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Robustness of schedules for project scheduling problem with cash flow optimisation

M. KLIMEK[1]* and P. ŁEBKOWSKI[2]

[1] Institute of Informatics Pope John Paul II State School of Higher Education, 95-97 Sidorska St., 21-500 Biała Podlaska, Poland
[2] AGH University of Science and Technology, Department of Operations Research and Information Technology, 10 Gramatyka St., 30-067 Kraków, Poland

Abstract. The paper presents the resource-constrained project scheduling with cash flow optimisation. New project models with bonus-penalty system and payoff in stages have been proposed. For the models presented, the application of proactive scheduling is analysed, as designed to improve project execution under uncertainty. Next, schedule robustness is discussed and measures of robustness are proposed.

Key words: resource-constrained project scheduling with cash flow optimisation, proactive scheduling, milestones.

1. Introduction

Any schedule being developed is most often assessed against time-related criteria, such as minimising project completion time, meeting milestone deadlines etc. However, it often proves to be insufficient to use exclusively time-related criteria in planning production or investment projects. Economic criteria (i.e. maximising discounted cash flows) are particularly material in the assessment of business activity. It is advisable that the comprehensive evaluation of the effects of project execution in line with an adopted schedule should include economic criteria. It is already in the phase of making a decision concerning the project execution schedule that the analysis of economic effects may mitigate the project failure risk.

The paper analyses the Resource-Constrained Project Scheduling Problem (RCPSP) with predefined milestones [1]. The assessment of a schedule to be developed includes economic criteria (maximising discounted cash flows). Additionally, the assessment includes the effect of uncertainty accompanying the execution of real-life projects (and relating to customers' changing expectations, errors in estimating activity execution times, temporary unavailability of resources, equipment failures etc.) on the cash flow levels and quality of schedules selected for execution. The objective of this paper is to find measures of robustness and set rules for the construction of a schedule which would be robust to disturbances (the variability of activity durations) and which would support cash flow maximising from the project contractor's perspective.

2. Literature review

In project scheduling with cash flow optimisation not only activity starting times, but also forecasts of the related cash flows are determined. The financial aspects of project execution may be analysed with static (simple) methods assuming the constant value of money in time or dynamic (discount) methods assuming a variable value of money in time. The use of dynamic methods is particularly advisable in countries with high inflation ratios and/or in planning long-term projects (e.g., large construction, building or IT projects). One of practical solutions to the high inflation problem is arranging for the contract between the contractor and the client to stipulate that each cash flow under the contract should be adjusted for an inflation ratio [2].

Discount methods are more commonly used in research. Discounting consists in the computation of the present value of future cash flows based on the current value of money determined with the use of the adopted rate of interest on capital (discount rate). Russell [3] was first to propose the Max-NPV (Net Present Value) model, in which discounted cash inflows (payments received for the activities executed) and discounted cash outflows are analysed during project execution. The Net Present Value is the most common assessment criterion including economic aspects into project scheduling. Recent papers devoted to this subject include Mika et al. [4], Vanhoucke et al. [5] and Waligóra [6].

NPV reflects the change in the value of money in time and includes total cash flows related to the project being executed. The use of NPV for the evaluation of projects under execution is reasonable in the event of large projects, where material changes in the value of money are observed. In the event of minor, short-term projects, cash flow discounting is not necessary [2].

The net present value, see formula (1) is the aggregate of net cash flows, discounted separately for each period, at the predefined discount rate α.

$$NPV = \sum_{i=1}^{H} \frac{CF_i}{(1+\alpha)^{t_i}}, \tag{1}$$

where H is the total number of individual cash flows recorded within the period being analysed (from the project start to

*e-mail: marcin_kli@interia.pl

finish), CF_i is the value of the i-th cash flow, t_i is the time of occurrence of the i-th cash flow, counted in capitalization periods (months or days) for the given discount rate α.

From the project contractor's perspective, various types of cash flows are considered: cash inflows (positive cash flows), that is payments made by the client to the contractor, and cash outflows (negative cash flows), that is payments expensed by the contractor to the client (e.g., contractual penalties), suppliers or employees. Expenses relate to task completion and use of resources. Inflows relate to payments for milestone completion. The contractor's outflows are as a rule more frequent than the contractor's inflows, with the value of outflows depending on the cost incurred.

The NPV is affected by numerous factors, including the schedule of the client's payments to the contractor. The research performed [4, 7–8] includes various Payment Project Scheduling (PPS) models. In PPS problems, the payment project scheduling is developed with a view to achieving the maximum NPV. The predefined values include the client's total payment under the project, number of tranches and amounts of individual tranches. In practice, the amounts and dates for individual cash flows are determined in negotiations between the client and the contractor, taking into consideration the project execution progress, cost incurred by the contractor, duration of individual activities etc. The following intuitive principle governs the determination of cash flow dates from the contractor's perspective: cash inflows should be received as soon as possible, while cash outflows should be incurred as late as possible. The client, however, prefers making payments to the contractor as late as possible, with the best option being the payment to the contractor upon the final completion of the project. The research reports discuss the PPS problem from the perspective of the both contractor [4, 8] and client [9]. Ulusoy & Cebelli [10] strive to find a solution which would be satisfactory for the both client and contractor.

In the literature [4, 8] the following four PPS models are considered:

1. Lump-Sum Payment (LSP) – the client pays the entire payment to the contractor immediately after project completion; the payment model most favourable to the client, feasible for small projects, as it requires the contractor to arrange financing for the execution of the entire project;
2. Payments at Event Occurrences (PEO) – events most often relate to the execution of individual tasks; for instance, payments are triggered by milestone execution or execution of each individual activity, which is referred to as Payments at Activities' Completion (PAC) times [4], with the last payment made upon project completion, while the value of earlier payments depends on the scope of executed milestones (scope of executed activities);
3. Equal Time Intervals (ETI) – the contractor receives H payments from the client, with (H-1) payments made at predefined time intervals and the last payment made upon project completion;
4. Progress Payments (PP) – payments are made at predefined equal time intervals, as in the ETI model, but the number of payments is not known, because payments are made until project is not finished; payments made to employees on a monthly basis is an example of payments in the PP model.

These payment models may be described by formulae(2–4).

$$NPV_{LSP} = \frac{\sum_{i=1}^{H} CF_i}{(1+\alpha)^{C_{\max}}}, \tag{2}$$

$$NPV_{PAC} = \sum_{i=1}^{n} \frac{CF_i}{(1+\alpha)^{FT_i}}, \tag{3}$$

$$NPV_{PP}(NPV_{ETI}) = \sum_{i=1}^{H-1} \frac{CF_i}{(1+\alpha)^{t_i}} + \frac{CF_H}{(1+\alpha)^{C_{\max}}}, \tag{4}$$

where $C_{\max}$ is the project duration time (makespan), counted in capitalization periods for the given discount rate α, n stands for the number of activities in the project, CF_i is the value of the i-th cash flow, generally for the execution of the i-th activity (e.g., in PAC model), FT_i is the finish time for the i-th activity, counted in capitalization periods for the given discount rate α.

The problem of maximising NPV with inflows (positive cash flows) and with the payment model LSP (see formula (2)) is equivalent to the problem of minimising the project makespan ($C_{\max}$).

A practically useful payment model is PAC (see formula (3)), in which the client pays a predefined amount to the contractor for each executed activity. Cash flows relating to activity may occur at various times of activity execution. However, the most common option consists in projecting cash flows prior to the start or upon the completion of an activity; then, as a rule, expenses related to activity are assumed to be incurred at the start, while the related income is assumed to be received at the completion. Activity-related income/expenses may also be aggregated into a single cash flow recorded at the start or completion of the activity.

In the ETI and PP models (see formula (4)), fixed time intervals of the length t occur between consecutive payment times and the last payment is made at the project makespan ($t_H = C_{\max}$).

$$t_i - t_{i-1} = t, \quad \text{for} \quad i = 2, 3, ..., H-1. \tag{5}$$

Payments at fixed time intervals occur in the PP model, but the number H of payments is not known (H is the least positive integer larger than or equal to the quotient of the project makespan $C_{\max}$ and the length t of the time interval between consecutive payments).

An interesting research direction taking into consideration economic criteria is a problem in which capital is assumed to be among non-renewable resources, and thus a constrain in schedule construction; it is referred to as the Capital Constrained Project Scheduling Problem (CCPSP) [11]. In the CCPSP problem, cash expenses and income should set each other off at any time t. Task execution requires expenses which may be made if funds are available received for the execution of earlier tasks or milestones of the project. Activity execution may not start at a given time if the related expenses exceed funds available [11]. The cash available may be negative, which is interpreted as debt. Objective functions used here include (maximised) discounted cash available in each time period. Maximum cash availability at various times favourably affects the contractor's operations in other areas of its business.

Researchers also analyse additional financial aspects of project execution. They include a bonus-penalty system, whose components include penalties for delays in project execution and bonuses for activity completion before the agreed date. He & Xu [12] analyse various payment models with a bonus-penalty system, from the both contractor's and client's perspective. Time windows are defined for individual activities, that is time intervals in which neither a bonus nor a penalty is applicable with respect to a completed activity. For completing an activity (milestone) before the time window starts the contractor receives a bonus, while for a delayed completion (after the time window) the contractor is charged with a penalty. A bonus-penalty system should be designed with a view to motivating the contractor to execute project activities as quick as possible. In the absence of additional incentive and taking into consideration economic criteria, the contractor could purposefully delay the execution of project milestones (start of activity involves the employment of resources and materials, and thus incurring expenses). For instance, the contractor could prefer paying a minor penalty to incurring cost of earlier employment of resources and materials. In terms of the dates and amounts of cash flows, the client's and the contractor's preferences are inconsistent. It is these inconsistency that renders the use of a bonus-penalty system advisable and reasonable. For a bonus-penalty system to be effective from the contractor's perspective, a bonus should exceed the contractor's expenditure resulting from earlier execution of an activity (group of activities), while a penalty for a delay should exceed the contractor's gain on a delayed execution of a project task (milestone). From the client's perspective, an effective bonus-penalty system is one in which a bonus for the contractor for earlier execution does not exceed the client's gain on the earlier execution of a project task (milestone), while a penalty for a delay should be higher than the client's profit lost as a result of a delayed execution of a project task (milestone).

The majority of research papers concerning project scheduling with cash flow optimisation refer to the deterministic problem, while uncertainty-related aspects, frequently present in the execution of actual projects, are omitted. Including uncertainty in project planning is crucial and the number of research reports and papers covering the issue has been growing. However, these papers primarily analyse models with time-related criteria rather than with economic criteria optimisation. One might say that project scheduling with cash flow optimisation under uncertainty has been neglected, despite its major practical importance and its significant effect on the quality of orderings executed [13].

The models considered for the project scheduling problem with cash flow optimisation include models with stochastic [14–15] or fuzzy [16–17] activity durations. Optimisation process includes the determination of expected values of the objective function (i.e., expected accumulated cash flows EPV – Expected Present Value) computed for various project progress scenarios (various task durations) [13–14].

Numerous problems and models concerning project scheduling with economic criteria optimisation still wait to be tackled. Implementation of effective methods mitigating the adverse effect of disturbances might significantly enhance financial benefits of project execution.

A review of research into project scheduling including economic criteria and cash flows may be found in review papers [2, 18]. These papers also discuss other aspects not covered herein.

3. Problem statement

A project is a unique set of activities executed in order to achieve predefined objectives with use of specified resources (human resources, machinery and materials). An activity (task) is an element of a project, representing a separable entity, for which starting and/or completion times are defined. Nonpreemptive scheduling is considered and a single-mode RCPSP problem is analysed. A project is presented as an Activity On Node (AON) network. In an AON, a project is depicted as an acyclic simple directed graph $G(V, E)$, where the nodes (elements of the set V) represent activities (arranged topologically in the increasing order of numbers assigned to the activities) and edges (elements of the set E) represent ordering relations between activities. The project comprises n activities. The graph $G(V, E)$ additionally includes two activities with zero durations and zero demand for resources: the initial node numbered 0 and the final node numbered $n+1$.

Finish-start, zero-lag precedence relations occur between activities in the project: the successor may start immediately upon the predecessor having ended (see formula (6)).

$$ST_i + d_i \leq ST_j \qquad \forall (i,j) \in E, \tag{6}$$

where ST_i is the starting time of the activity i, d_i is the duration of the activity i.

Renewable resources are used to execute an activity; they also represent constraints to project execution. The number of available resources is fixed at a_k for each resource type $k = 1, \ldots, K$ (K is the number of resource types) at any time t. During the execution of an activity, the use of resources must not exceed their respective available quantities (see formula (7)).

$$\sum_{i \in A_t} r_{ik} \leq a_k, \qquad \forall t,\ \forall k, \tag{7}$$

where A_t is the set of activities performed in the time interval $[t1, t]$, r_{ik} is the demand of the activity i for the resources of type k.

For a deterministic RCPSP problem, the most commonly used objective functions are: minimum project makespan and minimum delay in project execution. In models including cash flows, the problem most often considered is that of maximising NPV.

In this paper, the authors consider a new RCPSP model with contractual milestone deadlines, taking into consideration cash flows occurring during task execution [1]. It is a common practice that the client and contractor agree on settling individual milestones of a project. The client pays agreed amounts to the contractor for timely execution of specified activities. Delays, if any, usually trigger penalties, for instance, contractual penalties. On the other hand, the parties may agree that a timely execution of a milestone triggers payment for the execution thereof. As a rule, several milestones are defined over project makespan; such milestones are particularly material from the project execution perspective and support monitoring and control of work progress over the entire project makespan.

Settlement by milestones is primarily favourable to the contractor, who may then receive payments for the execution of individual tasks and use the funds thus raised to execute further activities, purchase materials etc. instead of employ its own funds. While earlier payments are not favourable to the client, the latter may then monitor and control the project execution progress at the crucial moments thereof and thus mitigate the risk of project failure.

Milestones are defined by way of setting time constraints for individual activities [1]. For each task i, a deadline $\delta_i \neq 0$ is defined. A milestone is a set of activities with the same deadline. Let *MA_m* denote the set of tasks (activities) directly connected with the m-th milestone of the project, containing all activities with the same deadline δ_i (see formula (8)).

$$MA_m = \{i : \delta_i = MT_m, i \in V\}, \tag{8}$$

$$MT_m < MT_{m+1}, m \in <1, M), \tag{9}$$

where δ_i is the contractual execution deadline for the i-th activity, as defined by classifying the activity in a given project milestone, M is the number of agreed milestones, *MT_m* is the contractual deadline for the m-th milestone.

The control of work progress over the entire makespan of the project may be achieved by, for instance, even arrangement of milestone deadlines *MT_m* over the project execution period.

For the RCPSP problem with predefined milestones, this paper analyses the problem of optimising cash flows. The following assumptions have been included in the problem:

- The client makes payment to the contractor each time the contractor has completed a predefined milestone, at predefined dates stipulated in the contract.
- A milestone is a set of tasks to be completed by a specified date assigned to that milestone.
- Delayed execution of a milestone may generate cost to be incurred by the contractor in the form of contractual penalty, which reduces the amount to be received by the contractor for the execution of a given milestone.
- The contractor's expenditure relates to commitment of resources and starting of individual tasks, as this involves using funds to purchase and transport materials and other resources necessary to execute the task. Expenditure required for a given task is incurred at the time provided for the execution start in the baseline schedule. The commitment of resources generates the contractor's direct expenses; for instance, if human resources are involved in project execution, the contractor's expenses include remuneration for the employees.
- Delayed start of a task (relative to the baseline schedule) may trigger the contractor's additional financial liabilities relating to i.e. the need to store materials.

The proposed objective is maximising functions described by formulae (1)–(13).

$$F_1 = \sum_{i=1}^{n} \frac{CFA_i}{(1+\alpha)^{RST_i}} + \sum_{m=1}^{M} \frac{CFM_m}{(1+\alpha)^{RMT_m}} + \sum_{p=1}^{H_p} \frac{CFP_p}{(1+\alpha)^{t_p}}, \tag{10}$$

$$F_2 = \sum_{i=1}^{n} \frac{CFA_i}{(1+\alpha)^{RST_i}} + \sum_{m=1}^{M} \frac{CFM_m}{(1+\alpha)^{RMT_m}}, \tag{11}$$

$$F_3 = \sum_{i=1}^{n} CFA_i + \sum_{m=1}^{M} CFM_m + \sum_{p=1}^{H_p} CFP_p\,, \tag{12}$$

$$F_4 = \sum_{i=1}^{n} CFA_i + \sum_{m=1}^{M} CFM_m\,, \tag{13}$$

where *RST_i* is the real or expected starting time for the i-th activity, *RMT_m* is the real or expected finish time for the m-th milestone, *CFA_i* stands for the contractor's payments (cash outflows from the contractor's perspective) connected with i-th activity execution, *CFM_m* stands for the client's payments (cash inflows from the contractor's perspective) for the implementation of the m-th milestone, *CFP_p* stands for the periodic contractor's payments (cash outflows from the contractor's perspective) in period p connected, for instance, with the use of human resources in this period (as in the PP model), H_p is the number of periodic contractor's payments.

The objective functions F_2 and F_4 do not include a component connected with periodic payments (*CFP*). These functions are applicable if all cost incurred may be attributed to the tasks being executed (e.g. in a task-based remuneration system). In the models governed by the objective functions F_3 and F_4, cash flows are not subject to discounting. These functions are applicable if the change in the value of money in time over the project makespan is not relevant (e.g., for short-term projects).

The contractor's outflows of the type *CFA* include the cost of engaging resources and materials in the execution of activities. To simplify the calculation of discounted cash flows, it has been assumed that the contractor incurs the expenses (outflows) *CFA* at the starting time for a given activity:

$$CFA_i = -C_i - CL_i \cdot \max(RST_i - ST_i, 0), \tag{14}$$

where C_i is the contractor-incurred cost of executing the i-th activity, *CL_i* is the cost triggered by a delayed start of executing the i-th activity.

The cash flows CFM_m (see formula (15)) are the client's payment to the contractor for the execution of the m-th milestone. To simplify the calculation of discounted cash flows, it has been assumed that the transfers *CFM* are performed exactly at the time of completing a given milestone.

$$CFM_m = PM_m - CM_m \cdot \max(RMT_m - MT_m, 0), \quad (15)$$

where PM_m is the client's payment to the contractor for executing the m-th milestone, CM_m is the unit cost of delay payable by the contractor for a delayed execution of the m-th milestone.

For each milestone deadline, cost of delayed execution (marginal cost) CM_m may be defined per time unit, as such cost would be incurred by the contractor upon payment of a contractual penalty to the client. Such cost, if any, reduces the payment due to the contractor for the execution of the m-th milestone. It has been assumed here that the client makes the payment (generates cash flows CFM_m) due to the customer for the execution of the m-th milestone upon the completion of that milestone, that is all activities included in the set MA_m having been finished. If milestones are evenly distributed over the project duration, the customer may receive steady financing in the form of those cash flows.

In the planning phase, the actual activities starting times RST_i and milestone completion times RMT_m are not known. In the nominal scheduling phase, it may be assumed that the schedule being developed is executed in line with the plan, that is $RST_i = ST_i$ for each activity $i = 1, \ldots, n$. Milestone completion times RMT_m are determined based on the baseline schedule, without any analysis of possible lengthening of activities durations. The value of the objective function F (F_1, F_2, F_3 or F_4) for RST_i and RMT_m values thus computed is the decisive criterion in the selection of the best baseline arrangement. Where multiple solutions yield the same value of the objective function, the proposed decisive criterion is schedule robustness to disturbances in project execution.

4. Proactive scheduling

Many actual projects are executed under uncertainty (resulting from clients' changing requirements, dynamic environment, unpredictable disturbances etc.). Research into project scheduling under uncertainty is described in review papers [19–21].

The majority of research into project scheduling under uncertainty focuses on guaranteeing a timely completion of the project. Many contractors implement Critical Chain Project Management (CCPM) [22] method in executing real-life projects. The overriding objective of CCPM is to protect project due date: by inserting project buffer and protecting activities included in the critical chain. The critical chain method is effective, but it is not advisable to use it for any project. The CCPM method is not appropriate for projects for which it is relevant to timely start of activities. Control of the project based on accurately implemented stable schedule streamlines the organisation of project work [1, 20]. Easier is the coordination of the enterprise's own resources (personnel, machines etc.) and transfer of resources among projects (in a multiple project environment). The larger the number of deviations from the planned schedule, the larger the disturbance in the project execution process. For a less disturbed schedule, lower storage cost is observed and a lower cost of loan financing the purchase of materials supplied Just-In-Time (JIT).

Proactive scheduling is one of the most effective approaches reducing the adverse effect of disturbances on project execution stability [19–20]. A proactive schedule, known also as a robust schedule, is developed in the project planning phase. According to the definition, a proactive schedule is a schedule robust against production disturbances, able to remedy, *inter alia*, the effects of minor increases in task duration, which may be caused by uncontrollable factors.

Project planning starts with the generation of a nominal schedule, which takes into consideration exclusively performance criteria and qualitative criteria at then current system parameters; uncertainty is not analysed. tasks (activities) are arranged in a nominal schedule so that it is not possible to start any one of them earlier than scheduled (no left shift in the Gantt graph is feasible without violating ordering or resource constraints).

A nominal schedule becomes a baseline schedule in the proactive scheduling phase, in which a robust schedule is created, preventing instabilities of nominal schedules, as the volatility and uncertainty of system parameters are taken into consideration. Tools for the creation of a proactive schedule include the insertion of time buffers at critical points of the schedule. In creating a schedule, available statistics may be used concerning possible disturbances of project execution plan; such statistics are gathered by way of analysing earlier executed projects.

Research indicates that using a proactive schedule brings measurable benefits and mitigates the effect of disturbances on the planned schedule [19–20]. System stability is enhanced, while the performance of the original schedule deteriorates insignificantly only, if at all. The research into proactive scheduling includes the analysis of problem of guaranteeing the timely execution of the entire project or its milestones [1], as well as the problem of minimizing the weighted cost of instability of individual tasks [20]. However, no research reports have been published covering the problem of the timely execution of milestones with cash flow optimisation.

The main factors rendering proactive scheduling with predefined milestones usable in practice include [1]:

- Separation, within a project, of contractual milestones with predefined execution deadlines (benefits milestone definition brings);
- Uncertainty observed during project execution, variability of task durations, difficulties in estimating the activities duration;
- Importance of stability over the project makespan, financial and organisational benefits of practically accurate execution of a proactive schedule.

The objective of proactive scheduling is to minimise changes in the schedule caused by the changes in task durations. In the case of the problem considered herein, the application of proactive scheduling may increase the projected value of accumulated cash flows.

Proactive scheduling of an RCPSP problem may include the following two optimising phases:

1. robust allocation of resources – an appropriate allocation of resources to individual tasks in the nominal schedule;
2. robust allocation of buffers – insertion of time and resource buffers.

Resource allocation problem is most often reduced to the problem of minimising the number of additional ordering relations between tasks [23–24]. Each additional ordering constraint reduces schedule robustness. Robust resource allocation should also take into consideration cost of instability of individual tasks. Allocation rules included in the resource allocation algorithms covered by the research conducted include: allocating the same resources to tasks linked to each other with ordering relations, maximising aggregate flows between individual tasks, minimizing the number of additional edges in the task network (e.g. with use of integer programming).

The next phase, that is buffer allocation, is performed after resources have been allocated to individual tasks. It consists in inserting resource and/or time buffers before (or after) tasks in order to make the schedule robust against such phenomena as temporary unavailability of resources (caused by, e.g., machinery failure) or variability of task durations [20]. Appropriate distribution of time buffers is material in minimising project execution cost. Time buffers are designed to prevent minor increases (more generally, fluctuations) in activity durations. Other disturbances of project execution need not be considered, as it has been assumed that the majority of disturbances results in activity duration fluctuations. Buffer insertion often lengthens project makespan. For an RCPSP problem with cash flow optimisation, postponing task execution starting time (postponing outflows) may increase the value of the objective function, that is the discounted aggregate of inflows and outflows.

Real-life project management solutions, such as the Critical Chain Project Management method, strive to avoid re-estimating planned task durations. Adopting safe estimates of task duration (longer planned durations) proves less effective than adopting aggressive estimates of task duration (shorter planned durations). If safe estimates have been used, than, nevertheless, task execution takes, as a rule, the entire (prolonged) time allocated – Parkinson's law operates, and even delays occur almost as often as with aggressive estimates (a manifestation of the student syndrome) [22]. While project scheduling for the problem considered, the authors assume the planned task durations to be aggressive estimates. Project planning with shorter estimated task durations enables the available time reserve to be used to protect the schedule (e.g., by inserting time and/or resource buffers) at points most exposed to disturbances and/or at which cost of disturbance would be highest.

5. Measures of robustness

Schedule robustness is the criterion used to assess arrangements being made robust in the proactive scheduling phase. A material problem here is to identify appropriate robustness measures for the proposed model of project scheduling with predefined milestones and cash flow optimisation.

Depending on the type of robustness being analysed, robustness measures used in the research may be divided into quality robustness measures and solution robustness measures [25].

Quality robustness measures are strictly connected with the optimisation criterion, that is the objective function of the problem analysed, such as minimum project duration, timely completion of the project, maximum cash flows etc. Such measures do not measure the conformity of all details of the project (e.g., timely commencement of individual tasks) to the schedule, efficiency indicators play the key role here.

Solution robustness measures cover the conformity of numerous various details of the project (e.g., timely commencement of individual tasks) to the schedule. They are often defined as stability indicators and computed as the distance $\Delta(S^R, S^0)$ from the schedule planned S^0 to the one actually executed S^R. The distance is determined as, for instance, the sum of deviations of actual task starting times from the respective starting times provided for in the scheduled arrangement. The distance $\Delta(S^R, S^0)$ has to be estimated, as the schedule parameters S^R are not known in the planning phase; such estimation is difficult and uncertain. The schedule parameters S^R are obtained by simulation (for various project progress scenarios) or with use of approximation techniques.

Those robustness measures appear to be more useful which may be computed as early as in the schedule development phase, without time-consuming estimation or experiment-driven determination of the schedule parameters S^R.

Research papers also cover complex robustness measures with objective functions reflecting both solution robustness and quality robustness. An example of a complex measure is a bi-objective function maximising the probability of all tasks having been executed by the project execution deadline and simultaneously minimising the sum of weighted deviations of actual task starting times from the respective planned times (relative importance of individual criteria have to be defined).

Robustness measures include:

- for the problem of minimising project execution duration: maximising the Timely Project Completion Probability (TPCP measure);
- maximising the sum of Free Slacks FS_i [26] (a free slack is defined as free time buffer – measured as the number of time units – by which a given task execution may be extended without any delay in starting the execution of the task's successors) for all project tasks $i = 1, \ldots, n$;
- maximising the minimum free slack FS_i or maximising the minimum FS_i/d_i ratio [25] – the use of those measures results in an even distribution of time buffers, which is not always achieved, while maximising the sum of free slacks FS_i;

- for a problem with weighted instability costs [20], minimising the sum of weighted deviations of the actual task starting times from their respective planned counterparts; the distance from the planned schedule to the actual one may also be computed as the maximum distance between the arrangements in various disturbance occurrence scenarios.

For the authors' models, in the planning phase, during proactive scheduling, the real start times of activities and finish times of project milestones are not known. It is only possible to project or simulate them. As the problem of project scheduling under uncertainty is considered, maximised are exclusively the expected values of the objective functions F_1, F_2, F_3, F_4 for an established disturbance generation rule.

An objective function (F_1, F_2, F_3 or F_4) may be used to measure robustness, with values determined by simulation for randomly generated disturbances (changes is activities durations). The use of an experimentally determined objective function is time consuming and depends on simulation parameters (disturbances), whose generation prior to the project execution phase makes sense primarily if task duration distributions are known. However, it is not always that such statistics are available. Accordingly, it is justified to develop such rules for determining values of objective functions which would support reflecting the effect of lengthening activities duration without simulation. In developing such rules, the authors assume that all tasks share the same probability of duration lengthening. Under the authors' proposed approach, a modified, right-shifted schedule is created (what is known as right-shift rescheduling), including the changed durations of activities. The value of the relevant objective function (F_1, F_2, F_3 or F_4) is computed for the modified schedule. For each activity i ($i = 1, \ldots, n$) a modified duration may be computed in one of the following ways:

- $d_i + \gamma$ – lengthening the duration of each task by γ time units (in the numeric example included herein, this way has been used, with $\gamma = 1$) irrespective of task parameters, that is duration or resource intensiveness;
- $d_i + \gamma\% \cdot d_i$ – lengthening of the duration of each task is proportional to the duration and amounts to $\gamma\% \cdot d_i$, where $\gamma\%$ is the parameter defining the percentage increase in the task duration;
- $d_i + \gamma\% \cdot d_i \cdot sumr_i$ – lengthening of the duration of each task is proportional to the task's both duration and aggregate demand for resources ($sumr_i$) and amounts to $\gamma\% \cdot d_i \cdot sumr_i$, where $\gamma\%$ is the parameter defining the increase in the task duration as percentage of the product of duration d_i and aggregate demand $sumr_i$.

6. An illustrative instance

In this paper authors suggest that the activity arrangement should be subject to a two-phase assessment based on the objective functions F_1, F_2, F_3 or F_4. In the first assessment phase, the objective function is computed assuming the project is executed as scheduled, without any delay (cost is omitted of a delayed execution of project tasks or milestones). The computed value of the objective function is the decisive criterion for the selection of the best basic schedule. The second assessment phase is performed if multiple schedules share the same best value of the objective function. Then the robustness of those schedules (with the same value of the objective function computed in the first phase) against lengthening of activity durations is computed. It is this robustness that serves as the decisive criterion in the selection of a preferred schedule from among the nominal schedules identified in the first phase.

It is not always possible to identify optimum nominal arrangements within the acceptable time, because the resource-constrained project scheduling problem is NP-hard [27]. For NP-hard problems, exact algorithms are used to projects composed of a rather low number of tasks, and approximate algorithms (heuristics) for larger projects. A review of effective algorithms for the RCPSP problem is included in review papers [28, 29].

The cash flow optimisation problem considered herein may better be described by a numerical example. To this end, let us define an instance of a project executed with the use of a single resource, whose availability equals 8 and with the optimisation criterion being maximising the objective function F_4 (as a short-term makespan for the project has been assumed, the analysis does not cover cash flow discounting or periodic payments *CFP*). Table 1 sets forth information on the tasks included in the project, while Table 2 – the information on the project milestones.

Table 1
Activities of the project analysed

i	d_i	r_i	δ_i	N_i	C_i	CL_i
0	0	0	–	1, 2, 4	–	–
1	3	6	4	10	40	2
2	2	4	10	3, 8	20	1
3	3	3	10	9	30	1
4	2	4	10	5	20	1
5	2	4	10	6	20	1
6	3	3	15	10	30	2
7	2	3	15	10	10	1
8	4	1	10	9	10	0
9	3	2	15	10	10	2
10	0	0	15	–	–	–

i is activity number; N_i is the set of direct successors of the activity i.

Table 2
Milestones of the project analysed

m	MT_m	MA_m	PM_m	CM_m
1	4	1	100	5
2	10	2, 3, 4, 5, 8	100	5
3	15	6, 7, 9, 10	200	20

m is milestone number.

Figure 1 presents the AON network of the project (tasks included in the same milestone are shaded in the same hue).

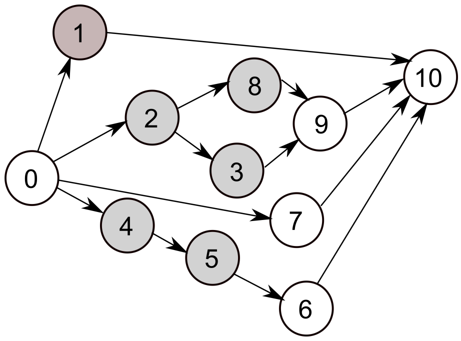

Fig. 1. *AON* network for the project analysed

The minimum makespan of the project is 10 time units, as results, for instance, from the aggregate time and resource intensiveness of the tasks:

$$\left\lceil \frac{\sum_{i=1}^{n}(d_i \cdot r_i)}{a} \right\rceil = \left\lceil \frac{76}{8} \right\rceil = 10.$$

Figure 2 presents an example of a schedule providing for makespan covering 10 time units, in the form of a Gantt chart.

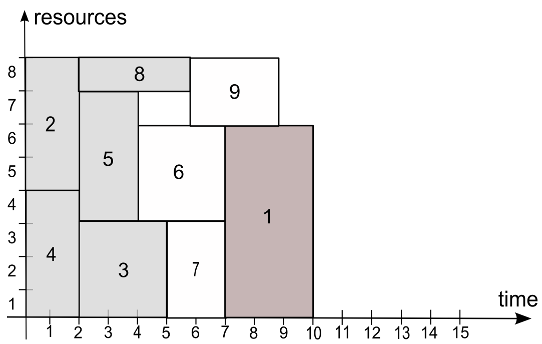

Fig. 2. Gantt chart with the minimum makespan

For the considered nominal scheduling with cash flow optimisation (maximising the objective function F_4), the schedule described in Fig. 1 is not appropriate, because it does not provide for milestones. The execution of the first milestone is delayed by six time units relative to the deadline, which triggers contractual expenditure of 30 currency units:

$$CM_1 \cdot (RMT_1 - MT_1) = 5 \cdot (10 - 4) = 30.$$

The value of the objective function F_4 may be computed as follows:

$$\sum_{i=1}^{n} CFA_i = -\sum_{i=1}^{n} C_i - \sum_{i=1}^{n} [CL_i \cdot \max(RST_i - ST_i, 0)]$$
$$= -190 - 0 = -190,$$
$$\sum_{m=1}^{M} CFM_m = \sum_{m=1}^{M} PM_m$$
$$- \sum_{m=1}^{M} [CM_m \cdot \max(RMT_m - MT_m, 0)]$$
$$= 400 - 30 = 370,$$

$$F_4 = \sum_{i=1}^{n} CFA_i + \sum_{m=1}^{M} CFM_m = -190 + 370 = 180.$$

For the purposes of computation, the project has been assumed to be executed precisely in line with the nominal schedule. For instance, it has been assumed that the time of completion of first milestone RMT_1 is such that it ends on the date planned in the nominal schedule (10 time units).

In computing values of the objective function F_4 for the nominal scheduling (assuming the schedule is implemented as planned), the only variable component are reductions in the client's payments triggered by a delayed completion of milestones. For the project illustrated in Fig. 2, these reductions total 30 currency units. It is, though, possible to develop a schedule in which this cost (reductions) are 0, that is all milestone deadlines are met. Such a schedule will maximise aggregate cash flows at 210 currency units.

It is not feasible to build a nominal schedule with the value of the objective function F_4 at 210 currency units and makespan of 10 time units. The contractual deadline for the first milestone can be bet only if Task 1 starts at the time $t = 0$. As Task 2 may not be executed concurrently with Task 1, the minimum makespan of the project with the function F_4 maximised equals the aggregate time required to execute Tasks 1, 2, 8 and 9, that is 12 time units. Figure 3 presents examples of nominal schedules $H1$ and $H2$ with makespan of 12 time units, taking into consideration milestone deadlines, rendering the value of 210 for the objective function F_4.

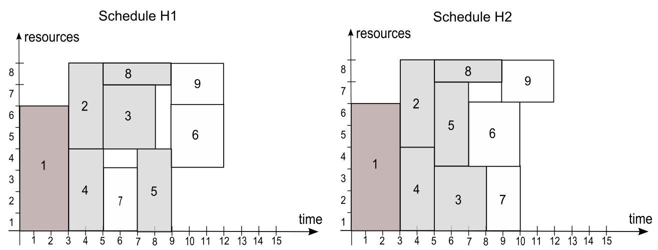

Fig. 3. Schedules $H1$ and $H2$ with aggregated cash flows of 210 currency units

For the schedules $H1$ and $H2$, the objective function F_4 takes the same value of 210 currency units. Multiple nominal schedules with the same value of the objective function F_4 may exist, that is schedules providing for achieving all contractual milestone deadlines.

This being the case, the authors suggest that the selection of one of those schedules should be based on the comparison of their respective robustness measured as the value of F_4 after duration of each task has been lengthened by 1 time unit.

After such lengthening of task durations, in the $H1$, starting times of individual tasks are as follows: $ST_1 = 0$, $RST_1 = 0$, $ST_2 = 3$, $RST_2 = 4$, $ST_3 = 5$, $RST_3 = 7$, $ST_4 = 3$, $RST_4 = 4$, $ST_5 = 7$, $RST_5 = 10$, $ST_6 = 9$, $RST_6 = 13$, $ST_7 = 5$, $RST_7 = 7$, $ST_8 = 5$, $RST_8 = 7$, $ST_9 = 9$, $RST_9 = 12$ while milestones completion times are: $RMT_1 = 4$, $RMT_2 = 13$, $RMT_3 = 17$.

Thus:

$$\sum_{i=1}^{n}[CL_i \cdot \max(RST_i - ST_i, 0)]$$
$$= 2\cdot 0 + 1\cdot 1 + 1\cdot 2 + 1\cdot 1$$
$$+1\cdot 3 + 2\cdot 4 + 1\cdot 2 + 0\cdot 2 + 2\cdot 3 = 23,$$

$$\sum_{m=1}^{M}[CM_m \cdot \max(RMT_m - MT_m, 0)]$$
$$= 5\cdot 0 + 5\cdot 3 + 20\cdot 2 = 55,$$

$$F_4 = \sum_{i=1}^{n} CFA_i + \sum_{m=1}^{M} CFM_m$$
$$= -190 - 23 + 400 - 55 = 132.$$

For the schedule $H2$, the same lengthening yields the following respective values for the activities: $ST_1 = 0$, $RST_1 = 0$, $ST_2 = 3$, $RST_2 = 4$, $ST_3 = 5$, $RST_3 = 7$, $ST_4 = 3$, $RST_4 = 4$, $ST_5 = 5$, $RST_5 = 7$, $ST_6 = 7$, $RST_6 = 10$, $ST_7 = 8$, $RST_7 = 11$, ST_8= 5, $RST_8 = 7$, $ST_9 = 9$, $RST_9 = 12$ and for the milestones: $RMT_1 = 4$, $RMT_2 = 12$, $RMT_3 = 16$.

Thus:

$$\sum_{i=1}^{n}[CL_i \cdot \max(RST_i - ST_i, 0)]$$
$$= 2\cdot 0 + 1\cdot 1 + 1\cdot 2 + 1\cdot 1$$
$$+1\cdot 2 + 2\cdot 3 + 1\cdot 3 + 0\cdot 2 + 2\cdot 3 = 21,$$

$$\sum_{m=1}^{M}[CM_m \cdot \max(RMT_m - MT_m, 0)]$$
$$= 5\cdot 0 + 5\cdot 2 + 20\cdot 1 = 30,$$

$$F_4 = \sum_{i=1}^{n} CFA_i + \sum_{m=1}^{M} CFM_m$$
$$= -190 - 21 + 400 - 30 = 159.$$

Thus the schedule $H2$ is more robust against disturbances than $H1$, as revealed by comparing the respective projected cash flows after the virtual lengthening. The value of F_4 is for $H2$ by 27 currency units higher than for $H1$.

The authors use the schedule $H2$ as the basic solution for proactive scheduling, in the course of which robust allocation of resources to individual tasks is carried out, followed (in the phase of robust buffer allocation) by the insertion of time and/or resource buffers.

The robust allocation of resources may be omitted if, for instance, it is possible to allocate resources on an ongoing basis during project execution with no delaying effect on work progress. One way to describe resource allocation is the resource flow network. Figure 4 illustrates the schedule $H2$ with an instance of resource allocation. On resource flow network arrows marked with numbers defining how many resource items are forwarded from a task to the next one.

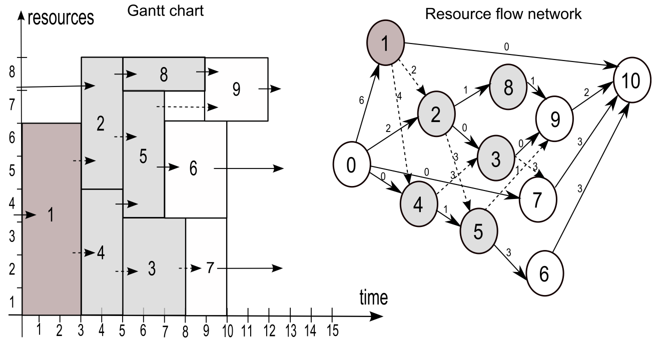

Fig. 4. Schedule $H2$ with an instance of resource allocation for that schedule

Each additional edge (x, y) in the resource flow network means another ordering constraint (not forced by technological considerations), which reduces schedule robustness, as a delay, if any, in completing the task x postpones commencing the task y. In Fig. 4, additional edges are drown with broken arrows. They are the edges: (1, 2), (1, 4), (2, 5), (3, 7), (4, 3) and (5, 9). Among them, the edges unavoidable [23] for the schedule $H2$ are (1, 2), (1, 4) and (3, 7). Theoretically, the other additional edges might be removed upon changing the allocation of resources to tasks. However, it is not possible to remove both (5, 7) and (5, 9). Thus the minimum number of additional arcs is 4. Figure 5 illustrates an example of a schedule and resource flow network with the minimum number of additional edges.

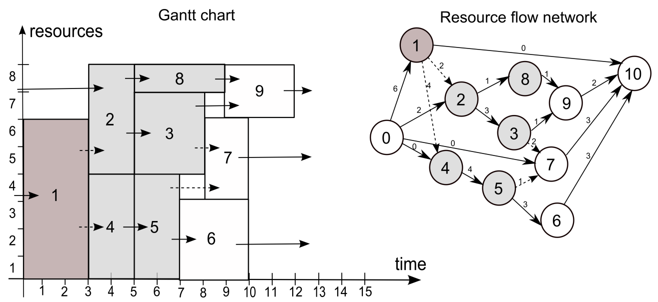

Fig. 5. Schedule $H2$ with the minimum number of additional edges and the related resource flow network

The schedule depicted in Fig. 5 is more robust against the lengthening of task durations that that in Fig. 4 is, the reason being the lower number of additional ordering constraints. For more elaborate analyses of the problem, along with the description of algorithms used, refer to works about resource allocation in project scheduling [23, 24].

Resources having been allocated to tasks (as in Fig. 5), time and/or resource buffers are inserted (use of resource buffers is not considered herein). In the problem under analysis, buffer allocation is designed to improve robustness through securing the schedule against possible disturbances (task duration variability), while simultaneously guaranteeing timely completion of milestones, with a view to maximising the aggregate forecast cash flows (measured as the value of the objective function F_4).

Figure 6 presents an example of a schedule with one-unit buffers inserted before Tasks 3, 5, 7 and 9, as well as a two-unit buffer before Task 6 (all buffers are designed to secure timely start of activity execution).

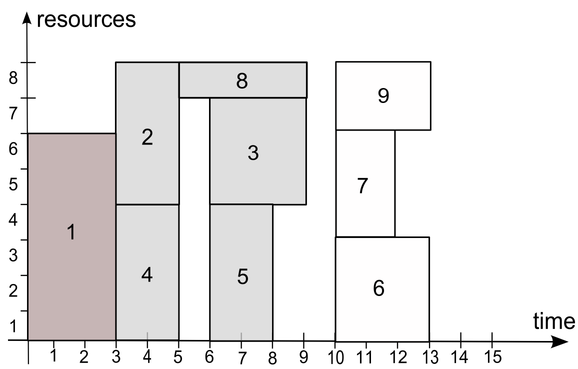

Fig. 6. Schedule with time buffers

Assuming project execution in line with the schedule, the value of the objective function F_4 for the schedule with time buffers is the same as for the nominal schedule, that is 210 currency units. Delaying outflows, connected with delayed start of individual tasks, would increase the value of an objective function (F_1 or F_2) for a model with discounted cash flows.

The use of proactive approach to scheduling reveals its benefits upon computing schedule robustness measured as the value of the objective function F_4 assuming lengthening the duration of each task by one time unit. For the schedule with time buffers, starting times of activities are as follows: $ST_1 = 0$, $RST_1 = 0$, $ST_2 = 3$, $RST_2 = 4$, $ST_3 = 6$, $RST_3 = 7$, $ST_4 = 3$, $RST_4 = 4$, $ST_5 = 6$, $RST_5 = 7$, $ST_6 = 10$, $RST_6 = 10$, $ST_7 = 10$, $RST_7 = 11$, $ST_8 = 5$, $RST_8 = 7$, $ST_9 = 10$, $RST_9 = 12$ and milestones completion times are: $RMT_1 = 4$, $RMT_2 = 12$, $RMT_3 = 16$.

Thus:

$$\sum_{i=1}^{n} [CL_i \cdot \max(RST_i - ST_i, 0)]$$
$$= 2 \cdot 0 + 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1$$
$$+1 \cdot 1 + 2 \cdot 0 + 1 \cdot 1 + 0 \cdot 2 + 2 \cdot 2 = 9,$$

$$\sum_{m=1}^{M} [CM_m \cdot \max(RMT_m - MT_m, 0)]$$
$$= 5 \cdot 0 + 5 \cdot 2 + 20 \cdot 1 = 30,$$

$$F_4 = \sum_{i=1}^{n} CFA_i + \sum_{m=1}^{M} CFM_m$$
$$= -190 - 9 + 400 - 30 = 171.$$

The value of the objective function F_4 with the above lengthening, computed for the schedule with time buffers, is by 12 currency units higher than for the nominal schedule $H2$. The computation confirms the rationality of using proactive scheduling to the problem considered.

For reports on research into buffer allocation, time and/or resource buffer insertion algorithms and robustness metrics used, refer to papers about robust project scheduling [19–20].

7. Conclusions

The paper analyses the problem of maximising cash flows from the contractor's perspective. A new model is proposed for scheduling a project with defined milestones. For this model, robustness measures are proposed taking into consideration the effect of uncertainty (changes in activity durations) on the value of aggregate cash flows. An example of project is discussed for the development of a schedule appropriate for the model and the objective functions defined.

The models analysed herein and presented methods of solving the problem (proactive scheduling with robust allocation of resources and buffers) are the authors' own development, not earlier considered for the problem of project scheduling with cash flow optimisation. Further research will include the development of effective proactive scheduling algorithms for the model presented in this paper.

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