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SIMILARITY-BASED FAILURE THRESHOLD DETERMINATION FOR SYSTEM RESIDUAL LIFE PREDICTION

OKREŚLANIE PROGU AWARII NA PODSTAWIE PODOBIEŃSTWA JAKO METODA POZWALAJĄCA NA PRZEWIDYWANIE TRWAŁOŚCI RESZTKOWEJ SYSTEMU

An accurate determination of the system failure threshold is an essential requirement in achieving an appropriate system residual life prediction and a reasonable planned maintenance strategy optimization afterward for degradation systems. This paper proposes a failure threshold determination method based on quantitative measurement of the similarity between the operating system and the historical systems. The similarity is formulated by a weighted average function and then calculated by a convex quadratic formulation to minimizing the variance between the operating system and the historical systems. With an accurate determination of the system failure threshold in real-time, a better prediction of the residual life for the operating system is achieved. Finally, a real case study for several power-shift steering transmission systems monitored using oil spectral analysis is adopted to illustrate and numerically compare the improved performance of the proposed method.

Keywords: *system failure threshold; residual life; similarity; prognostics; oil field data.*

W przypadku systemów podlegających degradacji, dokładne określenie progu awarii systemu stanowi niezbędny warunek dokonania trafnej prognozy jego trwałości resztkowej oraz późniejszej optymalizacji strategii konserwacji rutynowych. W artykule zaproponowano metodę wyznaczania progu awarii opartą na ilościowym pomiarze podobieństwa między systemem użytkowanym obecnie a systemami użytkowanymi uprzednio. Podobieństwo formułuje się na podstawie funkcji średniej ważonej, a następnie oblicza na podstawie wypukłej formy kwadratowej w celu zminimalizowania wariancji między obecnie użytkowanym systemem a uprzednimi systemami. Dzięki dokładnemu określeniu progu awarii systemu w czasie rzeczywistym uzyskuje się lepszą prognostykę trwałości resztkowej obecnie użytkowanego systemu. W końcowej części pracy, w celu zilustrowania i numerycznego porównania ulepszonej wydajności proponowanej metody, zaprezentowano studium przypadku obejmujące kilka układów przeniesienia napędu monitorowanych przy użyciu analizy spektralnej oleju.

Słowa kluczowe: *próg awarii systemu; trwałość resztkowa; podobieństwo; prognostyka; dane z badań oleju.*

Abbreviations and acronyms

PSST	Power-shift steering transmission
PHM	Prognostics and health management
RL	Residual life
CDF	Cumulative distribution function
Mh	Motorhour

Nomenclature

$L_{j,t}$	Degradation data for system i at time t
φ_i	Random effects of the degradation model for system i
η	Functional form of the degradation model
$\varepsilon_{i,t}$	Random noise for system i at time t
$\omega_{i,j}$	Weight parameter corresponding to the historical system i and the operating system j
$\tilde{L}_{j,t}$	Reconstructed degradation model for system j at time t using the weighted average method
$\boldsymbol{L}_{j,\cdot}$	Collected real-time degradation data until time t
$\hat{\varphi}_i$	Estimated model parameters of historical systems
$\hat{L}_{j,t}$	Reconstructed degradation data for system j at time t using the Bayesian updating method
$\boldsymbol{\theta}_j^{(1)}$	Updated random effects of the degradation model for system j
D_j	Failure threshold for system j
L_{i,n_i}	Degradation data for system i at time n_i
n_j	Number of data collections of operating system j
λ	Tuning coefficient

$R(\cdot)$	Regularization function
$\boldsymbol{X}_t$	Time-dependent covariates at time t
p	Order of the polynomial model
$\boldsymbol{\mu}_j^0, \boldsymbol{\mu}_j^1$	Prior and posterior mean of random effects $\boldsymbol{\theta}_j$
Σ_j^0, Σ_j^1	Prior and posterior variance of random effects $\boldsymbol{\theta}_j$
ψ_j	Design matrix for operating system j
u_j^d	Mean of failure threshold for system j
v_j^d	Variance of failure threshold for system j
T_j	Actual RL for testing PSST system j
$\tilde{T}_j$	Estimated RL for testing PSST system j
$\Phi(\cdot)$	CDF of the standard normal function
RL_e	Mean prediction error of system RL
FT_e	Mean prediction error of system failure threshold
N	Number of testing PSST systems
D_j^{True}	Actual failure threshold for PSST system j

1. Introduction

Degradation-induced failure is an inevitable and natural phenomenon for various industrial devices and systems. After a certain extent of degradation, a system will run to failure such that it no longer functions, which may result in production downtime, severe economic loss, and safety problems afterward. For example, for power-shift steering transmission (PSST) systems are the most vital components in military vehicles, mining machines, and heavy industry, and they always degrade to failure frequently than other components [14, 15]. Their failures are critically hazardous and often lead to catastrophic consequences, and therefore, should be avoided. One useful approach to avoid unexpected failures is to conducting prognostics and health management (PHM), in which machine condition monitoring plays a foundational role and has aroused intensive concerns in academia and industry [27, 31]. With the collected degradation data (e.g., vibration signals, temperature information, and oil field data) from condition monitoring in real-time, the residual life (RL) of an operating system can then be estimated by conducting prognostic analysis.

In the area of PHM, it is commonly assumed that a system failure will occur once its degradation data cross the predetermined failure threshold [13, 17, 24]. Therefore, the RL can be estimated by comparing the degradation profile with the failure threshold. Accordingly, to accurately evaluate the RL distribution, two challenges must be addressed [21]: (1) A reasonable degradation model that can characterize the degradation profile of an operating system, so that the degradation mechanism can be accurately captured. (2) A reliable failure threshold that can reflect the failure mechanism of the system, so that the moment when the system no longer fulfills its functions can be accurately predicted. For many years, while numerous researches have been carried out focusing on the challenge (1) [2, 12, 26, 36], the existing research still insufficient to address the challenge (2), i.e., find a methodology that can precisely determine the failure threshold of the system. Therefore, the purpose of this paper is to address a failure threshold determination problem for the system with collected degradation data for the estimation of the system residual technical life.

Extensive work has been done in the application of different methods and techniques in degradation modeling, from which most of the current literature assumes that the collected degradation information can precisely characterize the underlying degradation mechanism of the system [1, 8]. In most cases, it is simply assumed that the failure threshold is known a priori, and, as a result, the failure threshold is always used as a fixed value for all systems. In practice, however, some recent research has shown that it may be irrational to use a fixed failure threshold [3, 4, 9]. For instance, as the work was done in [18, 22], the random failure threshold assumption has been adopted. Recently, the work in [29] presents a system failure threshold determination method based on the statistic characteristics of the last degradation data collections from multiple historical systems, and the uncertainty in the failure threshold distribution is also considered. However, the estimated failure threshold is obtained using an average of historical systems, which may not adequately consider the unique characteristics of each system. It is known that the degradation process of a system is stochastic, and is under the influence of many known and unknown factors. Therefore, the last collected degradation data always quite different, as shown in many research and applications [33, 37]. Besides, for some systems that are governed by multiple failure modes [19, 28], the failure threshold often different for each failure mode. As a result, the population-wide characteristic-based failure threshold determination method in [29] may be unable to fully consider the unique properties of an individual system. Therefore, a reliable failure threshold determination method that can extract the unique property in each individual system should be developed for the RL distribution estimation of a system when the failure threshold is not predetermined as a priori.

Motivated by the above observations, this paper proposes a failure threshold determination method based on quantitative measurement of the similarity between the operating system and historical systems. The similarity is formulated by using a weighted average function, and a convex quadratic formulation is then developed to minimizing the variance between the operating system and the historical systems. Unlike the existing PHM researches that assume the failure threshold as a deterministic value, in the proposed method, a random failure threshold is considered for different systems. Based on the proposed method, the failure threshold can be determined for an operating system with the collected real-time degradation information. This is of practical significance to attaining a more reasonable and accurate RL distribution estimation and, thus, is the main contribution of this paper. Finally, to illustrate the proposed method, a case study is provided for power-shift steering transmission systems.

The rest of this paper is organized as follows. Section II describes the details of the proposed method, which includes problem motivation, problem formulation, and the following residual life prediction procedures. Section III provides an illustrative real case study that involves a spectral oil data set from several power-shift steering transmission systems to show the effectiveness of the proposed method and its improved performance when used for system failure threshold determination and residual life prediction. Finally, Section IV draws the conclusion of this work and provides some future research directions.

2. Development of the methodology

This paper considers systems that degrade over time in working conditions, and condition monitoring techniques are conducted to collect the degradation data during the whole lifecycle (from an initial state to failure). Once the degradation data is collected, an associated degradation process $\{L(t),\ t \geq 0\}$ can be periodically analyzed to evaluate the degradation severity of the system. In engineering practice, many research efforts have been made on modeling the evaluation of the system degradation process and its relationship with the collected degradation data. For example, a polynomial degradation model introduced by Chinnam in [5], which has been commonly used in many applications [16, 23, 30, 32], can be written as follows:

$$L_{i,t} = \eta(\varphi_i, t) + \varepsilon_{i,t} \tag{1}$$

where $L_{i,t}$ represents the measurement of degradation data for system i at time t, η represents the functional form of the introduced degradation model, $\varphi_i \in R^{p\times 1}$ represents the random effects of the degradation model for system i, and $\varepsilon_{i,t}$ is the random noise for system i at time t. With the introduced degradation model in Eq. (1), the degradation model of a system can be constructed based on the collected degradation data from condition monitoring during the lifecycle.

In the literature of PHM, the system degradation profiles (e.g., the failure modes, the failure threshold, and the constructed degradation model) of the same series of systems under the same working conditions (e.g., cycle condition, environment condition) are commonly assumed to be the same under some random variations [34, 35]. Under this assumption, our innovative idea is to reconstruct the operating system's degradation profile by the weighted average of the historical system degradation profiles. Specifically, the weight parameter measures the relative similarity between the degradation profiles of operating system j and historical system i compared to the other historical systems. Note that if the weight parameter $\omega_{i,j}$ is obtained, the failure threshold of the operating system j can then be determined by the weighted sample moments of historical systems. By doing this, the system failure threshold can be online estimated by effectively using the condition monitoring data from a group of historical systems.

2.1. Description of the problem

According to the above mentioned innovative idea, a general description of the concerned system failure threshold determination problem is first provided in this section. Recall that the degradation model of an operating system can be reconstructed using the weighted average of the established historical system degradation models, and, if the weight parameter $\omega_{i,j}$ is obtained, then the degradation model of an operating system can be reconstructed based on Eq. (1), which is given as:

$$E\left(\tilde{L}_{j,t}|\boldsymbol{L}_{j,\cdot}\right) = \sum_{i=1}^{m}\left(\omega_{i,j}\eta(\hat{\varphi}_i, t)\right) \tag{2}$$

where $\tilde{L}_{j,t}$ represents the reconstructed degradation model for operating system j at time t, and $L_{j,\cdot} = \left[L_{j,1},\ldots,L_{i,n_j}\right]^T$ represents the collected real-time degradation data during the lifecycle; $\omega_{i,j} \geq 0$ represents the proposed weight parameter corresponding to historical system i, and $\sum_{i=1}^{m}\omega_{i,j} = 1$; $\hat{\varphi}_i$ represents the estimated model parameters, m represents thenumber of historical systems.

On the other hand, when the real-time condition monitoring data are observed from an operating system, the Bayesian updating methods [7, 10] has also been widely used by many researchers to update the posterior distribution of the random effects for an operating system. In the perspective of the update process, the degradation model for operating system j can be updated using the Bayesian updating method based on the collected real-time condition monitoring data, which is given as:

$$E\left(\hat{L}_{j,t}|\boldsymbol{L}_{j,\cdot}\right) = \eta\left(E\left(\theta_j^{(1)}\right), t\right) \tag{3}$$

where $\hat{L}_{j,t}$ represents the reconstructed degradation data for operating system j at time t using the Bayesian updating method, $\boldsymbol{L}_{j,\cdot}$ represents the collected real-time condition monitoring data until time t, and $\theta_j^{(1)} \in R^{p\times 1}$ represents the estimated model parameters using the Bayesian updating method, which represents the updated random effects at time t.

Recall that the failure threshold of the same series of systems under the same operating conditions are expected to be the same, and the innovative idea is to reconstruct the degradation profile of an operating system by the weighted average of the historical system degradation profiles. In particular, if we consider the approximation of the Bayesian updated degradation model in Eq. (3) by using the reconstructed degradation model in Eq. (2), then, the system failure threshold distribution can be accurately estimated by using the information of historical systems with the real-time data of the operating system. In other words, the failure threshold for operating system j at the k th condition observation moment, D_j^k, can be determined by the weighted average of the last observation moments:

$$E\left[D_j^k\right] = \sum_{1}^{m}\left(\omega_{i,j}L_{i,n_i}^k\right) \tag{4}$$

where $\omega_{i,j} \geq 0$ for $i = 1,2,\ldots,m$, and $\sum_{i=1}^{m}\omega_{i,j} = 1$; L_{i,n_i} represents the degradation data of the last observation times before the failure of historical system i. Note that if the weight parameter $\omega_{i,j}$ is obtained, the failure threshold of operating system j can then be online determined. Therefore, the challenge here is to find the proper weight parameter, which will be specifically solved in the rest section.

2.2. Formulation of the methodology

Recall that the weight parameter $\omega_{i,j}$ is proposed to measure the relative similarity between the degradation profile of operating system j and the degradation profile of historical system i compared to other historical systems. As a result, the aim of this section is to find the optimal weight parametersfor the operating system to maximize the goodness-of-fit between the reconstructed degradation model in Eq. (2) and the Bayesian updated degradation model in Eq. (3). In particular, the optimization model is formulated as a programming problem to minimize the sum of squared errors to estimate the optimal weight parameter $\omega_{i,j}$, which is written as follows:

$$\begin{gathered}\min_{\omega_{i,j}}\sum_{t=1}^{n_j}\left(E\left[\hat{L}_{j,t}\right] - E\left[\tilde{L}_{j,t}\right]\right)^2 \\ \text{s.t. } \sum_{i=1}^{m}\omega_{i,j} = 1,\ \omega_{i,j} \geq 0,\ \text{for } i = 1,2,\ldots,m\end{gathered} \tag{5}$$

where n_j represents the number of data collections from operating system j, $E\left[\hat{L}_{j,t}\right]$ represents the degradation degree obtained by using the Bayesian updating method in Eq. (3), $E\left[\tilde{L}_{j,t}\right]$ represents the degradation degree obtained by using the weighted average method in Eq. (2) and $\sum_{i=1}^{m}\omega_{i,j}=1$ represents that the reconstructed degradation model is the weighted average of the historical system degradation profiles.

By solving the optimization problem in Eq. (5), the optimal weight parameter $\omega_{i,j}$ can be obtained and, the Bayesian updated degradation model can then be fitted by using the reconstructed degradation model of historical systems. It is worth note that overfitting problem may appear when the optimization model in Eq. (5) was directly implemented, especially when the operating system is in the initial stage that the number of available degradation data collections n_j is small. To solve this problem, a regularization function is introduced as:

$$R\left(\omega_{1,j},\ldots,\omega_{m,j}\right)=\sum_{i=1}^{m}\left(\omega_{i,j}\left\{E\left[\hat{L}_{j,n_i}\right]-L_{i,n_i}\right\}^2\right) \tag{6}$$

where $E\left[\hat{L}_{j,n_i}\right]$ represents the expected degradation data measurement for operating system j at the failure moment of historical system i, which can be obtained by using the Bayesian updating method in Eq. (3) up to the time n_i, L_{i,n_i} represents the measurement of degradation data at the failure moment of historical system i.

Remark 1: The proposed regularization function can be understood in this perspective: Recall that the main idea of our proposed failure threshold determination method is using the weighted average of the last measurements of degradation data of historical systems to determine the expected failure threshold of operating system j, as shown in Eq. (4). In this way, if the Bayesian updated degradation data for an operating system j at time n_i is obviously different from the measurement of degradation data for historical system i at time n_i, the historical system i has less impact on determining the failure threshold of operating system j. In other words, as shown in the regularization function of Eq. (6), we think a large penalty is supposed to be added to historical system i.

In summary, the concerned optimization model can be formulated by combining the regularization function in Eq. (6) with the formulation Eq. (5):

$$\begin{gathered}\min_{\omega_{i,j}}\sum_{t=1}^{n_j}\frac{\left(E\left[\hat{L}_{j,t}\right]-E\left[\tilde{L}_{j,t}\right]\right)^2}{n_j}+\lambda R\left(\omega_{1,j}\ldots,\omega_{m,j}\right)\\ \text{s.t. }\sum_{i=1}^{m}\omega_{i,j}=1,\ \omega_{i,j}\geq 0\text{ , for }i=1,2,\ldots,m\end{gathered} \tag{7}$$

where λ represents the tuning coefficient that measures the relative importance of the regularization function compared with the sum of squared errors between the reconstructed degradation model and the Bayesian updated degradation model. The tuning coefficient λ can be calculated with cross-validation [11], and in engineering practice, the value of λ often determined according to the importance assigned to each item.

Remark 2: Note that the optimization model in the Eq. (7) considers both the differences between the reconstructed degradation data for operating system j and the measurement of degradation data of historical systems at their failure moments, and the means squared error of the collected degradation data in the time domain for operating system j.

As noted above, the failure threshold for operating system j can be determined by using the optimal solution of weight parameters $\boldsymbol{\omega}_j$ with Eq. (4). Once the failure threshold D_j^k is determined, the RL for operating system j can then be evaluated. Therefore, in the next section, the RL distribution derivation method of an operating system will be investigated based on the proposed model in Eq. (7).

2.3. Estimation of the residual life

Recall that a system failure will occur once the degradation data cross the failure threshold. Without loss of generality, given the real-time degradation data $\boldsymbol{L}_{j,\cdot}=\left[L_{j,1},\ldots,L_{j,n_j}\right]^T$ of system j collected up to the current sampling moment n_j, the RL distribution of system j, $\tilde{T}_j$ can be estimated by:

$$P\left(\tilde{T}_j\leq \mathrm{t}|\boldsymbol{L}_{j,\cdot}\right)=P(L_{j,n_j+t}\geq D_j\mid \boldsymbol{L}_{j,\cdot}) \tag{8}$$

where D_j represents the failure threshold of system j, L_{j,n_j+t} represents the measurement of project degradation data at the future sampling time n_j+t.

Therefore, to accurately evaluate the RL distribution of the operating system, a degradation model that can characterize the system degradation profile should be first established. In this paper, the polynomial function form of the degradation model is used for its useful mathematical properties [5]. To be specific, a widely used p th-order polynomial degradation model is given as:

$$L_{j,t}=\boldsymbol{X}_t\boldsymbol{\theta}_j+\varepsilon_{j,t}=\sum_{k=0}^{p}\theta_{j,k}t^k+\varepsilon_{j,t} \tag{9}$$

where $\boldsymbol{X}_t=\left[1,t,\ldots,t^p\right]$, and p represents the order of the degradation model; $\boldsymbol{\theta}_j$ is the vector that represents the random effects of the model that follows a multivariate normal distribution $N_{p+1}\left(\boldsymbol{\mu}_j,\Sigma_j\right)$, $\varepsilon_{j,t}$ represents the random noise and it is assumed to follow $N\left(0,\sigma_j^2\right)$. Note that after the degradation profiles of historical systems are fitted, then the prior distribution can be estimated:

$$\boldsymbol{\theta}_j^{(0)}\sim \boldsymbol{N}_{p+1}\left(\boldsymbol{\mu}_j^0,\Sigma_j^0\right), \tag{10}$$

where $\boldsymbol{\mu}_j^0$ is the prior mean value of the random effects, and Σ_j^0 is the prior variance value of the random effects. Then, the posterior distribution, $\boldsymbol{\theta}_j^{(1)}$, can be calculated by using the collected real-time degradation data $\boldsymbol{L}_{j,\cdot}$ of operating system j. Specifically, using the Bayesian updating approach that introduced in Makis *et al.* [36], the updated random effects $\boldsymbol{\theta}_j^{(1)}$ will be obtained as the following normal distribution:

$$\boldsymbol{\theta}_j^{(1)}=\boldsymbol{\theta}_j\mid \boldsymbol{L}_{j,\cdot}\sim N_{p+1}\left(\boldsymbol{\mu}_j^1,\Sigma_j^1\right) \tag{11}$$

where $\boldsymbol{\mu}_j^1=\left(\frac{\boldsymbol{\psi}_j^T\boldsymbol{\psi}_j}{\sigma_j^2}+\left(\Sigma_j^0\right)^{-1}\right)^{-1}\left(\frac{\boldsymbol{\psi}_j^T\boldsymbol{L}_{j,\cdot}}{\sigma_j^2}+\left(\Sigma_j^0\right)^{-1}\boldsymbol{\mu}_j^0\right)$,

$\Sigma_j^1=\left(\frac{\boldsymbol{\psi}_j^T\boldsymbol{\psi}_j}{\sigma_j^2}+\left(\Sigma_j^0\right)^{-1}\right)^{-1}$, and

$$\boldsymbol{\emptyset}_j \in R^{n_j \times (p+1)} = \begin{bmatrix} 1 & \dots & 1 \\ \dots & \dots & \dots \\ 1 & \dots & t^p \\ \dots & \dots & \dots \\ 1 & \dots & n_j^p \end{bmatrix} \boldsymbol{\psi}_j \in R^{n_j \times (p+1)} = \begin{bmatrix} 1 & \dots & 1 \\ \dots & \dots & \dots \\ 1 & \dots & t^p \\ \dots & \dots & \dots \\ 1 & \dots & n_j^p \end{bmatrix}.$$

Remark 3: Note that other distributions or simulation methodologies can also be used to calculate the posterior distribution and the RL of an operating system. Here, the use of the normal distribution for the model parameter characterizations is to take the convenience of its closed-form solution results. In fact, for many degradation models introduced in the existing literature, a normal distribution assumption for the random effects has been widely used [13, 17, 24, 31].

Remark 4: Note that other degradation models can also be utilized to build the degradation profiles and to determine the failure threshold. Here, the use of the polynomial form for the system degradation model is to take advantage of the convenient when calculating the parameters and the system RL. In fact, many other degradation models can also be transformed into the polynomial form [30,32], such as the random coefficient growth model [8], and the exponential form model [34].

Therefore, for the convenience of illustration, we only focus on the polynomial form for degradation models and the normal distribution for the random effects. Consequently, given the updated random effects $\boldsymbol{\theta}_j^{(1)}$, the degradation model of operating system j at time twill be obtained:

$$L_{j,t} \mid \boldsymbol{\theta}_j^{(1)} \sim N\left(\boldsymbol{X}_t \boldsymbol{\mu}_j^1, \boldsymbol{X}_t \Sigma_j^1 \boldsymbol{X}_t^T + \sigma_j^2\right) \tag{12}$$

Recall that the mean value u_j^d and variance value v_j^d of the failure threshold D_j for an operating system j can be calculated by $u_j^d = \sum_{i=1}^{m}\left(\omega_{i,j} L_{i,n_i}\right)$ and $v_j^d = \sum_{i=1}^{m}\left(\omega_{i,j} L_{i,n_i}^2\right) - \left(\sum_{i=1}^{m}\left(\omega_{i,j} L_{i,n_i}\right)\right)^2$ using Eq. (4). We further assume the system failure threshold D_j follows a normal distribution as well, i.e., $D_j \sim N\left(u_j^d, v_j^d\right)$. Note that this normal assumption for the system failure threshold distribution has been widely used in much existing research [13, 17, 24, 31]. Then, using the determined system failure threshold, the CDF of the RL $\tilde{T}_j$ for operating system j based on the collected real-time degradation data $\boldsymbol{L}_{j,.}$ can be calculated as:

$$P\left(\tilde{T}_j \le t \mid \boldsymbol{L}_{j,.}\right) = P\left(L_{j,n_j+t} \ge D_j \middle| \boldsymbol{L}_{j,.}\right) = \Phi\left(\frac{\boldsymbol{X}_{n_j+t}\boldsymbol{\mu}_j^1 - \mu_j^d}{\sqrt{\boldsymbol{X}_{n_j+t}\Sigma_j^1 \boldsymbol{X}_{n_j+t}^T + \sigma_j^2 + v_j^d}}\right) = \Phi(\mathrm{g}(\mathrm{t})) \tag{13}$$

Given that the RL for operating system j should be greater than 0, the truncated CDF was further considered conditioning on $\tilde{T}_j \ge 0$:

$$P\left(\tilde{T}_j \le t \mid \tilde{T}_j \ge 0, \boldsymbol{L}_{j,.}\right) = \frac{P\left(0 \le \tilde{T}_j \le t \middle| \boldsymbol{L}_{j,.}\right)}{P\left(\tilde{T}_j \ge 0 \middle| \boldsymbol{L}_{j,.}\right)} = \frac{\Phi(\mathrm{g}(\mathrm{t})) - \Phi(\mathrm{g}(0))}{1 - \Phi(\mathrm{g}(0))} \tag{14}$$

Since the truncated CDF in Eq. (14) is skewed, the median can be used as the point estimator for the RL prediction.

2.4. Flowchart of the methodology

The flowchart of the proposed system failure threshold determination method is shown in Fig. 1. A convex quadratic formulation is developed to combine the information from the degradation data of historical systems and the information from the real-time condition monitoring data of an operating system. Using the degradation data from historical systems, the degradation models of each historical system is first established and then reconstructed using a weighted average method to fit the degradation model of an operating system, as shown in Eq. (2). In addition, the information about the last condition monitoring data collections ($L_{i,t}$) is extracted, and in other words, the failure threshold of historical system i is estimated using the last condition monitoring data collection ($L_{i,t}$). Then, to established the real-time update model, the prior distribution of the parameters in the Bayesian updated model is also obtained using the historical systems. Then, using the degradation data from the operating system, the degradation model for the operating system is updated based on the Bayesian approach using the collected real-time condition monitoring data, as shown in Eq. (11). Particularly, the method focuses on using the degradation profiles of historical systems to fit the degradation profile of an operating system. And then, using the updated degradation model and the reconstructed degradation model of historical systems, the failure threshold of the operating system is finally determined by solving Eq. (7). Finally, using the determined failure threshold and the

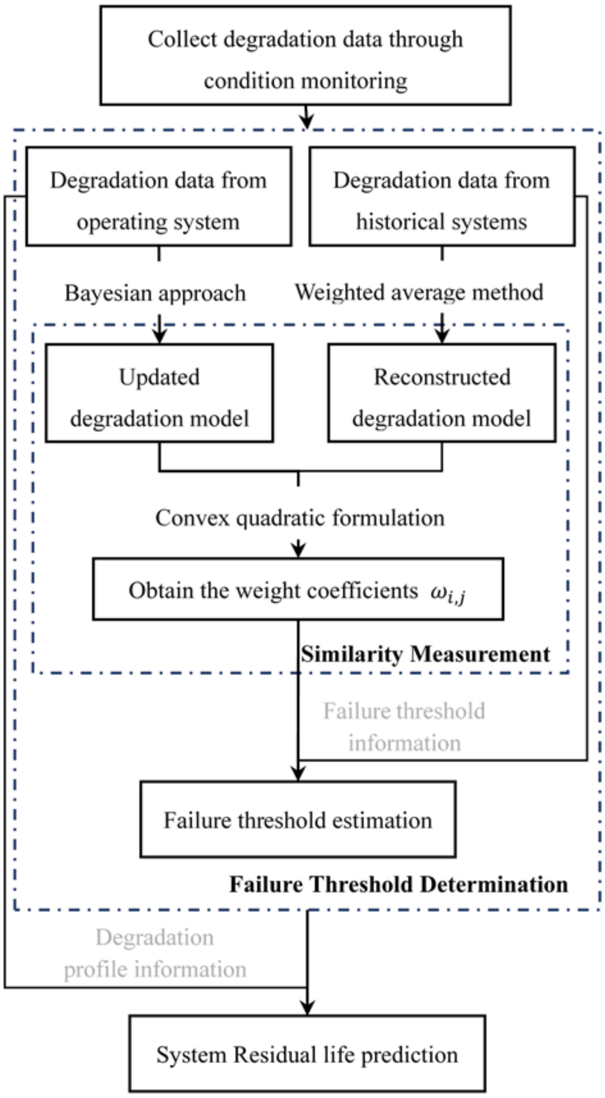

Fig. 1. Failure threshold determination framework for RL prediction

collected real-time condition monitoring data, the degradation model of an operating system can be established, and the system RL can then be estimated by Eq. (13) and Eq. (14).

3. Imperfect PM model considering system aging

In this section, the performance of the proposed system failure threshold determination method is investigated using a degradation dataset from several PSST systems. Specifically, we compare the prognostic performance of the proposed method with the existing population-wide characteristic-basedsystem failure threshold determination method. In fact, the method in [29] can be regarded as a special case of the proposed method in which each historical system is treated with equal importance, i.e., the weight coefficient is set as $\omega_{i,j}=1/\mathrm{m}$, for $i=1,2,\ldots,\mathrm{m}$. On the contrary, when determining the failure threshold of the operating system j , the proposed method will calculate the optimal weight coefficient $\omega_{i,j}$ for $i=1,2,\ldots,\mathrm{m}$ by solving Eq. (7).

3.1. Origin of the data

The dataset is collected from several PSST systems (Fig.2), which is a widely used mechanical powertrain system in heavy tracked vehicles. With PSST operating, metal debris mixed in lubricating oil accelerates the wear of every mechanical component, consequently leads to the degradation of the PSST system. However, the underlying degradation is not directly observable and can be only indirectly assessed via oil spectral analysis, which is a commonly used technique to monitor the underlying degradation conditions in oil-lubricated machines [24, 25, 28, 32]. In addition, the underlying failure mechanism in PSSTis not explicitly revealed. Therefore, researchers have to rely solely on the spectral oil data collected in the oil field dataset to establish the PSST degradation model and predict the RL.

In particular, the oil field dataset obtained from the reliability testbed (Fig. 3) for the PSST system is used for the case study illustration. Each system was run to failure under the same operating condition: cyclic multi-gear, load-varying, and multispeed. After a period of more than 10 years, we have collected oil field data for more than one thousand samples, and a detailed description of the dataset can be found in [31, 32]. The dataset we used in this paper contains $m=45$ training systems and 15 testing systems, of which each system contains more than 30 oil samples that were collected from 0 **Mh** to up to 284**Mh** according to the sample periodof nearly 5 **Mh**. Each sample contains 15 types of spectral oil data, among which 6 types, namely, Cr, Ni, Cu,

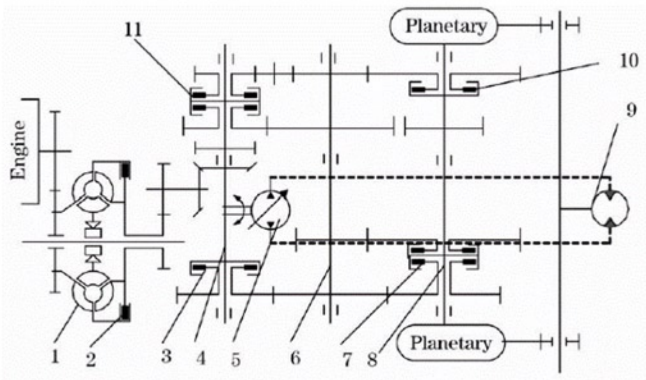

Fig. 2. Sketch of the PSST
1: Hydraulic torque convertor. 2: CV clutch.3: CH clutch. 4: First-shaft. 5: Steering pump. 6: Second shaft. 7: C1C2 clutch.8: Thirdshaft. 9: Steering motor. 10: C3 clutch. 11: CLCR clutch

Mn, Fe, and Mo, are selected in [30] and shown to be highly related to the degradation mechanism. Due to space restrictions, the selected spectral oil data of one PSST system are shown in Fig. 4.

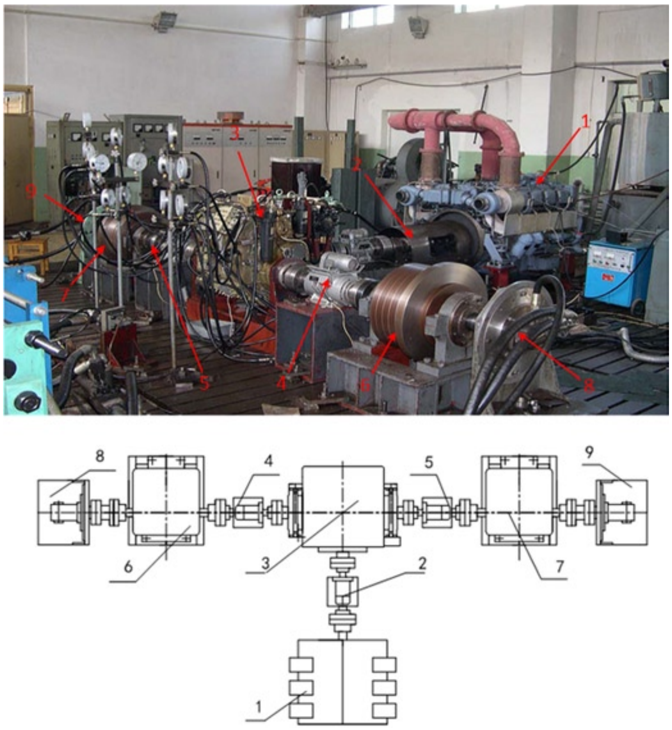

Fig. 3. A life-cycle testbed of the PSST
1: Diesel engine. 2, 4, 5: Torque and speed sensors.3: PSST. 6, 7: Inertia discs. 8, 9: Loading piston pump

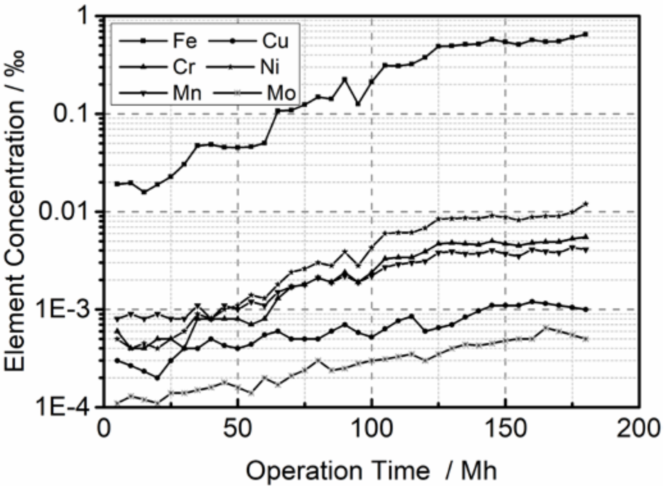

Fig. 4. Spectral oil data for one PSST system

3.2. System degradation modeling

For each PSST system, there are 6 types of spectral oil data that monitor the degradation process of the PSST system performance. Using these spectral oil data, the system degradation model can be established. The concerned PSST systems are subjected to three potential failure modes: 1) a fault at the transmission gears, or 2) a fault at the wet clutches, or 3) a fault at the rotary sealings. According to the previous research in [20], the spectral oil data following an exponential functional form. Thus, the following exponential degradation model is considered to model the degradation profiles of PSST systems:

$$y_{i,j,t} = \gamma_j + e^{\theta_{i,j,0}+\theta_{i,j,1}t+\theta_{i,j,2}t^2+\epsilon_{i,j,t}} \quad (15)$$

where $y_{i,j,t}$ represents the degradation data measurement for PSST system i, spectral oil data j at time t; γ_j represents the initial-effect coefficient for spectral oil data j, $\epsilon_{i,j,t}$ represents the random noise; and $\boldsymbol{\theta}_{i,j,0}$, $\boldsymbol{\theta}_{i,j,1}$ and $\boldsymbol{\theta}_{i,j,2}$ represent the random effects for PSST system i, spectral oil data j. Similar to Makis *et al*. [6] and Liu *et al*. [20], a log-transformation is used to process the original spectral oil data, and then the logged spectral oil data is modeled as follows:

$$L_{i,j,t} = \ln\left(y_{i,j,t} - \gamma_j\right) = \theta_{i,j,0} + \theta_{i,j,1}t + \theta_{i,j,2}t^2 + \epsilon_{i,j,t} \quad (16)$$

Using the established degradation model in Eq. (16), the PSST system RL can be then determined. It is noted that all the selected 60 PSST systems are degraded and failed according to one of the three possible failure modes. In addition, it is assumed that all PSST systems within the same failure modes following the assumptions: 1) have the same expected failure threshold subject to the same variations; and 2) the random effects have the same distribution.

To challenge our proposed method, it is further assumed that the failure modes of the training PSST system and the operating PSST system are unknown, which is usually the case in the real application. This assumption is to show the superiority of our proposed method in inferring the failure threshold and the RL of an operating PSST system, even if the failure mode of the training PSST system is unknown. In this way, the performance of our proposed system failure determination method can be evaluated and compared with the existing method [29]. Specifically, two metrics listed in the following are used for evaluation:

1) The mean prediction error of the system RL (RL_e), which is defined as follows:

$$RL_e(\%) = \frac{100}{N} * \sum_{j=1}^{N} \frac{\left|\left(T_j + n_j\right) - \left(\tilde{T}_j + n_j\right)\right|}{T_j + n_j} = \frac{100}{N} * \sum_{j=1}^{N} \frac{T_j - \tilde{T}_j}{T_j + n_j} \quad (17)$$

where N represents the number of testing PSST systems, n_j represents the number of collected spectral oil data for testing PSST system j, and $T_j + n_j$ represents the lifetime for testing PSST system j; T_j represents the actual RL for testing PSST system j, and $\tilde{T}_j$ represents the estimated system RL for the same testing PSST system.

2) The mean prediction error of the system failure threshold (FT_e), which is defined as follows:

$$FT_e(\%) = \frac{100}{N} * \sum_{j=1}^{N} \frac{\left|E\left[D_j\right] - D_j^{\text{True}}\right|}{D_j^{\text{True}}} \quad (18)$$

where $E\left[D_j\right]$ represents the estimated system failure threshold for testing PSST j using our proposed method in Eq. (4), and D_j^{True} represents the actual failure threshold for the same testing PSST system.

Matric (1) measures the mean prediction error between the predicted system residual life and the actual system residual life for the testing PSST system, and matric (2) measures the mean prediction error between the estimation value and the corresponding actual value of the system failure threshold for the testing PSST system. Note that these two metrics measure the concerned two aspects, namely, the failure threshold and system RL [4], when using collected condition monitoring data for system RL prediction.

3.3. System Failure Threshold Determination

For the system failure threshold determination, the procedures described in Section 2.4 are adopted. First, the system failure threshold of training PSST system i is estimated by the last observation (s_{i,n_i}) of the spectral oil data. Then the degradation model parameters are denoted as $\varphi_i = \left[\varphi_{i,0}, \varphi_{i,1}, \varphi_{i,2}\right]^T$ and the degradation model for each training PSST system i is reconstructed based on a second-order polynomial model. Next, the model parameters of testing system j are assumed to follow a normal distribution $\theta_j^{(0)} \sim N_{p+1}\left(\boldsymbol{u}_j^0, \Sigma_j^0\right)$, where $\boldsymbol{u}_j^0$ is estimated by the mean value of the parameters φ_i of training PSST systems, and Σ_j^0 is estimated by the variance value of the parameters φ_i of training PSST systems. Then the posterior distribution $\boldsymbol{\theta}_j^1$ is calculated using the real-time collected spectral oil data $\boldsymbol{L}_{j,.}$ from the testing PSST system as shown in Eq. (11). Finally, the weight coefficient $\omega_{i,j}$ is calculated by solving Eq. (7) with five-fold cross-validation and consequently, the failure threshold of testing PSST system j can be estimated with the mean value $u_j^d = \sum_{i=1}^{m}\left(\omega_{i,j}s_{i,n_i}\right)$ and variance value $u_j^d = \sum_{i=1}^{m}\left(\omega_{i,j}s_{i,n_i}^2\right) - \left(\sum_{i=1}^{m}\left(\omega_{i,j}s_{i,n_i}\right)\right)^2$. Please note that Henze-Zircller's test is conducted in this case study based on the spectral oil data sets from the training PSST systems, and the calculation results show that using the normal distribution as the prior distribution for the random effects is satisfactory, as mentioned in Eq. (10).

The average prediction error of the failure threshold of the PSST system, FT_e as defined in Eq. (18), is shown in Fig.5, of which the best performance spectral oil data sample (Fe) is used for illustration. The threshold is modeled as a function of the operating time of the testing PSST systems. To be specific, the point corresponding to the "0" label is the average prediction error of the testing PSST systems in the initial state, while the point corresponding to the "50" label is the average prediction error when the testing PSST systems operating to 50% of the whole life. Here, for the convenience of illustration, the existing population-wide characteristic-based system failure threshold determination method in [29] is denoted as the "benchmark method".

From Fig.5, we can observe that our proposed method has an excellent performance, and the average prediction error is consistently reduced with the increase of the operating time. On the contrary, the benchmark method has poor performance, and the RL prediction accuracy does not improve with the increase of the operating time. It can be concluded that the proposed method provides a better result compared with the benchmark method when used for system failure

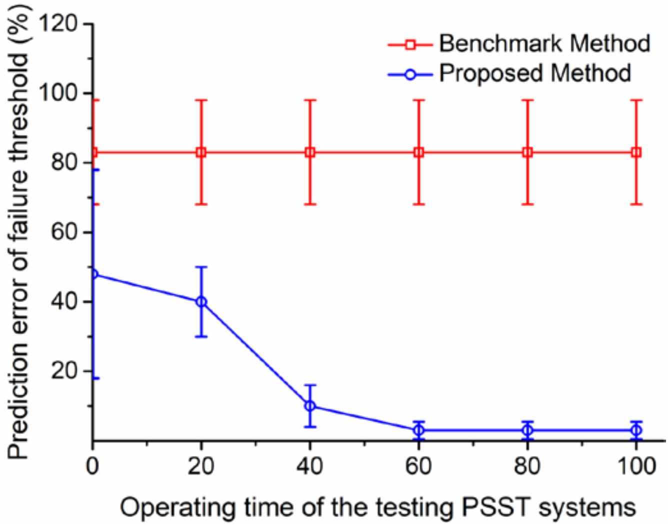

Fig. 5. Prediction error of the PSST system failure threshold using Fe

threshold determination. That is because that the threshold determination using the benchmark method just considering the population-wide characteristics from historical PSST systems, does not consider the individual characters from the operating PSST system. In addition, when using our proposed method, the average prediction error is approximately less than 3% when the operating PSST systems operate to half-life. To be specific, the result shows that the proposed method can determine the PSST system failure threshold accurately even the PSST system is operating in a stable condition. Moreover, from Fig.5, it can be seen that the average prediction error tends to be large when the testing PSST systems operate at an early stage (less than 20% of the lifetime). One possible reason is that the system degradation model in Eq. (3) is established using the Bayesian update method that may not fully capture the unique degradation features of the testing PSST systems when there are fewer spectral oil data samples available. However, when more spectral oil data samples are available, the accuracy of the PSST system failure threshold determination is improved significantly, which shows the rationality and superiority of the proposed system failure threshold determination method.

3.4. System residual life estimation

To further evaluate the performance of the proposed method for system RL prediction, the RL of the testing PSST systems is estimated by adopting the calculated system failure threshold in Section 3.3 and the proposed method in Section 2.3. Given the spectral oil data from all of the 15 testing PSST system, the average prediction error of the RL, as defined in Eq. (17), is then calculated by using the actual RL of the testing PSST system. The calculation result is summarized in Tab. 1.

From Tab. 1, we can observe that the accuracy of RL prediction using the proposed method is improved for all selected spectral oil data when compared with the benchmark method. Moreover, the average prediction error at different levels of actual RL for the testing PSST systems are calculated and shown in Fig.6, and the comparison between the proposed method and the benchmark method is shown in Fig.7. In Fig.6 and 7, the label "30" represents the average prediction error for the testing PSST system that has 30 or less actual RL. The minimum actual RL among all the testing PSST systems is 150. The label "150" represents the average prediction error using all the testing PSST systems. Thus, the same sets of testing PSST systems are used to calculate the accuracy of RL prediction as different levels of the actual RL.

From Fig.6, it can be seen that the RL prediction becomes accurate in general with the testing PSST system operating from the initial state to failure. This may because more spectral oil data samples are collected when the actual RL becomes less, and thus, the estimated system failure threshold, as well as the updated Bayesian degradation model for the operating PSST system, becomes more confidential.

Fig. 7 further shows the comparison by using the best performance spectral oil data sample (Fe) and the worst performance spectral oil data sample (Mn) according to the result in Tab.1. Based on Fig. 7, it can be clearly seen that the proposed system failure threshold determination method outperforms the original benchmark method in both spectral oil data. This observation further shows the rationality and superiority of our proposed method for system RL estimation.

Table 1. The average prediction error of the RL

Spectral oil data	Cr	Ni	Cu	Mn	Fe	Mo	Average
The proposed method	9.75%	10.6%	8.32%	15.85%	7.5%	12.6%	10.77%
The benchmark method	15.78%	14.55%	14.52%	20.21%	13.8%	18.06%	16.15%
Improvement	6.03%	3.95%	6.2%	4.36%	6.3%	5.46%	5.38%

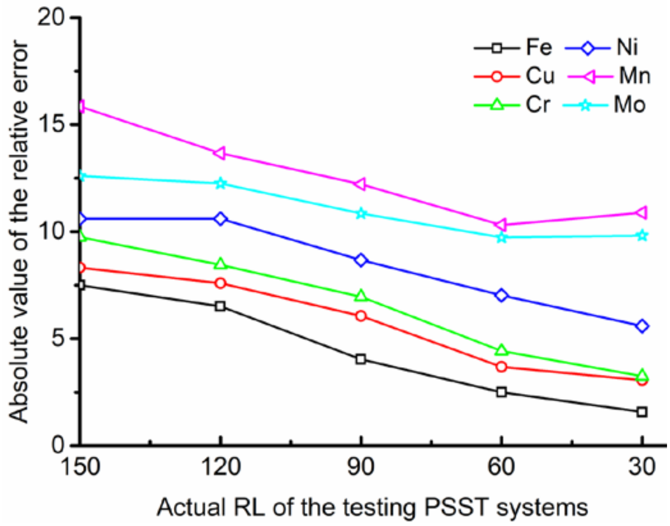

Fig. 6. Prediction error for the testing PSST systems using the proposed method

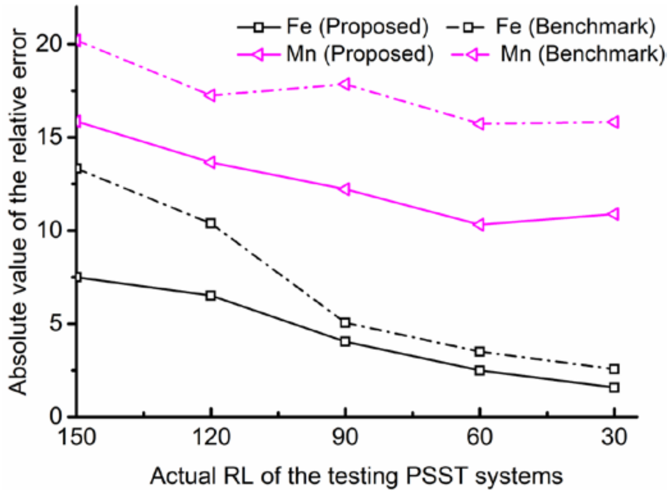

Fig. 7. Prediction error between the proposed method and the benchmark method

By comparing Fig. 6 and 7, it can be seen that the accuracy of RL prediction is highly related to the accuracy of system failure threshold determination. To be specific, with the improvement of the accuracy of system failure threshold determination for the operating PSST system, the accuracy of RL prediction using the proposed method also improves compared with the benchmark method.

4. Conclusion and discussion

This paper proposes a similarity-based failure threshold determination method for the system residual life prediction. To be specific, the similarity between the operating system and historical systems is measured by a weighted average function and calculated by using a convex quadratic formulation, and thus, the corresponding system failure threshold can be determined. The novelty of the proposed method is that the degradation profiles of historical systems and the real-time condition monitoring data of an operating system are combined for real-time estimating the system failure threshold of an operating system when the failure threshold is unknown.

The validity of the proposed method was demonstrated using selected oil field data collected from several PSST systems. The results not only show the effectiveness of the proposed method but also reveal the importance of real-time estimating the failure threshold for an operating system.

Compared with the existing failure threshold determination method which using population-wide characteristics, the proposed method provides an accurate determination of the system failure threshold, as illustrated in the case study. With the improved determination of the failure threshold, a more accurate system residual life estimation can then be obtained; even the monitored system is at an early moment of the degradation. Such advantages can be effectively used to obtain a better prognostic application and the following planned maintenance optimization. Following the results of the proposed method, the case study described in the existing works of literature, for example, Vališ et al. [24], Yan et al. [32] and Liu et al. [20], might be complemented when these methods are utilized for system soft failure prediction, health index extraction and other condition monitoring applications.

The main contribution of this paper is not only a new direction in the failure threshold determination for degradation systems but also open up possibilities for combining the real-time degradation data with the historical condition monitoring data. There are several possible directions for future research. First, other methods, such as a Bayesian framework, might be considered to update the failure threshold, as compared to the proposed frequentist method. Second, other degradation modeling method can fuse multiple condition monitoring data may have to be used when modeling other systems.

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