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PROPERTY IDENTIFICATION OF VISCOELASTIC SOLID MATERIALS IN NOMOGRAMS USING OPTIMIZATION TECHNIQUES

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Viscoelastic materials have been widely used as devices for vibration control in general. Frequently, dynamic properties of those materials are provided by manufacturers only in a graph form in the frequency domain. This is a recurring problem in industry and academia. Thereby, the goal of this work is to contribute to this important issue which is to obtain the properties of viscoelastic materials from nomograms supplied by the manufacturer. The methodology is based on the digitalization of the nomogram of the material and on the subsequent reading of a set of points from two curves in different temperatures. An optimization problem with simple restrictions is built having characteristic constants of the constitutive models as design variables. The problem is solved by applying a hybrid optimization technique. The results obtained are presented, and prove to be very promising.

Keywords: identification, optimization, viscoelastic materials, Zener fractional model, Wiechert model

1. Introduction

Viscoelastic materials (VEMs) have very common applications as structural components or as elements for mitigating vibrations in vehicular structures, aeronautic structures, rotating axes, etc. (Rao, 2002; Ribeiro *et al.*, 2015). Due to the great variety of applications of this type of materials, in the recent decades there has been an increase in the need for more precise models to describe their mechanical behavior. In this sense, a plenty of works have been presented in the recent years, not only in the frequency domain but also in the time domain.

In this context, among the works in the frequency domain, one can mention Park (2001), and Costa and Ribeiro (2011). In these works, the Wiechert generalized model is used to perform an interconversion of the relaxation modulus function (written in terms of Prony series) from the time domain to the frequency domain, obtaining a complex modulus. In this function, the real part represents the equivalent of the stored modulus in mechanical systems, while the imaginary part represents the equivalent of the loss modulus. Thereby, the authors use optimization techniques for fitting theoretical curves of the loss and storage modulus to the experimental curves. Following the same line, starting from a VEM composite with one-degree-of-freedom system, Lopes *et al.* (2004) proposes an identification methodology based on experimentally measured transmissibility curves of constitutive models based on fractional calculus. Agirre and Elejabarrieta (2010) present an inverse identification method using Laplace transforms, in which a four-parameter fractional model is converted from the time domain to the frequency domain. In order to identify properties of a material, an optimization problem is proposed aiming at minimization of the distance between the theoretical and the experimental values of dynamic response in a clamped beam, in a given frequency range. Likewise, Jrad *et al.* (2013) investigate the VEM's behavior in terms of the dynamic stiffness in the frequency domain using the

Wiechert model. The constants related to springs and dampers of the model are identified. More recently, Jalocha *et al.* (2015) propose a method to identify VEM material parameters, in which the relaxation times are optimized based on numerical integration involving the measured relaxation spectra.

From this brief review of literature, one can observe that different approaches to the characterization of VEMs are based on experimental data. However, one frequently faces the situation in which the user of such a material has access only to the nomogram provided by the manufacturer. That nomogram (exemplified in Fig. 1, for material EAR® C1002) provides a graphic representation for the dynamic modulus and loss factor in a reference temperature, and (using the concept of 'reduced frequency') the possibility of obtaining those variables in other temperatures.

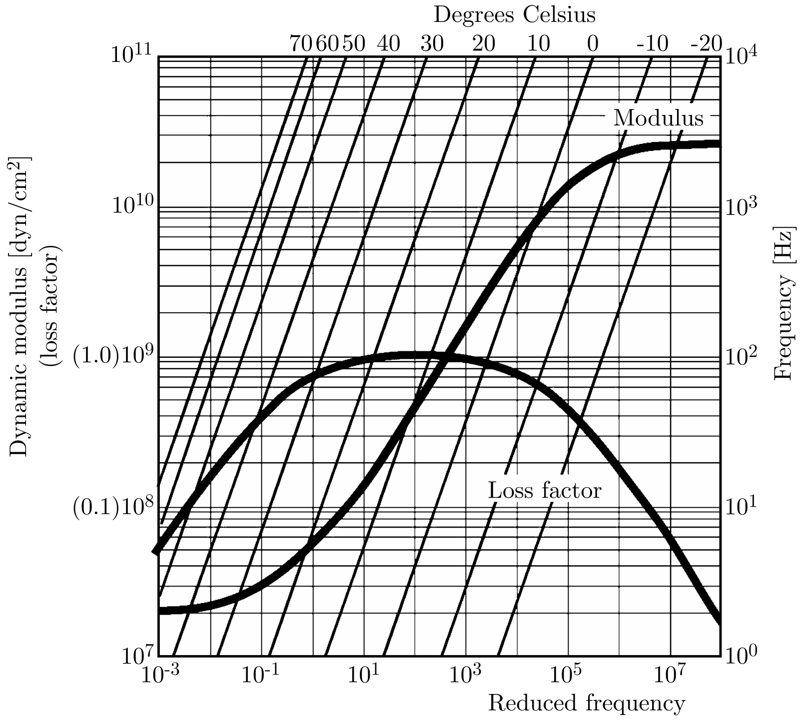

Fig. 1. Material: C1002 – nomogram provided by the EAR® manufacturer

The present work aims at developing and applying a numerical methodology to identify the properties of a linear and thermorheologically simple VEMs based exclusively on nomograms. Such a methodology is based on the digitalization of points of the curves from the nomogram, namely, the dynamic modulus and the loss factor, for different temperatures and frequencies. An objective function to be minimized is defined by taking into account the distance between the theoretical model and the digitized points. The design variables of the optimization process are parameters of the VEM constitutive model. Through mathematical optimization, one minimizes the objective function. However, due to the optimal point being strongly dependent of the initial point in the optimization process, we opt for a hybrid algorithm. In this case, initially a heuristic optimization method, based on the Genetic Algorithm, is used in order to approximate the probable optimum global point ($\mathbf{X}_{GA}$). This point is used as the starting point for an optimization process based on nonlinear programming. This last step aims at generating a better approximation of the global optimum.

2. Models for linear viscoelastic material response

VEMs can be defined as those which present elastic and viscous behavior simultaneously (Brinson and Brinson, 2008; Lemini, 2014). In order to describe their mechanical behavior, various works (Welch *et al.*, 1999; O'Brien *et al.*, 2001; Jrad *et al.*, 2013) make use of rheological models of linear viscoelasticity that consist of combinations of springs and dampers in which the springs represent the elastic part of the behavior and dampers represent its viscous part. Therewith, and associating those elements in series, parallel, series-parallel, etc., various constitutive equations appear, which can be written applying the classical mechanical model or the fractional calculus (Mainardi, 2010). In addition, by interconversion, those models can be defined in time or frequency domains.

2.1. Wiechert constitutive model (classical mechanical model)

Aiming at describing the mechanical behavior of VEMs, Brinson and Brinson (2008) discuss a classical physical model named 'Wiechert model' or 'generalized Maxwell model' (Fig. 2a). The mathematical modeling of this mechanical system in the time domain t, considering the influence of temperature T, creates the relaxation modulus function $E(t,T)$, written in terms of a series of decreasing exponentials, named 'Prony Series', as follows

$$E(t,T) = E_\infty + \sum_{i=1}^{NT} E_i \exp\Big(\frac{-t}{\alpha_T \tau_i}\Big) \tag{2.1}$$

where NT is the total number of terms of that series; E_∞ is a constant named as 'equilibrium modulus', representing the purely elastic response of the material; E_i and τ_i are the elastic constant and the relaxation time, respectively, associated to the i-th component of the Wiechert model (Soussou *et al.*, 1970; Brinson and Brinson, 2008; Suchocki *et al.*, 2013). Additionally, α_T is a constant defined as a shift factor that describes the dependence of the relaxation times in relation to temperature and, in the present work, follows the Williams-Landel-Ferry empirical equation (Williams *et al.*, 1955) given by

$$\log \alpha_T = -\frac{C_1^T (T - T_0)}{C_2^T + (T - T_0)} \tag{2.2}$$

where C_1^T and C_2^T are constants to be determined, which are related to the material properties (Ferry and Stratton, 1960; Ward and Sweeney, 2004; Brinson and Brinson, 2008).

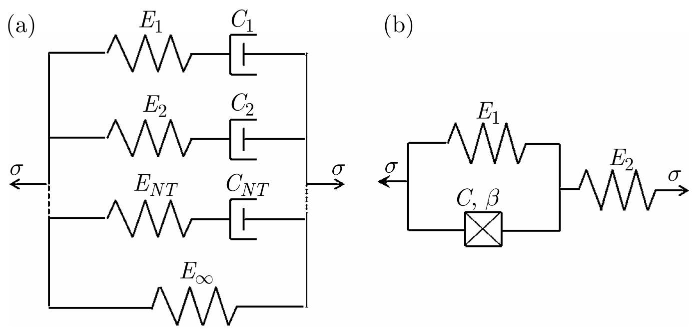

Fig. 2. (a) Wiechert model and (b) fractional Zener model

To analyze the VEMs behavior in the frequency domain, it is common in the literature (Nashif *et al.*, 1985; Honerkamp, 1989; Bavastri, 1997; Park, 2001; Lopes *et al.*, 2004; Jalocha *et al.*, 2015) to rewrite Eq. (2.1) as follows

$$E(\Omega_r) = E_{\mathrm{Re}}(\Omega_r) + \mathrm{i}E_{\mathrm{Im}}(\Omega_r) \tag{2.3}$$

defined as a complex dynamic modulus. The variable Ω_r, defined as the 'reduced frequency', groups temperature effects T and frequency Ω. The reduced frequency can be represented by

$$\Omega_r = \alpha_T \Omega \tag{2.4}$$

It is important to highlight that the real part of Eq. (2.3), defined as the 'storage modulus', is given by

$$E_{\mathrm{Re}}(\Omega_r) = E_\infty + \sum_{i=1}^{NT} \frac{(\Omega_r \tau_i)^2 E_i}{(\Omega_r \tau_i)^2 + 1} \tag{2.5}$$

and the imaginary part, defined as the 'loss modulus', is given by

$$E_{\mathrm{Im}}(\Omega_r) = \sum_{i=1}^{NT} \frac{\Omega_r \tau_i E_i}{(\Omega_r \tau_i)^2 + 1} \tag{2.6}$$

The storage modulus and the loss modulus indicate how much the material behavior approaches the elastic or viscous behavior, respectively. In this case, the loss factor η, is defined as the ratio between the loss and the storage, that is

$$\eta(\Omega_r) = \frac{E_{\mathrm{Im}}(\Omega_r)}{E_{\mathrm{Re}}(\Omega_r)} \tag{2.7}$$

This equation describes the ratio between energies dissipated and stored by the material in a cycle. Consequently, Eq. (2.3) can be written as follows

$$E(\Omega_r) = E_{\mathrm{Re}}(\Omega_r)[1 + \mathrm{i}\eta(\Omega_r)] \tag{2.8}$$

The real storage modulus and the corresponding loss factor are referred to as dynamic properties of the material at issue. Such information is useful, for example, for designing projects in which it is necessary to know the frequencies that results in a greater or lesser energy dissipation.

2.2. The fractional Zener constitutive model

Another mathematical model that describes VEMs behavior is the fractional Zener model (Fig. 2b) whose differential equation of a non-integer order (often named 'fractional order' in the literature) is given by

$$\sigma(t) + \frac{C}{E_1 + E_2} \frac{d^\beta \sigma(t)}{dt^\beta} = \frac{E_1 E_2}{E_1 + E_2} \varepsilon(t) + \frac{E_2 C}{E_1 + E_2} \frac{d^\beta \varepsilon(t)}{dt^\beta} \tag{2.9}$$

where E_1 and E_2 are the stiffness modulus of the elastic elements, C is the viscosity coefficient and β is the non-integer order of differentiation of the Scott-Blair model (Bagley and Torvik, 1986; Mainardi, 2010). Defining the parameters $\overline{E}_0 = E_1 E_2/(E_1 + E_2)$, $r_E = (E_1 + E_2)/E_1$ and $(\tau_a)^\beta = C/(E_1 + E_2)$, it is possible to rewrite Eq. (2.9) as follows

$$\sigma(t) + (\tau_a)^\beta \frac{d^\beta \sigma(t)}{dt^\beta} = \overline{E}_0 \varepsilon(t) + \overline{E}_0 r_E (\tau_a)^\beta \frac{d^\beta \varepsilon(t)}{dt^\beta} \tag{2.10}$$

This mathematical model is known in the literature as a 'four-parameter fractional constitutive model'. Consequently, the relaxation modulus, in the time domain, can be put as follows

$$E(t) = \overline{E}_0 \Big[1 + r_E E_\beta \Big(-\Big(\frac{t}{\tau_a}\Big)^\beta\Big)\Big] \tag{2.11}$$

where $E_\beta(\cdot)$ is the Mittag-Leffler (ML) function of an β order parameter (Mainardi and Spada, 2011).

The ML function, denoted by $E_\beta(z)$ with $\beta > 0$, can be defined by a representation in convergent series in the complex plane $\mathbb{C}$ as follows

$$E_\beta(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(1+\beta n)} \qquad \beta > 0, \quad z \in (C) \tag{2.12}$$

where Γ is the Euler gamma function. For the convergence of the series of potencies in Eq. (2.12), the parameter β can be complex, since $\mathrm{Re}(\beta) > 0$ (Mainardi, 2010).

To obtain the dynamic modulus in the frequency domain, one can apply the Fourier transform to all terms in Eq. (2.10), producing

$$\sigma(\Omega) + (\tau_a)^\beta (\mathrm{i}\Omega)^\beta \sigma(\Omega) = \overline{E}_0 \varepsilon(\Omega) + E_0 r_E (\tau_a)^\beta (\mathrm{i}\Omega)^\beta \varepsilon(\Omega) \tag{2.13}$$

From Eq. (2.13), one can define the complex modulus of the material, $\overline{E}_c(\Omega)$, as follows

$$\overline{E}_c(\Omega) = \frac{\sigma(\Omega)}{\varepsilon(\Omega)} = \frac{\overline{E}_0 + \overline{E}_0 r_E (\mathrm{i}\tau_a \Omega)^\beta}{1 + (\mathrm{i}\tau_a \Omega)^\beta} \tag{2.14}$$

Regarding the influence of temperature, Eq. (2.14) can be rewritten as a function of the reduced frequency (Pritz, 1996; Cruz, 2004; Lopes *et al.*, 2004; Jalocha *et al.*, 2015) according to

$$\overline{E}_c(\Omega_r) = \frac{\overline{E}_0 + \overline{E}_\infty b_1 (\mathrm{i}\Omega_r)^\beta}{1 + b_1 (\mathrm{i}\Omega_r)^\beta} \tag{2.15}$$

where $\overline{E}_\infty = \overline{E}_0 r_E$ and $b_1 = (\tau_a)^\beta$. Equation (2.15) is the classical model of a four-parameter fractional derivative in the frequency domain, and can be rewritten as follows

$$\overline{E}_c(\Omega_r) = E_{\mathrm{Re}}(\Omega_r) + \mathrm{i}E_{\mathrm{Im}}(\Omega_r) = E_{\mathrm{Re}}(\Omega_r)[1 + \mathrm{i}\eta(\Omega_r)] \tag{2.16}$$

In this case, the storage modulus can be expressed as

$$E_{\mathrm{Re}}(\Omega_r) = \frac{\overline{E}_0 + (\overline{E}_0 + \overline{E}_\infty) b_1 (\Omega_r)^\beta \cos\Big(\frac{\beta\pi}{2}\Big) + \overline{E}_\infty b_1^2 (\Omega_r)^{2\beta}}{1 + 2b_1 (\Omega_r)^\beta \cos\Big(\frac{\beta\pi}{2}\Big) + b_1^2 (\Omega_r)^{2\beta}} \tag{2.17}$$

and the loss factor as

$$\eta(\Omega_r) = \frac{(\overline{E}_\infty - \overline{E}_0) b_1 (\Omega_r)^\beta \sin\Big(\frac{\beta\pi}{2}\Big)}{\overline{E}_0 + (\overline{E}_0 + \overline{E}_\infty) b_1 (\Omega_r)^\beta \cos\Big(\frac{\beta\pi}{2}\Big) + \overline{E}_\infty b_1^2 (\Omega_r)^{2\beta}} \tag{2.18}$$

3. Methodology

Starting from the nomogram of a given material, the methodology is based on the digitalization of that image and in the subsequent reading of a set of points on the two characteristic curves of the material (storage modulus and loss factor) in different frequencies and temperatures. Next, the parameters of the theoretical model are identified through minimization, using a hybrid optimization technique, of the relative squared distance between that model and the curves obtained by digitalization. The standard optimization problem is solved based on the corresponding differences between the curves of the storage dynamic modulus R_{qk}^{Re} and the loss factor R_{qk}^{η} in the k-th point $1 \leqslant k \leqslant N_{pt}$ (N_{pt} is the total number of points of each analyzed curve) and in the q-th temperature $1 \leqslant q \leqslant N_{temp}$ (N_{temp} is the total number of evaluated curves in different temperatures). That is

$$R_{qk}^{\mathrm{Re}} = \frac{E_{\mathrm{Re}}^{Exp}(\Omega_k, T_q) - E_{\mathrm{Re}}(\Omega_k, T_q)}{E_{\mathrm{Re}}^{Exp}(\Omega_k, T_q)} \qquad R_{qk}^{\eta} = \frac{\eta^{Exp}(\Omega_k, T_q) - \eta(\Omega_k, T_q)}{\eta^{Exp}(\Omega_k, T_q)} \tag{3.1}$$

This way, the total relative squared distance R_{T2} is given by

$$R_{T2} = \frac{1}{N_{totalPts}} \sum_{q=1}^{N_{temp}} \sum_{k=1}^{N_{pt}} \Big((R_{qk}^{\mathrm{Re}})^2 + (R_{qk}^{\eta})^2 \Big) \tag{3.2}$$

where $N_{totalPts}$ is the sum of the total number of all curves. In this case, the goal is to minimize the total relative squared distance. Thus, the standard optimization problem for the Wiechert model is defined as

$$\begin{aligned} &\text{minimize} \quad R_{T2}(\mathbf{x}) : R^{NT+3} \to R \\ &\text{where} \quad \mathbf{x} = (E_\infty, E_i, C_1^T, C_2^T), \quad i = 1, 2, \ldots, NT \\ &\text{with restrictions:} \quad \mathbf{x}^{low} \leqslant \mathbf{x} \leqslant \mathbf{x}^{upp} \end{aligned} \tag{3.3}$$

where the superscripts "low" and "upp" indicate the upper and lower limit values of each design variable.

On the other hand, in the optimization procedure considering the four-parameter fractional Zener model, the optimization problem is written as follows

$$\begin{aligned} &\text{minimize} \quad R_{T2}(\mathbf{x}) : R^6 \to R \\ &\text{where} \quad \mathbf{x} = (\overline{E}_\infty, \overline{E}_0, b_1, \beta, C_1^T, C_2^T) \\ &\text{with restrictions:} \quad \mathbf{x}^{low} \leqslant \mathbf{x} \leqslant \mathbf{x}^{upp} \end{aligned} \tag{3.4}$$

The solution to those problems (Eq. (3.3) or (3.4)) provides the characteristic parameters of VEM under study according to the selected constitutive model.

3.1. Computational structure

The computational implementation of the proposed methodology has been carried out in a MATLAB® environment, as seen in the flowchart presented in Fig. 3.

The characterization procedure is based on a hybrid optimization technique. In that technique, an approximation of the optimal material parameters is initially obtained using Genetic Algorithms (GAs). Next, this result is improved by using a deterministic algorithm of Non-Linear Programming (NLP). In the process of optimization by GA, a sub-routine (ga.m) is used with a population of 5 000 individuals, 500 generations and a 1.0% mutation rate. Moreover, in NLP, a sub-routine (fmincon.m) is used with a maximum number of iterations of 1 000, 1E-11 tolerance and a maximum number of evaluation of the objective function of 10 000.

For the optimization procedure, the simple limits for parameters related to the WLF model, equilibrium modulus, terms of the Prony series and the fractional Zener model are listed in Table 1. Those limits are based on numerical experiments so that the upper and/or lower limits should not be obtained as optimal points.

Regarding the Wiechert model, the relaxation times τ_i, are fixed and defined dividing the interval between the minimum (τ_1) and the maximum (τ_{NT}) relaxation time into equal intervals on a logarithmic scale. This procedure is common in the literature (Honerkamp, 1989; Chen *et al.*, 2000; Soussou *et al.*, 1970; Jalocha *et al.*, 2015). Each material has specific properties and, thus, different time limits of relaxation. In this sense, Table 2 presents different intervals for each material.

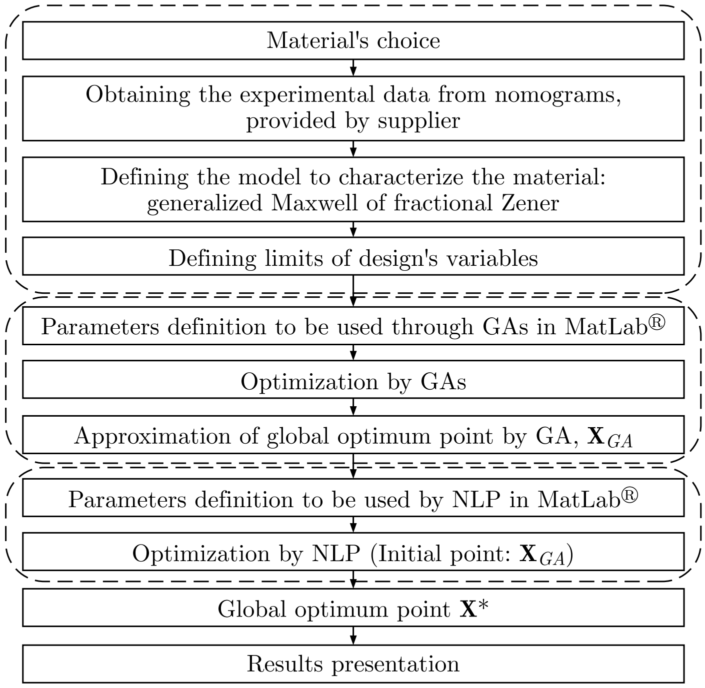

Fig. 3. Computational flowchart implemented in MatLab® environment

Table 1. Interval limits of the material properties used in the optimization process

Model	Variable	Nomenclature	Interval limits
WLF	WLF 1 material constant	C_1^T	$0 \leqslant C_1^T \leqslant 1000$
	WLF 2 material constant	C_2^T [°C]	$0 \leqslant C_2^T \leqslant 2000$
Wiechert	equilibrium modulus	E_∞ [MPa]	$0 \leqslant E_\infty \leqslant 10000$
	Prony series constants	E_i $(i = 1, \ldots, NT)$ [MPa]	$0 \leqslant E_i \leqslant 5000$
Zener fractional	equilibrium modulus	$\overline{E}_\infty$ [MPa]	$10^2 \leqslant \overline{E}_\infty \leqslant 10^5$
	instantaneous modulus	$\overline{E}_0$ [MPa]	$10 \leqslant \overline{E}_0 \leqslant 10^2$
	fractional derivative parameter	b_1	$10^{-5} \leqslant b_1 \leqslant 10^{-2}$
	fractional derivative order	β	$0 < \beta < 1$

Table 2. Relaxation time intervals for each material

Material	Minimum time τ_1	Maximum time τ_{NT}
C1002	10^{-8}	10^3
C2003	10^{-9}	10^3
ISODAMP	10^{-9}	10^3

4. Results

4.1. C1002 – material identification

The proposed methodology is applied to the identification of the EAR® C1002 material using Wiechert constitutive models with 8, 16, 32 Prony terms and the fractional Zener model. One can observe – according to Fig. 4 – that the more Prony terms, the better adjustments are obtained, so much so that above 16 terms there are no significant improvements, evidencing that from that quantity of terms on, it is already possible to describe the dynamic characteristics of the material under study with desired precision. On the other hand, the theoretical curves of the fractional model adjust almost perfectly to the data, evidencing a good identification of that material model. The results can be seen in Fig. 4d.

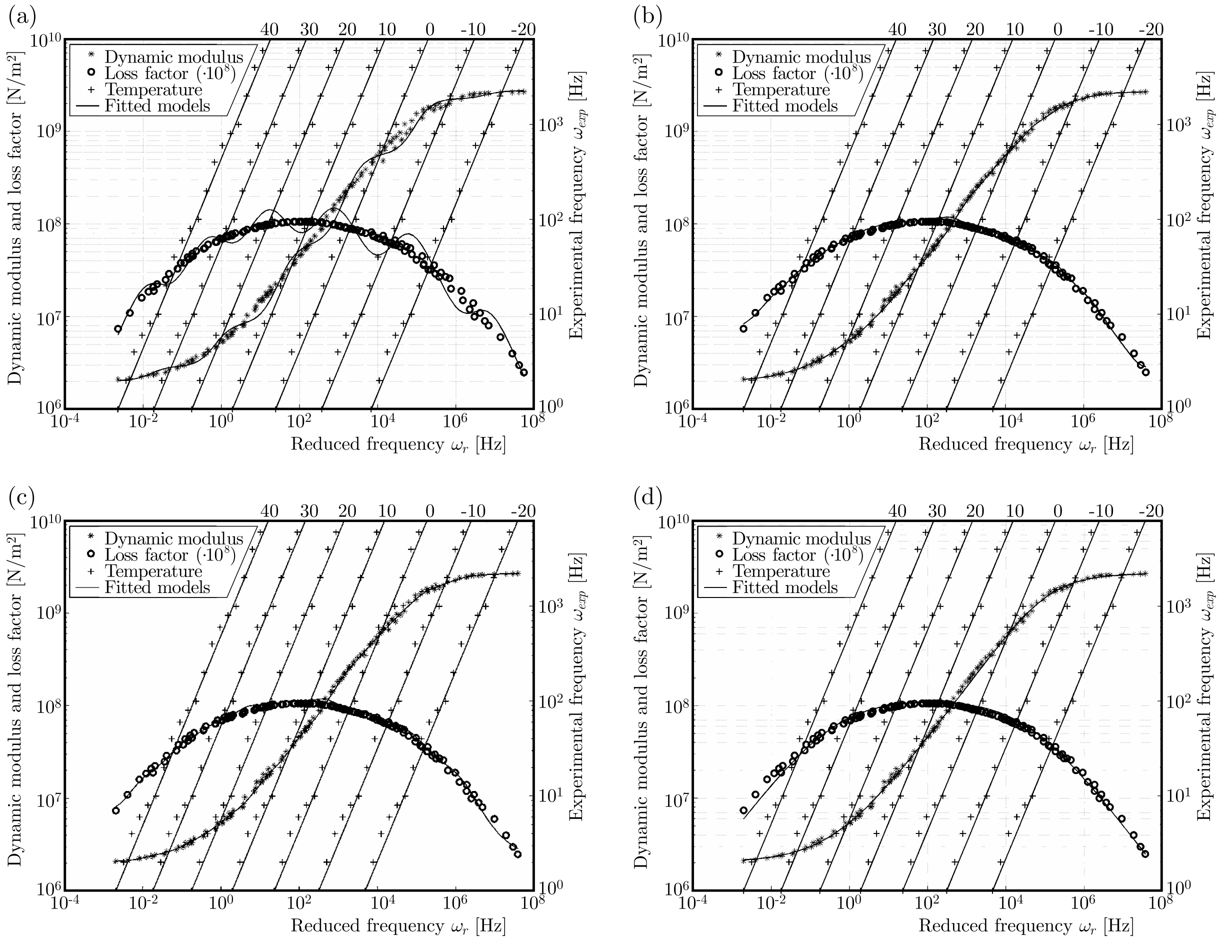

Fig. 4. C1002 identification: (a) 8 Prony terms, (b) 16 Prony terms, (c) 32 Prony terms, (d) fractional Zener model

The parameters obtained for each identification are presented in Table 3 and 4. Analyzing the optimal numerical values of constants C_1^T, C_2^T and E_0, one observes that the results – for the different analyses – are close, evidencing an adequate computational implementation.

Table 3. C1002 parameters obtained by hybrid optimization using the Wiechert model

Number of Prony terms	C_1^T [°C]	C_2^T [°C]	$E_0 = \sum_{i=1}^{NT} E_i$ [MPa]	E_∞ [MPa]	NLP error
8	33.40	317.23	2.54E+3	1.96	5.71E-2
16	35.27	340.77	2.67E+3	1.95	9.30E-3
32	34.75	336.15	2.69E+3	1.95	8.50E-3

Table 4. C1002 parameters using the fractional Zener model

Constants	C_1^T [°C]	C_2^T [°C]	$\overline{E}_\infty$ [MPa]	$\overline{E}_0$ [MPa]	b_1 [s$^\beta$]	β	τ_a [s]	NLP error
Numerical values	34.577	333.037	2.706E+3	1.927	2.089E-3	0.538	1.048E-5	1.225E-2

Furthermore, one must notice that the constant $\overline{E}_\infty$ of the fractional model corresponds to E_0 of the Wiechert model, and $\overline{E}_0$ corresponds to E_∞. Comparing the corresponding values, one notices a good correlation reaffirming that both models characterize precisely the mechanical behavior of that material.

4.2. C2003 – material identification

From the proposed methodology, similar identifications to those carried out with C1002 have been performed, and the graphic results are presented in Fig. 5. The numerical values that characterize the dynamics behavior of that material are shown in Table 5 and 6.

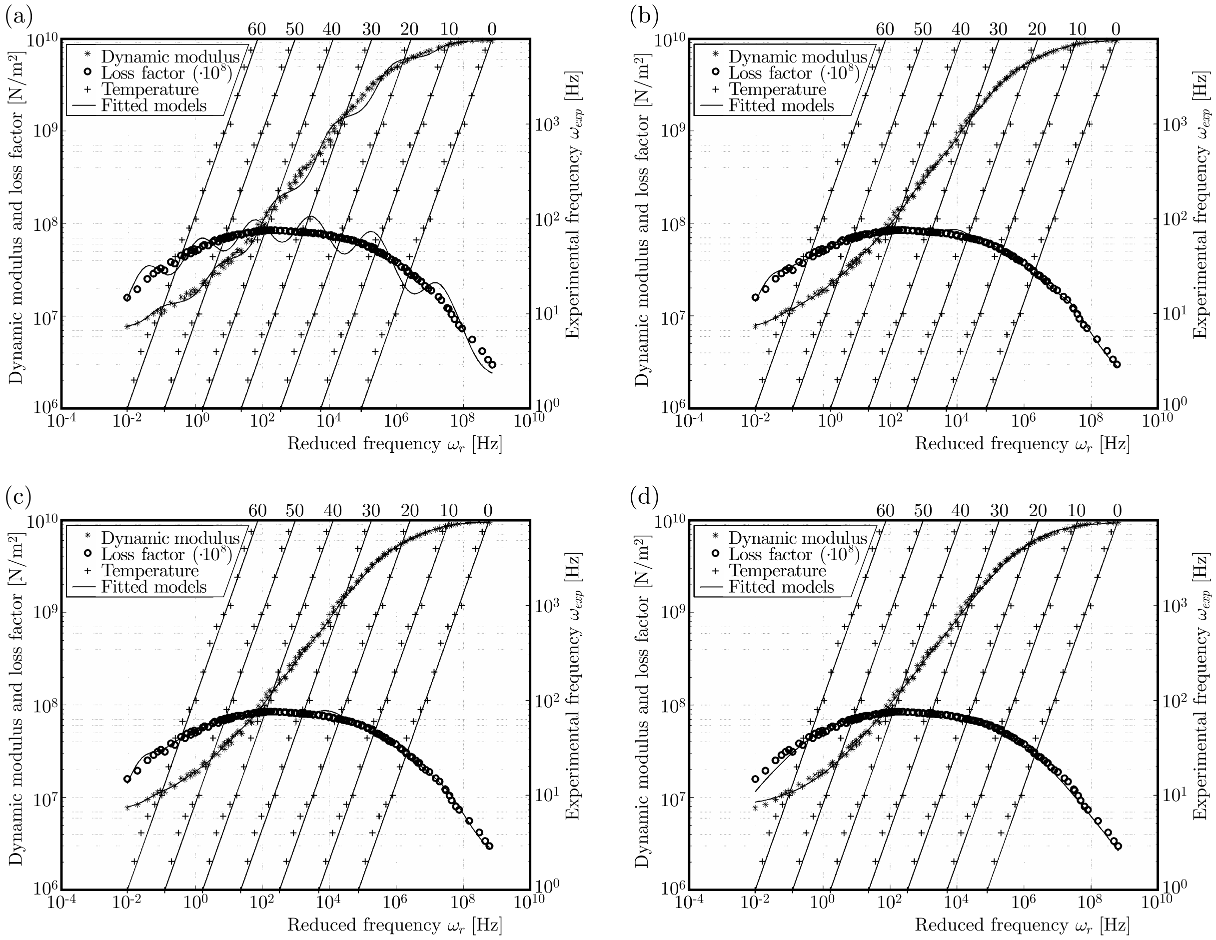

Fig. 5. C2003 identification: (a) 8 Prony terms, (b) 16 Prony terms, (c) 32 Prony terms, (d) fractional Zener model

Table 5. Parameters of C2003 material obtained by hybrid optimization using the Wiechert model

Number of Prony terms	C_1^T [°C]	C_2^T [°C]	$E_0 = \sum_{i=1}^{NT} E_i$ [MPa]	E_∞ [MPa]	NLP error
8	137.16	1165.16	9.30E+3	7.28	7.25E-2
16	178.62	1564.92	9.71E+3	7.17	4.70E-3
32	186.17	1632.68	9.70E+3	7.17	4.20E-3

Table 6. Parameters of material C2003 using the fractional Zener model

Constants	C_1^T [°C]	C_2^T [°C]	$\overline{E}_\infty$ [MPa]	$\overline{E}_0$ [MPa]	b_1 [s$^\beta$]	β	τ_a [s]	NLP error
Numerical values	146.012	1281.529	9.863E+3	6.989	1.593E-3	0.466	9.913E-7	6.54E-3

4.3. Identification of ISODAMP material

The characterization of the EAR® ISODAMP material is performed in a similar way the previous cases. The graphic results are illustrated in Fig. 6. The numerical values of each identification procedure are presented in Table 7 and 8.

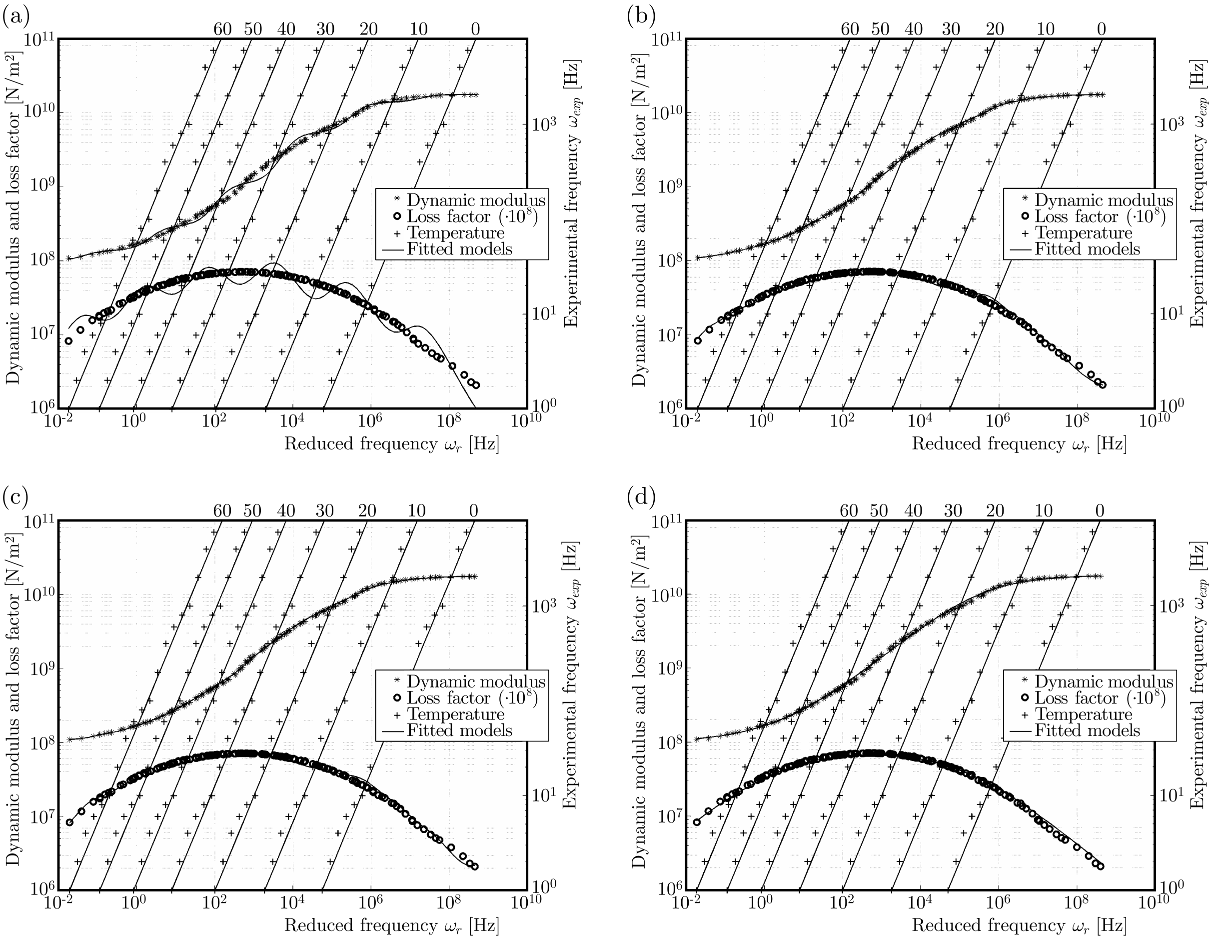

Fig. 6. ISODAMP identification: (a) 8 Prony terms, (b) 16 Prony terms, (c) 32 Prony terms, (d) fractional Zener model

Table 7. ISODAMP parameters obtained by hybrid optimization using the Wiechert model

Number of Prony terms	C_1^T [°C]	C_2^T [°C]	$E_0 = \sum_{i=1}^{NT} E_i$ [MPa]	E_∞ [MPa]	NLP error
8	14.34	153.35	1.68E+4	9.70E+1	6.63E-2
16	14.58	160.42	1.74E+4	9.72E+1	2.90E-3
32	14.56	160.13	1.75E+4	1.01E+2	2.60E-3

Table 8. ISODAMP parameters using the fractional Zener model

Constants	C_1^T [°C]	C_2^T [°C]	$\overline{E}_\infty$ [MPa]	$\overline{E}_0$ [MPa]	b_1 [s$^\beta$]	β	τ_a [s]	NLP error
Numerical values	14.785	162.518	1.757E+4	9.787E+1	4.704E-3	0.443	5.623E-6	3.52E-3

4.4. Relaxation modulus in the time domain

Previously, the parameters characterizing the mechanical behavior of EAR® C1002, C2003 and ISODAMP materials have been obtained. Replacing such parameters in the relaxation modulus, Eq. (2.1), it is possible having a graphic visualization, in the time domain, as illustrated in Fig. 7.

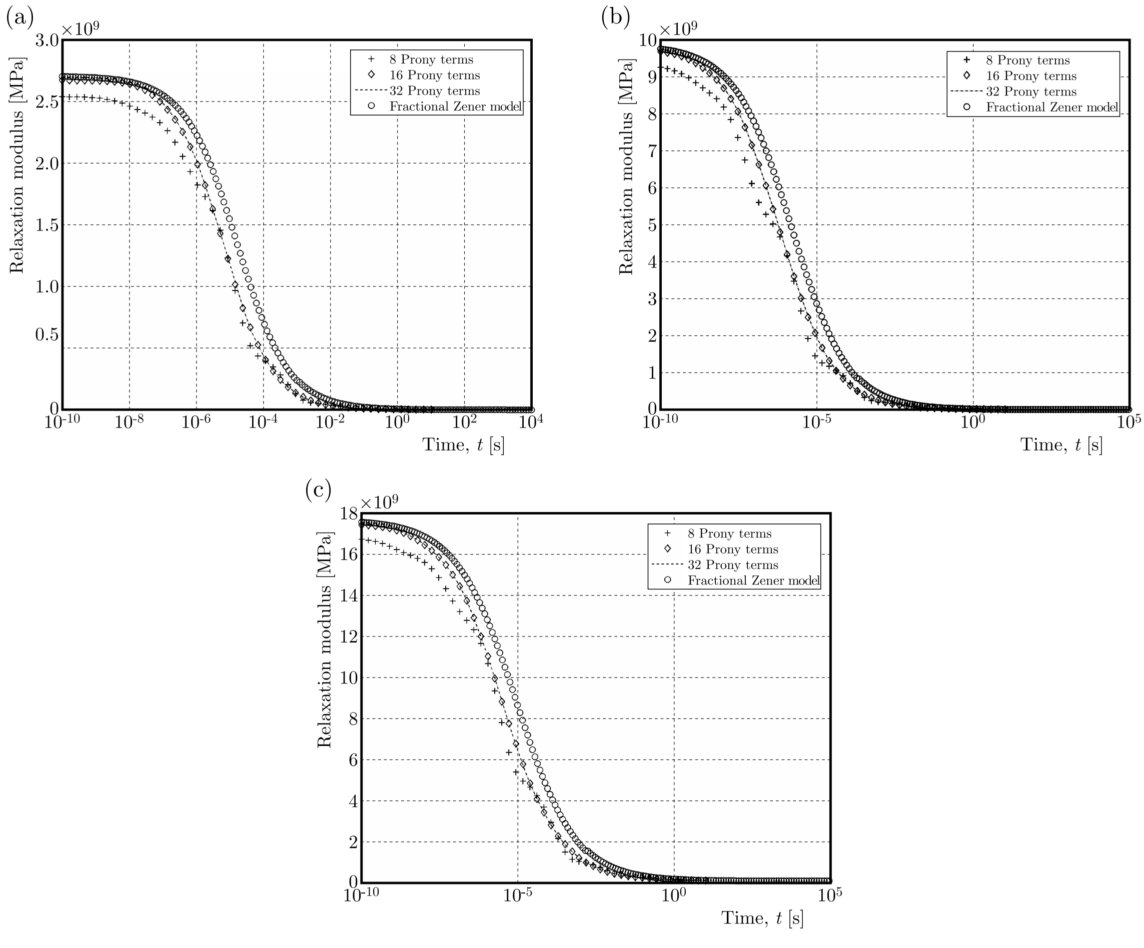

Fig. 7. Relaxation modulus by Wiechert model with increasing Prony terms and using fractional Zener model: (a) C1002, (b) C2003, (c) ISODAMP

5. Final remarks

The main goal of the present work is to obtain material parameters of constitutive models that seek to describe the dynamic modulus and the loss factor. Therefore, one starts from nomograms of EAR® C1002, C2003 and ISODAMP materials provided by their own manufacturer. Using a hybrid optimization technique, identifications are carried out involving the Wiechert model and the fractional Zener derivative model. Regarding the first model, various identifications are performed using increasing Prony terms. One can observe that the greater is the quantity of Prony terms, the better adjustments are obtained, so that for designs involving those VEMs under study, 16 Prony terms proved to be enough for their characterization. Regarding identification involving the fractional Zener model, more accurate adjustments are obtained not only for the dynamic modulus but also for the loss factor. Besides that, for having less parameters, the fractional Zener model leads to a shorter computational time of the optimization process.

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