Application of critical infrastructure safety modelling in port transport

Krzysztof Kołowrocki[✉], Joanna Soszyńska-Budny

Gdynia Maritime University
81–87 Morska St., 81-225 Gdynia, Poland, e-mails: k.kolowrocki@wn.am.gdynia.pl, joannas@am.gdynia.pl
[✉] corresponding author

Abstract

A new approach is proposed for safety investigations of complex multistate systems. These systems have dependent components, called critical infrastructures, with variable operating conditions. The safety function of the critical infrastructure system is defined and determined for an exemplary "*m* out of *l*" critical infrastructure. In the fully-developed model, it is assumed that system components have multistate exponential safety functions with interdependent departure rates from subsets of safety states. A critical infrastructure safety model is adopted for an oil pipeline transportation system operating in a maritime port.

Introduction

The newest trends in safety investigations of complex technical systems are directed at critical infrastructures. In general, a "critical infrastructure" is a single complex system of large scale, or a network of large, complex systems (a set of hard or soft structures) that function collaboratively and synergistically to ensure to a continuous flow of essential goods and services. These are complex systems with significant inside-system dependencies and outside-system dependencies that, if damaged, can have substantial destructive impacts on the health, safety and security, economics, and social conditions of large communities. These systems are made of a large number of interacting components, and even small perturbations can trigger large consequences that lead to multiple threats to human life and property. Because an extended failure within one of these infrastructures can incapacitate or destroy critical support systems for human life which can cascade across the boundaries of other critical infrastructures thereby leading to multi-infrastructural collapse with devastating, unprecedented and potentially transnational consequences.

Many technical systems belong to the class of complex critical infrastructure systems because of the large number of interacting components and subsystems they incorporate, and because their complicated operating processes have a significant impact on safety. This complexity and the inside- and outside-infrastructure dependencies impel the development of new and comprehensive methods of analyzing, identifying, predicting, improving and optimizing the safety of such systems. Complex critical infrastructure systems are found in pipelines for water, gas, oil and various chemical substances. Such pipelines are frequently encountered in ports in connection with maritime transportation. The analysis of critical infrastructures under variable operating conditions and as affected by interactions among subsystems is extremely complicated. Adding to the difficulties is the need impacts to guard against impacts from sources outside the critical infrastructure and the impacts of natural catastrophes.

From the perspective of ensuring the safety of critical infrastructures, monitoring methods should be based an approach designed to assess function under a variety of different operational states

(Amari & Misra, 1977; Xue, 1985; Kołowrocki, 2004; Kołowrocki & Soszyńska-Budny, 2011). Such an approach makes it possible to distinguish different critical infrastructures operating either inside or outside safety states, such that required levels of system effectiveness can be attained without risk of accidents affecting human populations or the environment.

Most safety analyses assume that the components of a system are independent. However, such an assumption is often erroneous, especially in the case of critical infrastructures, for which the opposite assumption is more often true. The dependence of safety states among the components of a subsystem should be assumed because interactions among the components may cause decreases in the safety states of elements that otherwise would be operating safely (Kołowrocki & Soszyńska-Budny, 2013). In reality, the interactions among subsystems of critical infrastructures can lead to the degradation of the safety state of the entire system.

To integrate the results of investigations of inside- and outside-dependencies, Blokus-Roszkowska (Blokus-Roszkowska, 2007) suggested the use of the semi-Markov models of Kołowrocki and Soszyńska-Budny (Kołowrocki & Soszyńska-Budny, 2011) to describe complex systems operation processes. This incorporation of inside and outside dependencies, including environmental hazards, into a multi-state, analytical model is the core of the methodology of assessing the safety of critical infrastructures (Kołowrocki & Soszyńska-Budny, 2013).

Multistate approach to safety analysis

In a multistate safety analysis of a system composed of n, $n \in N$, ageing components, we make the following assumptions:
- E_i, $i = 1,2,\dots,n$, are the components of the system;
- all components have the set of safety states $\{0,1,\dots,z\}$, $z \geq 1$;
- the safety states are ordered, with state 0 being the worst and state z the best;
- the component and the system safety states degrade over time t;
- $T_i(u)$, $i = 1,2,\dots,n$, $n \in N$, are independent random variables representing the lifetimes of components E_i in the safety state subset $\{u, u+1,\dots,z\}$, while they were in safety state z at moment $t = 0$;
- $T(u)$ is a random variable representing the lifetime of a system in the safety state subset $\{u, u+1,\dots,z\}$, while it was in the safety state z at the moment $t = 0$;

- $s_i(t)$ is a component E_i safety state at the moment t, $t \in \langle 0, \infty \rangle$, given that it was in the safety state z at the moment $t = 0$;
- $s(t)$ is the system safety state at the moment t, $t \in \langle 0, \infty \rangle$, given that it was in the safety state z at the moment $t = 0$.

The above assumptions imply that the safety states of an ageing system and its components can only decline over time.

Definition 1. A vector, defined as follows:

$$S_i(t,\cdot) = [S_i(t,0), S_i(t,1),\dots,S_i(t,z)]$$
$$\text{for } t \in \langle 0,\infty \rangle, \quad i = 1,2,\dots,n \tag{1}$$

where

$$S_i(t,u) = P(s_i(t) \geq u \mid s_i(0) = z) = P(T_i(u) > t) \tag{2}$$

for $t \in \langle 0, \infty \rangle$, $u = 0,1,\dots,z$, is the probability that the component E_i is in the safety state subset $\{u, u+1,\dots,z\}$, at the moment t, $t \in \langle 0, \infty \rangle$, while it was in the safety state z at the moment $t = 0$. This is called the multistate safety function of component E_i.

Definition 2. A vector, defined as:

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,0), \boldsymbol{S}(t,1),\dots,\boldsymbol{S}(t,z)], t \in \langle 0,\infty \rangle \tag{3}$$

where

$$\boldsymbol{S}(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t) \tag{4}$$

for $t \in \langle 0, \infty \rangle$, $u = 0,1,\dots,z$, is the probability that the system is in the safety state subset $\{u, u+1,\dots,z\}$ at the moment t, $t \in \langle 0, \infty \rangle$, while it was in the safety state z at the moment $t = 0$, is called the multi-state safety function of a system.

The safety functions $S_i(t,u)$ and $\boldsymbol{S}(t,u)$, $t \in \langle 0, \infty \rangle$, $u = 0,1,\dots,z$, defined by (2) and (4), are called the coordinates of the components of the system multistate safety functions $S_i(t,\cdot)$ and $\boldsymbol{S}(t,\cdot)$, given by (1) and (3). It is clear from Definition 1 and Definition 2 that, for $u = 0$, we have $S_i(t, 0) = 1$ and $\boldsymbol{S}(t, 0) = 1$.

Definition 3. A probability defined as:

$$r(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \leq t),$$
$$t \in \langle 0,\infty \rangle \tag{5}$$

is the probability that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,\dots,z\}$, while it was in the safety state z at the moment $t = 0$. This is called a "risk function" of the multi-state system (Kołowrocki & Soszyńska-Budny, 2011).

Under this definition, from (4), we have

$$r(t) = 1 - P(s(t) \geq r \mid s(0) = z) = 1 - \boldsymbol{S}(t,r),$$
$$t \in \langle 0,\infty \rangle \tag{6}$$

and if τ is the moment when the system risk exceeds a permitted level δ, then $\tau = r^{-1}(\delta)$, where

$r^{-1}(t)$ is the inverse function of the system risk function $r(t)$.

Safety of an "*m* out of *l*" system with dependent components

One of the basic multistate safety structures with components ageing in time is an "*m* out of *l*" system.

Definition 4. A multi-state system is called an "*m* out of *l*" system if its lifetime, $T(u)$, in the safety state subset $\{u, u+1,…,z\}$ is given by

$$T(u) = T_{(l-m+1)}(u), \ m = 1,2,...,l, \ u = 1,...,z,$$

where $T_{(l-m+1)}(u)$ is the $l-m+1$-th order statistic in the sequence of the component lifetimes $T_1(u)$, $T_2(u),…, T_l(u)$.

Definition 4 means that the multistate "*m* out of *l*" system is in the safety state subset $\{u, u+1,…,z\}$ if and only if at least *m* out of its *l* components are in this safety state subset.

In a multi-state "*m* out of *l*" system with dependent components, we may consider the dependency of the changes of their ageing safety states, and assume that after changing the safety state subset of one of the system components to a degraded safety state subset, the lifetimes of the remaining system components in the safety state subsets would also decrease. More precisely, we assume that if υ, $\upsilon = 0,1,2…,l-1$, components of the system are out outside the safety state subset $\{u, u+1,…,z\}$, then the mean values of the lifetimes $T_i'(u)$ in the safety state subset of the remaining components are given by:

$$E[T_i'(u)] = E[T_i(u)] - \frac{\upsilon}{l} E[T_i(u)] = \frac{l-\upsilon}{l} E[T_i(u)]$$
$$\text{for } i = 1,2,…,l, \ u = 1,2,…,z.$$

Hence, for the case when components have exponential safety functions given by:

$$S_i(t,\cdot) = [1, S_i(t,1),…,S_i(t,z)], \ t \in \langle 0,\infty),$$
$$i = 1,2,…,l, \qquad (7)$$

where

$$S_i(t,u) = \begin{cases} 1, & t < 0 \\ \exp[-\lambda(u)t], & t \geq 0, \lambda(u) \geq 0, i = 1,2,...,l \end{cases}$$
$$(8)$$

with the intensity of departure $\lambda(u)$ from the safety state subset $\{u, u+1,…,z\}$, we get the following formula for the intensities of departure from this safety state subset of the remaining components:

$$\lambda^{(\upsilon)}(u) = \frac{l}{l-\upsilon} \lambda(u)$$
$$\text{for } \upsilon = 0,1,2,...,l-1, \ u = 1,2,...,z \qquad (9)$$

Proposition 1 (Kołowrocki & Soszyńska-Budny, 2013). If, in a homogeneous multi-state "*m* out of *l*" system, the following is true:

(i) the components have exponential safety function given by (7)–(8);
(ii) the components are dependent;
(iii) the intensities of departure from the safety state subsets of the system components are given by (9);

then the multistate system safety function is given by the following formula:

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),…,\boldsymbol{S}(t,z)]$$

where

$$\boldsymbol{S}(t,u) = \sum_{\upsilon=0}^{l-m} \frac{[l\lambda(u)t]^{\upsilon}}{\upsilon!} \exp[-l\lambda(u)t], \ t \geq 0,$$
$$u = 1,…,z \qquad (10)$$

System operation under variable conditions

During its operation, we assume that the system takes v, $v \in N$, different operational states $z_1, z_2,…, z_v$. Further, we define the system operation process $Z(t)$, $t \in \langle 0,+\infty)$, with discrete operation states from the set $\{z_1, z_2,…, z_v\}$. Moreover, we assume that the system operation process $Z(t)$ is a semi-Markov process (Kołowrocki & Soszyńska-Budny, 2011), with conditional sojourn times θ_{bl} at operation states z_b when the next operation state is z_l, $b, l = 1,2,…,v, b \neq l$.

Under these assumptions, the system operation process may be described by parameters and characterized by the limiting values of the system operation process $Z(t)$ transient probabilities at the specific operation states given by Kołowrocki and Soszyńska-Budny (Kołowrocki & Soszyńska-Budny, 2011). These specific operation states are as follows:

$$p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \ b = 1,2,...,v \qquad (11)$$

where M_b, $b = 1,2,…,v$, are defined as in Kołowrocki and Soszyńska-Budny (Kołowrocki & Soszyńska-Budny, 2011), with the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfying the system of equations (2.23) reported in Kołowrocki and Soszyńska-Budny (Kołowrocki & Soszyńska-Budny, 2011).

Safety of multistate system at variable operation conditions

We assume that the changes of the system operation process $Z(t)$ have an influence on the system's multistate components E_i, $i = 1,2,…,n$, on

safety, and on system safety structure as well. We designate conditional lifetimes $T_1^{(b)}(u)$, $T_2^{(b)}(u)$,…, $T_n^{(b)}(u)$ for system components $E_1, E_2,…, E_n$ in the safety states subset $\{u, u+1, z\}$, $u = 1,2,…,z$; and $T^{(b)}(u)$ represents system conditional lifetimes in the safety states subset $\{u, u+1,…, z\}$, $u = 1,2,…,z$, while the system is at the operation state z_b, $b = 1,2,…,v$. Further, we define the conditional safety function of the system multi-state component E_i, $i = 1,2,…,n$, while the system is at the operation state z_b, $b = 1,2,…,v$, by the vector (5.4) (Kołowrocki & Soszyńska-Budny, 2011).

Next we define conditional safety function of the multistate system while the system is at operation state z_b, $b = 1,2,…,v$, by the vector (Kołowrocki & Soszyńska-Budny, 2011):

$$[\boldsymbol{S}(t,\cdot)]^{(b)} = [1, [\boldsymbol{S}(t,1)]^{(b)}, …, [\boldsymbol{S}(t,z)]^{(b)}] \quad (12)$$

where

$$[\boldsymbol{S}(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b)$$
$$\text{for } t \in \langle 0,\infty), \ u = 1,2,…,z, \ b = 1,2,…,v \quad (13)$$

The safety function $[\boldsymbol{S}(t,u)]^{(b)}$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the safety state subset $\{u, u+1,…, z\}$ is greater than t, while the process $Z(t)$ is at operation state z_b. Consequently, we mark by $T(u)$ the system unconditional lifetime in the safety states subset $\{u, u+1,…, z\}$, $u = 1,2,…,z$, and we define the system unconditional safety function by the vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),…, \boldsymbol{S}(t,z)] \quad (14)$$

where

$$\boldsymbol{S}(t,u) = P(T(u) > t) \text{ for } t \in \langle 0,\infty)$$
$$u = 1,2,…,z \quad (15)$$

When the system operation time θ is large enough, the system unconditional safety function coordinates are given by:

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)}$$
$$\text{for } t \geq 0, \ u = 1,2,…,z \quad (16)$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,…,z$, $b = 1,2,…,v$, are the coordinates of the system conditional safety functions defined by (12)–(13) and p_b, $b = 1,2,…,v$, are the system operation process limit transient probabilities given by (11).

Safety of multistate "*m* out of *l*" system with dependent components under variable conditions

Proposition 1, may be generalized in the following way (Kołowrocki & Soszyńska-Budny, 2013).

Proposition 2. If in a homogeneous multi-state an *m* out of *l* system with the shape parameters $m^{(b)}$, $l^{(b)}$ at the operation state z_b, $b = 1,2,…,v$,

(i) the components have at the operation state z_b, $b = 1,2,…,v$, the exponential safety function given by

$$[S_i(t,\cdot)]^{(b)} = [1, [S_i(t,1)]^{(b)}, …, [S_i(t,z)]^{(b)}]$$
$$\text{for } t \in \langle 0,\infty), \ i = 1,2,…,l^{(b)} \quad (17)$$

where

$$[S_i(t,u)]^{(b)} = \begin{cases} 1, & t < 0 \\ \exp[-[\lambda(u)]^{(b)}t], & t \geq 0, [\lambda(u)]^{(b)} \geq 0 \\ i = 1,2,…,l^{(b)} \end{cases}$$

$$(18)$$

with the intensity of departure $[\lambda(u)]^{(b)}$ from the safety state subset $\{u, u+1,…, z\}$;

(ii) the components are dependent in such a way that after the departure from the safety state subset $\{u, u+1,…, z\}$ by υ components of the "*m* out of *l*" system the intensities $[\lambda(u)]^{(b)}$ of departures from this safety states subset of this system remaining components at the operation stare z_b increase according to the formula

$$[\lambda^{(\upsilon)}(u)]^{(b)} = \frac{l^{(b)}}{l^{(b)} - \upsilon}[\lambda(u)]^{(b)}$$
$$\upsilon = 0,1,2,…,l^{(b)} - 1, \ u = 1,2,…,z \quad (19)$$

then the multistate system safety function is given by the formula

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),…,\boldsymbol{S}(t,z)] \quad (20)$$

where

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\sum_{\upsilon=0}^{l^{(b)}-m^{(b)}} \frac{[l^{(b)}\lambda(u)t]^{\upsilon}}{\upsilon!} \exp[-l^{(b)}\lambda(u)t]]$$
$$\text{for } t \geq 0, \ u = 1,…,z \quad (21)$$

Safety of port oil piping transport system

Piping system description

The piping transportation system described here is a Baltic oil terminal designed to receive crude oil from tankers and to ship out finished oil products by rail or truck, or to receive products from land transport and transfer it to tankers. The terminal has three parts *A*, *B* and *C*, linked by pipeline to the pier. The scheme of this terminal is presented in Figure 1.

The port oil pipeline system consists of three subsystems:

− subsystem S_1 is composed of two pipelines, each composed of 178 pipe segments and 2 valves;

- subsystem S_2 is composed of two pipelines, each of which consists of 717 pipe segments and 2 valves;
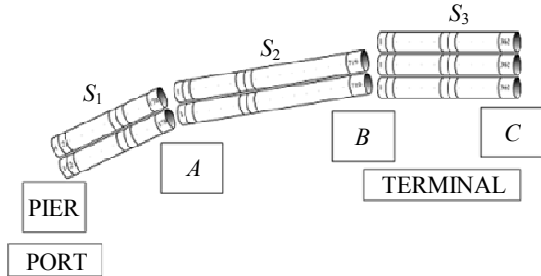- subsystem S_3 is composed of three pipelines, each of which consists of 360 pipe segments and 2 valves.

Figure 1. The scheme of the port oil transportation system

The subsystems S_1, S_2, S_3, in Figure 1 form the general series port safety structure schematically described in Figure 2.
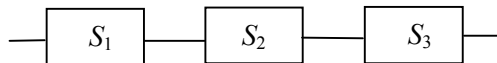

Figure 2. General scheme of the port oil pipeline system safety structure

The system is a series composed of two series-parallel subsystems, S_1 and S_2, each containing two pipelines; and one "2 out of 3" series, subsystem S_3.

Piping system operation process

The subsystems S_1, S_2 and S_3 form the general series port oil pipeline system safety structure presented in Figure 2. However, the structure of the safety systems and its subsystems depends on its changing operation states over time (Kołowrocki & Soszyńska-Budny, 2011).

From expert opinion it has been learned that there are eight operation states for this system (Kołowrocki & Soszyńska-Budny, 2011):

- an operation state z_1 – transport of one kind of medium from Terminal B to Terminal C using two out of three pipelines of the subsystem S_3;
- an operation state z_2 – transport of one kind of medium from the Terminal C to Terminal B using one out of three pipelines of subsystem S_3;
- an operation state z_3 – transport of one kind of medium from the Terminal B through Terminal A to the pier using one of two subsystem S_1 pipelines, and one of two subsystem S_2 pipelines;
- an operation state z_4 – transport of one kind of medium from the pier through Terminals A and B to Terminal C using one of two subsystem S_1 pipelines, one of two subsystem S_2 pipelines, and two of three subsystem S_3 pipelines;

- an operation state z_5 – transport of one kind of medium from the pier through Terminal A to Terminal B using one out of two pipelines in the S_1 subsystem, and one of two pipelines in subsystem S_2;
- an operation state z_6 – transport of one kind of medium from the Terminal B to Terminal C using two of three subsystem S_3 pipelines, while simultaneously transporting one kind of medium from the pier through Terminal A to Terminal B using one of the subsystem S_1 pipelines and one of two subsystem S_2 pipelines;
- an operation state z_7 – transport of one kind of medium from the Terminal B to Terminal C using one of three subsystem S_3 pipelines, and simultaneously transporting a second kind of medium from Terminal C to Terminal B using one of three pipelines of subsystem S_3.

The influence of the system operation states just described in terms of changes of the pipeline system safety structure is as follows.

At system operation states z_1 and z_7, the system consists of subsystem S_3, a "2 out of 3" system containing three series subsystems. At system operation state z_2, the system is consists of the series-parallel subsystem S_3, which contains three pipelines. At the system operation states z_3 and z_5, the system is in series and is composed of two series-parallel subsystems S_1, S_2, each containing two pipelines. At system operation states z_4 and z_6, the system is in series and is composed of two series-parallel subsystems S_1, and S_2, each containing two pipelines and one series-"2 out of 3" subsystem S_3.

To identify the unknown parameters of the port oil piping transportation system operation process, suitable statistical data coming from real realizations should be collected. All operation process parameters are estimated as described in Kołowrocki and Soszyńska-Budny (Kołowrocki & Soszyńska-Budny 2011).

This way, the port oil pipeline transportation system operation process is approximately defined, allowing us to predict it main characteristics. One such characteristic is the unconditional mean sojourn times of the piping system operation process at particular operation states:

$$M_1 \cong 1610.52, \ M_2 \cong 2640, \ M_3 = 575,$$
$$M_4 = 380, \ M_5 \cong 789.35, \ M_6 \cong 475.76,$$
$$M_7 \cong 1497.16 \qquad (22)$$

The limit values of the piping system operation process transient probabilities $p_b(t)$ at the operation states z_b, $b = 1,2,\ldots,7$, are given by:

$$p_1 = 0.395, \; p_2 = 0.060, \; p_3 = 0.003,$$
$$p_4 = 0.002, \; p_5 = 0.200, \; p_6 = 0.058,$$
$$p_7 = 0.282 \qquad (23)$$

Piping system safety

After considering the comments and opinions of experts as well as the effectiveness and safety aspects of the operation of the oil pipeline system, we identified the following three safety states ($z = 2$) of the system and its components:

- a safety state 2 – piping operation is fully safe;
- a safety state 1 – piping operation is less safe and more dangerous because of the possibility of environmental pollution;
- a safety state 0 – piping is destroyed.

Moreover, on the basis of recommendations from experts in the field, we assumed the only safety transitions possible are from better to worse, and that the critical safety state for the system and its components is $r = 1$.

As mentioned, the port oil transportation system safety structure depends on its changes with operation state. The influence of system operation states on the system safety structure and its component safety functions is as follows.

At system operation state z_1, the system is composed of the subsystem S_3 and its three series subsystems ($l^{(1)} = 3$), each composed of 362 components with the exponential safety functions given below. The system is a "2 out of 3" system ($m^{(1)} = 2$) of these subsystems. The subsystem S_3 consists of 3 pipelines and in each pipeline there are:

- 360 pipe segments with conditional three-state safety functions co-ordinates

$$[S^{(3)}(t,1)]^{(1)} = \exp[-0.0059t]$$
$$[S^{(3)}(t,2)]^{(1)} = \exp[-0.0074t]$$

- 2 valves with conditional three-state safety functions co-ordinates

$$[S^{(3)}(t,1)]^{(1)} = \exp[-0.0166t]$$
$$[S^{(3)}(t,2)]^{(1)} = \exp[-0.0181t]$$

Consequently, we determine the three-state safety functions of the system series subsystems/components E_i, $i = 1,2,3$, at the operation state z_1 in the form of the vector

$$[S_i(t,\cdot)]^{(1)} = [1, \, [S_i(t,1)]^{(1)}, [S_i(t,2)]^{(1)}]$$
$$t \geq 0 \qquad (24)$$

for $i = 1,2,3$, with the exponential coordinates

$$[S_i(t,1)]^{(1)} = \exp[-(360 \cdot 0.0059 + 2 \cdot 0.0166)t] =$$
$$= \exp[-2.1572t], \quad i = 1,2,3 \qquad (25)$$

$$[S_i(t,2)]^{(1)} = \exp[-(360 \cdot 0.0074 + 2 \cdot 0.0181)t] =$$
$$= \exp[-2.7002t], \quad i = 1,2,3 \qquad (26)$$

Considering (24)–(26) the subsystems dependence of the form (19) and applying either (10) or (21), we get the piping system conditional safety function at the operation state z_1 of the form:

$$[\boldsymbol{S}(t,\cdot)]^{(1)} = [1, [\boldsymbol{S}(t,1)]^{(1)}, \; [\boldsymbol{S}(t,2)]^{(1)}], \; t \geq 0 \qquad (27)$$

where

$$[\boldsymbol{S}(t,1)]^{(1)} = [\boldsymbol{S}^{(3)}(t,1)]^{(1)} =$$
$$= \sum_{\upsilon=0}^{1} \frac{[3 \cdot 2.1572t]^{\upsilon}}{\upsilon!} \exp[-3 \cdot 2.1572t] = \qquad (28)$$
$$= \exp[-6.4716t] + 6.4716t \exp[-6.4716t]$$

$$[\boldsymbol{S}(t,2)]^{(1)} = [\boldsymbol{S}^{(3)}(t,2)]^{(1)} =$$
$$= \sum_{\upsilon=0}^{1} \frac{[3 \cdot 2.7002t]^{\upsilon}}{\upsilon!} \exp[-3 \cdot 2.7002t] = \qquad (29)$$
$$= \exp[-8.1006t] + 8.1006t \exp[-8.1006t]$$

Proceeding in an analogous way in the remaining operation cases, we get similar results and finally, considering (23), (28)–(29), other similar results, and applying formula (16), we get the following piping system unconditional safety function:

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \, \boldsymbol{S}(t,2)], \; t \geq 0 \qquad (30)$$

where

$$\boldsymbol{S}(t,1) =$$
$$= 0.395[\exp[-6.4716t] + 6.4716t \exp[-6.4716t]] +$$
$$+ 0.060[\exp[-6.4716t] + 6.4716t \exp[-6.4716t] +$$
$$+ \frac{[6.4716t]^2}{2} \exp[-6.4716t]] +$$
$$+ 0.003[\exp[-11.2064t] + 11.2064t \exp[-11.2064t] +$$
$$+ 20.1465t^2 \exp[-11.2064t]] +$$
$$+ 0.002[\exp[-17.6780t] + 17.6780t \exp[-17.6780t] +$$
$$+ 92.6692t^2 \exp[-17.6780t] +$$
$$+ 130.3801t^3 \exp[-17.6780t]] +$$
$$+ 0.200[\exp[-11.2064t] + 11.2064t \exp[-11.2064t] +$$
$$+ 20.1465t^2 \exp[-11.2064t]] +$$
$$+ 0.058[\exp[-17.6780t] + 17.6780t \exp[-17.6780t] +$$
$$+ 92.6692t^2 \exp[-17.6780t] +$$
$$+ 130.3801t^3 \exp[-17.6780t]] +$$
$$+ 0.282[\exp[-6.4716t] + 6.4716t \exp[-6.4716t]]$$

for $t \geq 0$

$$(31)$$

$S(t,2) =$

$= 0.395[\exp[-8.1006t] + 8.1006t \exp[-8.1006t]] +$

$+ 0.060[\exp[-8.1006t] + 8.1006t \exp[-8.1006t] +$

$+ \dfrac{[8.1006t]^2}{2} \exp[-8.1006t]] +$

$+ 0.003[\exp[-15.8620t] + 15.8620t \exp[-15.8620t] +$

$+ 40.2374t^2 \exp[-15.8620t]] +$

$+ 0.002[\exp[-23.9626t] + 23.9626t \exp[-23.9626t] +$

$+ 168.7291t^2 \exp[-23.9626t] +$

$+ 326.4331t^3 \exp[-23.9626t]] +$

$+ 0.200[\exp[-15.8620t] + 15.8620t \exp[-15.8620t] +$

$+ 40.2374t^2 \exp[-15.8620t]] +$

$+ 0.058[\exp[-23.9626t] + 23.9626t \exp[-23.9626t] +$

$+ +168.7291t^2 \exp[-23.9626t] +$

$+ 326.4331t^3 \exp[-23.9626t]] +$

$+ 0.282[\exp[-8.1006t] + 8.1006t \exp[-8.1006t]]$

for $t \geq 0$ (32)

As the critical safety state is $r = 1$, then the system risk function, according to (6), is given by

$$r(t) = 1 - S(t,1) \tag{33}$$

where $S(t,1)$ is given by (31), the moment when the system risk exceeds a permitted level $\delta = 0.05$ is $\tau = r^{-1}(0.05) = 0.049$.
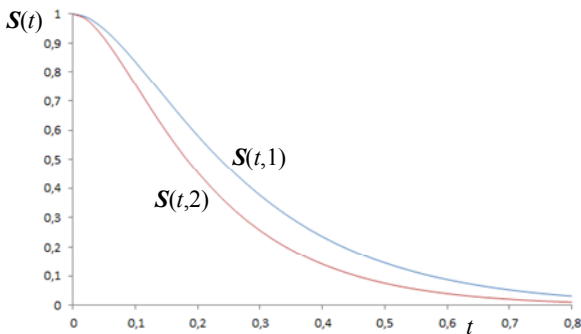

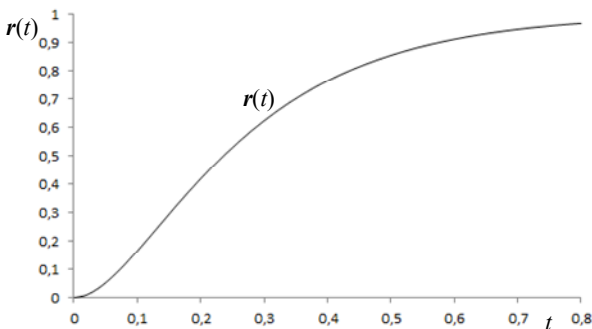**Figure 3.** The graph of the piping system unconditional safety function

Figure 4. The graph of the piping system risk function

Conclusions

This paper describes the results of applying general analytical models to complex, technical multi-state safety systems and their applications to a safety analysis of critical infrastructures (Kołowrocki & Soszyńska-Budny, 2013). The material presented here describes the procedures and algorithms that define the safety characteristics of complex technical systems with dependent components under variable operational conditions. The safety characteristics of the port oil transportation system with dependent components predicted in this paper differ from those described by Kołowrocki & Soszyńska-Budny (Kołowrocki & Soszyńska-Budny, 2011) for this system with independent components. This fact justifies considering the complex technical systems with dependent components at the variable operation conditions that is appearing out in a natural way from practice. This approach, assuming the accuracy of the systems' operation processes and knowledge of their components' safety parameters, increases the precision of predicting their safety characteristics.

Acknowledgments

References

1. AMARI, S.V. & MISRA R.B. (1977) Dynamic reliability analysis of coherent multistate systems. *IEEE Transactions on Reliability*. 46. pp. 460–461.
2. BLOKUS-ROSZKOWSKA, A. (2007) On component failures' dependency influence on system's lifetime. *International Journal of Reliability, Quality and Safety Engineering*. Special Issue: System Reliability and Safety. 14(6). pp. 1–19.
3. KOŁOWROCKI, K. & SOSZYŃSKA-BUDNY, J. (2011) *Reliability and Safety of Complex Technical Systems and Processes: Modeling – Identification – Prediction – Optimization*. Springer.
4. KOŁOWROCKI, K. & SOSZYŃSKA-BUDNY, J. (2013) On safety of critical infrastructures modelling with application to port oil transportation system. *Journal of Polish Safety and Reliability Association*. Summer Safety and Reliability Seminars. 4, 2. pp. 189–204.
5. KOŁOWROCKI, K. (2004) *Reliability of Large Systems*. Elsevier.
6. XUE, J. (1985) On multi-state system analysis. *IEEE Transactions on Reliability*. 34. pp. 329–337.