damping force necessary to suppress the waves traveling along the axially moving string is considered. The presented vibration suppression algorithm uses the idea of wave cancellation – the active distributed force is the source of the superimposed secondary wave which is to cancel the primary wave.

The numerical results presented demonstrate the effectiveness of the distributed force damper and predominance over the concentrated force damper – a relatively narrow segment, compared to the wavelength, can provide over 90% vibration suppression efficiency.

The paper does not present the technical realization of the damping segment. In practice, a continuous active damping segment could be implemented by a system of discrete forces applied at such a small distance from each other (in relation to the incident wavelength) that they could be treated as a distributed force.

The presented method can also be used for the axially moving finite string, in the damping algorithm it is then necessary to take into account the waves reflected from the borders.

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